The Car Sharing Economy: Interaction of Business Model Choice and Product Line Design

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In what follows we provide details on the proofs of our results. The analytical expressions are explicitly given unless they hinder manuscript readability in which case only the shorthand notation is provided. For instance, instead of providing the complete expression of the optimal profit under the (O,M) equilibrium we use $\Pi^*_{(O,M)}$ to denote it. The complete forms are available from the authors upon request. When comparing across different equilibria we utilize the notation (h,l) with $h \in \{O\}$ and $l \in \{O,M,\emptyset\}$. For instance, $e_H{}^*(O,M)$ denotes the optimal fuel efficiency of the vehicles sold to the High segment under the (O,M) equilibrium.

Proof of Remark 1. Assume that each customer requests a vehicle according to a Poisson process with rate λ' and that the mean duration of each vehicle use is τ. Set $\lambda'\tau \doteq d$, where d denotes customers' transportation needs. We model the service time at the $\bullet/M/1$ using a Poisson process with rate $\lambda = n_i\lambda'$, where n_i is the size of the segment served through *Membership*. Given that the idle time of the node represents the time when customer requests cannot be satisfied because no cars are available, the utilization $\rho_{\bullet/M/1}$ provides the probability that a customer finds a vehicle available.

We assume that customer usage times are independent and identically distributed according to a general probability distribution $\mathbb{G}(\cdot)$ with mean τ. Therefore, we capture the service process through the $\bullet/G/\infty$ node whose service time distribution is $\mathbb{G}(\cdot)$.

The OEM guarantees an exogenously determined service level a, by choosing S such that $a=\rho_{\bullet/M/1}$. Based on the FPM approximation (Whitt 1984), we construct the open counterpart of the closed queueing network and equate the expected equilibrium population of the open network to S. The open network comprises an $M/M/1$ and an $M/G/\infty$ node. The $M/M/1$ node is characterized by external Poisson arrivals with rate λ_c and exponential service times with parameter λ. The service rate of each server at the $M/G/\infty$ node is $1/\tau$ and the expected number of jobs (i.e., vehicles) in the open network is $\mathbb{E}\left[N_c\right]=\mathbb{E}\left[N_{M/M/1}\right]+\mathbb{E}\left[N_{M/G/\infty}\right]=\frac{\lambda_c}{\lambda-\lambda_c}+\lambda_c\tau$. For the open network to be equivalent to the closed network, the external rate λ_c must satisfy $\mathbb{E}\left[N_c\right]=S$. In this case, $\rho_{M/M/1}=\frac{\lambda_c}{\lambda}$ determines the service level, which must equal a. Substituting $\lambda_c=a\lambda$ in $\mathbb{E}\left[N_c\right]=S$ results in $S\left(a\right)\approx\frac{a}{1-a}+a\lambda\tau$. Given that $\lambda\doteq n_i\lambda'$ and $d\doteq\lambda'\tau$, the fleet size that achieves an availability level a is $S\left(a\right)\approx\frac{a}{1-a}+an_id$.$\square$

Proof of Propositions 1 and 2. For each possible equilibrium we first determine the optimal prices and then the optimal fuel efficiencies.

Under $(O,\emptyset)$ and for given efficiencies, the OEM determines the selling price F based on $\max_F\Pi_{(O,\emptyset)}=\left(F-c_w\left(1-e_H\right)^2-c_e{e_H}^2\right)n_H$ subject to the individual rationality constraints $d\big(\nu+\left(1-e_H\right)\left(\theta_H-g\right)\big)-F\geq 0$, and $d\big(\nu-\left(1-e_H\right)\left(g-\theta_L\right)\big)-F\leq 0$ which can be rewritten as $d\big(\nu-\left(1-e_H\right)\left(g-\theta_L\right)\big)\leq F\leq d\big(\nu+\left(1-e_H\right)\left(\theta_H-g\right)\big)$. The profit $\Pi_{(O,\emptyset)}$ is linear increasing in F. Therefore, for a given fuel efficiency the optimal selling price is $\tilde{F}=d\big(\nu+\left(1-e_H\right)\left(\theta_H-g\right)\big)$. Define $\tilde{\Pi}_{(O,\emptyset)}\doteq\Pi_{(O,\emptyset)}\left(F=\tilde{F}\right)$. Then, the OEM determines the optimal fuel efficiency based on $\max_{e_H}\tilde{\Pi}_{(O,\emptyset)}$ such that $0\leq e_H\leq 1$. $\tilde{\Pi}_{(O,\emptyset)}$ is concave in the fuel efficiency because $\partial^2\tilde{\Pi}_{(O,\emptyset)}/\partial^2 e_H=-2\left(c_w+c_e\right)n_H<0$, therefore, after solving $\partial\tilde{\Pi}_{(O,\emptyset)}/\partial e_H=0$ we obtain ${e_H}^*=\frac{2c_w-d(\theta_H-g)}{2(c_w+c_e)}$. It is straightforward to show that ${e_H}^*\geq 0$ iff $c_w\geq\bar{c}_w\doteq\frac{d(\theta_H-g)}{2}>0$ and ${e_H}^*\leq 1$ iff $c_e\geq\bar{c}_e\doteq-\frac{d(\theta_H-g)}{2}$, which is always true as $\bar{c}_e<0$. Following Chen (2001) we focus on interior values of ${e_H}^*\in(0,1)$. Hence, we assume that $c_w>\bar{c}_w$. Based on ${e_H}^*$ we also calculate $F^*=d\nu+\frac{d(\theta_H-g)\left(2c_e+d(\theta_H-g)\right)}{2(c_w+c_e)}$ and $\Pi^*_{(O,\emptyset)}=n_H\left(d\nu+\frac{d(\theta_H-g)\left(4c_e+d(\theta_H-g)\right)-4c_wc_e}{4(c_w+c_e)}\right)$.

Under (O,O) and for given efficiencies, the OEM determines the selling prices F_H, and F_L (we use the subscript $i=H$ and $i=L$ to indicate the selling price charged to the High and Low segment, respectively) based on $\max_{F_H,F_L}\Pi_{(O,O)}=\big(F_H-c_w\left(1-e_H\right)^2-c_e{e_H}^2\big)n_H+\big(F_L-c_w\left(1-e_L\right)^2-c_e{e_L}^2\big)n_L$ subject to the individual rationality constraints

$d\big(\nu+(1-e_H)(\theta_H-g)\big)-F_H \geq 0$ and $d\big(\nu+(1-e_L)(\theta_L-g)\big)-F_L \geq 0$, and the incentive compatibility constraints $d\big(\nu+(1-e_H)(\theta_H-g)\big)-F_H \geq d\big(\nu+(1-e_L)(\theta_H-g)\big)-F_L$ and $d\big(\nu+(1-e_L)(\theta_L-g)\big)-F_L \geq d\big(\nu+(1-e_H)(\theta_L-g)\big)-F_H$. The individual rationality constraints can be rewritten as $F_H \leq d\big(\nu+(1-e_H)(\theta_H-g)\big)$ and $F_L \leq d\big(\nu+(1-e_L)(\theta_L-g)\big)$. Similarly, the incentive compatibility constraints are rewritten as $F_L+d(e_H-e_L)(g-\theta_L) \leq F_H \leq F_L-d(e_H-e_L)(\theta_H-g)$, which hold iff $e_L \geq e_H$ (we show below that at optimality $e_L > e_H$ holds). The profit $\Pi_{(O,O)}$ is linear increasing in both F_H and F_L. Therefore, for given vehicle efficiencies, the optimal selling prices are $\tilde{F}_L = d\big(\nu+(1-e_L)(\theta_L-g)\big)$ and $\tilde{F}_H = \min\Big\{\tilde{F}_L+d(e_L-e_H)(\theta_H-g), d\big(\nu+(1-e_H)(\theta_H-g)\big)\Big\}$, from which is simple to show that $\tilde{F}_H = d(\nu+(1-e_L)\theta_L+(e_L-e_H)\theta_H-(1-e_H)g)$. Define $\tilde{\Pi}_{(O,O)} \doteq \Pi_{(O,O)}\left(F_H=\tilde{F}_H, F_L=\tilde{F}_L\right)$. Then the OEM determines the optimal vehicle efficiencies based on $\max_{e_H,e_L}\tilde{\Pi}_{(O,O)}$ such that $0 \leq e_H \leq 1$ and $0 \leq e_L \leq 1$. The profit $\tilde{\Pi}_{(O,O)}$ is jointly concave in the vehicle efficiencies because $\partial^2\tilde{\Pi}_{(O,O)}/\partial^2 e_H = -2(c_w+c_e)n_H < 0$, $\partial^2\tilde{\Pi}_{(O,O)}/\partial^2 e_L = -2(c_w+c_e)n_L < 0$, and $\left(\partial^2\tilde{\Pi}_{(O,O)}/\partial^2 e_H\right)\left(\partial^2\tilde{\Pi}_{(O,O)}/\partial^2 e_L\right) - \left(\partial^2\tilde{\Pi}_{(O,O)}/\partial e_H\partial e_L\right)^2 = 4(c_w+c_e)^2 n_H n_L > 0$. Therefore, after solving $\partial\tilde{\Pi}_{(O,O)}/\partial e_H = 0$ and $\partial\tilde{\Pi}_{(O,O)}/\partial e_L = 0$ we obtain ${e_H}^* = \frac{2c_w-d(\theta_H-g)}{2(c_w+c_e)}$ and ${e_L}^* = \frac{2c_w n_L - d\left(gn_L+n_H\theta_H-(n_H+n_L)\theta_L\right)}{2(c_w+c_e)n_L}$. The efficiency ${e_H}^* \geq 0$ iff $c_w \geq \bar{c}_w \doteq \frac{d(\theta_H-g)}{2} > 0$ and ${e_H}^* \leq 1$ iff $c_e \geq \bar{c}_e \doteq -\frac{d(\theta_H-g)}{2}$, which is always true as $\bar{c}_e < 0$. Similarly, ${e_L}^* \geq 0$ iff $c_w \geq \tilde{c}_w \doteq -\frac{d\left(n_H(\theta_H-\theta_L)+n_L(g-\theta_L)\right)}{2n_L}$, which is always true as $\tilde{c}_w < 0$ and ${e_L}^* \leq 1$ iff $c_e \geq \tilde{c}_e \doteq \frac{d\left(n_H(\theta_H-\theta_L)+n_L(g-\theta_L)\right)}{2n_L} > 0$. Once again, we focus on interior values ${e_H}^*, {e_L}^* \in (0,1)$ and for that reason we assume that $c_w > \bar{c}_w$ and $c_e > \tilde{c}_e$. It is straightforward to show that ${e_L}^* - {e_H}^* = \frac{d(n_H+n_L)(\theta_H-\theta_L)}{2(c_w+c_e)n_L} > 0$, which means that ${e_L}^* > {e_H}^*$ holds. Based on the optimal efficiencies, we also obtain $F_L^* = d\nu - \frac{d(g-\theta_L)\left(2c_e n_L - d\left((\theta_H-\theta_L)n_H+(g-\theta_L)n_L\right)\right)}{2(c_w+c_e)n_L}$, $F_H^* = F_L^* + \frac{d^2(n_H+n_L)(\theta_H-g)(\theta_H-\theta_L)}{2(c_w+c_e)n_L}$, and $\Pi_{(O,O)}^* = (n_H+n_L)\left(d\nu - \frac{4c_w c_e n_L - d\left(d\left(n_L(g-\theta_L)^2+n_H(\theta_H-\theta_L)^2\right)\right)+4c_e n_L(g-\theta_L)}{4(c_w+c_e)n_L}\right)$.

Under (O,M) and for given efficiencies, the OEM determines the selling price F and the per-unit-of-time price p based on $\max_{F,p}\Pi_{(O,M)} = \left(F-c_w(1-e_H)^2-c_e{e_H}^2\right)n_H + a\big(p-g(1-e_L)\big)n_L d - \left(c_w(1-e_L)^2+c_e{e_L}^2\right)\left(\frac{a}{1-a}+an_L d\right)$ subject to the individual rationality constraints $d\big(\nu+(\theta_H-g)(1-e_H)\big)-F \geq 0$ and $ad\big(\nu+\theta_L(1-e_L)-p\big) \geq 0$, and the incentive compatibility constraints $d\big(\nu+(\theta_H-g)(1-e_H)\big)-F \geq ad\big(\nu+\theta_H(1-e_L)-p\big)$ and $ad\big(\nu+\theta_L(1-e_L)-p\big) \geq d\big(\nu+(\theta_L-g)(1-e_H)\big)-F$. The individual rationality constraints can be rewritten as $F \leq d\big(\nu+(\theta_H-g)(1-e_H)\big)$ and $p \leq \nu+\theta_L(1-e_L)$. Similarly, the incentive compatibility constraints are rewritten as $d\big(ap+(1-a)\,\nu-(1-e_H)\,(g-\theta_L)-a\,(1-e_L)\big)\theta_L \leq F \leq d\big(ap+(1-a)\,\nu+(1-e_H)\,(\theta_H-g)-a\,(1-e_L)\big)\theta_H$, which hold iff $e_L \geq e_H$ (we show below that at optimality $e_L > e_H$ holds). The profit $\Pi_{(O,M)}$ is linear increasing in both F and p. Therefore, for given vehicle efficiencies the optimal per-unit-of-time price is $\tilde{p}=\nu+(1-e_L)\,\theta_L$ and the optimal selling price is $\tilde{F}=d\big(a\tilde{p}+(1-a)\,\nu+(1-e_H)\,(\theta_H-g)-a\,(1-e_L)\big)\theta_H$. Define $\tilde{\Pi}_{(O,M)} \doteq \Pi_{(O,M)}\left(F=\tilde{F}, p=\tilde{p}\right)$. Then, the OEM determines the optimal vehicle efficiencies based on $\max_{e_H,e_L} \tilde{\Pi}_{(O,M)}$ such that $0 \leq e_H \leq 1$ and $0 \leq e_L \leq 1$. The profit $\tilde{\Pi}_{(O,M)}$ is jointly concave in the vehicle efficiencies because $\partial^2\tilde{\Pi}_{(O,M)}/\partial^2 e_H = -2\,(c_w+c_e)\,n_H < 0$, $\partial^2\tilde{\Pi}_{(O,M)}/\partial^2 e_L = -2\,(c_w+c_e)\,a\left(dn_L+\frac{1}{1-a}\right) < 0$, and $\left(\partial^2\tilde{\Pi}_{(O,M)}/\partial^2 e_H\right)\left(\partial^2\tilde{\Pi}_{(O,M)}/\partial^2 e_L\right)-\left(\partial^2\tilde{\Pi}_{(O,M)}/\partial e_H \partial e_L\right)^2 = \frac{4(c_w+c_e)^2 n_H a\left(1+(1-a)dn_L\right)}{1-a} > 0$. Therefore, after solving $\partial\tilde{\Pi}_{(O,M)}/\partial e_H = 0$ and $\partial\tilde{\Pi}_{(O,M)}/\partial e_L = 0$ we obtain ${e_H}^* = \frac{2c_w-d(\theta_H-g)}{2(c_w+c_e)}$ and ${e_L}^* = \frac{2c_w\left(1+(1-a)dn_L\right)-d(1-a)\left((g-\theta_L)n_L+(\theta_H-\theta_L)n_H\right)}{2(c_w+c_e)(1+(1-a)dn_L)}$. The efficiency ${e_H}^* \geq 0$ iff $c_w \geq \bar{c}_w \doteq \frac{d(\theta_H-g)}{2} > 0$ and ${e_H}^* \leq 1$ iff $c_e \geq \bar{c}_e \doteq -\frac{d(\theta_H-g)}{2}$, which is always true as $\bar{c}_e < 0$. With respect to ${e_L}^*$, in this case we have that ${e_L}^* \geq 0$ iff $c_w \geq \hat{c}_w \doteq -\frac{(1-a)d\left((g-\theta_L)n_L+(\theta_H-\theta_L)n_H\right)}{2(1+(1-a)dn_L)}$, which is always true as $\hat{c}_w < 0$. In addition, ${e_L}^* \geq 1$ iff $c_e \geq \hat{c}_e \doteq \frac{(1-a)d\left((g-\theta_L)n_L+(\theta_H-\theta_L)n_H\right)}{2(1+(1-a)dn_L)} > 0$. As before, our focus is on interior values ${e_H}^*, {e_L}^* \in (0,1)$ and for that reason we assume that $c_w > \bar{c}_w$ and $c_e > \hat{c}_e$. Based on the optimal efficiencies we also obtain the optimal prices F^* and p^* along with the optimal profit $\Pi^*_{(O,M)}$ (analytical expressions available upon request).

Note that the difference $\hat{c}_e - \tilde{c}_e = \frac{d\left(-1+(1-a)n_L(1-d)\right)\left((g-\theta_L)n_L+(\theta_H-\theta_L)n_H\right)}{2n_L(1+(1-a)dn_L)} > 0$ due to $n_L > \underline{n}_L \doteq \frac{1}{(1-d)(1-a)} > 0$.[1] This implies that $e_i^*(h,l) \in (0,1)$ for all $i \in \{H,L\}$, $h \in \{O\}$, and $l \in \{O,M\}$ when $c_w > \bar{c}_w$ and $c_e > \max\{\hat{c}_e, \tilde{c}_e\} = \hat{c}_e$.

By comparing the fuel efficiencies under the different market equilibria we find that ${e_H}^*(O,\emptyset) = {e_H}^*(O,O) = {e_H}^*(O,M) = \frac{2c_w-d(\theta_H-g)}{2(c_w+c_e)}$ and ${e_L}^*(O,O) - {e_H}^*(O,O) = \frac{d(n_H+n_L)(\theta_H-\theta_L)}{2(c_w+c_e)n_L} > 0$, therefore, ${e_L}^*(O,O) > {e_H}^*(O,O)$. Similarly, given that $n_L > \underline{n}_L$, ${e_L}^*(O,M) - {e_H}^*(O,M) = \frac{d\left(\theta_H\left(1+dn_L+n_H-a(dn_L+n_H)\right)+g\left((1-a)(1-d)n_L-1\right)-(1-a)\theta_L(n_H+n_L)\right)}{2(c_w+c_e)(1+(1-a)dn_L)} > 0$ and ${e_L}^*(O,M) - {e_L}^*(O,O) = \frac{d\left(-1+(1-d)n_L(1-a)\right)\left((g-\theta_L)n_L+(\theta_H-\theta_L)\theta_H\right)}{2(c_w+c_e)n_L(1+(1-a)dn_L)} > 0$, hence, ${e_L}^*(O,M) > {e_H}^*(O,M)$ and ${e_L}^*(O,M) > {e_L}^*(O,O)$.

In terms of comparative statics we obtain: $\partial {e_H}^*/\partial\theta_H = -\frac{d}{2(c_w+c_e)} < 0$, $\partial {e_H}^*/\partial d = -\frac{\theta_H-g}{2(c_w+c_e)} < 0$, $\partial {e_L}^*(O,O)/\partial\theta_H = \frac{dn_H}{2(c_w+c_e)n_L} > 0$, $\partial {e_L}^*(O,O)/\partial d = \frac{(\theta_H-\theta_L)n_H+(g-\theta_L)n_L}{2(c_w+c_e)n_L} > 0$,

[1] As we have already stated in §4.2 of the main paper, throughout the analysis we assume that $n_L > \underline{n}_L$, otherwise (O,M) is always dominated (the derivation of the condition is provided in the proof of Proposition 4).

$\partial e_L{}^*(O,O)/\partial n_H = \frac{d(\theta_H-\theta_L)}{2(c_w+c_e)n_L} > 0$, $\partial e_L{}^*(O,O)/\partial n_L = -\frac{dn_H(\theta_H-\theta_L)}{2(c_w+c_e)n_L^2} < 0$, $\partial e_L{}^*(O,M)/\partial a = -\frac{d\left((\theta_H-\theta_L)n_H+(g-\theta_L)n_L\right)}{2(c_w+c_e)(1+dn_L(1-a))^2} < 0$, $\partial e_L{}^*(O,M)/\partial \theta_H = \frac{dn_H(1-a)}{2(c_w+c_e)(1+dn_L(1-a))} > 0$, $\partial e_L{}^*(O,M)/\partial d = \frac{(1-a)\left((\theta_H-\theta_L)n_H+(g-\theta_L)n_L\right)}{2(c_w+c_e)(1+dn_L(1-a))^2} > 0$, $\partial e_L{}^*(O,M)/\partial n_H = \frac{d(1-a)(\theta_H-\theta_L)}{2(c_w+c_e)(1+dn_L(1-a))} > 0$, and $\partial e_L{}^*(O,M)/\partial n_L = \frac{d(1-a)\left(g-\theta_L-dn_H(1-a)(\theta_H-\theta_L)\right)}{2(c_w+c_e)(1+dn_L(1-a))^2} > 0$ iff $n_H < \frac{g-\theta_L}{d(1-a)(\theta_H-\theta_L)}$.

Using a similar notation to distinguish between the prices under different equilibria we have, $F_H^*(O,O) - F_L^*(O,O) = \frac{d^2(n_H+n_L)(\theta_H-g)(\theta_H-\theta_L)}{2(c_w+c_e)n_L} > 0$, $F_H^*(O,M) - F_H^*(O,O) = \frac{d(\theta_H-\theta_L)\left(2(1-a)c_en_L\left(1+(1-a)dn_L\right)+d\left((1-a)n_L(a-d)-1\right)\left((\theta_H-\theta_L)n_H+(g-\theta_L)n_L\right)\right)}{2n_L(c_w+c_e)((1-a)dn_L+1)} > 0$ due to $c_e > \hat{c}_e$ and $n_L > \underline{n}_L$, and $F_H^*(O,\emptyset) - F_H^*(O,M) = \frac{da(\theta_H-\theta_L)\left(2c_e\left(1+(1-a)dn_L\right)-d(1-a)\left((\theta_H-\theta_L)n_H+(g-\theta_L)n_L\right)\right)}{2(c_w+c_e)(1+(1-a)dn_L)} > 0$ due to $c_e > \hat{c}_e$. Therefore, $F_H^*(O,\emptyset) > F_H^*(O,M) > F_H^*(O,O) > F_L^*(O,O)$.□

Proof of Proposition 3. We start by calculating the CAFE level under the different equilibria. Specifically, $r(O,\emptyset) = e_H{}^* = \frac{2c_w-d(\theta_H-g)}{2(c_w+c_e)}$, $r(O,O) = \frac{n_H}{n_H+n_L}e_H{}^* + \frac{n_L}{n_H+n_L}e_L{}^* = \frac{2c_w-d(g-\theta_L)}{2(c_w+c_e)}$, and $r(O,M) = \frac{n_H}{n_H+\left(\frac{a}{1-a}+an_Ld\right)}e_H{}^* + \frac{\frac{a}{1-a}+an_Ld}{n_H+\left(\frac{a}{1-a}+an_Ld\right)}e_L{}^* = \frac{2c_w(a+(1-a)(adn_L+n_H))+(1-a)d(n_H(a(\theta_H-\theta_L)+g-\theta_H)+an_L(g-\theta_L))}{2(c_w+c_e)(a+(1-a)(adn_L+n_H))}$. We continue by calculating the difference $r(O,O) - r(O,\emptyset) = \frac{d(\theta_H-\theta_L)}{2(c_w+c_e)} > 0$, which implies that $r(O,O) > r(O,\emptyset)$. With respect to the CAFE under (O,O) and (O,M) we have $r(O,O) - r(O,M) = \frac{d\left(ag(1-(1-a)(1-d)n_L)-a\theta_L(n_L((a-1)(1-d))+1)+(1-a)^2n_H(\theta_H-\theta_L)\right)}{2(c_w+c_e)(a+(1-a)(adn_L+n_H))}$, which can be positive or negative. However, $\partial\left(r(O,O) - r(O,M)\right)/\partial\theta_H = \frac{dn_H(1-a)^2}{2(c_w+c_e)(a+(1-a)(n_H+adn_L))} > 0$, therefore, $r(O,O) > r(O,M)$ for all $\theta_H > \bar{\theta}_H \doteq \{\theta_H : r(O,O) - r(O,M) = 0\} = \frac{\theta_L(a-(1-a)(a(-dn_L+n_H+n_L)-n_H))+ag(-1+(1-a)n_L(1-d))}{(1-a)^2n_H}$, where $\bar{\theta}_H > \theta_L$ because $\bar{\theta}_H - \theta_L = \frac{a(g-\theta_L)\left(-1+(1-d)n_L(1-a)\right)}{(1-a)^2n_H} > 0$ for all $\nu > \bar{\nu}$.□

Proof of Proposition 4. We observe that the OEM's profit always increases in ν as $\partial\Pi^*_{(O,\emptyset)}/\partial\nu = dn_H > 0$, $\partial\Pi^*_{(O,O)}/\partial\nu = d(n_H+n_L) > 0$ and $\partial\Pi^*_{(O,M)}/\partial\nu = d(n_H+an_L) > 0$, with $\partial\Pi^*_{(O,\emptyset)}/\partial\nu > \partial\Pi^*_{(O,M)}/\partial\nu > \partial\Pi^*_{(O,O)}/\partial\nu$. Define $\tilde{\nu} \doteq \left\{\nu : \Pi^*_{(O,O)} - \Pi^*_{(O,\emptyset)} = 0\right\} = \frac{4c_wc_en_L^2+d((g-\theta_L)n_L+(\theta_H-\theta_L)n_H)(4c_en_L-((g-\theta_L)n_L+(\theta_H-\theta_L)n_H))}{4(c_w+c_e)dn_L^2} > 0$ for $c_e > \tilde{c}_e$ and $\hat{\nu} \doteq \left\{\nu : \Pi^*_{(O,\emptyset)} = 0\right\} = \frac{4c_wc_e-d\left(4c_e+d(\theta_H-g)\right)(\theta_H-g)}{4(c_w+c_e)d}$. Given that $\partial\Pi^*_{(O,\emptyset)}/\partial\nu > \partial\Pi^*_{(O,O)}/\partial\nu$ and that $\tilde{\nu} - \hat{\nu} = \frac{(n_H+n_L)(\theta_H-\theta_L)\left(4c_en_L+d\left(n_H(\theta_L-\theta_H)+n_L(\theta_H+\theta_L-2g)\right)\right)}{4(c_w+c_e)n_L^2} > 0$ for $c_e > \tilde{c}_e$, if focused only on selling, the OEM prefers to induce (O,O) for all $\nu \geq \tilde{\nu}$, $(O,\emptyset)$ for all $\nu \in [\hat{\nu}^+, \tilde{\nu})$, and $(\emptyset,\emptyset)$ for all $\nu\,[0,\hat{\nu}^+)$. Define $\bar{\nu} \doteq \left\{\nu : \Pi^*_{(O,M)} - \Pi^*_{(O,O)} = 0\right\}$ and $\underline{\nu} \doteq \left\{\nu : \Pi^*_{(O,M)} - \Pi^*_{(O,\emptyset)} = 0\right\}$

(analytical expressions available upon request).We also calculate $\frac{d\left(\Pi^*_{(O,M)}-\Pi^*_{(O,\emptyset)}\right)}{d\nu}=adn_L>0$ and $\frac{d\left(\Pi^*_{(O,M)}-\Pi^*_{(O,O)}\right)}{d\nu}=-\left(1-a\right)dn_L<0$. Therefore, with car sharing, the OEM prefers to induce (O,M) over $(O,\emptyset)$ for all $\nu\geq\underline{\nu}$ and (O,M) over (O,O) for all $\nu\leq\bar{\nu}$. For (O,M) to exist it is necessary that $\bar{\nu}-\tilde{\nu}=\frac{a(-1+(1-a)(1-d)n_L)\left(4c_wc_en_L+\frac{(1-a)d^2((g-\theta_L)n_L+(\theta_H-\theta_L)n_H)^2}{1+(1-a)dn_L}\right)}{4(1-a)^2dn_L^2(c_w+c_e)}>0$ and $\tilde{\nu}-\underline{\nu}=\frac{(-1+(1-a)(1-d)n_L)\left(\frac{4c_wc_en_L}{1-a}+\frac{d^2((g-\theta_L)n_L+(\theta_H-\theta_L)n_H)^2}{1+(1-a)dn_L}\right)}{4dn_L^2(c_w+c_e)}>0$, both of which are true iff $n_L>\underline{n}_L\doteq\frac{1}{(1-d)(1-a)}$. When $n_L>\underline{n}_L$, the thresholds $\underline{\nu}$, and $\bar{\nu}$ are guaranteed to exist as $\underline{\nu},\bar{\nu}>0$ for $c_e>\hat{c}_e$. Hence, the OEM induces i) $(\emptyset,\emptyset)$ for all $\nu\in[0,\hat{\nu}^+)$, ii) $(O,\emptyset)$ for all $\nu\in[\hat{\nu}^+,\underline{\nu})$, iii) (O,M) for all $\nu\in[\underline{\nu},\bar{\nu}]$ and iv) (O,O) for all $\nu>\bar{\nu}$.$\square$

Proof of Corollary 1. For all $\nu\in[\underline{\nu},\tilde{\nu})$ the OEM replaces $(O,\emptyset)$ with (O,M). We have $E\left(O,\emptyset\right)=\zeta_pn_H+\zeta_u\left(e_H{}^*\right)dn_H$ and $E\left(O,M\right)=\zeta_p\left(n_H+\frac{a}{1-a}+an_Ld\right)+\zeta_u\left(e_H{}^*\right)dn_H+\zeta_u\left(e_L{}^*\right)adn_L$, where $\zeta_u\left(e_L{}^*\right)<\zeta_u\left(e_H{}^*\right)$ because $e_L{}^*>e_H{}^*$. The change in the impact is given by $E\left(O,M\right)-E\left(O,\emptyset\right)=a\left(\frac{1}{1-a}\zeta_p+d\left(\zeta_p+\zeta_u\left(e_L{}^*\right)\right)n_L\right)>0$, therefore, $E\left(O,M\right)>E\left(O,\emptyset\right)$. For all $\nu\in[\tilde{\nu},\bar{\nu}]$ the OEM replaces (O,O) with (O,M). We have $E\left(O,O\right)=\zeta_p\left(n_H+n_L\right)+\left(\zeta_u\left(e_H{}^*\right)n_H+\zeta_u\left(e_L{}^*\left(O,O\right)\right)n_L\right)d$ and $E\left(O,M\right)=\zeta_p\left(n_H+\frac{a}{1-a}+an_Ld\right)+\zeta_u\left(e_H{}^*\right)dn_H+\zeta_u\left(e_L{}^*\left(O,M\right)\right)adn_L$, where $\zeta_u\left(e_L{}^*\left(O,M\right)\right)<\zeta_u\left(e_L{}^*\left(O,O\right)\right)<\zeta_u\left(e_H{}^*\right)$ because $e_L{}^*\left(O,M\right)>e_L{}^*\left(O,O\right)>e_H{}^*$. The change in the impact is given by $E\left(O,M\right)-E\left(O,O\right)=\zeta_p\left(\frac{a}{1-a}-\left(1-ad\right)n_L\right)+\left(a\zeta_u\left(e_L{}^*\left(O,M\right)\right)-\zeta_u\left(e_L{}^*\left(O,O\right)\right)\right)dn_L<0$ for $n_L>\underline{n}_L$, hence, $E\left(O,M\right)<E\left(O,O\right)$.

The majority of the generated insights throughout the paper are not contingent on the OEM selecting a specific equilibrium. The reason is that the optimal prices and efficiencies are characterized for given equilibria and compared across them. Including more *Membership* equilibria may only impact the presentation of Proposition 4 and Corollary 1 without however, changing any of the major points we made based on these results.

Extending the proofs of Proportions 1 and 2 to the (M,M) and $(M,\emptyset)$ equilibria, we obtain: i) $e_L{}^*\left((M,M)\right)=\frac{2c_w(1+(1-a)dn_L)+(1-a)d(\theta_Hn_H-\theta_L(n_H+n_L)+n_Lg)}{2(c_w+c_e)(1+(1-a)dn_L)}$, $e_H{}^*\left((M,M)\right)=\frac{2c_w(1+(1-a)dn_H)+(1-a)dn_H(g-\theta_H)}{2(c_w+c_e)(1+(1-a)dn_H)}$, $e_H{}^*\left((M,\emptyset)\right)=\frac{2c_w+g-\frac{g}{1+dn_L(1-a)})}{2(c_w+c_e)}$ ii) $p^*\left((M,\emptyset)\right)=\nu+\theta_H-\frac{\left(2c_w+g-\frac{g}{1+dn_L(1-a)}\right)\theta_H}{2(c_w+c_e)}$, $p^*_H\left((M,M)\right)$, $p^*_L\left((M,M)\right)$ and iii) $\Pi^*_{(M,M)}$, and $\Pi^*_{(M,\emptyset)}$ (analytical expressions for $p^*_H\left((M,M)\right)$, $p^*_L\left((M,M)\right)$, $\Pi^*_{(M,M)}$, and $\Pi^*_{(M,\emptyset)}$ are available upon request).

Based on these derivations, it is straightforward to obtain the following: $\frac{\partial\Pi^*_{(O,\emptyset)}}{\partial\nu}=dn_H$, $\frac{\partial\Pi^*_{(O,O)}}{\partial\nu}=d\left(n_H+n_L\right)>\frac{\partial\Pi^*_{(O,\emptyset)}}{\partial\nu}$, $\frac{\partial\Pi^*_{(O,M)}}{\partial\nu}=d\left(n_H+an_L\right)<\frac{\partial\Pi^*_{(O,O)}}{\partial\nu}$, $\frac{\partial\Pi^*_{(M,\emptyset)}}{\partial\nu}=adn_L<\frac{\partial\Pi^*_{(O,M)}}{\partial\nu}$, $\frac{\partial\Pi^*_{(M,M)}}{\partial\nu}=ad\left(n_H+n_L\right)<\frac{\partial\Pi^*_{(O,M)}}{\partial\nu}$, and $\frac{\partial\Pi^*_{(M,M)}}{\partial\nu}>\frac{\partial\Pi^*_{(M,\emptyset)}}{\partial\nu}$. The comparisons of $\frac{\partial\Pi^*_{(\cdot,\cdot)}}{\partial\nu}$ under the different equilibria indicate that for moderate values of n_L (i.e., when the Low segment is not very large or very small compared to the High segment) the OEM may prefer to induce the following equilibria (from smaller to larger values of ν): $(M,\emptyset) \to (O,\emptyset) \to (M,M) \to (O,M) \to (O,O)$; in contrast to $(O,\emptyset) \to (O,M) \to (O,O)$, which is what Proposition 4 indicates. However, the inclusion of more *Membership* equilibria does not refute any of the insights provided in Proposition 4 and Corollary 1. In particular, Proposition 4 makes two major points: i) In markets with high valuation of vehicle use, the OEM prefers to sell to both segments; ii) Car sharing is not necessarily associated with low valuations of vehicle use. On the contrary, car sharing can be the optimal choice in a medium-valuation market. Our additional analysis indicates that these two conclusions continue to hold regardless of whether we include *Membership* only equilibria. These insights remain valid even when accounting for $(M,\emptyset)$ and (M,M).

Along the same lines, the primary contribution of Corollary 1 is to show that environmental impact reduction and CAFE level compliance may be at odds, and that there are market conditions where environmental impact increases while CAFE compliance improves and vice versa. After including (M,M) and $(M,\emptyset)$ we obtain the following: $\frac{\partial\left(r(M,M)-r(O,O)\right)}{\partial n_L} = \frac{d(1-a)(g-\theta_L)}{(c_w+c_e)(2+d(n_H+n_L)(1-a))^2} > 0$ and $\frac{\partial\left(E(M,M)-E(O,O)\right)}{\partial n_L} = -(1-ad)\,\zeta_p - d\left(\zeta_u\left(e_L^*(O,O)\right) - a\zeta_u\left(e_L^*(M,M)\right)\right) < 0$ (it is straightforward to show that $\zeta_u\left(e_L^*(O,O)\right) > \zeta_u\left(e_L^*(M,M)\right)$ as $e_L^*(O,O) < e_L^*(M,M)$) implying that for $\hat{n}_L < n_L < \tilde{n}_L$, where $\hat{n}_L \doteq \{n_L : r(M,M) - r(O,O) = 0\}$ and $\tilde{n}_L \doteq \{n_L : E(M,M) - E(O,O) = 0\}$, $r(M,M) > r(O,O)$ and $E(M,M) > E(O,O)$. Similarly, $\frac{\partial\left(r(M,M)-r(O,\emptyset)\right)}{\partial n_L} = \frac{d(1-a)(g-\theta_L)}{(c_w+c_e)(2+d(n_H+n_L)(1-a))^2} > 0$ and $\frac{\partial\left(E(M,M)-E(O,\emptyset)\right)}{\partial n_L} = ad\left(\zeta_p + \zeta_u\left(e_H^*(O,\emptyset)\right)\right) > 0$ imply that for $n_L > \max\{\bar{n}_L, \acute{n}_L\}$, where $\bar{n}_L \doteq \{n_L : r(M,M) - r(O,\emptyset) = 0\}$ and $\acute{n}_L \doteq \{n_L : E(M,M) - E(O,\emptyset) = 0\}$, $r(M,M) > r(O,O)$ and $E(M,M) > E(O,\emptyset)$.

The above analysis indicates that the tension between environmental impact and CAFE level continues to exist even when we account for additional *Membership* equilibria.□

Proof of Proposition 5. In the calculation of $r(O,M)$ we now include an incentive multiplier m as follows: $r((O,M),m) = \frac{n_H}{n_H+m\left(\frac{a}{1-a}+adn_L\right)} e_H^* + \frac{m\left(\frac{a}{1-a}+adn_L\right)}{n_H+m\left(\frac{a}{1-a}+adn_L\right)} e_L^*(O,M)$. It is straightforward to show that $\partial r((O,M),m)/\partial m = \frac{\left(e_L^*(O,M)-e_H^*\right)n_H\left(\frac{a}{1-a}+adn_L\right)}{\left(n_H+m\left(\frac{a}{1-a}+adn_L\right)\right)^2} > 0$ as $e_L^*(O,M) > e_H^*$. Hence, for any $m > \bar{m} \doteq \{m : r((O,M),m) - r(O,O) = 0\}$, where $\bar{m} = \frac{n_H(1-a)(\theta_H-\theta_L)}{a(g(1-(1-d)n_L(1-a))-\theta_L+(1-a)(\theta_L(-dn_L+n_H+n_L)-\theta_H n_H))} > 1$ for any $n_L >$

$\underline{n}_L$ and $\theta_H > \bar{\theta}_H$, we obtain $r\left(\left(O,M\right),m\right) > r\left(O,O\right)$. Similarly, we obtain $\partial\bar{m}/\partial g = -\frac{n_H(-1+(1-d)n_L(1-a))(1-a)(\theta_H-\theta_L)}{a(g(-1+(1-d)n_L(1-a))+\theta_L-(1-a)(-n_H\theta_H+(n_H+(1-d)n_L)\theta_L))^2} < 0$ for any $n_L > \underline{n}_L$, $\partial\bar{m}/\partial n_L = -\frac{(1-d)nH(1-a)^2(po-\theta_L)(\theta_H-\theta_L)}{a(g(-1+(1-d)n_L(1-a))+\theta_L-(1-a)(-n_H\theta_H+(n_H+(1-d)n_L)\theta_L))^2} < 0$, whereas $\partial\bar{m}/\partial\theta_H = \frac{n_H(-1+(1-d)n_L(1-a))(1-a)(g-\theta_L)}{a(g(-1+(1-d)n_L(1-a))+\theta_L-(1-a)(-n_H\theta_H+(n_H+(1-d)n_L)\theta_L))^2} > 0$ for any $n_L > \underline{n}_L$, $\partial\bar{m}/\partial d = \frac{n_H n_L(1-a)^2(g-\theta_L)(\theta_H-\theta_L)}{a(g(-1+(1-d)n_L(1-a))+\theta_L-(1-a)(-n_H\theta_H+(n_H+(1-d)n_L)\theta_L))^2} > 0$, $\partial\bar{m}/\partial n_H = \frac{n_H(-1+(1-d)n_L(1-a))(1-a)(g-\theta_L)(\theta_H-\theta_L)}{a(g(-1+(1-d)n_L(1-a))+\theta_L-(1-a)(-n_H\theta_H+(n_H+(1-d)n_L)\theta_L))^2} > 0$ for any $n_L > \underline{n}_L$, and $\partial\bar{m}/\partial a = -\frac{n_H(\theta_H-\theta_L)\left(g\left(-1+(1-d)n_L(1-a)^2+\theta_L+(1-a)^2(n_H(\theta_H-\theta_L)-(1-d)n_L\theta_L)\right)\right)}{a^2(g(-1+(1-d)n_L(1-a))+\theta_L-(1-a)(-n_H\theta_H+(n_H+(1-d)n_L)\theta_L))^2} > 0$ iff $n_H < \frac{\left(1-(1-d)n_L(1-a)^2\right)(g-\theta_L)}{(1-a)^2(\theta_H-\theta_L)}$.□

Proof of Proposition 6. The range of ν values for which the OEM prefers to induce $(O,\emptyset)$ over (O,O) increases in θ_H because $\partial\tilde{\nu}/\partial\theta_H = \frac{n_H\left(2c_e n_L-d\left((g-\theta_L)n_L+(\theta_H-\theta_L)n_H\right)\right)}{2(c_w+c_e)n_L^2} > 0$ for $c_e > \tilde{c}_e$. Similarly, the range of ν values for which the OEM induces (O,M) increases in θ_H because $\partial\left(\bar{\nu}-\underline{\nu}\right)/\partial\theta_H = \frac{dn_H\left(-1+(1-d)n_L(1-a)\right)\left((g-\theta_L)n_L+(\theta_H-\theta_L)n_H\right)}{2(c_w+c_e)n_L^2\left(1+dn_L(1-a)\right)(1-a)} > 0$ for $n_L > \underline{n}_L$.□

Proof of Proposition 7. It follows the proof of Propositions 1 and 2 with the difference that when optimizing $\tilde{\Pi}_{(h,l)}$ the CAFE constraint $r\left(h,l\right) \geq R$ is binding (i.e., the regulation R exceeds the CAFE levels calculated in Proposition 3). As before, we focus on interior values of fuel efficiencies.

Under $(O,\emptyset)$ the OEM determines the optimal efficiency based on $\max_{R\leq e_H\leq 1}\tilde{\Pi}_{(O,\emptyset)}$. The profit $\tilde{\Pi}_{(O,\emptyset)}$ is concave in e_H, therefore, if R is larger than the unconstrained optimal efficiency calculated in Proposition 1, the OEM chooses ${e_H}^* = R$, which results in ${F_H}^* = d\left(\left(\theta_H-g\right)\left(1-R\right)+\nu\right)$ and $\partial F_H^*/\partial R = -d\left(\theta_H-g\right) < 0$.

Under (O,O) the OEM determines the optimal efficiencies based on $\max_{e_H,e_L}\tilde{\Pi}_{(O,O)}$ such that $0 \leq e_H \leq 1$, $0 \leq e_L \leq 1$ and $\frac{n_H}{n_H+n_L}e_H + \frac{n_L}{n_H+n_L}e_L \geq R$. We form the Lagrangean $\mathcal{L} = \tilde{\Pi}_{(O,O)} + \psi\left(\frac{n_H}{n_H+n_L}e_H + \frac{n_L}{n_H+n_L}e_L - R\right)$. Given that $\tilde{\Pi}_{(O,O)}$ is jointly concave in e_H and e_L we solve $\partial\mathcal{L}/e_H = 0$, $\partial\mathcal{L}/e_L = 0$ and $\partial\mathcal{L}/\psi = 0$ to obtain ${e_H}^* = R - \frac{d(\theta_H-\theta_L)}{2(c_w+c_e)}$, ${e_L}^* = R + \frac{d(\theta_H-\theta_L)n_H}{2(c_w+c_e)n_L}$, and $\psi = \left(n_H+n_L\right)\left(2\left(c_w+c_e\right)R - 2c_w - d\left(g-\theta_L\right)\right) > 0$ for all $R > r\left(O,O\right)$. Based on these values we also obtain ${F_H}^* = d\left(\left(g-\theta_L\right)R + \frac{2(c_w+c_e)n_L(\theta_L-g+\nu)+d(\theta_H-\theta_L)((\theta_H-\theta_L)+(\theta_H-g)n_L)}{2n_L(c_w+c_e)}\right)$, ${F_L}^* = {F_H}^* - \frac{d^2(n_H+n_L)(\theta_H-g)(\theta_H-\theta_L)}{2(c_w+c_e)n_L}$ and $\partial F_H^*\left(O,O\right)/\partial R = \partial F_L^*\left(O,O\right)/\partial R = d\left(g-\theta_L\right) > 0$.

Under (O,M) the OEM determines the optimal efficiencies based on $\max_{e_H,e_L}\tilde{\Pi}_{(O,M)}$ such that $0 \leq e_H \leq 1$, $0 \leq e_L \leq 1$ and $\frac{n_H}{n_H+\left(\frac{a}{1-a}+an_Ld\right)}e_H + \frac{\frac{a}{1-a}+an_Ld}{n_H+\left(\frac{a}{1-a}+an_Ld\right)}e_L \geq R$. The Lagrangean is given by $\mathcal{L} = \tilde{\Pi}_{(O,M)} + \psi\left(\frac{n_H}{n_H+\left(\frac{a}{1-a}+an_Ld\right)}e_H + \frac{\frac{a}{1-a}+an_Ld}{n_H+\left(\frac{a}{1-a}+an_Ld\right)}e_L - R\right)$. Once again, given that $\tilde{\Pi}_{(O,M)}$ is jointly concave in e_H and e_L we solve $\partial\mathcal{L}/e_H = 0$, $\partial\mathcal{L}/e_L = 0$ and $\partial\mathcal{L}/\psi = 0$ to obtain ${e_H}^* = R - \frac{ad(\theta_H(-a(dn_L+n_H)+dn_L+n_H+1)+g((1-a)(1-d)n_L-1)-(1-a)\theta_L(n_H+n_L))}{2(c_w+c_e)(a+(1-a)(adn_L+n_H))}$, ${e_L}^* = R + \frac{(1-a)dn_H(\theta_H((1-a)dn_L+(1-a)n_H+1)-g(n_L(a-ad+d-1)+1)-(1-a)\theta_L(n_H+n_L))}{2(c_w+c_e)((1-a)dn_L+1)(a+(1-a)(adn_L+n_H))}$, and $\psi = \frac{2c_w(1-R)((a-1)(adn_L+n_H)-a)-(1-a)d(an_L(-2c_eR-\theta_L+g)+n_H((a-1)\theta_H-a\theta_L+g))+2c_eR((1-a)n_H+a)}{1-a} > 0$ for all $R > r(O,M)$. The analytical expressions for ${F_H}^*$ and p^* are available upon request. By differentiating we obtain $\partial p^*/\partial R = -\theta_L < 0$, and $\partial F_H^*(O,M)/\partial R = d\left(g - \left(1-a\right)\theta_H - a\theta_L\right) < 0$ iff $a < \frac{\theta_H - g}{\theta_H - \theta_L} \in (0,1)$.$\square$

Proof of Proposition 8. For brevity define $\bar{\Pi}^*_{(O,\emptyset)}$, $\bar{\Pi}^*_{(O,O)}$, and $\bar{\Pi}^*_{(O,M)}$ to be the unconstrained optimal profits we developed in the Proof of Propositions 1 and 2 for the $(O,\emptyset)$, (O,O), and (O,M) equilibrium, respectively. Using the optimal prices and fuel efficiencies we calculated in Proposition 7 we obtain the constrained optimal profits $\Pi^*_{(O,\emptyset)} = \bar{\Pi}^*_{(O,\emptyset)} - \frac{n_H\left(2c_w-2(c_w+c_e)R-d(\theta_H-g)\right)^2}{4(c_w+c_e)}$, $\Pi^*_{(O,O)} = \bar{\Pi}^*_{(O,O)} - \frac{(n_H+n_L)\left(2c_w-2(c_w+c_e)R-d(g-\theta_L)\right)^2}{4(c_w+c_e)}$, and $\Pi^*_{(O,M)} = \bar{\Pi}^*_{(O,M)} - \frac{\left(2c_w(1-R)((1-a)(adn_L+n_H)+a)+(1-a)d(an_L(-2c_eR-\theta_L+g)+n_H((a-1)\theta_H-a\theta_L+g))-2c_eR((1-a)n_H+a)\right)^2}{4(1-a)(c_w+c_e)(a+(1-a)(adn_L+n_H))}$. Paralleling Proposition 4, we calculate $\tilde{\nu} \doteq \left\{\nu : \Pi^*_{(O,O)} - \Pi^*_{(O,\emptyset)} = 0\right\}$, $\underline{\nu} \doteq \left\{\nu : \Pi^*_{(O,M)} - \Pi^*_{(O,\emptyset)} = 0\right\}$ and $\bar{\nu} \doteq \left\{\nu : \Pi^*_{(O,O)} - \Pi^*_{(O,M)} = 0\right\}$. When selling only $r(O,O) > r(O,\emptyset)$ (see Proposition 3), hence, the CAFE standard is binding for either equilibrium when $R \geq r(O,O)$. $\tilde{\nu}$ is convex in R because $\partial^2\tilde{\nu}/\partial R^2 = \frac{2(c_w+c_e)}{d} > 0$. In addition, $\partial\tilde{\nu}/\partial R|_{R=e^*_L(O,O)} = 0$. Therefore, $\partial\tilde{\nu}/\partial R > 0$ for all $R \geq e^*_L(O,O)$ and $\partial\tilde{\nu}/\partial R < 0$ for all $R \in \left[r(O,O), e^*_L(O,O)\right)$. With car sharing $r(O,M) < r(O,O)$ for $\theta_H > \bar{\theta}_H$ and $r(O,M) > r(O,O)$ for $\theta_H < \bar{\theta}_H$ (see Proposition 3). Hence, for $\theta_H > \bar{\theta}_H$ the CAFE standard is binding when $R \geq r(O,O)$. The range $\bar{\nu} - \underline{\nu}$ is convex in R because $\partial^2\left(\bar{\nu} - \underline{\nu}\right)/\partial^2 R = \frac{2(c_w+c_e)\left(-1+(1-d)n_L(1-a)\right)}{dn_L(1-a)^2} > 0$ for $n_L > \underline{n}_L$. Furthermore, $\partial\left(\bar{\nu} - \underline{\nu}\right)/\partial R|_{R=r(O,O)} = \frac{d(g-\theta_L)\left(-1+(1-d)n_L(1-a)\right)}{2(1-a)^2dn_L} > 0$ for $n_L > \underline{n}_L$. Therefore, $\partial\left(\bar{\nu} - \underline{\nu}\right)/\partial R > 0$ for all $R > r(O,O)$. Similarly, for $\theta_H < \bar{\theta}_H$ the CAFE standard is binding when for $R \geq r(O,M)$. In this case, $\partial\left(\bar{\nu} - \underline{\nu}\right)/\partial R|_{R=r(O,M)} = \frac{\left(-1+(1-a)n_L(1-d)\right)\left(n_H\left(a(\theta_H-\theta_L)-(\theta_H-g)\right)+an_L(g-\theta_L)\right)}{(1-a)n_L\left((1-a)(adn_L+n_H)+a\right)} > 0$ for $n_L > \underline{n}_L$ and $\theta_H < \bar{\theta}_H$. Therefore, $\partial^2\left(\bar{\nu} - \underline{\nu}\right)/\partial^2 R = \frac{2(c_w+c_e)\left(-1+(1-d)n_L(1-a)\right)}{dn_L(1-a)^2} > 0$ implies that $\partial\left(\bar{\nu} - \underline{\nu}\right)/\partial R > 0$ for all $R > r(O,M)$. Thus, $\partial\left(\bar{\nu} - \underline{\nu}\right)/\partial R > 0$ for all θ_H.$\square$

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