Appendix A: Summary of the Notation

Table 1 Notation

Parameter	Units	Definition
f	\$	fixed (annual) membership fee
Q_{ik}	Scalar	Population of customer group k in area i
g_i	\$	Fixed cost of serving area i
r	\$ per unit time per EV	Usage based price of the service
t_{ij}	unit of time	Expected travel time to destination j from origin i
c	\$	Average charging cost each time
η	\$ per unit of time	Repositioning cost
τ_{ij}	unit of time	Expected reposition time to destination j from origin i
h	\$	Annually amortized EV cost
a_{ij}	Scalar random variable	Utility of serving destination j for a customer in area i
$\bar{a}_{ij}$	Scalar	Expected value of a_{ij}
$\mathbf{\Gamma}_i$	Scalar	Covariance matrix of a_{ij} for area i
b_k	Scalar	Aspirational level for a customer in group k to adopt the service
μ_i	Trips per unit of time	Outbound trip rate from area i
P_{ij}	[0,1]	Probability of trip destination j given the origin i
w_{ik}	[0,1]	Weight (in trip) of customer group k in area i
α	(0,1)	Service level
P_c	[0,1]	Probability of an arrival EV needs recharge
t_c	unit time per EV	Average charging time per EV

Decision Variables	Units	Definition
x_i	$\{0,1\}$	1 if area i is served; 0 otherwise.
q_{ik}	[0,1]	Adoption rate of customer group k in area i
N	EVs	Fleet size
Ψ_{ij}	Trips per unit of time	Trip demand rate from i to j
ψ'_{ij}	Trips per unit of time	Satisfied trip rate if j is served
ψ_{ij}	Trips per unit of time	Satisfied trip rate: $\psi_{ij} = \psi'_{ij} x_j$
ϕ_{ij}	Trips per unit of time	Repositioning trip rate from i to j
λ_i	Trips per unit of time	EV arrival rate at charging stations in area i

Appendix B: Proofs of Analytical Results

B.1. Proof of Lemma 1

We begin the proof with deriving the formulation for the worst-case adoption rate in (4) following a similar approach as in El Ghaoui et al. (2003). For ease of exposition, it is safe to temporarily drop the index $i \in I$ and $k \in K$. Given the service region decision $\mathbf{x}$, solving the following optimization problem yields the worst-case non-adoption rate $V(\mathbf{x}) = \sup \text{Prob}(\sum_{j\in J} a_{ij} x_j \le b_k)$:

$$\max \mathbb{E}[I(\mathbf{a})]$$

$$\text{s.t.} \quad \int_{\mathbb{R}^n_+} \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix}^T p(\mathbf{a}) d\mathbf{a} = \mathbf{\Sigma}$$

where p is the probability density function and $I(\mathbf{a})$ is the indicator function defined as

$$I(\mathbf{a}) = \begin{cases} 1, \text{ if } \sum_{j\in J} a_j x_j \le b \\ 0, \quad \text{otherwise} \end{cases}$$

We write the Lagrange function with symmetric multiplier matrix $M \in \mathcal{S}_{n+1}$ as:

$$
\begin{aligned}
L(p,\mathbf{M}) &= \int_{\mathbb{R}^n_+} I(\mathbf{a})p(\mathbf{a})d\mathbf{a} + \left\langle \mathbf{M}, \boldsymbol{\Sigma} - \int_{\mathbb{R}^n_+} \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix}^T p(\mathbf{a})d\mathbf{a} \right\rangle \\
&= \langle \mathbf{M}, \boldsymbol{\Sigma} \rangle + \int_{\mathbb{R}^n_+} \big(I(\mathbf{a}) - l(\mathbf{a})\big)p(\mathbf{a})d\mathbf{a}
\end{aligned}
$$

where $l(\mathbf{a}) = [\,\mathbf{a}\ 1\,]\mathbf{M}[\,\mathbf{a}\ 1\,]^T$. Since $\boldsymbol{\Sigma} \succ 0$, strong duality holds. Therefore, we have

$$
V(\mathbf{x}) = \inf_{\mathbf{M}=\mathbf{M}^T} \sup_p L(p,\mathbf{M})
$$

where

$$
\begin{aligned}
\sup_p L(p,\mathbf{M}) &= \langle \mathbf{M}, \boldsymbol{\Sigma} \rangle + \sup_{p \in \mathcal{P}} \int_{\mathbb{R}^n_+} \big(I(\mathbf{a}) - l(\mathbf{a})\big)p(\mathbf{a})d\mathbf{a} \\
&= \begin{cases} \langle \mathbf{M}, \boldsymbol{\Sigma} \rangle, \text{ if } I(\mathbf{a}) - l(\mathbf{a}) \le 0, \forall \mathbf{a} \in \mathbb{R}^n_+ \\ +\infty, \qquad\qquad\qquad\qquad \text{otherwise.} \end{cases}
\end{aligned}
$$

$V(\mathbf{x})$ is finite if and only if $I(\mathbf{a}) - l(\mathbf{a}) \le 0, \forall \mathbf{a} \in \mathbb{R}^n_+$. There are two cases:

1. $l(\mathbf{a}) \ge 0, \forall \mathbf{a} \in \mathbb{R}^n_+$.
2. $l(\mathbf{a}) \ge 1, \forall \mathbf{a} \in \mathbb{R}^n_+$ such that $\sum_{j\in J} a_j x_j \le b$.

Case 1 is equivalent to $\mathbf{M} \succeq_{co} 0$ and Case 2 holds if there exist a scalar $\tau \ge 0$ such that, $l(\mathbf{a}) \ge 1 - 2\tau(\mathbf{a}^T\mathbf{x} - b)$. In addition, the Slater's condition is satisfied as there exist an $\mathbf{a}$ such that $\sum_{j\in J} a_j x_j < b$. As a result, the above constraint for Case 2 is also sufficient. Equivalently, it can be written as $\mathbf{M} + \begin{bmatrix} 0 & \tau\mathbf{x} \\ \tau\mathbf{x}^T & -1-2\tau b \end{bmatrix} \succeq_{co} 0$.

$V(\mathbf{x})$ is then the solution to the following copositive program (CP):

$$
\begin{aligned}
V(\mathbf{x}) = \min &\langle \mathbf{M}, \boldsymbol{\Sigma} \rangle \\
\text{s.t.} & \\
&\tau \ge 0 \\
&\mathbf{M} \succeq_{co} 0 \\
&\mathbf{M} + \begin{bmatrix} 0 & \tau\mathbf{x} \\ \tau\mathbf{x}^T & -1-2\tau b \end{bmatrix} \succeq_{co} 0.
\end{aligned}
$$

We then complete the proof by restoring the indices $i \in I$ and $k \in K$ and replacing $V(\mathbf{x}) \le 1-q$ (as $V(\mathbf{x})$ is the non-adoption rate) with the above CP:

$$
\begin{aligned}
&\langle \mathbf{M}_{ik}, \boldsymbol{\Sigma}_i \rangle \le 1-q \\
&\tau_{ik} \ge 0 \\
&\mathbf{M}_{ik} \succeq_{co} 0 \\
&\mathbf{M}_{ik} + \begin{bmatrix} 0 & \tau_{ik}\mathbf{x} \\ \tau_{ik}\mathbf{x}^T & -1-2\tau_{ik} b_k \end{bmatrix} \succeq_{co} 0
\end{aligned}
$$

The last step is to linearize the term $\tau_{ik}\mathbf{x}$. Since $\mathbf{x}$ is a binary vector and τ_{ik} is continuous, we can replace the term $\tau_{ik}\mathbf{x}$ by vector $\mathbf{d}_{ik}$ with the following constraints:

$$
\begin{aligned}
&-\rho\mathbf{x} \le \mathbf{d}_{ik} \\
&\mathbf{d}_{ik} \le \rho\mathbf{x} \\
&\tau_{ik}\mathbf{e} + \rho(\mathbf{x} - \mathbf{e}) \le \mathbf{d}_{ik} \\
&\mathbf{d}_{ik} \le \tau_{ik}\mathbf{e} + \rho(\mathbf{e} - \mathbf{x})
\end{aligned}
$$

where ρ is a large scalar and $\mathbf{e}$ is the vector of ones. This completes the proof.

B.2. Proof of Proposition 1

By restricting the copositive constraints with semidefinite constraints, we obtain a lower bound to the optimal solution to the formulation with copositive constraints in Lemma 1. When $\Theta(\mathbf{q},\mathbf{x})$ is linear, the following formulation is a mixed integer semidefinite program (MISDP):

$$\max_{q_{ik},x_i,\mathbf{M}_{ik},\tau_{ik},\mathbf{d}_{ik}} \sum_{i\in I}\sum_{k\in K} fQ_{ik}q_{ik} - \sum_{i\in I} g_i x_i + \Theta(\mathbf{q},\mathbf{x})$$

s.t.

$$\langle \mathbf{M}_{ik}, \mathbf{\Sigma}_{ik}\rangle \le 1 - q_{ik}, \forall i\in I, \forall k \in K \tag{1}$$

$$\mathbf{M}_{ik} \succeq 0, \forall i\in I, \forall k \in K \tag{2}$$

$$\mathbf{M}_{ik} + \begin{bmatrix} 0 & \mathbf{d}_{ik} \\ \mathbf{d}_{ik}^T & -1-2\tau_{ik}b_k \end{bmatrix} \succeq 0, \forall i\in I, \forall k \in K \tag{3}$$

$$-\rho\mathbf{x} \le \mathbf{d}_{ik}, \forall i\in I, \forall k \in K \tag{4}$$

$$\mathbf{d}_{ik} \le \rho\mathbf{x}, \forall i\in I, \forall k \in K \tag{5}$$

$$\tau_{ik}\mathbf{e} + \rho(\mathbf{x}-\mathbf{e}) \le \mathbf{d}_{ik}, \forall i\in I, \forall k \in K \tag{6}$$

$$\mathbf{d}_{ik} \le \tau_{ik}\mathbf{e} + \rho(\mathbf{e}-\mathbf{x}), \forall i\in I, \forall k \in K \tag{7}$$

$$\tau_{ik} \ge 0, \forall i\in I, \forall k \in K \tag{8}$$

$$q_{ik} \le x_i, \forall i\in I, \forall k \in K$$

$$x_i \in \{0,1\}, \forall i \in I.$$

From Appendix B.1, we note that constraints (4)-(7) are imposed to linearize the term $\tau_{ik}\mathbf{x}$. Let us temporarily restore the nonlinear term $\tau_{ik}\mathbf{x}$, and the constraints (1)-(8) become:

$$\begin{aligned}
&\langle \mathbf{M}_{ik}, \mathbf{\Sigma}_{ik}\rangle \le 1 - q_{ik}, \forall i\in I, \forall k \in K\\
&\mathbf{M}_{ik} \succeq 0, \forall i\in I, \forall k \in K\\
&\mathbf{M}_{ik} + \begin{bmatrix} 0 & \tau_{ik}\mathbf{x} \\ \tau_{ik}\mathbf{x}^T & -1-2\tau_{ik}b_k \end{bmatrix} \succeq 0, \forall i\in I, \forall k \in K\\
&\tau_{ik} \ge 0, \forall i\in I, \forall k \in K.
\end{aligned}$$

Moreover, it can be shown that τ_{ik} in the optimal solution is strictly positive (as long as an optimal solution exists), as $\tau_{ik}=0$ leads to $q_{ik}\le 0$. We are then able to rewrite the above constraints as follows, by defining new variables $\tilde{\tau}_{ik} = \frac{1}{\tau_{ik}} \ge 0$ and $\tilde{\mathbf{M}}_{ik} = \mathbf{M}_{ik}\tilde{\tau}_{ik}$:

$$\langle \tilde{\mathbf{M}}_{ik}, \mathbf{\Sigma}_{ik}\rangle \le \tilde{\tau}_{ik}(1 - q_{ik}), \forall i\in I, \forall k \in K \tag{9}$$

$$\tilde{\mathbf{M}}_{ik} \succeq 0, \forall i\in I, \forall k \in K \tag{10}$$

$$\tilde{\mathbf{M}}_{ik} + \begin{bmatrix} 0 & \mathbf{x} \\ \mathbf{x}^T & -\tilde{\tau}_{ik}-2b_k \end{bmatrix} \succeq 0, \forall i\in I, \forall k \in K \tag{11}$$

$$\tilde{\tau}_{ik} \ge 0, \forall i\in I, \forall k \in K \tag{12}$$

We now show that constraints (9)-(12) can be reformulated in second-order conic form. Let $\mathcal{P}$ be the set of probability distributions with mean μ and covariance matrix $\mathbf{\Gamma} \succ 0$. Let $\epsilon \in (0,1]$ and $\gamma \in \mathbb{R}$ given. The following propositions are equivalent (El Ghaoui et al. 2003):

1. $\sqrt{\frac{1-\epsilon}{\epsilon}}\sqrt{\mathbf{x}^T\mathbf{\Gamma}_i\mathbf{x}} - \mu^T\mathbf{x} \leq \gamma$
2. There exist a symmetric matrix $\mathbf{M}$ and $\tau \in \mathbb{R}$ such that

$$\begin{aligned}
&\langle \mathbf{M}, \mathbf{\Sigma}\rangle \leq \tau\epsilon \\
&\mathbf{M} \succeq 0 \\
&\mathbf{M} + \begin{bmatrix} 0 & \mathbf{x} \\ \mathbf{x}^T & -\tau + 2\gamma \end{bmatrix} \succeq 0 \\
&\tau \geq 0
\end{aligned}$$

where $\mathbf{\Sigma}$ is the second-moment matrix.

By viewing $1-q_{ik}$ as ϵ and $-b_k$ as γ in the above result, the semidefinite constraint (9)-(12) can be expressed as

$$\begin{aligned}
&\sqrt{\frac{q_{ik}}{1-q_{ik}}}\sqrt{X^T\mathbf{\Gamma}_i X} - \bar{\mathbf{a}}_i^T\mathbf{x} + b_k \leq 0 \\
&\equiv \begin{cases} 1 - q_{ik} \geq \frac{\mathbf{x}^T\mathbf{\Gamma}_i\mathbf{x}}{(\bar{\mathbf{a}}_i^T\mathbf{x} - b_k)^2 + \mathbf{x}^T\Gamma_i\mathbf{x}} \\ \bar{\mathbf{a}}_i^T\mathbf{x} - b_k \geq 0. \end{cases}
\end{aligned} \tag{13}$$

In the case of $\bar{\mathbf{a}}_i^T\mathbf{x} - b_k \leq 0$, the worst case probability is 1, because there exists a feasible two-point distribution of $\bar{\mathbf{a}}_i^T\mathbf{x}$ with both points less than b_k. To allow such possibility, we further introduce a set of disjunctive constraints. When $\bar{\mathbf{a}}_i^T\mathbf{x} - b_k \leq 0$, we set q_{ik} to 0. Therefore, we have either $\bar{\mathbf{a}}_i^T\mathbf{x} \geq b_k$ or $q_{ik} = 0$. By introducing new variables, we can express the disjunctive constraints as the feasible set:

$$\mathcal{X}_{ik} = \left\{ (\mathbf{x}, q_{ik}) : \begin{array}{l} b_k u_{ik} \leq \bar{\mathbf{a}}_i^T\mathbf{x} \\ q_{ik} \leq u_{ik} \\ u_{ik} \in \{0,1\} \end{array} \right\}. \tag{14}$$

Moreover, we linearize the term $x_{j_1}x_{j_2}$ with $z_{j_1j_2}$ defined in

$$\mathcal{Z} = \left\{ (z_{j_1j_2}, x_{j_1}, x_{j_2}) : \begin{array}{l} z_{j_1j_2} \leq x_{j_1} \\ z_{j_1j_2} \leq x_{j_2} \\ x_{j_1} + x_{j_2} - 1 \leq z_{j_1j_2} \\ z_{j_1j_2} \geq 0 \end{array} \right\}. \tag{15}$$

Lastly, the first inequality in constraint (13) can be expressed as $(1 - q_{ik} + (\bar{\mathbf{a}}_i^T\mathbf{x} - b_k)^2 + \mathbf{x}^T\mathbf{\Gamma}_i\mathbf{x})^2 \geq 4\mathbf{x}^T\mathbf{\Gamma}_i\mathbf{x} + (1 - q_{ik} - (\bar{\mathbf{a}}_i^T\mathbf{x} - b_k)^2 - \mathbf{x}^T\mathbf{\Gamma}_i\mathbf{x})^2$. Since $z_{j_1j_2} = x_{j_1}x_{j_2}$ in (15), we further linearize the terms $(\bar{\mathbf{a}}_i^T\mathbf{x})^2$ and $\mathbf{x}^T\mathbf{\Gamma}_i\mathbf{x}$ as $\sum_{(j_1,j_2)\in I\times I}\bar{a}_{ij_1}\bar{a}_{ij_2}z_{j_1j_2}$ and $\sum_{(j_1,j_2)\in I\times I}\sigma_{ij_1j_2}z_{j_1j_2}$ respectively. Let $v_{ik} = b_k^2 + \sum_{(j_1,j_2)\in I\times I}(\bar{a}_{ij_1}\bar{a}_{ij_2} + \sigma_{ij_1j_2})z_{j_1j_2} - 2b_k\sum_{j\in I}\bar{a}_{ij}x_j$, it is reorganized as $\left\| \begin{matrix} 1 - q_{ik} - v_{ik} \\ 2\mathbf{\Gamma}_i^{\frac{1}{2}}\mathbf{x} \end{matrix} \right\|_2 \leq 1 - q_{ik} + v_{ik}$ and leads to the second-order cone constraint (5).

B.3. Detailed Formulation of the MISOCP (22)

$$\begin{aligned}
\max_{\substack{x_i, q_{ik}, N, \Psi_{ij} \\ \psi_{ij}, \psi'_{ij}, \phi_{ij}, \lambda_i}} \quad & \sum_{i\in I}\sum_{k\in K} fQ_{ik}q_{ik} - \sum_{i\in I} g_i x_i + \xi\left(\sum_{j\in I}\sum_{i\in I} rt_{ij}\psi_{ij} - \sum_{i\in I}\sum_{j\in I}\eta\tau_{ij}\phi_{ij} - \sum_{i\in I} c\lambda_i\right) - hN \\
\text{s.t.} \quad & \\
& \left\| \begin{matrix} 1 - q_{ik} - v_{ik} \\ 2\mathbf{\Gamma}_i^{\frac{1}{2}}\mathbf{x} \end{matrix} \right\|_2 \leq 1 - q_{ik} + v_{ik}, \forall i \in I, \forall k \in K \\
& v_{ik} = b_k^2 + \sum_{(j_1,j_2)\in I\times I}(\bar{a}_{ij_1}\bar{a}_{ij_2} + \sigma_{ij_1j_2})z_{j_1j_2} - 2b_k\sum_{j\in I}\bar{a}_{ij}x_j, \forall i \in I, \forall k \in K
\end{aligned}$$

$$q_{ik} \le x_i, \forall i \in I, \forall k \in K$$
$$\Psi_{ij} = P_{ij}\mu_i \sum_{k\in K} w_{ik}q_{ik}, \forall i,j \in I$$
$$\psi'_{ij} \ge \alpha\Psi_{ij}, \forall i,j \in I$$
$$\psi'_{ij} \le \Psi_{ij}, \forall i,j \in I$$
$$\psi'_{ij} = P_{ij}\sum_{l\in I}\psi'_{il}, \forall i,j \in I$$
$$\psi_{ij} \le \psi'_{ij}, \forall i,j \in I$$
$$\psi_{ij} \le P_{ij}\mu_i x_j, \forall i,j \in I$$
$$\psi'_{ij} + P_{ij}\mu_i(x_j - 1) \le \psi_{ij}, \forall i,j \in I$$
$$\sum_j \psi_{ji} + \sum_j \phi_{ji} = \sum_j \psi_{ij} + \sum_j \phi_{ij}, \forall i \in I$$
$$\lambda_i = P_c \sum_{j\in J}\psi_{ji}, \forall i \in I$$
$$\sum_{i\in I}\frac{\alpha}{1-\alpha}x_i + \sum_{j\in I}\sum_{i\in I} t_{ij}\psi_{ij} + \sum_{i\in I}\sum_{j\in I}\tau_{ij}\phi_{ij} + \sum_{i\in I} t_c\lambda_i \le N$$
$$(z_{j_1j_2}, x_{j_1}, x_{j_2}) \in \mathcal{Z}, \forall j_1, j_2 \in I$$
$$(\mathbf{x}, q_{ik}) \in \mathcal{X}_{ik}, \forall i \in I, \forall k \in K$$
$$N, q_{ik}, \psi_{ij}, \psi'_{ij}, \Psi_{ij}, \phi_{ij}, \lambda_i \ge 0, \forall i,j \in I, k \in K$$
$$x_i \in \{0,1\}, \forall i \in I.$$

B.4. Time-varying Travel Pattern

The customer adoption decision depends only on the service region coverage and thus the constraints (3) and (5)-(7) from the adoption rate model still hold. To deal with time-varying travel patterns, we partition the day into T periods (indexed by t), within each of which the travel pattern is assumed (approximated) to be stationary, i.e., represented by P_{ij}^t and μ_i^t for each period t. Let $l(t)$ be the length of period t, the problem in the presence of time-varying travel patterns, following the pointwise stationary approximation (Green and Kolesar 1991), is then formulated as:

$$\max_{\substack{x_i, q_{ik}, N, \Psi_{ij}^t \\ \psi_{ij}^t, \psi_{ij}^{\prime t}, \phi_{ij}^t, \lambda_i^t}} \sum_{i\in I}\sum_{k\in K} fQ_{ik}q_{ik} - \sum_{i\in I} g_i x_i + \xi\sum_{t\in T} l(t)\left(\sum_{j\in I}\sum_{i\in I} rt_{ij}\psi_{ij}^t - \sum_{i\in I}\sum_{j\in I}\eta\tau_{ij}\phi_{ij}^t - \sum_{i\in I} c\lambda_i^t\right) - hN$$

s.t.

$$\text{Constraint } (3)\&(5)-(7)$$
$$\Psi_{ij}^t = P_{ij}^t\mu_i^t\sum_{k\in K} w_{ik}q_{ik}, \forall i,j \in I, t \in T$$
$$\psi_{ij}^{\prime t} \ge \alpha\Psi_{ij}^t, \forall i \in I, t \in T$$
$$\psi_{ij}^{\prime t} \le \Psi_{ij}^t, \forall i,j \in I, t \in T$$
$$\psi_{ij}^{\prime t} = P_{ij}^t\sum_{l\in I}\psi'_{il}, \forall i,j \in I, t \in T$$
$$\psi_{ij}^t \le \psi_{ij}^{\prime t}, \forall i,j \in I, t \in T$$

$$
\begin{aligned}
&\psi_{ij}^t \leq P_{ij}^t \mu_i^t x_j, \forall i,j \in I, t \in T \\
&\psi_{ij}^{\prime t} + P_{ij}^t \mu_i^t (x_j - 1) \leq \psi_{ij}^t, \forall i,j \in I, t \in T \\
&\sum_j \psi_{ji}^t + \sum_j \phi_{ji}^t = \sum_j \psi_{ij}^t + \sum_j \phi_{ij}^t, \forall i \in I, t \in T \\
&\lambda_i^t = P_c \sum_{j \in J} \psi_{ji}^t, \forall i \in I, t \in T \\
&\sum_{i \in I} \frac{\alpha}{1-\alpha} x_i + \sum_{j \in I} \sum_{i \in I} t_{ij} \frac{\psi_{ij}^t}{l(t)} + \sum_{i \in I} \sum_{j \in I} \tau_{ij} \frac{\phi_{ij}^t}{l(t)} + \sum_{i \in I} t_c \frac{\lambda_i^t}{l(t)} \leq N, \forall t \in T \\
&N, q_{ik}, \psi_{ij}^t, \psi_{ij}^{\prime t}, \Psi_{ij}^t, \phi_{ij}^t, \lambda_i^t \geq 0, \forall i,j \in I, \forall k \in K, \forall t \in T \\
&x_i \in \{0,1\}, \forall i \in I.
\end{aligned}
$$

B.5. Proof of Proposition 2

Given a service region design $I' \subseteq I$, let q'_{ik} be the realized adoption rate, e.g. $q'_{ik} = \text{Prob}\big(\sum_{j \in I'} a_{ij} x_j \geq b_k\big)$. Note that the fleet size constraint (21) is binding, the formulation (22) becomes a linear program below:

$$
\begin{aligned}
\max_{q_{ik}, \psi_{ij}, \phi_{ij}} & \sum_{i \in I'} \sum_{k \in K} f Q_{ik} q_{ik} - \sum_{i \in I'} g_i x_i + \xi \left(\sum_{j \in I'} \sum_{i \in I'} r t_{ij} \psi_{ij} - \sum_{i \in I'} \sum_{j \in I'} \eta \tau_{ij} \phi_{ij} - \sum_{i \in I'} c P_c \sum_{j \in I'} \psi_{ji} \right) \\
& - h \left(\sum_{i \in I'} \frac{\alpha}{1-\alpha} x_i + \sum_{j \in I'} \sum_{i \in I'} t_{ij} \psi_{ij} + \sum_{i \in I'} \sum_{j \in I'} \tau_{ij} \phi_{ij} + \sum_{i \in I'} t_c P_c \sum_{j \in I'} \psi_{ji} \right) \\
\text{s.t.} & \\
& q_{ik} \leq q'_{ik}, \forall i \in I', \forall k \in K \\
& \psi_{ij} \geq \alpha P_{ij} \mu_i \sum_{k \in K} w_{ik} q_{ik}, \forall i \in I' \\
& \psi_{ij} \leq P_{ij} \mu_i \sum_{k \in K} w_{ik} q_{ik}, \forall i,j \in I' \\
& \psi_{ij} = P_{ij} \sum_{l \in I'} \psi_{il}, \forall i,j \in I' \\
& \sum_{j \in I'} \psi_{ji} + \sum_{j \in I'} \phi_{ji} = \sum_{j \in I'} \psi_{ij} + \sum_{j \in I'} \phi_{ij}, \forall i \in I' \\
& q_{ik}, \psi_{ij}, \phi_{ij} \geq 0, \forall i,j \in I', \forall k \in K.
\end{aligned}
$$

Since the service region design I' is given, the terms with x_i are constant and can be omitted. We define an equivalent objective function

$$
\begin{aligned}
\mathbf{H}(q_{ik}, \psi_{ij}, \phi_{ij}) &= \sum_{i \in I'} \sum_{k \in K} f Q_{ik} q_{ik} + \sum_{j \in I'} \sum_{i \in I'} \left[(\xi r - h) t_{ij} - P_c (\xi c + h t_c) \right] \psi_{ij} - \sum_{j \in I'} \sum_{i \in I'} (\xi \eta + h) \tau_{ij} \phi_{ij} \\
&= \sum_{i \in I'} \sum_{k \in K} f Q_{ik} q_{ik} + \sum_{j \in I'} \sum_{i \in I'} (u_{ij} \psi_{ij} - v_{ij} \phi_{ij})
\end{aligned}
$$

where $u_{ij} = (\xi r - h) t_{ij} - P_c(\xi c + h t_c)$ and $v_{ij} = (\xi \eta + h) \tau_{ij}$.

Let $y_{ik}, m_{ij}, n_{ij}, s_{ij}$ and z_i be the dual variables. We write the dual formulation of the above linear program as

$$
\min_{y_{ik}, m_{ij}, n_{ij}, s_{ij}, z_i} \sum_{i \in I'} \sum_{k \in K} q'_{ik} y_{ik}
$$

s.t.

$$fQ_{ik}+\sum_{j\in I'}(m_{ij}-\alpha n_{ij})P_{ij}\mu_i w_{ik}\le y_{ik},\forall i\in I',\forall k\in K \tag{16}$$

$$u_{ij}-m_{ij}+n_{ij}+\sum_{l\in I'}P_{il}s_{il}-s_{ij}+z_j-z_i\le 0,\forall i,j\in I' \tag{17}$$

$$-v_{ji}\le z_j-z_i,\forall i,j\in I' \tag{18}$$

$$y_{ik},m_{ij},n_{ij}\ge 0,\forall i,j\in I'$$

From inequalities (17) and (18), we have

$$u_{ij}-v_{ji}+\sum_{l\in I'}P_{il}s_{il}-s_{ij}\le m_{ij}-n_{ij}\le m_{ij}-\alpha n_{ij},\forall i,j\in I'$$

$$\Rightarrow\sum_{j\in I'}P_{ij}(u_{ij}-v_{ji})\le\sum_{j\in I'}P_{ij}(m_{ij}-\alpha n_{ij}),\forall i\in I'$$

This inequality provides a lower bound to the LHS of constraint (16). That is,

$$fQ_{ik}+\mu_i w_{ik}\sum_{j\in I'}P_{ij}(u_{ij}-v_{ji})\le y_{ik},\forall i\in I',\forall k\in K$$

To have the adoption rate constraint $q_{ik}\le q'_{ik}$ tight, we need the associated dual variable $y_{ik}>0$. Therefore, by noting that $v_{ii}=0$, the corresponding sufficient condition is to have

$$fQ_{ik}+\mu_i w_{ik}P_{ii}u_{ii}+\mu_i w_{ik}\sum_{\substack{j\in I'\\ j\ne i}}P_{ij}(u_{ij}-v_{ji})>0,\forall i\in I',\forall k\in K.$$

□

Appendix C: Estimation of Key Parameters

C.1. Utility Parameters

We use the trip distributions that describe customer preferences over destinations as proxies to utility parameters **a**. Our study focuses on the 61 candidate zip codes in San Diego County, excluding remote and military areas. We estimate the origin-destination trip distributions for the 61 candidate zip codes by extrapolating, using a gravity model, from the Car2Go operations data for the 18 zip codes they currently cover. The sample daily trips for each origin-destination pair of the 18 zip codes are counted and the outbound (inbound) trips from (to) each zip code are summarized in Table 2.

Table 2 Sample Daily Outbound(Inbound) Trips in Current Car2Go Service Region

91910	**91911**	**92101**	**92102**	**92103**	**92104**
7.92 (8.19)	3.19 (3.27)	334.04 (335.23)	55.69 (54.77)	144.04 (144.04)	81.69 (80.5)
92105	**92106**	**92107**	**92108**	**92109**	**92110**
11.23 (10.35)	45.89 (45.65)	54.23 (54.92)	50.77 (51.92)	84.08 (83.96)	44.92 (46)
92111	**92113**	**92115**	**92116**	**92120**	**92123**
0.35 (0.35)	1.23 (1.19)	14.62(14.85)	60.08 (58.73)	1.12 (1.19)	0.12 (0.08)

Table 2 suggests a large variation in trip demands of the 18 zip codes. The majority of the trips were generated in the downtown San Diego with zip codes 92101 and 92103, while few trip demands are observed from zip codes 92111 and 92123. To better capture the demand pattern, we exclude zip codes with very low demands, e.g., 91911, 92111, 92113, 92120 and 92123, in the estimation of the gravity model, since inclusion of these outliers led to significantly worse fits.

We first fit a gravity model for the sample trip rates for one-way trips among 18 zip codes (i.e., trips where origins and destinations are in distinct zip codes). Trips that originate and end within the same zip code are handled later. Besides the (working-age) population factor in the classic gravity model, we also test other socioeconomic factors, such as per capita income, business establishments, students enrollments and workplace population, that may affect the trip distributions. The only statistical significant factors we find are working-age population and per capita income. Similarly, Wills (1986) integrates income as a destination-attribute variable in his trip distribution models. Hence, we fit the gravity model for one-way trips as follows:

$$T_{ij} = \beta_1 \frac{Pop_i Pop_j Inc_i^{\beta_2} Inc_j^{\beta_3}}{dist_{ij}^{\beta_4}} \tag{19}$$

where Pop_i is the working-age population and Inc_i is the per capita income in i. The number of trips generated from i is proportional to Pop_i while Pop_j and Inc_j are indicators of the attractiveness of destination j.

Let $\hat{T}_{ij}^{Car2Go}$ be the observed trips from Car2Go's operations data from i to j. The sample daily trip are adjusted by the current adoption rate (current regular members divided by market size in the current service region), e.g., $\hat{T}_{ij}^{Car2Go}$/(Current Adoption Rate). We then apply log-linear regression to $\hat{T}_{ij}$ and obtain the gravity model in (19) for the aggregated daily trip distribution below with adjusted R-squared of 0.7515 and residual standard error of 0.9227.

$$T_{ij} = \exp(-62.212) \frac{Pop_i Pop_j Inc_i^{2.253} Inc_j^{2.249}}{dist_{ij}^{2.013}}. \tag{20}$$

For trips that begin and end within the same zip code (round trips), we fit the following model using the same socialeconomic factors as independent variables:

$$T_{ii} = \beta_1 Pop_i Inc_i^{\beta_2}.$$

The fitted model, as shown below, has adjusted R-squared of 0.7845 and residual standard error:

$$T_{ii} = \exp(-43.194) Pop_i Inc_i^{3.413}. \tag{21}$$

With the residual standard errors provided by the regressions, we randomly generate 1000 sample trip distributions for the 61 candidate zip codes using gravity models in (20) and (21). In each sample n, the sample utility parameter $\hat{a}_{ij}^n$ is estimated by normalizing the outbound trips by their destinations:

$$\hat{a}_{ij}^n = \frac{\hat{T}_{ij}^n}{\sum_{j\in I} \hat{T}_{ij}^n}.$$

The mean of utility parameter, $\bar{a}_{ij}$, is then calculated as the sample average of $\hat{a}_{ij}^n$. We also construct the estimated diagonal covariance matrix Γ_i for each i with σ_{ij}^2 set to be sample variance of $\hat{a}_{ij}^n$ for all origin i and destination j.

Lastly, the outbound demand rate μ_i is estimated by averaging the total outbound trips $\hat{\mu}_i^n = \sum_{j\in I} \hat{T}_{ij}^n$ over all samples. The average travel speed in San Diego is assumed to be 31 mph (John Kleint 2016) for conversion of travel time to travel distance as $d_{ij}(\text{mile}) = t_{ij}(\text{min}) \times \frac{31(\text{mph})}{60(\text{min/h})}$.

C.2. Adoption Requirement

The aspirational level b is evaluated based on the mode choice data in CHTS. We summarize the mode choice distribution in 4 categories: non-moto (including walking and cycling, etc.), private car (including driver, passenger, car rental and carpool, etc,.), bus (including bus and shuttle, etc.), and rail (including subways and light rail, etc.). We focus on motorized trips, and group bus and rail modes to public transportation. Using K-means clustering approach (e.g., Hartigan and Wong (1979)), we group the individuals into 3 clusters with different mode choice distributions as shown in Table 3.

Table 3 Mode Choice Distribution

Group	Car	Public
1	0.01	0.99
2	0.53	0.47
3	1.00	0.00

For instance, the potential travelers from Group 2 choose to drive for 53% of their trips. We hence set $b = 53\%$ for these customers, i.e., they would be willing to switch to car sharing if it covers at least 53% of their travel needs. After adoption, customers will use car sharing service regularly with destinations that are in the service region. Following the same reasoning, Groups 1 and 3 are not target segments for Car2Go because these customers either never drive or rely too much on their own cars that they would require 100% service coverage. As a result, the target Group 2 account for 4.05% of the entire population and we set $Q_{ik} = 20\% \times 4.05\% \times Pop_i$, where Pop_i is the population in area i, by assuming 20% local market share in car sharing business. Due to the limited sample sizes at zip code level, we assume all zip codes share the same mode choice distribution.

C.3. Coverage Costs

The fixed coverage cost associated with serving region i includes investments in charging infrastructure, in partnership with charging service providers. As planned in 2011, Car2Go's fleet in San Diego can be recharged at 1000 Blink EV charging stations (Engadget 2014). We use the EV charging station data from U.S. Department of Energy (2014) to estimate the number of chargers required in each zip code to support the EV sharing system. For each zip code in the current service region, we compute the charger density (CD) by dividing the number of chargers (CG) with the land area (LA). From linear regression with various socialeconomic factors, we find that number of business establishment (BE) is the only significant factor to CD with the fitted model as below

$$CD_i = 3.19\exp(-11)BE_i.$$

The number of chargers needed for each candidate area is then approximated by

$$CG_i = \max\{CD_i, 0\} \times LA_i.$$

Suppose the investment in each charger by Car2Go through partnership with Blink is h_c (e.g., \$800, about half of the typical cost to install a household Level 2 charger in California (California Plug-in Electric Vehicle Collaborative 2016)), then the coverage cost for region i follows $g_i = h_c CG_i$.

Appendix D: Heuristic Formulation

D.1. H1 Formulation

The heuristic H_1 solves the following mixed integer program based on a stylized view on adoption behavior (assuming $q_s = b$):

$$\max_{\substack{x_i, N, \Psi_{ij} \\ \psi_{ij}, \psi'_{ij}, \phi_{ij}, \lambda_i}} \sum_{i \in I} f Q_i q_s - \sum_{i \in I} g_i x_i + \xi \left(\sum_{j \in I} \sum_{i \in I} r t_{ij} \psi_{ij} - \sum_{i \in I} \sum_{j \in I} \eta \tau_{ij} \phi_{ij} - \sum_{i \in I} c \lambda_i \right) - hN$$

s.t.

$$\text{Constraint } (9) - (21)$$
$$\Psi_{ij} = \mu_i P_{ij} q_s, \forall i \in I, j \in J$$
$$N, \psi_{ij}, \psi'_{ij}, \Psi_{ij}, \phi_{ij}, \lambda_i \geq 0$$
$$x_i \in \{0, 1\}, \forall i \in I.$$

D.2. H2 Formulation

The heuristic H_2 solves the following mixed integer SOCP formulated based on a stylized view on fleet operations:

$$\max_{\substack{x_i, q_i, N, \Psi_{ij} \\ \psi_{ij}, \psi'_{ij}, \phi_{ij}, \lambda_i}} \sum_{i \in I} f Q_i q_i - \sum_{i \in I} g_i x_i + \xi \left(\sum_{j \in I} \sum_{i \in I} r t_{ij} \psi_{ij} - \sum_{i \in I} \sum_{j \in I} \eta \tau_{ij} \phi_{ij} - \sum_{i \in I} c \lambda_i \right) - hN$$

s.t.

$$\text{Constraint } (3)\&(5) - (19)$$
$$N = \bar{t} \times \sum_{i \in I} x_i \mu_i q_i$$
$$N, \psi_{ij}, \psi'_{ij}, \Psi_{ij}, \phi_{ij}, \lambda_i \geq 0$$
$$x_i \in \{0, 1\}, \forall i \in I.$$

D.3. H3 Formulation

The heuristic H_3 solves the following mixed integer program formulated based on a stylized view on both adoption behavior and fleet operations:

$$\max_{\substack{x_i, N, \Psi_{ij} \\ \psi_{ij}, \psi'_{ij}, \phi_{ij}, \lambda_i}} \sum_{i \in I} f Q_i q_s - \sum_{i \in I} g_i x_i + \xi \left(\sum_{j \in I} \sum_{i \in I} r t_{ij} \psi_{ij} - \sum_{i \in I} \sum_{j \in I} \eta \tau_{ij} \phi_{ij} - \sum_{i \in I} c \lambda_i \right) - hN$$

s.t.

$$\text{Constraint } (9) - (19)$$
$$\Psi_{ij} = \mu_i P_{ij} q_s, \forall i \in I, j \in J$$
$$N = \bar{t} \times \sum_{i \in I} x_i \mu_i q_i$$
$$N, \psi_{ij}, \psi'_{ij}, \Psi_{ij}, \phi_{ij}, \lambda_i \geq 0$$
$$x_i \in \{0, 1\}, \forall i \in I.$$

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