Online Appendix A: Proof of Results

Proof of Lemma 1: Part a: π_2 is concave in p_2, i.e., $\frac{\partial^2\pi_2}{\partial p_2^2} < 0$. The result can be obtained from solving the first order condition (FOC) $\frac{\partial\pi_2}{\partial p_2} = 0$. Part b: π_2 is concave in p_2 and Π_2 is concave in p_{C2}. The results can be obtained from solving the FOCs $\frac{\partial\pi_2}{\partial p_2} = 0$ and $\frac{\partial\Pi_2}{\partial p_{C2}} = 0$. □

Proof of Corollary 1: Solving the subgame profit for the case of $E = 1$ gives $x_1^s = \frac{2a(2-\delta+\delta\gamma)-b(1-2\delta+2\delta\gamma)}{a(2a(2-2\delta+\delta\gamma)-b(1-\delta+2\gamma\delta))} p_1$ and $x_1^m = \frac{p_1}{a}$. Note that $2a(2-2\delta+\delta\gamma)-b(1-\delta+2\gamma\delta) > 0$ because $\delta < 1$ and $a > b$. $\pi_2^* = \frac{4(a-b)\delta(1-\delta(1-\gamma))^2}{((4a-b)(1-\delta)+2\gamma\delta(a-b))^2} p_1^2 M(i)$ and $\Pi_2^* = \frac{(a-b)b\delta(1-\delta(1-\gamma))^2}{a((4a-b)(1-\delta)+2\gamma\delta(a-b))^2} p_1^2 M(i) - F$. Then $\Pi_2^* > 0 \Leftrightarrow p_1^2 M(i) > \mathit{\Lambda}$. □

Proof of Proposition 1: First, consider sub-problem (11a). Define $X^{E=0} = \frac{(1-\delta+\delta\gamma)(\gamma(4-3\delta)+(1-\gamma)(4-\delta)(1-\delta))}{a(2-(2-\gamma)\delta)^2}$. Solving the FOCs $\frac{\partial\pi}{\partial p_1} = 0$ and $\frac{\partial\pi}{\partial i} = 0$ gives $p_1^{E=0} = \frac{1}{2X^{E=0}}$ and $i^{E=0} = \{i: M'(i) = 4X^{E=0}\}$. Since $M''(i) < 0$, this solution satisfies the second-order condition (SOC) because $\frac{\partial^2\pi}{\partial p_1^2} = -2X^{E=0} < 0$ and $\frac{\partial^2\pi}{\partial i^2}\frac{\partial^2\pi}{\partial p_1^2} - \left(\frac{\partial^2\pi}{\partial i\partial p_1}\right)^2\Big|_{i^{E=0},p_1^{E=0}} = -\frac{1}{2}M''(i^{E=0}) > 0$. The solution is unique because $M'(i)$ is a decreasing function. To guarantee the existence of the solution, define $i^{E=0} = 0$ when $M'(0) > 4X^{E=0}$. Then the unconstrained solution of sub-problem (11a) has optimal profit $\pi^{E=0} = \frac{M(i^{E=0})}{4X^{E=0}} - i^{E=0}$. The constraint of sub-problem (11a) is satisfied if and only if $M(i^{E=0})[p_1^{E=0}]^2 \leq \mathit{\Lambda} \Leftrightarrow F \geq \hat{F}_2 \equiv \frac{ab\delta(a-b)(2-(2-\gamma)\delta)^4 M(i^{E=0})}{4(\gamma(4-3\delta)+(1-\gamma)(4-\delta)(1-\delta))^2(4a-b-2a\delta(2-\gamma)+b\delta(1-2\gamma))^2}$. $\hat{F}_2$ exists and is unique because $i^{E=0}$ exists and is unique. Therefore, when $F \geq \hat{F}_2$, the manufacturer would set $i = i^{E=0}$ and $p_1 = p_1^{E=0}$, and firm C does not enter.

Consider $F < \hat{F}_2$. The constrained solution to sub-problem (11a) can be obtained from substituting $p_1 = \sqrt{\frac{\Lambda}{M(i)}}$ in the profit function, which gives $\bar{\pi} = \sqrt{\Lambda M(i)} - X^{E=0}\Lambda - i$. (The bar in $\bar{\pi}$ is used to denote the constraint is binding.) $\frac{\partial^2 \bar{\pi}}{\partial i^2} = \frac{\sqrt{\Lambda}}{2}\left(\frac{-M'(i)}{2M^{3/2}(i)} + \frac{M''(i)}{M^{1/2}(i)}\right) < 0$. Define $g(i) \equiv \frac{\sqrt{M(i)}}{M'(i)}$. Then solving the FOC $\frac{\partial \bar{\pi}}{\partial i} = 0$ gives $\bar{\iota}^{E=0} = \left\{i : g(i) = \frac{\sqrt{\Lambda}}{2}\right\}$. This solution exists and is unique because $g(0) = 0$ and $g(i)$ is an increasing function. The constrained solution to sub-problem (11a) has optimal profit $\bar{\pi}^{E=0} = \sqrt{M(\bar{\iota}^{E=0})\Lambda} - X^{E=0}\Lambda - \bar{\iota}^{E=0}$.

Similar to the derivation of the unconstrained solution to sub-problem (11a), the unconstrained solution to sub-problem (11b) is $p_1^{E=1} = \frac{1}{2X^{E=1}}$, $i^{E=1} = \{i : M'(i) = 4X^{E=1}\}$ and $\pi^{E=1} = \frac{M(i^{E=1})}{4X^{E=1}} - i^{E=1}$, where $X^{E=1} = \frac{(1-\delta+\delta\gamma)\left(\gamma(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)+(1-\gamma)(1-\delta)(16a^2-8ab+b^2-4a^2\delta+4ab\delta)\right)}{a\left(4a-b-2a\delta(2-\gamma)+b\delta(1-2\gamma)\right)^2}$. To guarantee the existence of the solution, define $i^{E=1} = 0$ when $M'(0) > 4X^{E=1}$. Note that it is not necessary to consider the constrained solution for sub-problem (11b) because $\forall i, p : \pi^{E=1}(i,p) < \pi^{E=0}(i,p)$. Therefore, if the constraint to sub-problem (11b) is violated, then $E = 0$ because $\pi^{E=1}(i^{E=1}, p_1^{E=1}) < \bar{\pi}^{E=0}$.

Lastly, we will show below that, when $F < \hat{F}_2$, there exists a unique $\hat{F}_1$ such that $\pi^{E=1} > \bar{\pi}^{E=0} \Leftrightarrow F < \hat{F}_1$. (1): There exists at least one $\hat{F}_1$ for $\pi^{E=1} = \bar{\pi}^{E=0}$ because $\pi^{E=1} > \bar{\pi}^{E=0}$ when $F = 0$ and $\pi^{E=1} < \bar{\pi}^{E=0}$ when $F = \hat{F}_2$. The second inequality is true because $\bar{\pi}^{E=0} = \pi^{E=0}$ when $F = \hat{F}_2$, and $\forall i, p : \pi^{E=1}(i,p) < \pi^{E=0}(i,p)$. (2): $\pi^{E=1}$ is independent of F. (3): $\frac{\partial \bar{\pi}^{E=0}}{\partial F} = \frac{\sqrt{\Lambda}}{2F}\left(\sqrt{M(\bar{\iota}^{E=0})} - 2X^{E=0}\sqrt{\Lambda}\right)$, so $\frac{\partial \bar{\pi}^{E=0}}{\partial F} > 0 \Leftrightarrow M(\bar{\iota}^{E=0}) > 4\Lambda(X^{E=0})^2$. Since $\bar{\iota}^{E=0} = g^{-1}\left(\frac{\sqrt{\Lambda}}{2}\right)$, so $M(\bar{\iota}^{E=0}) > 4\Lambda(X^{E=0})^2 \Leftrightarrow \frac{\sqrt{\Lambda}}{2} > g[M^{-1}(4\Lambda(X^{E=0})^2)] \Leftrightarrow \frac{\sqrt{\Lambda}}{2} > \sqrt{4\Lambda(X^{E=0})^2}/M'[M^{-1}(4\Lambda(X^{E=0})^2)] \Leftrightarrow M'[M^{-1}(4\Lambda(X^{E=0})^2)] > 4X^{E=0} \Leftrightarrow M'^{(-1)}(4X^{E=0}) > M^{-1}(4\Lambda(X^{E=0})^2)$. Note that there is a change in sign in the last part because $M'(i)$ is a decreasing function. This is true because $i^{E=0} = M'^{(-1)}(4X^{E=0})$, so $F < \hat{F}_2 \Leftrightarrow$

$M(i^{E=0}) > 4\Lambda(X^{E=0})^2 \Leftrightarrow M'^{(-1)}(4X^{E=0}) > M^{-1}(4\Lambda(X^{E=0})^2)$. Therefore, $\bar{\pi}^{E=0}$ is an increasing function, so there exists a unique $\hat{F}_1$. In summary, the unique optimal i and p_1 are:

$$(i^*, p_1^*) = \begin{cases} (i^{E=1}, p_1^{E=1}) & \text{if } F < \hat{F}_1 \\ (\bar{\iota}^{E=0}, \bar{p}_1^{E=0}) & \text{if } \hat{F}_1 \leq F < \hat{F}_2 \\ (i^{E=0}, p_1^{E=0}) & \text{otherwise} \end{cases}. \square$$

Optimal solution for the case where $\boldsymbol{M(i) = \sqrt{i}}$: $i^{E=0} = \frac{1}{64(X^{E=0})^2}$, $p_1^{E=0} = \frac{1}{2X^{E=0}}$ and $\pi^{E=0} = \frac{1}{64(X^{E=0})^2}$, $\bar{\iota}^{E=0} = \left(\frac{\Lambda}{16}\right)^{\frac{2}{3}}$, $\bar{p}_1^{E=0} = \sqrt[3]{4\Lambda}$, $\bar{\pi}^{E=0} = \left(\frac{3\sqrt{3}}{16}\Lambda\right)^{2/3} - X^{E=0}\Lambda$, $i^{E=1} = \frac{1}{64(X^{E=1})^2}$, $p_1^{E=1} = \frac{1}{2X^{E=1}}$, $\pi^{E=1} = \frac{1}{64(X^{E=1})^2}$ and $\hat{F}_2 = \frac{a^2(a-b)b\delta(2-(2-\gamma)\delta)^6}{32(1-\delta+\gamma\delta)\left(4a-b-2a\delta(2-\gamma)+b\delta(1-2\gamma)\right)^2\left(\gamma(4-3\delta)+(1-\gamma)(4-\delta)(1-\delta)\right)^3}$. Let $\tilde{F} = \sqrt[3]{F}$. Then $\tilde{\pi} \equiv \pi^{E=1} - \bar{\pi}^{E=0}$ is a cubic polynomial in $\tilde{F}$, with the coefficient for $\tilde{F}^3$ is positive. The two solutions to $\frac{\partial\tilde{\pi}}{\partial\tilde{F}} = 0$ are $\tilde{F} = 0$ and $\tilde{F} = \sqrt[3]{\hat{F}_2}$. Moreover, $\tilde{\pi} > 0$ when $\tilde{F} = 0$ and $\tilde{\pi} < 0$ when $\tilde{F} = \sqrt[3]{\hat{F}_2}$. Therefore, $\hat{F}_1$ is the cube of the second root to $\tilde{\pi} = 0$.

Below, it is necessary to show that a polynomial, $g(a, b, \delta)$, is positive when $a > b > 0$ and $0 < \delta < 1$. We prove this using the technique of Pun (2015). Specifically, since a polynomial is a continuous and well-defined function, the minimum is either at the solutions of the first-order conditions (FOCs) or at the boundaries. Note that the solutions to the FOCs are numeric values; thus, it can be demonstrated that $g(a, b, \delta) > 0$ at these solutions. Furthermore, the boundaries are polynomials of two variables, which can also be shown to be positive using similar technique. To illustrate, consider the proof of $\frac{\partial\pi^{E=1}}{\partial a} = \frac{a(4a-b-2a\delta-b\delta)^3 X}{32(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)^3} > 0$, where $X(a, b, \delta) = 64a^3 - 48a^2b + 12ab^2 - b^3 - 80a^3\delta + 52a^2b\delta + 6ab^2\delta - 2b^3\delta + 24a^3\delta^2 - 20a^2b\delta^2 - 6ab^2\delta^2 - b^3\delta^2$. We need to show that $X(a, b, \delta) > 0$ when $a > b > 0$ and $0 < \delta < 1$. There is no root to the FOCs $\frac{\partial X(a,b,\delta)}{\partial a} = 0$, $\frac{\partial X(a,b,\delta)}{\partial b} = 0$ and $\frac{\partial X(a,b,\delta)}{\partial \delta} = 0$, so the minimum value must be at the boundary. However, the boundary is positive because (1) $X(a, b, 0) = (4a - b)^3 > 0$, (2) $X(a, b, 1) = 4(a - b)(2a^2 - 2ab + b^2) > 0$, (3)

$X(a,0,\delta) = 8a^3(2-\delta)(4-3\delta) > 0$ and (4) $X(a,a,\delta) = 3a^3(1-\delta)(9+\delta) > 0$. Therefore, $X(a,b,\delta) > 0$ is true.

Proof of Lemma 2 and Propositions 2: Define $\hat{a}_1$ and $\hat{a}_2$ to be the inverse of $\hat{F}_1$ and $\hat{F}_2$ with respect to a, i.e., $\hat{a}_1 = \hat{F}_1^{-1}$ and $\hat{a}_2 = \hat{F}_2^{-1}$. Moreover, it can be shown that $\hat{F}_1$ increases and then decreases in b, so there are two roots to the inverse of $\hat{F}_1$ with respect to b, i.e., $b = \hat{F}_1^{-1}$. We define $\hat{b}_{1a}$ to be smaller root and $\hat{b}_{1b}$ to be the larger root. Similarly, $\hat{F}_2$ increases and then decreases in b, so we define $\hat{b}_{2a}$ to be smaller root and $\hat{b}_{2b}$ to be the larger root. Part a: If $a > \hat{a}_1$, $\frac{\partial p_1^{E=1}}{\partial a} = \frac{(4a-b-2a\delta-b\delta)X}{2(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)^2} > 0$, where X is defined above and this is true using the method presented above. If $a < \hat{a}_2$, $\frac{\partial p_1^{E=0}}{\partial a} = \frac{(2-\delta)^2}{2(4-3\delta)} > 0$. If $\hat{a}_2 < a < \hat{a}_1$, $\frac{\partial \bar{p}_1^{E=0}}{\partial a} = \sqrt[3]{\frac{4}{\Lambda^2}} \frac{F(4a-b-2a\delta-b\delta)(-8a^2+12ab-b^2+4a^2\delta-6ab\delta-b^2\delta)}{3b(a-b)^2\delta} < 0 \Leftrightarrow a < \frac{6-3\delta+\sqrt{28-40\delta+13\delta^2}}{4(2-\delta)}b$. The function p_1^* is continuous at $a = \hat{a}_2$, but there is a jump at $a = \hat{a}_1$. The proof for $\frac{\partial p_2^*}{\partial a}$ is similar to the proof for $\frac{\partial p_1^*}{\partial a}$, so we omit the details to avoid redundancy. Part b: If $a > \hat{a}_1$, $\frac{\partial \pi^{E=1}}{\partial a} = \frac{a(4a-b-2a\delta-b\delta)^3 X}{32(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)^3} > 0$. If $a < \hat{a}_2$, $\frac{\partial \pi^{E=0}}{\partial a} = \frac{a(2-\delta)^4}{32(4-3\delta)^2} > 0$. Next, if $\hat{a}_2 < a < \hat{a}_1$, $\frac{\partial \bar{\pi}^{E=0}}{\partial a} = \frac{F(4a-b-2a\delta-b\delta)}{b(a-b)^2\delta}\left(\frac{(4-3\delta)(7b-4a-5b\delta+2a\delta)}{(2-\delta)^2} - \frac{-b^2+12ab-8a^2-b^2\delta-6ab\delta+4a^2\delta}{2\sqrt[3]{4\Lambda}}\right) < 0 \Leftrightarrow \frac{(4-3\delta)(7b-4a-5b\delta+2a\delta)}{(2-\delta)^2} < \frac{-b^2+12ab-8a^2-b^2\delta-6ab\delta+4a^2\delta}{2\sqrt[3]{4\Lambda}}$. Lastly, π^* is continuous at $a = \hat{a}_1$ and at $a = \hat{a}_2$. Part c: $\frac{\partial p_1^{E=0}}{\partial b} = 0$ when $F \geq \hat{F}_2 \Leftrightarrow (b \leq \hat{b}_{2a} \text{ or } b \geq \hat{b}_{2b})$, $\frac{\partial \bar{p}_1^{E=0}}{\partial b} > 0 \Leftrightarrow b > \frac{2a(2-\delta)}{7-5\delta}$ when $\hat{F}_1 \leq F < \hat{F}_2 \Leftrightarrow (\hat{b}_{2a} < b \leq \hat{b}_{1a} \text{ or } \hat{b}_{1b} \leq b < \hat{b}_{2b})$ and $\frac{\partial p_1^{E=1}}{\partial b} = \frac{-a^2\delta(4a-b-2a\delta-b\delta)(16a-b-14a\delta-b\delta)}{(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)^2} < 0$ when $F < \hat{F}_1 \Leftrightarrow \hat{b}_{1a} < b < \hat{b}_{1b}$. The function p_1^* is continuous at $F = \hat{F}_2$, but there is a jump at $F = \hat{F}_1$. The proof for $\frac{\partial p_2^*}{\partial b}$ is similar to the proof for $\frac{\partial p_1^*}{\partial b}$, so we omit the details to avoid redundancy. Part d: When $F \geq \hat{F}_2$, $\frac{\partial \pi^{E=0}}{\partial b} = 0$. When $F < \hat{F}_1$, $\frac{\partial \pi^{E=1}}{\partial b} = \frac{-a^3\delta(4a-2a\delta-b-\delta b)^3(16a-b-14a\delta-b\delta)}{16(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)^3} < 0$. When $\hat{F}_1 \leq F < \hat{F}_2$,

$\frac{\partial \bar{\pi}^{E=0}}{\partial b} = \left(\sqrt[3]{\frac{1}{32\Lambda}} - \frac{4-3\delta}{a(2-\delta)^2}\right)\frac{a^2(-4a+7b+2a\delta-5b\delta)(4a-2a\delta-b-\delta b)F}{b^2(a-b)^2\delta^2}$. Note that $\sqrt[3]{\frac{1}{32\Lambda}} > \frac{4-3\delta}{a(2-\delta)^2} \Leftrightarrow \Lambda < \frac{a^3(2-\delta)^6}{32(4-3\delta)^3} \Leftrightarrow F < \hat{F}_2$, which is true. Therefore, $\frac{\partial \bar{\pi}^{E=0}}{\partial b} > 0 \Leftrightarrow b > \frac{2a(2-\delta)}{7-5\delta}$. □

When the market does not have firm C, the manufacturer's profit is simply $\pi^B = \pi^{E=0}$. Therefore:

$$CoC = \begin{cases} \pi^{E=0} - \pi^{E=1} & \text{if } F < \hat{F}_1 \\ \pi^{E=0} - \bar{\pi}^{E=0} & \text{if } \hat{F}_1 \leq F < \hat{F}_2 . \\ 0 & \text{otherwise} \end{cases}$$

Proof of Lemma 3 and Proposition 3: We define $\hat{\delta}_1$ and $\hat{\delta}_2$ to be the inverse of $\hat{F}_1$ and $\hat{F}_2$ with respect to δ, i.e., $\hat{\delta}_1 = \hat{F}_1^{\ -1}$ and $\hat{\delta}_2 = \hat{F}_2^{\ -1}$. Part a: If $\delta < \hat{\delta}_2$, $\frac{\partial p_1^{E=0}}{\partial \delta} = \frac{a(2-\delta)(3\delta-2)}{2(4-3\delta)^2} > 0 \Leftrightarrow \delta > \frac{2}{3}$. If $\hat{\delta}_1 < \delta < \hat{\delta}_2$, $\frac{\partial \bar{p}_1^{E=0}}{\partial \delta} = -\sqrt[3]{\frac{4}{\Lambda^2}}\frac{aF(4a-b+2a\delta+b\delta)(4a-b-2a\delta-b\delta)}{3b(a-b)\delta^2} < 0$. If $\delta > \hat{\delta}_1$, $\frac{\partial p_1^{E=1}}{\partial \delta} > 0 \Leftrightarrow (2a+b)(12a^2-2ab-b^2)\delta - (4a-b)(4a^2+6ab-b^2) > 0 \Leftrightarrow \delta > \frac{(4a-b)(4a^2+6ab-b^2)}{(2a+b)(12a^2-2ab-b^2)}$. Lastly, the function p_1^* is continuous at $\delta = \hat{\delta}_2$, but there is a jump at $\delta = \hat{\delta}_1$. Part b: The proof uses similar methodology, and we do not show it here because of redundancy. Part c: If $\delta < \hat{\delta}_2$, $\frac{\partial \pi^{E=0}}{\partial \delta} = \frac{a^2(2-\delta)^3(3\delta-2)}{32(4-3\delta)^3} > 0 \Leftrightarrow \delta > \frac{2}{3}$. When $\hat{\delta}_1 < \delta < \hat{\delta}_2$, $\frac{\partial \bar{\pi}^{E=0}}{\partial \delta} = \frac{(4a-b-2a\delta-b\delta)F}{4b(a-b)(2-\delta)^3\delta^2\sqrt[3]{\Lambda}}X_1$, where $X_1 \equiv 8\sqrt[3]{\Lambda}(16a-4b-16a\delta+10b\delta+4a\delta^2-7b\delta^2) - \sqrt[3]{2}a(2-\delta)^3(4a-b+2a\delta+b\delta)$. Thus, $\frac{\partial \bar{\pi}^{E=0}}{\partial \delta} > 0 \Leftrightarrow X_1 > 0 \Leftrightarrow F > \Gamma_\pi \equiv \frac{a^2(a-b)b(2-\delta)^9\delta(4a-b+2a\delta+b\delta)^3}{256(4a-b-2a\delta-b\delta)^2(16a-4b-16a\delta+10b\delta+4a\delta^2-7b\delta^2)^3}$. If $\delta > \hat{\delta}_1$, $\frac{\partial \pi^{E=1}}{\partial \delta} > 0 \Leftrightarrow (2a+b)(12a^2-2ab-b^2)\delta - (4a-b)(4a^2+6ab-b^2) > 0 \Leftrightarrow \delta > \frac{(4a-b)(4a^2+6ab-b^2)}{(2a+b)(12a^2-2ab-b^2)}$. Lastly, π^* is continuous at $\delta = \hat{\delta}_1$ and at $\delta = \hat{\delta}_2$. □

Proof of Lemma 4 and Proposition 4: $\Pi^* = 0$ for Regions 1 and 2 (cf. $F > \hat{F}_1$). At Region 3, $\frac{\partial \Pi^*}{\partial \gamma} = \frac{(a-b)b\delta a^2}{32} \frac{(2a+b)(1-\delta)\delta\left((4a-b)(1-\delta)+2\gamma\delta(a-b)\right)^3\left(2(a-b)\delta(-4a+b+8a\delta)\gamma-(1-\delta)\left(8a(2a-b)(1-\delta)+b(b+8a\delta)\right)\right)}{(1-\delta+\gamma\delta)^2\left(\gamma(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta)+(1-\gamma)(1-\delta)(16a^2-8ab+b^2-4a^2\delta+4ab\delta)\right)^4}$. The denominator is positive. So $\frac{\partial \Pi^*}{\partial \gamma} < 0 \Leftrightarrow 2(a-b)\delta(-4a+b+8a\delta)\gamma - (1-\delta)\left(8a(2a-b)(1-\delta) + b(b+8a\delta)\right) < 0$. Then, the result can easily be obtained. □

Online Appendix B: Production Cost Differences

We illustrate the robustness of our results and to obtain additional insight with respect to the cost difference in this Appendix. We assume that the unit production cost of firm C's product is Δc, and the unit production cost of the manufacturer's product is zero. Therefore, the manufacturer has a cost advantage when $\Delta c > 0$, and firm C has a cost advantage when $\Delta c < 0$. We consider these three cost differences: $\Delta c = -0.005$, $\Delta c = 0$ and $\Delta c = 0.005$. We present the impacts of this cost difference in Tables A1-A4, using the same parameter settings as those presented in the main text, i.e., $\gamma = 1$, $a = 1.5$, $b = 1$, $\delta = 0.6$ and $F = 0.00015$. Sometimes a strategy shift leads to the results, and we present the magnitude of the jump at those points. For example, Figure 3 shows that p_2 decreases in a at $a = 1.35$ when $\Delta c = 0$ because the strategy shifts from Region 2 ($E = 0$ and binding solution) to Region 3 ($E = 1$ and non-binding solution), and the decrease in p_2 because of this strategy shift is 33% (cf. Table A1). We have also varied a wide-array of parameters to confirm the robustness of our results.

In summary, we have confirmed that the structural insights presented in the paper are robust to different cost differences, but the cost difference affects the critical values where the counter-intuitive results occur. Specifically, when the manufacturer has a cost advantage, the critical values where prices and profit decrease in the quality of the manufacturer's product (cf. Proposition 2a) occur when the manufacturer's product has a higher quality. This is because prices decrease when the manufacturer deters firm C from entering by adjusting the prices and marketing investment (Region 2), and when firm C is indifferent between entering or not (strategy shift between Regions 2 and 3). Firm C is less likely to enter when the manufacturer has a cost advantage, so the critical value increases in quality when the manufacturer has a cost advantage. Moreover, the degree of change because of the cost difference at this critical value can be significant. For example, the decrease in the price of the manufacturer's product in period 2 can be up to 15% smaller when the manufacturer has a cost advantage. Similarly, the values where the price increases in the quality of firm C's product are closer to the two extremes ($b = 0$ and $b = 1$) when firm C has a cost advantage (cf. Proposition 2b). This is because the price increases occur at the strategy shift between Regions 2 and 3, and the area of Region 3 is larger when firm C has a cost advantage.

The critical values for which the price in the first period increases while the price in the second period decreases in the degree of customer's patience (cf. Proposition 3) are closer to $\delta = 1$ when the manufacturer has a cost advantage. Similar to the rationale presented earlier, this is because a change occurs when firm C is indifferent to entering or not (strategy shift between Region 2 and 3), and firm C is less likely to enter when the manufacturer has a cost advantage. Finally, firm C is more likely worse off from the customers anticipating the possibility of firm C's product (cf. Proposition 4) when the manufacturer has a cost disadvantage.

Table A1 – Impact of the quality of the manufacturer's product (a)

	$\Delta c = -0.005$	$\Delta c = 0$	$\Delta c = 0.005$
p_1 decreases in a when	$1 < a < 1.2$	$1.02 < a < 1.25$	$1.05 < a < 1.3$
p_2 decreases in a when	$1 < a < 1.2$ $a = 1.3$ at -43%	$1.02 < a < 1.25$ $a = 1.35$ at -33%	$1.05 < a < 1.3$ $a = 1.5$ at -29%
π^* decreases in a when	$1 < a < 1.2$	$1.02 < a < 1.21$	$1.05 < a < 1.22$

Table A2 – Impact of the quality of the competing product (b)

	$\Delta c = -0.005$	$\Delta c = 0$	$\Delta c = 0.005$
p_1 increases in b when	$b = 0.1$ at 12% $b > 0.7$	$b = 0.2$ at 23% $b > 0.7$	$b > 0.7$
p_2 increases in b when	$b = 0.1$ at 13% $b > 0.7$	$b = 0.2$ at 10% $b > 0.7$	$b > 0.7$
π^* increases in b when	$b > 0.7$	$b > 0.7$	$b > 0.7$

Table A3 – Impact of the customer patience parameter (δ)

	$\Delta c = -0.005$	$\Delta c = 0$	$\Delta c = 0.005$
p_1 increases in δ when	$\delta > 0.925$	$\delta = 0.25$ at 28% $\delta > 0.925$	$\delta = 0.4$ at 37% $\delta > 0.925$
p_2 decreases in δ when	N.A.	$\delta = 0.25$ at -44%	$\delta = 0.4$ at -34%
π^* increases in δ when	$\delta > 0.925$	$\delta > 0.925$	$\delta > 0.925$

Table A4 – Impact of customers' strategic behavior

	$\Delta c = -0.005$	$\Delta c = 0$	$\Delta c = 0.005$
$\Pi^*(\gamma = 0) > \Pi^*(\gamma = 1)$ when	$0.1 < \delta < 0.95$	$0.15 < \delta < 0.95$	$0.2 < \delta < 0.95$

Online Appendix C: Proof for Section 6

6.1 Loyal Customers / Regular Shoppers: First, assume $E = 1$. Then in the second period, we have $U_2 > 0 \Leftrightarrow x > x_2^L \equiv \frac{p_2}{a\delta}$, $U_2 > U_{C2} \Leftrightarrow x > x_2^B \equiv \frac{p_2 - p_{C2}}{\delta(a-b)}$ and $U_{C2} > 0 \Leftrightarrow x > x_{C2}^B \equiv \frac{p_{C2}}{b\delta}$. These two functions are jointly concave in p_2 and p_{C2}, and solving the FOC gives $p_2 = \frac{2a(a-b)\delta\left(gx_1^L+(1-g)x_1^B\right)}{4a-(1+3g)b}$ and $p_{C2} = \frac{b(a-b)\delta\left(gx_1^L+(1-g)x_1^B\right)}{4a-(1+3g)b}$. In the first period, solving for the two indifferent customers gives $p_2 = \frac{2\delta(a-b)}{Z_{LOY}^{E=1}}p_1$, $p_{C2} = \frac{\delta b(a-b)}{aZ_{LOY}^{E=1}}p_1$, $\pi_2 = \frac{4(a-b)(a-gb)\delta}{a\left(Z_{LOY}^{E=1}\right)^2}M(i)p_1^2$, and $\Pi_2 = \frac{(a-b)(1-g)b\delta}{a\left(Z_{LOY}^{E=1}\right)^2}M(i)p_1^2 - F$, where $Z_{LOY}^{E=1} = 2a(2-\delta) - b\big(1+\delta+3g(1-\delta)\big)$. Note that $\Pi_2 > 0 \Leftrightarrow M(i)p_1^2 > \Lambda_{LOY} \equiv \frac{a\left(Z_{LOY}^{E=1}\right)^2 F}{b\delta(1-g)(a-b)}$. After substituting for x_1^M and x_1^B, $\pi = p_1M(i) - X_{LOY}^{E=1}p_1^2M(i) - i$, where $X_{LOY}^{E=1} = \frac{(4a-(1+3g)b)^2 - \delta\left(12a^2-2ab(1+11g)-b^2(1-4g-9g^2)\right)}{a\left(Z_{LOY}^{E=1}\right)^2}$. The profit function is jointly concave in i and p_1. Solving the FOC gives $p_1^{E=1} = \frac{1}{2X_{LOY}^{E=1}}$, $i^{E=1} = \{i: M'(i) = 4X_{LOY}^{E=1}\}$ and $\pi^{E=1} = \frac{M\left(i^{E=1}\right)}{4X_{LOY}^{E=1}} - i^{E=1}$.

Second, consider the case where firm C would not enter. The derivation is the same as the one presented in the proof of Proposition 1. In summary, when firm C does not enter into the market ($E = 0$), the equilibrium solutions are $p_1^{E=0} = \frac{1}{2X_{LOY}^{E=0}}$, $i^{E=0} = \{i: M'(i) = 4X_{LOY}^{E=0}\}$, $\pi^{E=0} = \frac{M\left(i^{E=0}\right)}{4X_{LOY}^{E=0}} - i^{E=0}$, $\bar{p}_1^{E=0} = \sqrt[3]{4\Lambda_{LOY}}$, $\bar{\iota}^{E=0} = \left\{i: g(i) = \frac{\sqrt{\Lambda_{LOY}}}{2}\right\}$, $\bar{\pi}^{E=0} = \sqrt{M(\bar{\iota}^{E=0})\Lambda_{LOY}} - X_{LOY}^{E=1}\Lambda_{LOY} - \bar{\iota}^{E=0}$, and $\hat{F}_2 = \frac{a^2(a-b)b(1-g)(2-\delta)^6\delta}{32(4-3\delta)^3\left(Z_{LOY}^{E=1}\right)^2}$, where $X_{LOY}^{E=1} = \frac{(4-3\delta)}{a(2-\delta)^2}$ and $\Lambda_{LOY} = \frac{aF(2a(2-\delta)-b(1+\delta+3g-3g\delta))^2}{(a-b)b(1-g)\delta}$.

After examining the equilibrium solution under this subsection, it can be shown that all results hold, even when the market consists of some dedicated customer segments.

6.2. Targeted Advertising: First, assume $E = 1$. $\pi_2 = \frac{(x_1-x_2)(x_1+x_2)}{2}p_2M(i)$ and $\Pi_2 = \frac{(x_2-x_{C2})(x_2+x_{C2})}{2}p_2M(i) - F$. Solving the FOC gives $p_2 = \frac{(a-b)\left(2b+\sqrt{3(a-b)^2+b^2}\right)\delta x_1}{\sqrt{9(a-b)^2+8b^2+8b\sqrt{3(a-b)^2+b^2}}}$ and

$p_{C2} = \frac{b(a-b)\delta x_1}{\sqrt{9(a-b)^2+8b^2+8b\sqrt{3(a-b)^2+b^2}}}$. Next, solving for x_1 gives $\Pi = \frac{(a-b)b\delta\left((a-b)^2+b^2+b\sqrt{3(a-b)^2+b^2}\right)}{\Gamma_{TAR}^3} p_1^3 M(i) - F$, where $\Gamma_{TAR} = \delta(a-b)\left(2b + \sqrt{3(a-b)^2+b^2}\right) + a(1-\delta)\sqrt{9(a-b)^2+8b^2+8b\sqrt{3(a-b)^2+b^2}}$ and $\Pi > 0 \Leftrightarrow p_1^3 M(i) > \Lambda_{TAR} = \frac{\Gamma_{TAR}^3 F}{(a-b)b\delta\left((a-b)^2+b^2+b\sqrt{3(a-b)^2+b^2}\right)}$. Then, $\pi = \pi_2 + \frac{(1-x_1)(1+x_1)}{2} p_1 M(i) - i = \frac{p_1 M(i)}{2} - X_{TAR}^{E=1} p_1^3 M(i) - i$, where $X_{TAR}^{E=1} = \frac{(a-b)\delta\left[b\left(12(a-b)^2+6b^2\right)+3\left((a-b)^2+2b^2\right)\sqrt{3(a-b)^2+b^2}\right]+a\left(9(a-b)^2+8b^2+8b\sqrt{3(a-b)^2+b^2}\right)^{3/2}(1-\delta)}{2\Gamma_{TAR}^3}$. Solving the FOCs gives $p_1^{E=1} = \frac{1}{\sqrt{6X_{TAR}^{E=1}}}$, $i^{E=1} = \left\{i: M'(i) = 3\sqrt{6X_{TAR}^{E=1}}\right\}$ and $\pi^{E=1} = \frac{M(i^{E=1})}{3\sqrt{6X_{TAR}^{E=1}}} - i^{E=1}$.

Second, assume $E = 0$. Solving the FOC gives $p_2 = \frac{a\delta}{\sqrt{3}} x_1$. Then $\pi = \pi_2 + \frac{(1-x_1)(1+x_1)}{2} p_1 M(i) - i = \frac{p_1 M(i)}{2} - X_{TAR}^{E=0} p_1^3 M(i) - i$, where $X_{TAR}^{E=0} = \frac{3(9-9\delta+\sqrt{3}\delta)}{2a^2(3-3\delta+\sqrt{3}\delta)^3}$. Solving the FOCs gives $p_1^{E=0} = \frac{1}{\sqrt{6X_{TAR}^{E=0}}}$, $i^{E=0} = \left\{i: M'(i) = 3\sqrt{6X_{TAR}^{E=0}}\right\}$ and $\pi^{E=0} = \frac{M(i^{E=0})}{3\sqrt{6X_{TAR}^{E=0}}} - i^{E=0}$. Lastly, $p_1^3 M(i) = \Lambda_{TAR}$ at the boundary solution, so $\bar{\iota}^{E=0} = \left\{i: \frac{M^{\frac{1}{3}}(i)}{M'(i)} = \frac{\Lambda_{TAR}^{1/3}}{3}\right\}$ and $\bar{\pi}^{E=0} = \Lambda_{TAR}^{1/3} M^{\frac{2}{3}}(\bar{\iota}^{E=0})/2 - X_{TAR}^{E=0}\Lambda_{TAR} - \bar{\iota}^{E=0}$.

When $M(i) = \sqrt{i}$, the equilibrium solutions are $p_1^{E=0} = \frac{1}{\sqrt{6X_{TAR}^{E=0}}}$, $i^{E=0} = \frac{1}{216X_{TAR}^{E=0}}$, $\pi^{E=0} = \frac{1}{216X_{TAR}^{E=0}}$, $\bar{\iota}^{E=0} = \sqrt{\frac{\Lambda_{TAR}}{216}}$, $\bar{p}_1^{E=0} = \sqrt[4]{4\Lambda_{TAR}}$, $\bar{\pi}^{E=0} = \sqrt{\frac{\Lambda_{TAR}}{54}} - X_{TAR}^{E=0}\Lambda_{TAR}$, $p_1^{E=1} = \frac{1}{\sqrt{6X_{TAR}^{E=1}}}$, $i^{E=1} = \frac{1}{216X_{TAR}^{E=1}}$, $\pi^{E=1} = \frac{1}{216X_{TAR}^{E=1}}$ and $\hat{F}_2 = \frac{\delta(a-b)b\left((a-b)^2+b^2+b\sqrt{3(a-b)^2+b^2}\right)}{216\left(X_{TAR}^{E=0}\right)^2\Gamma_{TAR}^3}$. After examining the equilibrium solution under this subsection, it can be shown that all results hold whether advertising is generic for all customers, or targeted to a select group of customers.

6.3. Product Development Investment: When $M(i) = \sqrt{i}$, the manufacturer's subgame optimal profit before deciding the optimal quality is (cf. proof of Proposition 1) as follows:

$$\pi = \begin{cases} \frac{1}{64(X^{E=1})^2} - f(a) & \text{if } F < \hat{F}_1 \\ \left(\frac{3\sqrt{3}}{16}\Lambda\right)^{2/3} - X^{E=0}\Lambda - f(a) & \text{if } \hat{F}_1 \leq F < \hat{F}_2 \\ \frac{1}{64(X^{E=0})^2} - f(a) & \text{otherwise} \end{cases}$$

where $X^{E=0} = \frac{(1-\delta+\delta\gamma)\left(\gamma(4-3\delta)+(1-\gamma)(4-\delta)(1-\delta)\right)}{a(2-(2-\gamma)\delta)^2}$, $\Lambda \equiv \frac{aF((4a-b)(1-\delta)+2\gamma\delta(a-b))^2}{(a-b)b\delta(1-\delta(1-\gamma))^2}$ and $X^{E=1} = \frac{(1-\delta+\delta\gamma)\left(\gamma\left(16a^2-8ab+b^2-12a^2\delta+2ab\delta+b^2\delta\right)+(1-\gamma)(1-\delta)\left(16a^2-8ab+b^2-4a^2\delta+4ab\delta\right)\right)}{a\left(4a-b-2a\delta(2-\gamma)+b\delta(1-2\gamma)\right)^2}$.