# Managing Reliability and Stability Risks in Forest Harvesting

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## Appendix

### A.1 Proof of Theorem 4

**Proof.** a) Let $(\hat{x}, \hat{\lambda})$ be a feasible solution for (17)-(19). It follows from Theorem 3 that: $G = \left\{\omega^k \in \bar{\Omega} : \sum_{j\in J} \sum_{l=1}^{n_j} \hat{\lambda}_{j,l}\, \beta^k_{j,l} = |J|\right\} \subseteq \bar{\Omega}^+$. Let $L = \left\{(j,l) : \hat{\lambda}_{j,l} = 1, j \in J, l = 1, \ldots, n_j\right\}$. From (15), $\omega^k_j \geq \sum_{r=1}^{n_j} \hat{\lambda}_{j,r} c_{j,r} = c_{j,l}, (j,l) \in L, \omega^k \in G$. Further, the construction of $C$ (16) implies that one can always find $\omega^{k'} \in G$ such that:

$$\omega^{k'}_j = c_{j,l} \; (j,l) \in L \; . \tag{1}$$

Since $\omega^{k'} \in G \subseteq \bar{\Omega}^+_B$, we have $\mathbb{P}\left(\xi \leq \omega^{k'}\right) \geq p$, and

$$\mathbb{P}\left(\sum_{j\in J} x_{i,t}\xi_j \leq \sum_{j\in J} x_{i,t}\omega^{k'}_j \;,\; i \in I\right) \geq p \; . \tag{2}$$

Therefore,

$$\sum_{j\in J} \omega^{k'}_j \; x_{i,t} \leq -d_i, \; i \in I \quad \Rightarrow \quad \mathbb{P}\left(\sum_{j\in J} \xi_j \; x_{i,t} \leq -d_i, \; i \in I\right) \geq p \; . \tag{3}$$

Combining (1) and (3) gives:

$$\sum_{j\in J} x_{i,t} \left(\sum_{l=1}^{n_j} \hat{\lambda}_{j,l} \; c_{j,l}\right) \leq -d_i, \; i \in I \quad \Rightarrow \quad \text{(13) holds.} \tag{4}$$

This demonstrates that any solution $(\hat{x}, \hat{\lambda})$ feasible for **SL1** is also feasible for(13).

b) Any feasible solution $(\hat{x}, \hat{d})$ for (13) is feasible for **SL1**. As (13) permits to rewrite any $p$-sufficient recombination $\omega^k \in \bar{\Omega}^+$ as $\omega^k = [\omega^k_1, \ldots, \omega^k_j, \ldots, \omega^k_{|J|}] = [\sum_{l=1}^{n_1} \hat{\lambda}_{1,l}\, c_{1,l}, \ldots, \sum_{l=1}^{n_j} \hat{\lambda}_{j,l}\, c_{j,l}, \ldots, \sum_{l=1}^{n_{|J|}} \hat{\lambda}_{|J|,l}\, c_{|J|,l}]$, we have to show that there always exists a $p$-sufficient recombination $\omega^k \in \bar{\Omega}^+$ for which $(\hat{x}, \hat{d})$ is feasible. The set partitioning constraints (4) imply that any feasible solution is such that for any $i \in I$ one of the $|T|$ variables $x_{i,t}$ takes value 1 and all the other are equal to 0. Let $j^*_i = (i, t^*)$ and $\hat{x}_{i,t^*}$ denote the variable taking value 1. The constraint (13) becomes $\mathbb{P}(\xi_{j^*_i} \leq -d_i, i \in I) \geq p$, which holds true if $\omega^k_j \leq -d_i, i \in I$ $(j = (i,t))$ and $\mathbb{P}(\xi_{j^*_i} \leq \omega^k_j, i \in I) \geq p$. This can only be the case if $\omega^k \in \bar{\Omega}^+$ and any solution feasible for (13) satisfies the conditions of at least one $p$-sufficient recombination.

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Combining a) and b) shows that the probabilistic constraint (13) is equivalent to the system **SL1** of mixed-integer quadratic inequalities

$$\sum_{t \in T} x_{i,t} \left( \sum_{l=1}^{n_j} \lambda_{j,l}\, c_{j,l} \right) \leq -d_i \ , \ i \in I \ . \tag{5}$$

c) We use the Fortet inequalities to demonstrate the equivalence of the feasible sets respectively defined by (5) and (20)-(24). The Fortet inequalities linearize the binary bilinear terms $x_{i,t}\lambda_{j,l}$ in (5) by lifting: i) the decisional space by adding the auxiliary decision variables $z_{j,l}, j \in J, l = 1, \ldots, n_j$ and ii) the constraint space by adding four sets of linear constraints of form (20)-(23) for each bilinear term $x_{i,t}\lambda_{j,l}$, which guarantee that $z_{j,l} = x_{i,t}\lambda_{j,l}$. The constraints (20), (21), and (23) guarantee that $z_{j,l}$ is equal to 0 if $x_{i,t}$ or $\lambda_{j,l}$ is equal to 0, while (22) sets $z_{j,l}$ to 1 if both $x_{i,t}$ and $\lambda_{j,l}$ are both equal to 1. The Fortet inequalities allow us to transition from a set of mixed-integer quadratic inequalities to one with mixed-integer linear inequalities and to obtain the result that we set out to prove. □

## A.2 Proof of Proposition 7

**Proof.** The inequalities (20)-(23) guarantee that $z_{j,l} = \lambda_{j,l}x_{i,t}, l = 1, \ldots, n_j, j = (i,t) \in J$. Removing (20) does not ensure that $z_{j,l} = 0$ if $\lambda_{j,l} = 1$ and $x_{i,t} = 0, j = (i,t)$. The introduction of the new constraints (25) will compensate for that. Indeed, the combination of (18) and (25) ensures that exactly one of the $n_j$ variables $z_{j,l}$ is equal to one. The only variable $z_{j,l}$ taking value one must be the one for which both $\lambda_{j,l} = 1$ and $x_{i,t} = 1, j = (i,t)$. Thus, this precludes $z_{j,l} = 1$ when $\lambda_{j,l} = 1$ and $x_{i,t} = 0, j = (i,t)$. As the inequalities (25) do not cut any integer solution feasible for (20),they are valid and stronger than (20). □

## A.3 Proof of Proposition 8

**Proof.** Problem **M1** maximizes the sum of the non-negative variables $d_i$, which are upper-bounded by (24) in **SL1**. Each parameter $c_{j,l}, j \in J, l = 1, \ldots, n_j$ is negative as each $c_{j,l}$ represents a negative profit, see (15). Hence, $d_i$ grows larger as $z_{j,l}$ increases. It follows that each $z_{j,l}$ will systematically take value 1 if allowed by (20) and (21). Thus, the constraints (22), which lower bounds the variables $z_{j,l}$, are superfluous and can be removed. The replacement of (20) by (25) is due to Lemma 7. □

## A.4 Trade-Off Between Stability and Reliability Requirements

We cross-validate the managerial guidelines presented in Section 5.3 for the cases when (i) scenarios are generated from historical data and from a mixture of plausible future market conditions, (ii) the planning horizon comprises $|T| = 10, 8$ periods, and (iii) the reliability level $p$ is set to 0.99, 0.95, and 0.9. Results in Figures 1 and 2 and Tables 1 and 2 are obtained from experiments using 5000 scenarios.

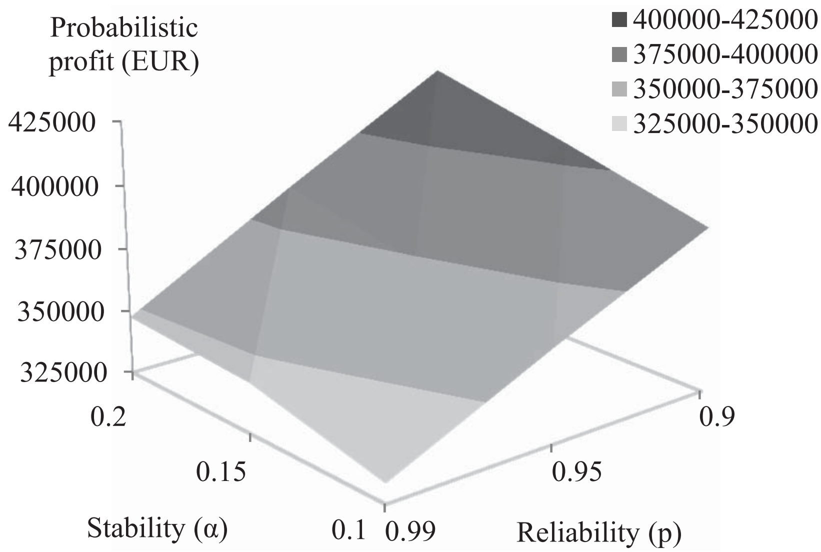


Figure 1: Tradeoffs among stability and reliability in 10-period problem using a mixture of scenarios

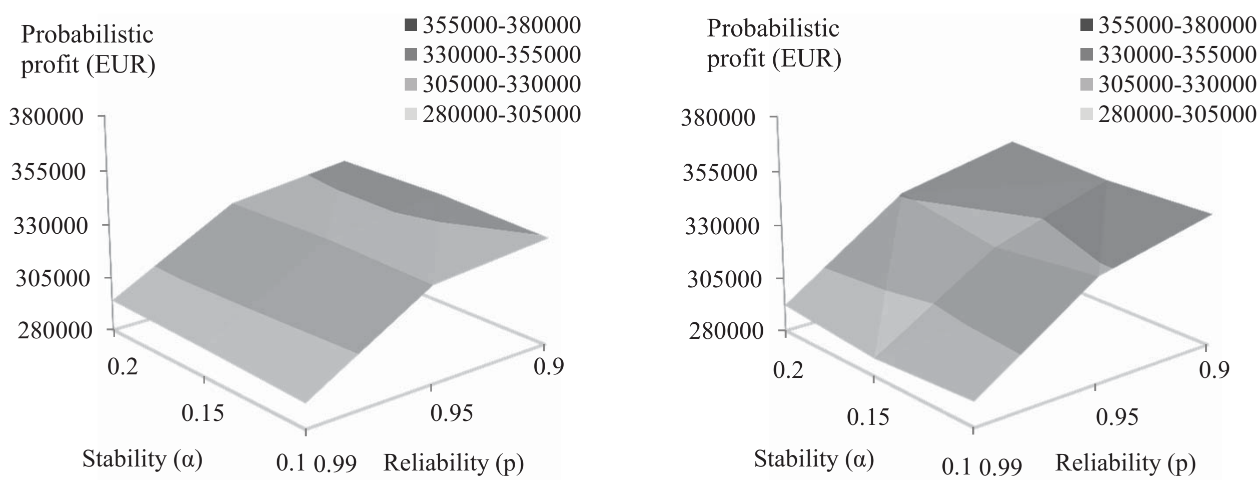


Figure 2: Tradeoffs among stability and reliability in 8-period problem, left: historical scenarios, right: mixture of scenarios

Table 1: Impact of increasing profit stability requirement on the number of stands harvested at early periods (periods 1-5 when $|T| = 10$ and periods 1-4 when $|T| = 8$)

| | | | Number of stands harvested during early periods | | | Similar to observed result |
|---|---|---|---|---|---|---|
| $\|T\|$ | Scenarios | $p$ | No stability constraint | $\alpha = 0.2$ | $\alpha = 0.1$ | |
| 10 | Historical | 0.99 | 5 | 21 | 25 | ✓ |
| | | 0.95 | 5 | 24 | 27 | ✓ |
| | | 0.9 | 5 | 23 | 23 | ✓ |
| | Mixed | 0.99 | 5 | 22 | 28 | ✓ |
| | | 0.95 | 5 | 25 | 25 | ✓ |
| | | 0.9 | 5 | 21 | 23 | ✓ |
| 8 | Historical | 0.99 | 5 | 13 | 25 | ✓ |
| | | 0.95 | 4 | 14 | 19 | ✓ |
| | | 0.9 | 4 | 19 | 22 | ✓ |
| | Mixed | 0.99 | 8 | 16 | 20 | ✓ |
| | | 0.95 | 4 | 19 | 23 | ✓ |
| | | 0.9 | 4 | 19 | 20 | ✓ |
| | | Average | 4.92 | 19.67 | 23.33 | ✓ |

Table 2: Impact of increasing profit stability requirement on the correlation between optimal harvesting times and growth rates (of main tree species) of stands

| | | | Correlation | | | Similar to observed result |
|---|---|---|---|---|---|---|
| $\|T\|$ | Scenarios | $p$ | No stability constraint | $\alpha = 0.2$ | $\alpha = 0.1$ | |
| 10 | Historical | 0.99 | 0.15 | 0.43 | 0.51 | ✓ |
| | | 0.95 | 0.14 | 0.45 | 0.51 | ✓ |
| | | 0.9 | 0.12 | 0.43 | 0.52 | ✓ |
| | Mixed | 0.99 | 0.17 | 0.41 | 0.50 | ✓ |
| | | 0.95 | 0.09 | 0.47 | 0.55 | ✓ |
| | | 0.9 | 0.08 | 0.44 | 0.45 | ✓ |
| 8 | Historical | 0.99 | 0.21 | 0.25 | 0.41 | ✓ |
| | | 0.95 | 0.08 | 0.29 | 0.36 | ✓ |
| | | 0.9 | 0.16 | 0.34 | 0.36 | ✓ |
| | Mixed | 0.99 | 0.26 | 0.33 | 0.37 | ✓ |
| | | 0.95 | 0.07 | 0.30 | 0.36 | ✓ |
| | | 0.9 | 0.19 | 0.28 | 0.35 | ✓ |
| | | Average | 0.14 | 0.37 | 0.44 | ✓ |