Improving Supplier Compliance Through Joint and Shared Audits with Collective Penalty

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ONLINE APPENDIX

Proof of Lemma 1: The first statement follows immediately from the fact that $2r > 1$. All other statements can be obtained from differentiating x^I and z^I given in (3) with respect to the corresponding parameter. However, to prove the last statement, one needs to account for the fact that $r = \frac{g+w}{2\gamma}$ and $m = (p-w)$. We omit the details. ■

Proof of Lemma 2: Observe from (15) that $z_1 = z_2 = z$ by symmetry and apply (15) to show that $x^S = 2rz^S(2 - z^S)$. This proves the second statement. Next, by substituting $x = 2r(2z - z^2)$ into (15) and by rearranging the terms, the buyer's audit level z is the solution to $V(z) = 0$. By showing that $V(0) < 0, V(1 - \sqrt{\frac{2r-1}{2r}}) > 0$ and $V(z)$ is concave over $[0, 1 - \sqrt{\frac{2r-1}{2r}}]$, we prove the first statement and that $z^S \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$. Next, observe that $x^S(z) = 2rz(2-z)$ is increasing in z when $z \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$, that $x^S(0) = 0$ and that $x^S(1 - \sqrt{\frac{2r-1}{2r}}) = 1$, we can use the fact that $z^S \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$ to show that $x^S \in (0,1)$. ■

Proof of Lemma 3: We have that $S^J - S^I = x^I(1-x^I) - x^J(1-x^J) = \dfrac{\alpha r(d-m)\,(2r^2(d-m)^2 - \alpha^2)}{(\alpha + 2r(d-m))^2(\alpha + r(d-m))^2}$. Hence, $S^J > S^I$ if and only if $\sqrt{2}\,r(d-m) > \alpha$. Similarly, $S^S - S^J = x^J(1-x^J) - x^S(1-x^S) = (x^J - x^S)(1 - x^J - x^S)$, so that $S^S - S^J \to 0^-$ as $\alpha \to 0^+$. Further, we know: (i) from Lemmas 4 and 5 that $\dfrac{dx^J}{d\alpha} < 0$ and $\dfrac{dx^S}{d\alpha} < 0$, (ii) from Equation (10) that $\lim_{\alpha\to\infty} x^J = 0$, and (iii) from Proposition 1 that $0 \leqslant x^S < x^J$, which indicates that $\lim_{\alpha\to\infty} x^S = 0$. Hence, we conclude that there exists a threshold α_J such that $S^S - S^J < 0$ if and only if $\alpha < \alpha_J$.

When comparing S^I and S^S, we have $S^S - S^I = x^I(1-x^I) - x^S(1-x^S) = (x^I - x^S)(1-x^I-x^S)$. By noting that $\dfrac{dx^I}{d\alpha} < 0$, $\dfrac{dx^S}{d\alpha} < 0$, $\lim_{\alpha\to\infty} x^I = 0$, and $\lim_{\alpha\to\infty} x^S = 0$, we conclude that there exists a threshold α_I such that $(1-x^I-x^S) < 0$ if and only if $\alpha < \alpha_I$. Further, by Proposition 1, $x^S > x^I$ if and only if $\alpha > \widetilde{\alpha} = \max\{(d-m)(\widetilde{r}-r), 0\}$. Therefore, if $\widetilde{r} \leqslant r$, then $x^S > x^I$ for all $\alpha \geqslant 0$, and hence $S^S - S^I > 0$ if and only if $\alpha < \alpha_I$.

When $\widetilde{r} > r$, as $\alpha \to 0^+$ we have $(x^I - x^S) \to 0^+$ and $(1-x^I-x^S) \to -1$. Thus, $S^S - S^I < 0$ when α is sufficiently small (i.e., when $\alpha < \min\{\alpha_I, \widetilde{\alpha}\}$) and sufficiently large (i.e., when $\alpha > \max\{\alpha_I, \widetilde{\alpha}\}$). On the other hand, when α is moderate (i.e., when $\min\{\alpha_I, \widetilde{\alpha}\} < \alpha < \max\{\alpha_I, \widetilde{\alpha}\}$) then $S^S - S^I > 0$. ■

Proof of Lemma 4: The proof follows the same approach as the proof for Lemma 1. We omit the details. ■

Proof of Lemma 5: To prove the first statement, we use the fact that $2r > 1$ and the fact that $z^S \in (0,1)$ to show that $x^S = 2rz^S(2-z^S) > z^s + z^S(1-z^S) > z^S$. To prove the second statement, we differentiate (16) with respect to $k \equiv \frac{2\alpha}{d-m}$ and apply the implicit function theorem, getting: $U(z^S)\cdot\frac{dz^S}{dk} + z^S = 0$, where $U(z) = [6rz^2 - 12rz + (1+4r+k)]$. By noting that $U(z)$ is increasing in z and that $U(0) > 0$ and $U(1-\sqrt{\frac{2r-1}{2r}}) > 0$, we can conclude that $U(z^S) > 0$. Hence, we can conclude that $\frac{dz^S}{dk} < 0$. Also, observe that $\frac{dx^S}{dk} = 4r(1-z^S)\cdot\frac{dz^S}{dk} < 0$. Combine these results with the fact that k in increasing in α and decreasing in d, we obtain the desirable properties about z^S and x^S as stated in the second statement.

To prove the third statement, differentiate (16) with respect to r and apply the implicit function theorem, getting: $U(z^S)\cdot\frac{dz^S}{dr} + W(z^S) = 0$, where $U(z)$ is defined above and $W(z) = 2z(z^2 - 3z + 2)$. By using the fact that both $U(z) > 0$ and $W(z) > 0$ for any $z \in (0,1)$, we can conclude that $\frac{dz^S}{dr} = -\frac{W(z^S)}{U(z^S)} < 0$. Next, observe that $\frac{dx^S}{dr} = 2z^S(2-z^S) + 4r(1-z^S)\cdot\frac{dz^S}{dr}$. By substituting $\frac{dz^S}{dr} = -\frac{W(z^S)}{U(z^S)}$ and by rearranging the terms and by using the fact that $V(z^S) = 0$, it can be shown that: $\frac{dx^S}{dr} = \frac{2z^S\cdot(4r(z^S)^2 - 4rz^S + 3 + 2k)}{U(z^S)} = \frac{2z^S\cdot(-2x^S + 4rz^S + 3 + 2k)}{U(z^S)} > 0$, where the last inequality is due to the fact that $x^S < 1$. Finally, by combining the result that $\frac{dz^S}{dr} < 0$ and $\frac{dz^S}{dr} > 0$ and by using the fact that $r = \frac{g+w}{2\gamma}$, we obtain the third statement.

To prove the fourth statement, implicitly differentiating the equation $V(z^S) = 0$ and the expression for x^S with respect to r, we get:

$$
\begin{aligned}
\frac{\partial z^S}{\partial r} &= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha - 6r(d-m)(2-z^S)z^S + (4r+1)(d-m)} \\
&= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha + (d-m)(1-2rz^S(2-z^S)) + 4r(d-m)(1-(2-z^S)z^S)} \\
&= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} < 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial x^S}{\partial r} &= 2(2-z^S)z^S + 4r(1-z^S)\frac{\partial z^S}{\partial r} \\
&= -\frac{2(z^S-2)z^S(2\alpha + 2r(d-m)(z^S-2)z^S + d - m)}{2\alpha + (d-m)(1-2rz^S(2-z^S)) + 4r(d-m)(1-(2-z^S)z^S)}
\end{aligned}
$$

$$= \frac{2(2-z^S)z^S(2\alpha+(d-m)(1-2r(2-z^S)z^S))}{2\alpha+(d-m)(1-x^S)+4r(d-m)(1-\frac{x^S}{2r})}$$
$$= \frac{2(2-z^S)z^S(2\alpha+(d-m)(1-x^S))}{2\alpha+(d-m)(1-x^S)+4r(d-m)(1-\frac{x^S}{2r})} > 0.$$

It remains to prove the last statement. By noting that $r=\frac{g+w}{2\gamma}$ and $m=(p-w)$, we differentiate (16) with respect to w and apply the implicit function theorem to get:

$$\frac{dz^S}{dw} = \frac{z^S\left(2\gamma\alpha-(2-z^S)(1-z^S)(d-p+w)^2\right)}{(d-p+w)\left[2\gamma\alpha+(d-p+w)(\gamma+2(g+w)-3(g+w)(2-z^S)z^S)\right]}$$
$$\Rightarrow \frac{dz^S}{dw} \geqslant 0 \Leftrightarrow 2\gamma\alpha-(2-z^S)(1-z^S)(d-p+w)^2 \geqslant 0$$

because, by using the fact that $x^S=2rz^S(2-z^S)\leqslant 1$ it can be easily verified that the denominator of the above expression is positive. Now,

$$2\gamma\alpha-(2-z^S)(1-z^S)(d-p+w)^2 \geqslant 0 \Leftrightarrow (2-z^S)(1-z^S) \leqslant \frac{2\gamma\alpha}{(d-p+w)^2}$$
$$\Leftrightarrow 2rz^S(2-z^S)(1-z^S) \leqslant \frac{4r\gamma\alpha z^S}{(d-p+w)^2} = \frac{2\alpha z^S(g+w)}{(d-p+w)^2}$$
$$\Leftrightarrow 2rz^{S3}-6rz^{S2}+4rz^S+z^S\left[1+\frac{2\alpha}{d-p+w}\right] \leqslant \frac{2\alpha z^S(g+w)}{(d-p+w)^2}+z^S\left[1+\frac{2\alpha}{d-p+w}\right]$$
$$\Leftrightarrow 1 \leqslant z^S\left[1+\frac{2\alpha}{d-p+w}+\frac{2\alpha(g+w)}{(d-p+w)^2}\right] \quad \text{by using (16)}$$
$$z^S \geqslant \left[1+\frac{2\alpha}{d-p+w}+\frac{2\alpha(g+w)}{(d-p+w)^2}\right]^{-1} = [f(w)]^{-1} \text{ (say)}$$

By noting that $\alpha>0$, $f'(w)<0$, $\lim_{w\to\infty} f(w)=1$, and $\lim_{w\to\infty} z^S=0$ (from (16)) we infer that there exists a threshold value of w above which $z^S<f(w)^{-1}$, that is, $\frac{dz^S}{dw}<0$.

Next, by noting that $x^S=2rz^S(2-z^S)$ and by using the expression for $\frac{dz^S}{dw}$, we get:

$$\frac{dx^S}{dw} = \frac{2(g+w)(1-z^S)\left(\frac{dz^S}{dw}\right)+(2-z^S)z^S}{\gamma},$$
$$= \frac{z^S}{\gamma}\left\{(2-z^S)+\frac{2(g+w)(1-z^S)\left[2\alpha\gamma-(2-z^S)(1-z^S)(d-p+w)^2\right]}{(d-p+w)\left[2\alpha\gamma+(d-p+w)\left(\gamma-3(g+w)(2-z^S)z^S+2(g+w)\right)\right]}\right\}$$

It follows from Assumptions 2 and the fact that $r=\frac{g+w}{2\gamma}>\frac{1}{2}$ and $(d-m)=(d-p+w)>0$, the denominator of the second term is also positive. Hence, the sign of the above term depends on the sign of the numerator alone. By expanding and rearranging the terms, the numerator can be simplified as: $\gamma\left(2\alpha\left[(d-m)(2-z^S)+4r\gamma(1-z^S)\right]+(d-m)^2(2-z^S)(1-x^S)\right)>0$, where the last inequality is due to the fact that both x^S and z^S are bounded above by 1. This completes our proof. ■

Proof of Lemma 6: For a given θ_2 of buyer 2, we show by contradiction that buyer 1's best response must satisfy $\theta_1\leqslant\theta_2$. Suppose buyer 1's best response has $\theta_1>\theta_2$. Then for every fixed compliance level x,

$$\Pi_1^J(\theta_1;\theta_2,x) = m(1-z_1(\theta_1)(1-x))-d(1-z_1(\theta_1))(1-x)-\theta_1\,\alpha\,z_1(\theta_1)^2$$
$$\Rightarrow \frac{\partial\Pi_1^J(\theta_1;\theta_2,x)}{\partial\theta_1} = -\alpha\,z_1(\theta_1)^2<0.$$

Hence, buyer 1 sets θ_1 such that $\theta_1 \leqslant \theta_2$. Similarly, buyer 2 sets θ_2 such that $\theta_2 \leqslant \theta_1$. Hence, $\theta_1 = \theta_2$ in equilibrium. ■

Proof of Lemma 7: Given the symmetry of the buyers, we drop the indexes in this proof. By Lemma 6 it is true that the equilibrium under J comprises of symmetric cost sharing. Let $\Pi^J(\theta;\theta,x)$ be the profit under J when $\theta_i = \theta_j = \theta$ as obtained from from (32) and let x^J be the equilibrium compliance level when $\theta = \frac{1}{2}$. We prove that $\theta = \frac{1}{2}$ is the payoff dominant equilibrium.

Suppose $\theta < \frac{1}{2}$, then by (32) the profit of each buyer under J is given by (4). Let z^I and $x^I = 2rz^I$ be the equilibrium audit and compliance levels in the independent mechanism. Then,

$$\Pi^I(z^I) < \Pi^I(z^I) + \frac{1}{2}\alpha {z^I}^2 = \Pi^J(\theta = \frac{1}{2}; \theta = \frac{1}{2}, x^I) \leqslant \Pi^J(\theta = \frac{1}{2}; \theta = \frac{1}{2}, x^J)$$

where x^J is the equilibrium compliance level in the joint mechanism with $\theta = \frac{1}{2}$, and the last inequality follows by noting that $\frac{\partial \Pi^J}{\partial x} = mz + d(1-z) > 0$ (obtained from using Envelope theorem on (8)) and $x^J \geqslant x^I$. Hence, the equilibrium with $\theta < \frac{1}{2}$ is dominated by $\theta = \frac{1}{2}$.

Now, suppose $\theta > \frac{1}{2}$. Let $z^J(\theta)$ and $x^J(\theta) = 2rz^J(\theta)$ be the equilibrium audit and compliance levels. Then,

$$\begin{aligned}
&z^J(\theta) = \frac{(d-m)}{2\alpha\theta + 2r(d-m)} \Rightarrow \frac{dz^J(\theta)}{d\theta} < 0 \text{ and} \\
&\Pi^J(\theta;\theta,x) = m(1 - z(\theta)(1-x)) - d(1-z(\theta))(1-x) - \theta\alpha z(\theta)^2 \\
&\Rightarrow \frac{d\Pi^J(\theta;\theta,x^J(\theta))}{d\theta} = -\alpha z^J(\theta)^2 + \frac{\partial \Pi^J}{\partial x} \cdot 2r\frac{dz^J(\theta)}{d\theta} < 0 \text{ since } \frac{\partial \Pi^J}{\partial x} > 0 \text{ and } \frac{dz^J(\theta)}{d\theta} < 0.
\end{aligned}$$

Hence, the equilibrium with $\theta > \frac{1}{2}$ is dominated by $\theta = \frac{1}{2}$. ■

Proof of Lemma 8: For a given θ_2 of buyer 2, we show by contradiction that buyer 1's best response must satisfy $\theta_1 \leqslant \theta_2 \left(\frac{d-m_1}{d-m_2}\right)$. Suppose buyer 1's best response has $\theta_1 > \theta_2 \left(\frac{d-m_1}{d-m_2}\right)$. Then,

$$\begin{aligned}
&\Pi_1^J(\theta_1;\theta_2,x) = m_1(1 - z_1(\theta_1)(1-x)) - d(1 - z_1(\theta_1))(1-x) - \theta_1\alpha z_1(\theta_1)^2 \\
&\Rightarrow \frac{\partial \Pi_1^J(\theta_1;\theta_2,x)}{\partial \theta_1} = -\alpha z_1(\theta_1)^2 < 0.
\end{aligned}$$

Hence, buyer 1 sets θ_1 such that $\theta_1 \leqslant \theta_2 \left(\frac{d-m_1}{d-m_2}\right)$. Similarly, buyer 2 sets θ_2 such that $\theta_2 \leqslant \theta_1 \left(\frac{d-m_2}{d-m_1}\right)$. Hence, $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$ in equilibrium.

Clearly, for every given compliance level x of the supplier,

$$\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2} \Rightarrow z_1(\theta_1) = \frac{(d-m_1)(1-x)}{2\alpha\theta_1} = \frac{(d-m_2)(1-x)}{2\alpha\theta_2} = z_2(\theta_2). \quad ■$$

Proof of Lemma 9: For ease of notation, let $\hat{\theta}_i = \frac{d-m_i}{2d-m_1-m_2}$. By Lemma 8 that in equilibrium $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$. Let $\Pi_i^J(\theta_i;\theta_j,x)$ be the profit of buyer i and x^J be the compliance level when $\theta_i = \hat{\theta}_i$ and $\theta_j = \hat{\theta}_j$. We prove that $\theta_i = \hat{\theta}_i$, $i = 1,2$ is the payoff dominant equilibrium. We argue for buyer i and the argument for buyer j is similar.

Suppose $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2} < \frac{1}{2d-m_1-m_2}$, then by (33) the profit of buyer i under J is given by

$$\Pi_i^I(z_i^I; z_j^I, x^I) = m_i(1 - z_i^I(1 - 2rz_i^I)) - d(1 - z_i^I)(1 - 2rz_i^I) - \alpha {z_i^I}^2. \tag{34}$$

Let z_i^I and $x^I = r_1 z_1^I + r_2 z_2^I$ be the equilibrium audit and compliance levels under independent audits. Then for buyer i we have

$$\begin{aligned}
\Pi_i^J(z^J) &= m_i(1 - z^J(1-x^J)) - d(1-z^J)(1-x^J) - \hat{\theta}_i \alpha {z^J}^2 \\
&\geqslant m_i(1 - z_i^I(1-x^J)) - d(1-z_i^I)(1-x^J) - \hat{\theta}_i \alpha {z_i^I}^2 \text{ since } z^J \text{ maximizes (25) for every fixed value of } x \\
&\geqslant m_i(1 - z_i^I(1-x^I)) - d(1-z_i^I)(1-x^I) - \hat{\theta}_i \alpha {z_i^I}^2 \text{ because } x^I < x^J \text{ and } \frac{\partial \Pi_i^J}{\partial x} = mz + d(1-z) > 0 \\
&\geqslant m_i(1 - z_i^I(1-x^I)) - d(1-z_i^I)(1-x^I) - \alpha {z_i^I}^2 \text{ because } \hat{\theta}_i \in [0,1] \\
&= \Pi_i^I(z_i^I),
\end{aligned}$$

and $x^J \geqslant x^I$ because

$$\begin{aligned}
\theta_i = \hat{\theta}_i \Rightarrow x^J &= \frac{(r_1+r_2)(2d-m_1-m_2)}{2\alpha + (r_1+r_2)(2d-m_1-m_2)} \\
\Rightarrow x^J - x^I &= \frac{(r_1+r_2)(2d-m_1-m_2)}{2\alpha + (r_1+r_2)(2d-m_1-m_2)} - \frac{r_1(d-m_1)+r_2(d-m_2)}{2\alpha + r_1(d-m_1)+r_2(d-m_2)} \\
&= \frac{2\alpha((d-m_1)r_2 + (d-m_2)r_1)}{(2\alpha + (r_1+r_2)(2d-m_1-m_2))(2\alpha + r_1(d-m_1)+r_2(d-m_2))} \geqslant 0.
\end{aligned}$$

Hence, the equilibrium with $\theta_i < \hat{\theta}_i (\Leftrightarrow \theta_j < \hat{\theta}_j)$ is dominated by $\theta_i = \hat{\theta}_i (\Leftrightarrow \theta_j = \hat{\theta}_j)$. Now, suppose $\theta_i > \hat{\theta}_i (\Leftrightarrow \theta_j > \hat{\theta}_j)$. Then

$$\begin{aligned}
& z^J(\theta_i) = \frac{(d-m_i)}{2\alpha\theta_i + (d-m_i)(r_1+r_2)} \Rightarrow \frac{dz^J(\theta_i)}{d\theta_i} < 0 \text{ and} \\
& \Pi_i^J(\theta_i;\theta_j,x) = m_i(1 - z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i \alpha z_i(\theta_i)^2 \\
& \Rightarrow \frac{d\Pi_i^J(\theta_i;\theta_j,x^J(\theta_i))}{d\theta_i} = -\alpha z^J(\theta_i)^2 + \frac{\partial \Pi_i^J}{\partial x} \cdot (r_1+r_2)\frac{dz^J(\theta_i)}{d\theta_i} < 0 \text{ since } \frac{\partial \Pi_i^J}{\partial x} > 0 \text{ and } \frac{dz^J(\theta_i)}{d\theta_i} < 0.
\end{aligned}$$

Hence, the equilibrium with $\theta_i > \hat{\theta}_i$ is dominated by $\theta_i = \hat{\theta}_i$. ■

Proof of Proposition 1: Observe from (3) and (10) that $z^I = \frac{d-m}{2(\alpha+r(d-m))} < \frac{d-m}{\alpha+2r(d-m)} = z^J$. Next, by sustituting $z^I = \frac{d-m}{2(\alpha+r(d-m))}$ into (16) and by rearranging the terms, one can show that $V(z^I) = 2(2r+1)\alpha^2 + 2r(d-m)(4r-1)\alpha + r(1-2r)^2(d-m)^2 > 0 = V(z^S)$. By using the fact that $V(z)$ is increasing in z, we can conclude that $z^I > z^S$. Therefore, we prove the first statement: $z^S < z^I < z^J$.

Noting that $x^J = 2rz^J$ and $x^I = 2rz^I$, it follows that $x^J > x^I$.

Before we proceed further, we define the function $L(z) = z(2-z)$, which is an inverted parabola with roots 0 and 2, and mode at 1, for better exposition and shorthand notation.

In the region $[0,1]$, we note that $L(z) > z^J \Leftrightarrow z > \overline{z}$ where $\overline{z}$ is the solution of $L(z) = z^J$. The solution is given by $\overline{z} = 1 - \frac{\sqrt{\alpha + (2r-1)(d-m)}}{\sqrt{\alpha + 2r(d-m)}}$ and, hence $\frac{(d-m)V(\overline{z})}{\alpha} = 2 - 2\sqrt{1-z^J} - z^J\sqrt{1-z^J} = 2 - (2 - z^J)\sqrt{1-z^J} > 0$. Thus, $\overline{z} > z^S \Leftrightarrow z^J = L(\overline{z}) > L(z^S) \Leftrightarrow x^J > x^S$.

Similarly, to compare x^I and x^S, we need to compare z^I and $z^S(2-z^S)$. To compare z^I and z^S we consider the solution of the equation $L(z) = z^I$ in the region $[0,1]$. On solving, we get $\underline{z} = 1 - \sqrt{\frac{2\alpha + (2r-1)(d-m)}{2(\alpha + r(d-m))}}$. Now, in order to compare $\underline{z}$ and z^S, we consider $V(\underline{z})$. On substituting the value of $\underline{z}$ in $V(z)$ we get $\frac{(d-m)V(\underline{z})}{\alpha} = 1 - (1+z^I)\sqrt{1-z^I}$. Hence,

$$V(\underline{z}) > 0 \Leftrightarrow z^I \geqslant \frac{\sqrt{5}-1}{2} \Leftrightarrow (d-m)\left[1 - r(\sqrt{5}-1)\right] \geqslant (\sqrt{5}-1)\alpha.$$

When $r \geqslant \frac{1}{\sqrt{5}-1}$, then $V(\underline{z}) < 0 \Leftrightarrow \underline{z} < z^S \Leftrightarrow z^I = L(\underline{z}) < L(z^S) = z^S(2 - z^S) \Leftrightarrow x^I < x^S$. On the other hand, if r is small (i.e., $r < \frac{1}{\sqrt{5}-1}$) and when α is sufficiently small then $V(\underline{z}) > 0 \Leftrightarrow \underline{z} > z^S \Leftrightarrow z^I = L(\underline{z}) > L(z^S) = z^S(2 - z^S) \Leftrightarrow x^I > x^S$. And, when r is small but α is sufficiently large, then $V(\underline{z}) < 0 \Leftrightarrow \underline{z} < z^S \Leftrightarrow z^I = L(\underline{z}) < L(z^S) = z^S(2 - z^S) \Leftrightarrow x^I < x^S$. This concludes the proof. ■

Proof of Proposition 2: First, it follows from (5) and (12) that $\pi_s^I(z^I) - \pi_s^J(z^J) = \gamma[(1 - 2r \cdot z^I) + (1 - 2r \cdot z^J)] \cdot [2r(z^J - z^I)]$. By applying the first statement of Proposition 1 (i.e., $z^J > z^I$), we prove the first statement. By using the same approach, we obtain the second statement. Finally, observe from (5) and (12) that $\pi_s^I(z^I) - \pi_s^S(z^S) = \gamma[(1 - x^I) + (1 - x^S)] \cdot (x^S - x^I)$. We prove the third statement by applying (2) and (3) of Proposition 1 (i.e., $x^S > x^I$ when α is sufficiently large). This completes our proof. ■

Proof of Proposition 3: First, we note that from (2), we note that for every fixed audit level z_i of buyer i, the buyer's profit is increasing in the supplier's compliance level x. That is:

$$\frac{\partial \Pi_i(z_i; z_j, x)}{\partial x} = m z_i + d(1 - z_i) > 0. \tag{35}$$

Now, the joint mechanism profits at the payoff-maximizing equilibrium $\theta_1 = \theta_2 = \frac{1}{2}$ is

$$\begin{aligned}
\Pi_i^J(z^J) &= m(1 - z^J(1 - x^J)) - d(1 - z^J)(1 - x^J) - \frac{1}{2}\alpha {z^J}^2 \\
&\geqslant m(1 - z^I(1 - x^J)) - d(1 - z^I)(1 - x^J) - \frac{1}{2}\alpha {z^I}^2 \\
&\qquad \text{since } z^J \text{ maximizes } \Pi_i^J(z; x^J) \\
&\geqslant m(1 - z^I(1 - x^I)) - d(1 - z^I)(1 - x^I) - \frac{1}{2}\alpha {z^I}^2 \quad \text{using (35) and } x^J \geq x^I \\
&= \Pi_i^I(z^I) + -\frac{1}{2}\alpha {z^I}^2 > \Pi_i^I(z^I)
\end{aligned}$$

Next, it follows from (4) and (17), we get: $\Pi^I(z^I) - \Pi^S(z^S) = \alpha\left({z^S}^2 - {z^I}^2\right) + (z^I - (2 - z^S)z^S)\, T_I(z^S)$, where $T_I(z^S) = 2r(d - m)(z^S)^2 - 4r(d - m)z^S + (d - m)(1 - 2rz^I) + 2dr > 0$. By noting that the term $T_I(z^S) > 0$ for $z^S \in (0, 1)$, we can prove our second statement by applying Proposition 1 to show that the terms $((z^S)^2 - (z^I)^2)$ and $(z^I - (2 - z^S)z^S)$ are both negative. This proves second statement. ■

Proof of Proposition 4:

$$[2\,\Pi^J + \pi_s^J] - [2\,\Pi^I + \pi_s^I] = \frac{\alpha(d - m)}{2(\alpha + 2r(d - m))^2(\alpha + r(d - m))^2}\, f(\alpha)$$

where $f(\alpha) = (d - m + 4r(d - \gamma))\,\alpha^2 + 6r^2(2d - \gamma)(d - m)\alpha + 2r^2(d - m)^2\,(4dr - (d - m))$, which is a quadratic in α. Note that $f(0) > 0$ always and $f(\alpha)$ is continuous in α. It follows that $f(\alpha) > 0$ for α sufficiently low. This proves the first statement. For the second statement: when $d > \gamma$, we have $f(\alpha) > 0$. Finally, for the third statement: if $2d > \gamma$, then $f'(0) > 0$. Further, if $2d > \gamma$ and $d - m > g + w$, then we get $f''(\alpha) = 2\,[d - m + 4r(d - \gamma)] \geqslant 2\,[d - m - 2r\gamma] = 2\,[(d - m) - (g + w)] > 0$, which indicates that f is convex. Thus, $f > 0$ for all positive values of α. This completes the proof. ■

Proof of Proposition 5:

$$[2\,\Pi^S + \pi_s^S] - [2\,\Pi^I + \pi_s^I] \quad = \left(\frac{d-m}{r} - \gamma\right)(x^S - x^I)(2 - x^I - x^S) + 2\left(d - \frac{d-m}{2r}\right)(x^S - x^I) + 2\alpha(z^I - z^S)(z^I + z^S)$$

From Proposition 1, the last term in the above expression is positive. If $\frac{d-m}{r} > \gamma\,(\Leftrightarrow d - m > \frac{g+w}{2})$ then the first term in brackets is always positive. Hence, if the compliance of supplier under S is higher than that under I, then $2\,\Pi^S + \pi_s^S > 2\,\Pi^I + \pi_s^I$. From Proposition 1, $x^S > x^I$ if and only if $\alpha \geq \widetilde{\alpha}$. This concludes the proof. ■