The Online Appendix

Proof of Lemma 1. From our requirements on $W_2(\cdot,\cdot)$, $\frac{\partial^2 W_2(v_2^p,q_k^p)}{\partial v_2^p \partial q_k^p} \leq 1$ and $W_2(\cdot,0)=0$, we have $\frac{\partial W_2(v_2^p,q_k^p)}{\partial v_2^p} \leq q_k^p$. Note that $\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},q_k,v_2^p) = \mathcal{T}_{k,a}\, S(q_k^{p*}(v_2^p)) - v_2^p\, q_k^{p*}(v_2^p) - w_k^p\, q_k = \mathcal{T}_{k,a}\, S(q_k^{p*}(v_2^p)) - v_2^p\, q_k^{p*}(v_2^p) - W_1(s_1^p,q_k) + W_2(v_2^p,q_k^{p*}(v_2^p)) - W_3(S(q_k^{p*}))$. In the case that $q_k^{p*}=q_k$ is a constant value (independent of v_2^p), the result holds, since $\frac{d\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},q_k,v_2^p)}{dv_2^p} = -q_k + \frac{\partial W_2(v_2^p,q_k)}{\partial v_2^p} \leq 0$.

Then, we consider the case that $q_k^{p*} < q_k$. Note that $\frac{d\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},q_k,v_2^p)}{dv_2^p} =$

$$\frac{\partial \mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},q_k,v_2^p)}{\partial q_k^{p*}} \cdot \frac{\partial q_k^{p*}}{\partial v_2^p} + \frac{\partial \mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},q_k,v_2^p)}{\partial v_2^p} = -q_k^{p*} + \frac{\partial W_2(v_2^p,q_k^{p*})}{\partial v_2^p} \leq 0. \qquad \square$$

Proof of Lemma 2. Note that $s_1^{p-1}=s_1^p$ for $p=2,4,6,8$. From (6), we have

$$\begin{aligned}
\mathbb{E}_{t_1}\bar{\pi}_{k,s}(q_k,Y_a^p,Y_a^{p-1},s_1^p) &= \Big[\sum_{i=p-1}^{p} A^i Y_a^i w_k^i\Big] q_k - s_1^p q_k + \sum_{i=p-1}^{p} A^i [Y_a^i \mathcal{T}_{k,s} S(q_k^{i*}) + \big(1-Y_a^i\big)\,\mathcal{H}_{k,a}] \\
&= W_1(s_1^p,q_k) - s_1^p q_k - [W_1(s_1^p,q_k) - \mathcal{H}_{k,a}] \sum_{i=p-1}^{p} A^i(1-Y_a^i) + \Gamma,
\end{aligned}$$

where $\Gamma = \sum_{i=p-1}^{p} A^i Y_a^i [\mathcal{T}_{k,s} S(q_k^{i*}) - W_2(v_2^i,q_k^{i*}) + W_3(S(q_k^{i*}))]$ independent of s_1^p. Also, $\sum_{i=p-1}^{p} A^i(1-Y_a^i) \leq 1$ is a constant value, q_k is determined at time t_0, and from (4), q_k^{i*} is independent of s_1^p for $i=p-1,p$. Since $\frac{\partial \mathcal{H}_{k,a}}{\partial s_1^p} \leq \frac{\partial W_1(s_1^p,q_k)}{\partial s_1^p} \leq q_k$, we have $\mathbb{E}_{t_1}\bar{\pi}_{k,s}(q_k,Y_a^p,Y_a^{p-1},s_1^p)$ decreases in s_1^p. $\square$

Proof of Lemma 3. Rewrite (4) into $\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},Q,v_2^p) = \mathcal{T}_{k,a}\, S(q_k^{p*}) - v_2^p\, q_k^{p*} - W_1(s_1^p,Q) + W_2(v_2^p,q_k^{p*}) - W_3(S(q_k^{p*}))$. From (3) and our requirements $\frac{\partial^2 W_2(v_2^p,q_k^p)}{\partial (q_k^p)^2} \leq 0$, $W_3'(\cdot) \leq \mathcal{T}_{k,a}$ and $W_3''(\cdot) \geq 0$, we have $\mathcal{T}_{k,a}\, S(q) - v_2^p\, q - w_k^p\, q_k$ is quasi-concave in q, and the maximal value is achieved at $\bar{q}_k^p$, which solves the first order condition $\mathcal{T}_{k,a} S'(q) - v_2^p + \frac{\partial W_2(v_2^p,q)}{\partial q} - W_3'(S(q))S'(q) = 0$. Then, for $Q \leq \bar{q}_k^p$, we have $q_k^{p*} = Q$. Note that $\frac{\partial \mathbb{E}_{t_2}\bar{\pi}_{k,a}(Q,Q,v_2^p)}{\partial Q} = \mathcal{T}_{k,a}\, S'(Q) - v_2^p - \frac{\partial W_1(s_1^p,Q)}{\partial Q} + \frac{\partial W_2(v_2^p,Q)}{\partial Q} - W_3'(S(Q))S'(Q)$ so that

$$\begin{aligned}
\frac{\partial^2 \mathbb{E}_{t_2}\bar{\pi}_{k,a}(Q,Q,v_2^p)}{\partial Q^2} &= \mathcal{T}_{k,a}\, S''(Q) - \frac{\partial^2 W_1(s_1^p,Q)}{\partial Q^2} + \frac{\partial^2 W_2(v_2^p,Q)}{\partial Q^2} - W_3''(S(Q))(S'(Q))^2 - W_3'(S(Q))S''(Q) \\
&= [\mathcal{T}_{k,a} - W_3'(S(Q))]S''(Q) - \frac{\partial^2 W_1(s_1^p,Q)}{\partial Q^2} + \frac{\partial^2 W_2(v_2^p,Q)}{\partial Q^2} - W_3''(S(Q))(S'(Q))^2 \leq 0,
\end{aligned}$$

since $S(Q)$ and $W_2(\cdot,Q)$ are concave functions of Q, $W_3'(\cdot) \leq \mathcal{T}_{k,a}$, $W_1(\cdot,Q)$ is convex in Q, and $W_3(\cdot)$ are convex functions. Therefore, $\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},Q,v_2^p)$ is concave in Q.

On the other hand, if $Q > \bar{q}_k^p$, then $q_k^{p*} = \bar{q}_k^p < Q$ and $\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},Q,v_2^p)$ is concave in Q since $W_1(s_1^p,Q)$ is partially convex in Q. Furthermore, $\frac{\partial \mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},Q,v_2^p)}{\partial Q}$ is continuous in Q since

$$\begin{aligned}
\left.\frac{\partial \mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},Q,v_2^p)}{\partial Q}\right|_{Q=\bar{q}_k^{p,-}} &= \mathcal{T}_{k,a}\, S'(Q) - v_2^p - \frac{\partial W_1(s_1^p,Q)}{\partial Q} + \frac{\partial W_2(v_2^p,Q)}{\partial Q} - W_3'(S(Q))S'(Q) \\
&= -\frac{\partial W_1(s_1^p,Q)}{\partial Q} = \left.\frac{\partial \mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^{p*},Q,v_2^p)}{\partial Q}\right|_{Q=\bar{q}_k^{p,+}}.
\end{aligned}$$

Finally, note that $\mathcal{H}_{k,s}$ is concave in Q and $-\mathcal{H}_{k,a}$ is concave in Q. $\square$

Proof of Proposition 1. We first consider the assembler's decisions at time t_2. In order that our coordination result is not influenced by the realizations of raw material prices, we focus on the case that $Q = \infty$, where Q is the assembler's production quantity at time t_0. For the centralized system, for $p = 1, 2, \cdots, 8$, since the production cost $s_1^p Q$ at time t_1 is sunk, the first order condition is

$$\frac{\partial \mathbb{E}_{t_2} \bar{\Pi}(q^{p*}, \infty, v_2^p)}{\partial q^{p*}} = S'(q^{p*}) - v_2^p = 0. \tag{1}$$

It is straightforward to show that $\mathbb{E}_{t_2} \bar{\Pi}(q^{p*}, \infty, v_2^p)$ decreases in v_2^p. Then, there exists a threshold, denoted by $\bar{v}_2^p$, for the production at time t_2 is

$$\mathbb{E}_{t_2} \bar{\Pi}(q^{p*}, \infty, \bar{v}_2^p) = S(q^{p*}(\bar{v}_2^p)) - \bar{v}_2^p \, q^{p*}(\bar{v}_2^p) = 0. \tag{2}$$

The centralized system will produce only when $v_2^p \leq \bar{v}_2^p$.

We next consider the decentralized system. From the facts that $S'(\cdot) \geq 0$, $S''(\cdot) \leq 0$, $W_2(\cdot, q_k^p)$ is concave in q_k^p, and $W_3(\cdot)$ is increasing and convex with $W_3'(\cdot) \leq \mathcal{T}_{k,a}$, we have q_k^{p*} can be solved from

$$\frac{\partial \mathbb{E}_{t_2} \bar{\pi}_{k,a}(q_k^{p*}, \infty, v_2^p)}{\partial q_k^{p*}} = [\mathcal{T}_{k,a} - W_3'(S(q_k^{p*}))] S'(q_k^{p*}) - v_2^p \left[1 - \frac{1}{v_2^p} \frac{\partial W_2(v_2^p, q_k^{p*})}{\partial q_k^{p*}} \right] = 0. \tag{3}$$

To make (3) equivalent to (1), let $1 - \frac{1}{v_2^p} \frac{\partial W_2(v_2^p, q_k^{p*})}{\partial q_k^{p*}} = \mathcal{T}_{k,a} - W_3'(S(q_k^{p*}))$, i.e.,

$$\frac{1}{v_2^p} \frac{\partial W_2(v_2^p, q_k^{p*})}{\partial q_k^{p*}} - W_3'(S(q_k^{p*})) = 1 - \mathcal{T}_{k,a} = \mathcal{T}_{k,s}. \tag{4}$$

For notational convenience, let $H(q) = \frac{1}{v_2^p} \frac{\partial W_2(v_2^p, q)}{\partial q} - W_3'(S(q))$. Note that $H'(q) = \frac{1}{v_2^p} \frac{\partial^2 W_2(v_2^p, q)}{\partial q^2} - W_3''(S(q)) S'(q) \leq 0$. If $H'(q) < 0$, then (4) holds for some specific value q_k^{p*} but not for arbitrary values. Then, we require $H'(q) = 0$ for any $q \geq 0$. As a result, $W_3(S(q)) = C_1 S(q)$ (since we require $W_3(0) = 0$) as in (9c) and $W_2(v_2^p, q) = (C_1 + \mathcal{T}_{k,s}) g(v_2^p) q$ with $g'(v_2^p) \leq 1$, since we assume $W_2(\cdot, 0) = 0$. Consequently, $H(q) = (C_1 + \mathcal{T}_{k,s}) \frac{g(v_2^p)}{v_2^p} - C_1$. Further notice that if we want (4) to hold for any value of v_2^p, we need to let $g(v_2^p) = v_2^p$ so that $W_2(v_2^p, q) = (C_1 + \mathcal{T}_{k,s}) v_2^p q$, which is (9b).

With $W_2(\cdot, \cdot)$ and $W_3(\cdot)$ specified as in (9b) and (9c), $q_k^{p*} = q^{p*}$. If the thresholds $\bar{v}_2^p$ of the centralized system are also the thresholds of the assembler in the decentralized system, from (2), $-\mathcal{T}_{k,s} \bar{v}_2^p \, q_k^{p*}(\bar{v}_2^p) - W_1(s_1^p, q_k) + W_2(\bar{v}_2^p, q_k^{p*}(\bar{v}_2^p)) - W_3(\bar{v}_2^p \, q_k^{p*}(\bar{v}_2^p)) + \mathcal{H}_{k,a} = \mathcal{H}_{k,a} - W_1(s_1^p, q_k) = 0$. Then,

$$\mathcal{H}_{k,a} = W_1(s_1^p, q_k). \tag{5}$$

We next consider the time t_1 decision of the supplier, for $p = 2, 4, 6, 8$. Note that the threshold of s_1^p for the centralized system not to produce at time t_1 is $\mathbb{E}_{t_1} \bar{\Pi}(q, Y^p, Y^{p-1}, s_1^p) = 0$, where $\mathbb{E}_{t_1} \bar{\Pi}(q, Y^p, Y^{p-1}, s_1^p) \equiv \sum_{i=p-1}^{p} A^i \left[Y^i \left(S(q^{i*}) - v_2^i q^{i*} \right) \right] - s_1^p q$ and $Y^i = 1$ if the centralized supply chain produces on path i at time t_2 and $Y^i = 0$ otherwise. For the decentralized system, (9b), (9c), and (9d) guarantee $Y_a^i = Y^i$ for $i = p-1$ and p. Substituting (9b), (9c), and (9d) into (6) yields

$$\mathbb{E}_{t_1} \bar{\pi}_{k,s}(q_k, Y_a^p, Y_a^{p-1}, s_1^p) = \sum_{i=p-1}^{p} A^i \left\{ Y_a^i \left[W_1(s_1^p, q_k) + (C_1 + \mathcal{T}_{k,s})(S(q_k^{i*}) - v_2^p q_k^{i*}) \right] + (1 - Y_a^i) \mathcal{H}_{k,a} \right\} - s_1^p \, q_k$$

$$= (C_1 + \mathcal{T}_{k,s}) \sum_{i=p-1}^{p} A^i \{Y_a^i [S(q_k^{i*}) - v_2^p q_k^{i*}] - s_1^p q_k\} + W_1(s_1^p, q_k) - (\mathcal{T}_{k,a} - C_1) s_1^p q_k. \quad (6)$$

The supplier's stopping threshold solves $\mathbb{E}_{t_1} \bar{\pi}_{k,s}(q_k, Y_a^p, Y_a^{p-1}, s_1^p) + \mathcal{H}_{k,s} = 0$. The decentralized system has the same threshold $\mathcal{S}_k(q_k)$ as long as $W_1(s_1^p, q_k) - (\mathcal{T}_{k,a} - C_1) s_1^p q_k + \mathcal{H}_{k,s} = 0,$. Then,

$$W_1(s_1^p, q_k) = (\mathcal{T}_{k,a} - C_1) s_1^p q_k - \mathcal{H}_{k,s}, \quad (7)$$

and $\mathcal{H}_{k,a} = (\mathcal{T}_{k,a} - C_1) s_1^p q_k - \mathcal{H}_{k,s}$ from (5). We need $\mathcal{H}_{k,s} \geq 0$, since otherwise, the supplier gets paid for not producing, which will create moral hazard issues. Substituting (7) into (6) yields

$$\begin{aligned} \mathbb{E}_{t_1} \bar{\pi}_{k,s}(q_k, Y_a^p, Y_a^{p-1}, s_1^p) &= (C_1 + \mathcal{T}_{k,s}) \sum_{i=p-1}^{p} A^i \{Y_a^i [S(q_k^{i*}) - v_2^p q_k^{i*}] - s_1^p q_k\} - \mathcal{H}_{k,s} \\ &= (C_1 + \mathcal{T}_{k,s}) \mathbb{E}_{t_1} \bar{\Pi}(q_k, Y_a^p, Y_a^{p-1}, s_1^p) - \mathcal{H}_{k,s}. \end{aligned}$$

We now consider the time t_0 decision of the optimal production quantity Q of the assembler. Substituting (7), (9b), (9c), and (5) into the objective of program (8), we obtain

$$\begin{aligned} \pi_{k,a}^p(q_k) &= Y_s^p \left[Y_a^p \left(\mathcal{T}_{k,a} S(q_k^{p*}) - v_2^p q_k^{p*} - w_k^p q_k \right) - (1 - Y_a^p) \mathcal{H}_{k,a} \right] + (1 - Y_s^p) \mathcal{H}_{k,s} \\ &= (\mathcal{T}_{k,a} - C_1) Y_s^p \left[Y_a^p \left(S(q_k^{p*}) - v_2^p q_k^{p*} \right) - s_1^p q_k \right] + \mathcal{H}_{k,s}. \end{aligned}$$

If $Y_s^p = 0$, then $\pi_{k,a}^p(q_k) = \mathcal{H}_{k,s}$. If $\mathcal{H}_{k,s} > 0$, then $\pi_{k,a}^p(q_k) > 0$. Again, the moral hazard issue will appear, since the assembler can decides very large q_k so that the supplier cannot produce in any path p and and thus is in a pure loss situation. Then, we need $\mathcal{H}_{k,s} = 0$ for a general coordination contract.

Finally, if the centralized system produces q_k as the decentralized system, (9b), (9c), and (9d) guarantee that q_k^{p*} is the optimal solution of the centralized system. Also, (9a) - (9e) ensure that the centralized and decentralized systems have the same thresholds $\mathcal{S}_k(q_k)$ and $\mathcal{V}_k(q_k)$, i.e., the Y_a^p and Y_s^p are the same for the centralized and decentralized systems. Consequently, $Y_s^p \left[Y_a^p \left(S(q_k^{p*}) - v_2^p q_k^{p*} \right) - s_1^p q_k \right]$ is the profit of the centralized system by producing q_k at time t_1 and q_k^{p*} at time t_2. Then, $C_1 + \mathcal{T}_{k,s}$ and $\mathcal{T}_{k,s} - C_1$ are the supplier's and assembler's shares of the total sales revenue, respectively. □

Proof of Proposition 2. Since we focus on the cases that the channel working capital is not enough so that the centralized supply chain may face bankruptcy risk on some paths, we consider the case that the assembler has the bankruptcy risk at time t_2, i.e., $\mathcal{L}^p > 0$ for some p. Recall that as long as a party can borrow, $G(\mathcal{K}_{k,c}^p, q_k^p) \geq 0$, for $c = a, s$, due to the competitiveness of the financing sector.

The requirements on $W_2(\cdot, \cdot)$ *and* $W_3(\cdot)$. For $\mathcal{K}_{k,a}^p \geq q_k^p$, let

$$A(\mathcal{K}_{k,a}^p, q_k^p) = \alpha \int_{2q_k^p}^{2\mathcal{K}_{k,a}^p} (\xi - 2q_k^p) f(\xi) d\xi, \quad (8a)$$

$$B(\mathcal{K}_{k,a}^p, q_k^p) = \int_{2q_k^p}^{2\mathcal{K}_{k,a}^p} (\xi - 2q_k^p) f(\xi) d\xi + 2(\mathcal{K}_{k,a}^p - q_k^p) \bar{F}(2\mathcal{K}_{k,a}^p) = \int_{2q_k^p}^{2\mathcal{K}_{k,a}^p} \bar{F}(\xi) d\xi, \quad (8b)$$

$$D(\mathcal{K}_{k,a}^p, q_k^p) = -B(\mathcal{K}_{k,a}^p, q_k^p) + A(\mathcal{K}_{k,a}^p, q_k^p) \leq 0. \quad (8c)$$

For notational convenience, let $M(q_k^p) = 1 - \frac{1}{v_2^p}\frac{\partial W_2(v_2^p, q_k^p)}{\partial q_k^p}$, and $\mathbb{1}_\Omega = 1$ if the condition Ω holds and 0 otherwise. From (A.1a), (A.1b), (A.4), and (A.6), note that $\Delta_{k,a}^p$ is constant at time t_2. Then,

$$\begin{aligned} \frac{d\mathcal{K}_{k,a}^p}{dq_k^p} &= \frac{[v_2^p\, M(q_k^p) + W_3'(S(q_k^p))S'(q_k^p)]/\mathcal{T}_{k,a} + D(\mathcal{K}_{k,a}^p, q_k^p)\mathbb{1}_{\mathcal{K}_{k,a}^p \geq q_k^p}}{2\min\{\mathcal{K}_{k,a}^p, q_k^p\}\bar{F}(2\mathcal{K}_{k,a}^p)G(\mathcal{K}_{k,a}^p, q_k^p)}, \\ \frac{dC_b(\mathcal{K}_{k,a}^p, q_k^p)}{dq_k^p} &= \frac{1 - G(\mathcal{K}_{k,a}^p, q_k^p)}{G(\mathcal{K}_{k,a}^p, q_k^p)} \cdot \frac{v_2^p M(q_k^p) + W_3'(S(q_k^p))S'(q_k^p)}{\mathcal{T}_{k,a}} + \left[\frac{D(\mathcal{K}_{k,a}^p, q_k^p)}{G(\mathcal{K}_{k,a}^p, q_k^p)} + B(\mathcal{K}_{k,a}^p, q_k^p)\right]\mathbb{1}_{\mathcal{K}_{k,a}^p \geq q_k^p}. \end{aligned} \tag{9}$$

From (19a) and noting that $\Delta_{k,a}^p$, $L_{k,a}^p$ and $W_1(s_1^p, q_k)$ are constant at time t_2, the first order condition of the assembler is $\frac{d\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^p, q_k, v_2^p)}{dq_k^p} =$

$$S'(q_k^p)\left[\mathcal{T}_{k,a} - \frac{W_3'(S(q_k^p))}{G(\mathcal{K}_{k,a}^p, q_k^p)}\right] - \frac{v_2^p M(q_k^p)}{G(\mathcal{K}_{k,a}^p, q_k^p)} - \mathcal{T}_{k,a}\left[\frac{D(\mathcal{K}_{k,a}^p, q_k^p)}{G(\mathcal{K}_{k,a}^p, q_k^p)} + B(\mathcal{K}_{k,a}^p, q_k^p)\right]\mathbb{1}_{\mathcal{K}_{k,a}^p \geq q_k^p}. \tag{10}$$

We next study the centralized system. After a similar analysis as the assembler, we have:

$$\mathbb{E}_{t_2}\bar{\Pi}(q^p, q, v_2^p) = S(q^p) - s_1^p q - v_2^p\, q^p - C_b(\mathcal{K}^p, q^p) - \Delta^p + L^p, \tag{11}$$

where Δ^p and L^p are constant at time t_2. The first order condition is $\frac{d\mathbb{E}_{t_2}\bar{\Pi}(q^p, q, v_2^p)}{dq^p} = 0$, where

$$\frac{d\mathbb{E}_{t_2}\bar{\Pi}(q^p, q, v_2^p)}{dq^p} = S'(q^p) - \frac{v_2^p}{G(\mathcal{K}^p, q^p)} - \left[\frac{D(\mathcal{K}^p, q^p)}{G(\mathcal{K}^p, q^p)} + B(\mathcal{K}^p, q^p)\right]\mathbb{1}_{\mathcal{K}^p \geq q^p}. \tag{12}$$

If $\mathcal{K}_{k,a}^p \leq q_k^p$, to equalize the first order conditions (10) and (11), let $M(q_k^p) = \mathcal{T}_{k,a} - \frac{W_3'(S(q_k^p))}{G(\mathcal{K}_{k,a}^p, q_k^p)} = C$, where $C \geq 0$ is a constant. From $W_3'(\cdot) \geq 0$ and $G(\mathcal{K}_{k,a}^p, q_k^p) \geq 0$, we have $C \leq \mathcal{T}_{k,a}$. Then,

$$W_3'(S(q_k^p)) = (\mathcal{T}_{k,a} - C)G(\mathcal{K}_{k,a}^p, q_k^p). \tag{13}$$

If $\alpha = 0$, we have $G(\mathcal{K}_{k,a}^p, q_k^p) = 1$ and (13) is reduced to (9c) for financially unconstrained supply chain. Now, Equation (9) can be simplified into

$$\frac{d\mathcal{K}_{k,a}^p}{dq_k^p} = \frac{(1 - C/\mathcal{T}_{k,a})[S'(q_k^p) - v_2^p/G(\mathcal{K}_{k,a}^p, q_k^p)] + v_2^p}{2\mathcal{K}_{k,a}^p\bar{F}(2\mathcal{K}_{k,a}^p)}.$$

From (A.2), $\frac{dG(\mathcal{K}_{k,a}^p, q_k^p)}{dq_k^p} = -\alpha[z(2\mathcal{K}_{k,a}^p) + 2\mathcal{K}_{k,a}^p z'(2\mathcal{K}_{k,a}^p)]\frac{d\mathcal{K}_{k,a}^p}{dq_k^p}$. If $S'(q_k^p) - \frac{v_2^p}{G(\mathcal{K}_{k,a}^p, q_k^p)} > 0$, then $\frac{d\mathcal{K}_{k,a}^p}{dq_k^p} > 0$ so that $G(\mathcal{K}_{k,a}^p, q_k^p)$ decreases in q_k^p. However, since $W_3''(S(q_k^p)) = W_3'(S(q_k^p))S'(q_k^p)) \geq 0$, $W_3'(S(q_k^p))$ always increases in q_k monotonically. To remove the conflict, we have to let $C = \mathcal{T}_{k,a}$. From Assumption (2.a), $W_3(S(q_k^p)) = 0$. If $\mathcal{K}_{k,a}^p > q_k^p$, the situation is more complex, but the above arguments still hold as long as $G(\mathcal{K}_{k,a}^p, q_k^p)$ is not monotonic in q_k^p. Moreover, the last two terms in (10) related to $\mathcal{K}_{k,a}^p > q_k^p$ have the coefficient $\mathcal{T}_{k,a}$. Again, we require $C = \mathcal{T}_{k,a}$ and $W_3(S(q_k^p)) = 0$, which is (20c).

For the requirement on $W_2(\cdot,\cdot)$, since $M(q_k^p) = 1 - \frac{1}{v_2^p}\frac{\partial W_2(v_2^p, q_k^p)}{\partial q_k^p} = C = \mathcal{T}_{k,a}$ and $\frac{1}{v_2^p}\frac{\partial W_2(v_2^p, q_k^p)}{\partial q_k^p} = \mathcal{T}_{k,s}$. To make it hold for any non-negative values of q_k^p and v_2^p, with a similar analysis as in Proposition 1, we have $W_2(v_2^p, q_k^p) = \mathcal{T}_{k,s}v_2^p q_k^p$, which is (20b). Then, the first order condition of the decentralized system will be the same as the centralized one, except a constant scalar $\mathcal{T}_{k,a}$.

Now, the expected profits of the assembler, supplier, and centralized system are as follows:

$$
\begin{aligned}
\mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^p, q_k, v_2^p) &= \mathcal{T}_{k,a}[S(q_k^p) - v_2^p q_k^p - C_b(\mathcal{K}_{k,a}^p, q_k^p)] - W_1(s_1^p, q_k) - \Delta_{k,a}^p + L_{k,a}^p, && (14)\\
\mathbb{E}_{t_2}\bar{\pi}_{k,s}(q_k^p, q_k, v_2^p) &= \mathcal{T}_{k,s}[S(q_k^p) - v_2^p q_k^p - C_b(\mathcal{K}_{k,s}^p, q_k^p)] - s_1^p q_k + W_1(s_1^p, q_k) - \Delta_{k,s}^p + L_{k,s}^p. && (15)\\
\mathbb{E}_{t_2}\bar{\pi}(q^p, q, v_2^p) &= S(q^p) - \Delta^p + L^p - q\, s_1^p - q^p\, v_2^p - C_b(\mathcal{K}^p, q^p). && (16)
\end{aligned}
$$

The requirement on $W_1(\cdot,\cdot)$ and working capital y_a and y_s. We first study the case $\Delta_{k,c}^p = L_{k,c}^p = 0$ for $c = a, s$. Let q_k and q_k^p be fixed. Let $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c}) = \mathcal{T}_{k,c} C_b(\mathcal{K}_{k,c}^p, q_k^p)$. Equation (A.4) defines $\mathcal{K}_{k,c}^p$ as a function of $\mathcal{L}_{k,c}^p$ and $\mathcal{T}_{k,c}$, and $\mathcal{K}_{k,c}^p(\delta\mathcal{L}_{k,c}^p, \delta\mathcal{T}_{k,c}) = \mathcal{K}_{k,c}^p(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ for $\delta > 0$. From (A.6),

$$
\begin{aligned}
H(\delta\mathcal{L}_{k,c}^p, \delta\mathcal{T}_{k,c}) &= \delta\mathcal{T}_{k,c}\alpha \begin{cases} \int_0^{2\mathcal{K}_{k,c}^p(\delta\mathcal{L}_{k,c}^p, \delta\mathcal{T}_{k,c})} \frac{\xi^2}{4} f(\xi) d\xi, & \mathcal{K}_{k,c}^p(\delta\mathcal{L}_{k,c}^p, \delta\mathcal{T}_{k,c}) \leq q_k^p, \\ \int_0^{2q_k^p} \frac{\xi^2}{4} f(\xi) d\xi + \int_{2q_k^p}^{2\mathcal{K}_{k,c}^p(\delta\mathcal{L}_{k,c}^p, \delta\mathcal{T}_{k,c})} q_k^p(\xi - q_k^p) f(\xi) d\xi, & \mathcal{K}_{k,c}^p(\delta\mathcal{L}_{k,c}^p, \delta\mathcal{T}_{k,c}) > q_k^p. \end{cases} \\
&= \delta H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c}).
\end{aligned}
$$

Therefore, $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ is positive homogeneous in $\mathcal{T}_{k,c}$ and $\mathcal{L}_{k,c}^p$ of degree 1. Then, $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ is sub-additive in $\mathcal{T}_{k,c}$ and $\mathcal{L}_{k,c}^p$, if any only if $H(\cdot,\cdot)$ is jointly convex of the two variables. From (A.4),

$$
\frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{L}_{k,c}^p} = \frac{1}{\mathcal{T}_{k,c}\frac{\partial S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)}{\partial \mathcal{K}_{k,c}^p}} \quad \text{and} \quad \frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{T}_{k,c}} = -\frac{S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)}{\mathcal{T}_{k,c}\frac{\partial S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)}{\partial \mathcal{K}_{k,c}^p}} = -S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)\frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{L}_{k,c}^p}. \tag{17}
$$

Then, we have

$$
\begin{aligned}
\frac{\partial H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p} &= \mathcal{T}_{k,c}\frac{\partial C_b(\mathcal{K}_{k,c}^p, q_k^p)}{\partial \mathcal{K}_{k,c}^p} \cdot \frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{L}_{k,c}^p} = \frac{1}{G(\mathcal{K}_{k,c}^p, q_k^p)} - 1, && (18)\\
\frac{\partial H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{T}_{k,c}} &= C_b(\mathcal{K}_{k,c}^p, q_k^p) - \mathcal{T}_{k,c}\frac{\partial C_b(\mathcal{K}_{k,c}^p, q_k^p)}{\partial \mathcal{K}_{k,c}^p} \cdot \frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{T}_{k,c}} = \frac{H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\mathcal{T}_{k,c}} - S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)\frac{\partial H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p}. \\
\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{L}_{k,c}^p)^2} &= -\frac{1}{G^2(\mathcal{K}_{k,c}^p, q_k^p)} \cdot \frac{\partial G(\mathcal{K}_{k,c}^p, q_k^p)}{\partial \mathcal{K}_{k,c}^p} \cdot \frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{L}_{k,c}^p} \geq 0, && (19)\\
\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p \partial \mathcal{T}_{k,c}} &= -\frac{1}{G^2(\mathcal{K}_{k,c}^p, q_k^p)} \cdot \frac{\partial G(\mathcal{K}_{k,c}^p, q_k^p)}{\partial \mathcal{K}_{k,c}^p} \cdot \frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{T}_{k,c}} = -S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{L}_{k,c}^p)^2}, \\
\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{T}_{k,c})^2} &= -\frac{H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{(\mathcal{T}_{k,c})^2} + \frac{1}{\mathcal{T}_{k,c}}\frac{\partial H}{\partial \mathcal{T}_{k,c}} - \frac{\partial S_b}{\partial \mathcal{K}_{k,c}^p} \cdot \frac{\partial \mathcal{K}_{k,c}^p}{\partial \mathcal{T}_{k,c}} \cdot \frac{\partial H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p} - S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p \partial \mathcal{T}_{k,c}} \\
&= -S_b(\mathcal{K}_{k,c}^p, \alpha, q_k^p)\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p \partial \mathcal{T}_{k,c}} = S_b^2(\mathcal{K}_{k,c}^p, \alpha, q_k^p)\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{L}_{k,c}^p)^2}.
\end{aligned}
$$

After some algebra, the Hessian matrix of the function $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ is

$$
\begin{bmatrix} \frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{L}_{k,c}^p)^2} & \frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p \partial \mathcal{T}_{k,c}} \\ \frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial \mathcal{L}_{k,c}^p \partial \mathcal{T}_{k,c}} & \frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{T}_{k,c})^2} \end{bmatrix} \cong \begin{bmatrix} \frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{L}_{k,c}^p)^2} & 0 \\ 0 & 0 \end{bmatrix},
$$

where two matrices $A \cong B$ means that A and B have the same positive/negative semi-definite properties. Consequently, the Hessian matrix of $H(\cdot,\cdot)$ is positive-definite if and only if $\frac{\partial^2 H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})}{\partial (\mathcal{L}_{k,c}^p)^2} \geq 0$, i.e., $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ is convex in $\mathcal{L}_{k,c}^p$ for a give $\mathcal{T}_{k,c}$, which can be seen from (19), and $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ is continuous in $\mathcal{L}_{k,c}^p$ at the boundary $\mathcal{K}_{k,c}^p = q_k^p$. Consequently, $H(\mathcal{L}_{k,c}^p, \mathcal{T}_{k,c})$ is sub-additive in $\mathcal{L}_{k,c}^p$ and

$\mathcal{T}_{k,c}$ for $\mathcal{L}^p_{k,c} > 0$ for $c = a, s$, since (A.4) is satisfied for $\mathcal{L}^p_{k,c} > 0$. Then,

$$H(\mathcal{L}^p_{k,a}, \mathcal{T}_{k,a}) + H(\mathcal{L}^p_{k,s}, \mathcal{T}_{k,s}) \geq H(\mathcal{L}^p_{k,a} + \mathcal{L}^p_{k,s}, \mathcal{T}_{k,a} + \mathcal{T}_{k,s}) = H(\mathcal{L}^p_{k,a} + \mathcal{L}^p_{k,s}, 1), \tag{20}$$

where the equality holds if and only if $\frac{\mathcal{L}^p_{k,a}}{\mathcal{L}^p_{k,s}} = \frac{\mathcal{T}_{k,a}}{\mathcal{T}_{k,s}}$. For $\mathcal{L}^p_{k,c} > 0$ for $c = a, s$, we have $\mathcal{L}^p_{k,a} = W_1(s^p_1, q_k) + \mathcal{T}_{k,a} q^p_k v^p_2 - y_a$, $\mathcal{L}^p_{k,s} = q_k\, s^p_1 - W_1(s^p_1, q_k) + \mathcal{T}_{k,s} q^p_k v^p_2 - y_s$, and $\mathcal{L}^p_{k,a} + \mathcal{L}^p_{k,s} = q_k\, s^p_1 + q^p_k v^p_2 - y_a - y_s$. Then, $\frac{\mathcal{L}^p_{k,a}}{\mathcal{L}^p_{k,s}} = \frac{\mathcal{T}_{k,a}}{\mathcal{T}_{k,s}}$ implies $W_1(s^p_1, q_k) = \mathcal{T}_{k,a} s^p_1 q_k + \mathcal{T}_{k,s} y_a - \mathcal{T}_{k,a} y_s$, which is (20a).

Note that $\mathcal{L}^p_{k,a} + \mathcal{L}^p_{k,s} \geq \mathcal{L}^p$. If the centralized system produces q_k at time t_1 and q^p_k at t_2, from (14), (15), and (16), $\mathbb{E}_{t_2} \bar{\pi}_{k,a}(q^p_k, q_k, v^p_2) + \mathbb{E}_{t_2} \bar{\pi}_{k,s}(q^p_k, q_k, v^p_2) = S(q^p_k) - s^p_1\, q_k - v^p_2 q^p_k - [H(\mathcal{L}^p_{k,a}, \mathcal{T}_{k,a}) + H(\mathcal{L}^p_{k,s}, \mathcal{T}_{k,s})]$ and $\mathbb{E}_{t_2} \bar{\pi}(q^p_k, q, v^p_2) = S(q^p_k) - s^p_1 q_k - v^p_2 q^p_k - H(\mathcal{L}^p, 1)$. Let $q^{p*} \leq q_k$ be the optimal production quantity at time t_2 of the centralized system if it produces q_k at time t_1. Then,

$$\mathbb{E}_{t_2} \bar{\pi}_{k,a}(q^p_k, q_k, v^p_2) + \mathbb{E}_{t_2} \bar{\pi}_{k,s}(q^p_k, q_k, v^p_2) \leq \mathbb{E}_{t_2} \bar{\pi}(q^p_k, q_k, v^p_2) \leq \mathbb{E}_{t_2} \bar{\pi}(q^{p*}, q_k, v^p_2). \tag{21}$$

The two equalities hold if and only if $\frac{\mathcal{L}^p_{k,a}}{\mathcal{L}^p_{k,s}} = \frac{\mathcal{T}_{k,a}}{\mathcal{T}_{k,s}}$, which guarantees the equality in (20) and $q^p_k = q^{p*}$. That is, the requirements on $W_1(\cdot, \cdot)$ and initial working capital are necessary conditions.

We now finish the case $L^p_{k,c} = 0$. Next, we study the general case $L^p_{k,c} \geq 0$, i.e., supply chain parties may have time t_1 bank loans. Again, let q_k and q^p_k be fixed for $p = 1, 2, \cdots, 8$. As we explain in Appendix A, the bank will arrange $\Delta^p_{k,a}$ and $\Delta^{p+1}_{k,a}$ so that they generate smallest bankruptcy costs. For $p = 1, 3, 5, 7$, recall that $A^p = \mathbb{P}\{p \mid (s^i_1, v^j_1)\}$ and $A^{p+1} = \mathbb{P}\{p+1 \mid (s^i_1, v^j_1)\}$ from Section 4.3, where $A^p + A^{p+1} = 1$. From (A.8) and (A.10), $L^p_{k,c} = \sum_{j=p}^{p+1} A^j \Delta^j_{k,c}$. If the loan obligation on path p is reduced by $d\Delta^p_{k,c}$, the bank has to reduce the loan obligation on path $p+1$ by $d\Delta^{p+1}_{k,c} = -\frac{A^p}{A^{p+1}} d\Delta^p_{k,c}$, to make $L^p_{k,c}$ unchanged. Based on (18), the expected bankruptcy costs on the two paths are

$$\begin{aligned}
\mathcal{T}_{k,c} \frac{\partial[\sum_{j=p}^{p+1} A^j C_b(\mathcal{K}^j_{k,c}, q^j_k)]}{\partial \Delta^p_{k,c}} &= \mathcal{T}_{k,c} \left[A^p \frac{\partial C_b(\mathcal{K}^p_{k,c}, q^p_k)}{\partial \Delta^p_{k,c}} + A^{p+1} \frac{\partial C_b(\mathcal{K}^{p+1}_{k,c}, q^{p+1}_k)}{\partial \Delta^{p+1}_{k,c}} \frac{d\Delta^{p+1}_{k,c}}{d\Delta^p_{k,c}} \right] \\
&= A^p \left[\frac{1}{G(\mathcal{K}^p_{k,c}, q^p_k)} - \frac{1}{G(\mathcal{K}^{p+1}_{k,c}, q^{p+1}_k)} \right].
\end{aligned}$$

Similar as (17), $\frac{\partial \mathcal{K}^p_{k,c}}{\partial \Delta^p_{k,c}} \geq 0$, while $\frac{\partial \mathcal{K}^{p+1}_{k,c}}{\partial \Delta^p_{k,c}} \leq 0$. Recall that q^p_k and q^{p+1}_k are fixed. Then, the expected bankruptcy cost is convex in $\Delta^p_{k,c}$, with the optimality condition $G(\mathcal{K}^p_{k,c}, q^p_k) = G(\mathcal{K}^{p+1}_{k,c}, q^{p+1}_k)$. Denote corresponding loan obligations as $\Delta^{p*}_{k,c}$ and $\Delta^{p+1*}_{k,c}$, and the bankruptcy thresholds as $\mathcal{K}^{p*}_{k,c}$ and $\mathcal{K}^{p+1*}_{k,c}$.

By making $W_1(\cdot, \cdot)$ and/or y_a and y_s deviate from (20a), let the time t_1 loan $L^p_{k,c}$ increase by $dL^p_{k,c}$ for party c, while the other party, denoted by $\bar{c}$, increase by $dL^p_{k,\bar{c}} = -dL^p_{k,c}$. Then,

$$\begin{aligned}
\sum_{i=c,\bar{c}} \mathcal{T}_{k,i} \frac{\partial[\sum_{j=p}^{p+1} A^j C_b(\mathcal{K}^{j*}_{k,i}, q^j_k)]}{\partial L^p_{k,c}} &= \sum_{i=c,\bar{c}} \sum_{j=p}^{p+1} \left[\mathcal{T}_{k,i} \frac{\partial C_b(\mathcal{K}^{j*}_{k,i}, q^j_k)]}{\partial \Delta^{j*}_{k,i}} \right] \left[A^j \frac{\partial \Delta^{j*}_{k,i}}{\partial L^p_{k,c}} \right] = \sum_{i=c,\bar{c}} \sum_{j=p}^{p+1} \left[\frac{1}{G(\mathcal{K}^{j*}_{k,i}, q^j_k)} \right] \left[A^j \frac{\partial \Delta^{j*}_{k,i}}{\partial L^p_{k,c}} \right] \\
&= \sum_{i=c,\bar{c}} \frac{1}{G(\mathcal{K}^{p*}_{k,i}, q^p_k)} \sum_{j=p}^{p+1} A^j \frac{\partial \Delta^{j*}_{k,i}}{\partial L^p_{k,c}} = \sum_{i=c,\bar{c}} \frac{\partial L^p_{k,i} / \partial L^p_{k,c}}{G(\mathcal{K}^{p*}_{k,i}, q^p_k)} = \frac{1}{G(\mathcal{K}^{p*}_{k,c}, q^p_k)} - \frac{1}{G(\mathcal{K}^{p*}_{k,\bar{c}}, q^p_k)}.
\end{aligned}$$

Again, it is straightforward to show $\frac{1}{G(\mathcal{K}^{p*}_{k,c}, q^p_k)} - \frac{1}{G(\mathcal{K}^{p*}_{k,\bar{c}}, q^p_k)}$ increases in $L^p_{k,c}$. Then, the total expected bankruptcy cost is convex in $L^p_{k,c}$, and the minimal costs are achieved at $G(\mathcal{K}^{p*}_{k,a}, q^p_k) = G(\mathcal{K}^{p*}_{k,s}, q^p_k)$, which further implies that $\mathcal{K}^{p*}_{k,a} = \mathcal{K}^{p*}_{k,s}$, and $\mathcal{K}^{p+1*}_{k,a} = \mathcal{K}^{p+1*}_{k,s}$. That is, the two parties have the same bankruptcy thresholds on the paths p and $p+1$, which is the case of condition (20a).

Finally, let q_k, q_k^p and q_k^{p+1} be fixed at the optimal quantities of the decentralized system. From the above analysis, the total expected profits increase when (20a) is satisfied, which is the centralized system ordering q_k, q_k^p and q_k^{p+1}. By letting the centralized system optimize, the expected profits improve further. That is, (21) still holds, and (20a) is necessary to achieve coordination.

The requirements on $\mathcal{H}_{k,a}$ and $\mathcal{H}_{k,s}$. The centralized system borrows $(s_1^p q_k - y_a - y_s)^+$ time t_1 bank loan. For the decentralized system, if the assembler does not produce at time t_2, the supplier receives $\mathcal{H}_{k,a}$ from her. Then, the supplier's loan is $(s_1^p q_k - \mathcal{H}_{k,a} - y_s)^+$. To make it proportional to the centralized system, let $(s_1^p q_k - \mathcal{H}_{k,a} - y_s)^+ = \mathcal{T}_{k,s}(s_1^p q_k - y_a - y_s)^+$. Then, $\mathcal{H}_{k,a} = \mathcal{T}_{k,a}\, s_1^p q_k + \mathcal{T}_{k,s} y_a - \mathcal{T}_{k,a} y_s$, which is (20d). For $\mathcal{H}_{k,s}$, after (20a), (20b), (20c), and (20d), the supplier should not be penalized for not producing, since it happens only if he gets negative expected profits or cannot borrow bank loans, which is also the case of the centralized system. Then, $\mathcal{H}_{k,s} = 0$, which is (20e). □

Proof of Proposition 3. For Part 1, $\gamma = 0$ means that the supplier is indifferent to producing or not. From Assumption 3, the tie-breaking rule, the supplier is forward-looking like the centralized system. Also, if $y_s = 0$, then y_a equals the total channel working capital. Therefore, coordination is achieved.

Similarly, if $y_s > 0$ but $y_a - y_s > \bar{y}$, then the assembler still have the enough working capital to behavior the same as the centralized system, i.e., removing y_s from it does not reduce its profitability.

Part 2 follows directly from Proposition 2, since (20) are reduced to (18a) if $y_s = y_a = 0$. □

Proof of Proposition 4. For concise, let $\mathbb{E}_{t_2}\bar{\Pi}_k(q_k^p, q_k, v_2^p) = \mathbb{E}_{t_2}\bar{\pi}_{k,a}(q_k^p, q_k, v_2^p) + \mathbb{E}_{t_2}\bar{\pi}_{k,s}(q_k^p, q_k, v_2^p)$ be the total profits of the decentralized system on path p. Let q^{p*} and q^* be optimal quantities of the centralized system. We first show Part 1. Let q_{pt}^{p*} and q_{pt}^* ($q_{pt,0}^{p*}$ and $q_{pt,0}^*$) be the optimal quantities to the assembler in decentralized system (with $\gamma = 0$), and let $\mathbb{E}_{t_2}\bar{\Pi}_{pt,0}$ be the expected profit of the decentralized system with $\gamma = 0$. Note that $\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p) \le \sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt,0}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)$, since the former is the expected profit of the decentralized system with a general γ but with ordering quantities that are optimal to the system with $\gamma = 0$. Then, $e_{pt}(\gamma, y_a, y_s) = \frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt}^{p*}, q_{pt}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\pi}(q^{p*}, q^*, v_2^p)\mathbb{P}\{p\}} =$

$$\frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt}^{p*}, q_{pt}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}} \cdot \frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt,0}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}} \cdot \frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt,0}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\pi}(q^{p*}, q^*, v_2^p)\mathbb{P}\{p\}}$$

$$= \frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt}^{p*}, q_{pt}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}} \cdot \frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt,0}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}} \cdot \bar{e}_{pt}(y_a, y_s).$$

Let $\underline{e}'(y_a, y_s) = \frac{\underline{e}(y_a, y_s)}{\bar{e}_{pt}(y_a, y_s)} \le 1$. Note that $\frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt}^{p*}, q_{pt}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}} \ge \sqrt{\underline{e}'(y_a, y_s)}$ can be satisfied within a limited scope of γ, denoted by $[0, \bar{\gamma}_1]$. Furthermore, as γ increases, either the time t_1 loan $L_{pt,a}^p$ or time t_2 loan $\mathcal{L}_{pt,a}^p$ increases. The (19) in the Appendix indicates that the assembler's expected bankruptcy cost is increasing and convex in the loan size, and thus, the expected profit is concave in the loan size. Then, there exists a scope of γ, denoted by $[0, \bar{\gamma}_2]$, such that $\frac{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^8 \mathbb{E}_{t_2}\bar{\Pi}_{pt,0}(q_{pt,0}^{p*}, q_{pt,0}^*, v_2^p)\mathbb{P}\{p\}} \ge \sqrt{\underline{e}'(y_a, y_s)}$. Let $\bar{\gamma} = \min\{\bar{\gamma}_1, \bar{\gamma}_2\}$. Then, for $\gamma \in [0, \bar{\gamma}]$, $e_{pt}(\gamma, y_a, y_s) \ge \underline{e}'(y_a, y_s)\bar{e}_{pt}(y_a, y_s) = \underline{e}(y_a, y_s)$.

For Part 2, let q_{rs}^{p*} and q_{rs}^* be the optimal solutions to the assembler in decentralized system.

Then,

$$e_{rs}(\lambda, y_a, y_s) = \frac{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q_{rs}^{p*}, q_{rs}^{*}, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\pi}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}} = \frac{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q_{rs}^{p*}, q_{rs}^{*}, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}} \cdot \frac{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\pi}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}}.$$

Since $e_{rs}(\frac{y_s}{y_a+y_s}, y_a, y_s) = 1$, $\frac{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q_{rs}^{p*}, q_{rs}^{*}, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}} \geq \sqrt{\underline{e}(y_a, y_s)}$ can be satisfied within a limited scope of λ around $\frac{y_s}{y_a+y_s}$, denoted by $[\underline{\lambda}_1, \bar{\lambda}_1]$. From the proof of Proposition 2, when the quantities are fixed at q^{p*} and q^*, the total expected profit of the decentralized system is concave in λ, with $\lambda^* = \frac{y_s}{y_a+y_s}$. Then, there exists a scope of λ around $\frac{y_s}{y_a+y_s}$, denoted by $[\underline{\lambda}_2, \bar{\lambda}_2]$, so that $\frac{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\Pi}_{rs}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}}{\sum_{p=1}^{8} \mathbb{E}_{t_2} \bar{\pi}(q^{p*}, q^{*}, v_2^p)\mathbb{P}\{p\}} \geq \sqrt{\underline{e}(y_a, y_s)}$. Let $\underline{\lambda} = \max\{\underline{\lambda}_1, \underline{\lambda}_2\}$ and $\bar{\lambda} = \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$. For $\lambda \in [\underline{\lambda}, \bar{\lambda}]$, $e_{rs}(\lambda, y_a, y_s) \geq \underline{e}(y_a, y_s)$. □

Proof of Proposition 5. The total expected profit of the decentralized system is $\mathbb{E}_{t_2} \bar{\pi}_{pt,s}(q_{pt}^p, q_{pt}, s_1^p, v_2^p) + \mathbb{E}_{t_2} \bar{\pi}_{pt,a}(q_{pt}^p, q_{pt}, s_1^p, v_2^p) = S(q_{pt}^p) - s_1^p q_{pt} - q_{pt}^p v_2^p - C_b'(\mathcal{K}_{pt,a}^p, q_{pt}^p)$ under the pass-through contract specified in (21), and $\mathbb{E}_{t_2} \bar{\pi}_{rs,s}(q_{rs}^p, q_{rs}, s_1^p, v_2^p) + \mathbb{E}_{t_2} \bar{\pi}_{rs,a}(q_{rs}^p, q_{rs}, s_1^p, v_2^p) = S(q_{rs}^p) - s_1^p q_{rs} - q_{rs}^p v_2^p - [\lambda_a C_b'(\mathcal{K}_{rs,a}^p, q_{rs}^p) + \lambda_s C_b'(\mathcal{K}_{rs,s}^p, q_{rs}^p)]$ under the revenue-sharing contract specified in (18a). Then, if

$$C_b'(\mathcal{K}_{pt,a}^p, q_{pt}^p) \leq \lambda_a C_b'(\mathcal{K}_{rs,a}^p, q_{rs}^p) + \lambda_s C_b'(\mathcal{K}_{rs,s}^p, q_{rs}^p), \tag{22}$$

it is possible that the total supply chain profit is larger under the pass-through contract.

To see such situation can exist for sure, we consider the situation the supplier has the reservation value $\pi_{0,s}$, i.e., $\mathbb{E}_{t_2} \bar{\pi}_{k,s}(q_k^p, q_k, s_1^p, v_2^p) \geq \pi_{0,s} > 0$, for $k = pt, rs$. Under the revenue-sharing contract, let the supplier's profit be $\pi_{0,s}$, i.e., $\mathbb{E}_{t_2} \bar{\pi}_{rs,s}(q_{rs}^p, q_{rs}, s_1^p, v_2^p) = \lambda_s [S(q_{rs}^p) - s_1^p q_{rs} - q_{rs}^p v_2^p - C_b'(\mathcal{K}_{rs,s}^p, q_{rs}^p)] = \pi_{0,s}$, where $\lambda_s = 1 - \lambda$ is the supplier's share of revenue. Then, λ_s is no smaller than a lower bound $\bar{\lambda}_s > 0$. Now, if the supplier's working capital amount is small, i.e., $y_s \leq \bar{y}_s$, then $\mathcal{K}_{rs,s}^p > 0$ and thus $\lambda_s C_b'(\mathcal{K}_{rs,s}^p, q_{rs}^p) > 0$. Under the pass-through contract, for the supplier to receive exactly $\pi_{0,s}$ profits, his profit margin has to be $\frac{\pi_{0,s}}{q_{pt}}$ (essentially a quantity discount), or equivalently, $\gamma q_{pt} = \pi_{0,s}$.

Consider the case that the assembler's working capital is large, i.e., $y_a \geq \bar{y}_a \geq s_1^h q_{pt} + \gamma q_{pt} + v_2^h q_{pt}^p = s_1^h q_{pt} + v_2^h q_{pt}^p + \pi_{0,s}$. (This is only a sufficient condition. As long as the assembler borrow a ban loan that is not that big, the conclusion still holds.) Then, $C_b'(\mathcal{K}_{pt,a}^p, q_{pt}^p) = 0$, since $\mathcal{K}_{pt,a}^p = 0$ (the assembler does not need to borrow). Therefore, the strict inequality in (22) holds.

Now, let $q_{pt} = q_{rs}$ and $q_{pt}^p = q_{rs}^p$. We can do so since the assembler does not borrow under the pass-through contract, and thus is not restricted by the bank's loan decision and can choose all quantities that the assembler chooses under revenue-sharing contract. Then, the inequality of (22) holds, and the pass-through contract Pareto dominates the revenue-sharing contract. Finally, the assembler can further optimize over q_{pt} and q_{pt}^p (instead of choosing the values under the revenue-sharing contract) under the constraint $\pi \geq \pi_{0,s}$, and obtain even better profits under the pass-through contract. □