Appendix

Lemma EC.1. *For $r_s \geq \delta_r(C_r)$, we have the following derivatives:*

$$\begin{aligned}\partial \bar{k}_b(r_s;C_r)/\partial r_s &= 1/[(1+r_s)z(\bar{k}_b(r_s;C_r))], & \text{(EC.1)}\\ \partial \bar{y}_b(r_s;C_r)/\partial r_s &= 1/[(1+r_s)^2 z(\bar{k}_b(r_s;C_r))], & \text{(EC.2)}\\ \partial \bar{r}_b(r_s;C_r)/\partial r_s &= [r_s - \bar{r}_b(r_s;C_r)]/[(1+r_s)^2 \bar{y}_b(r_s;C_r) z(\bar{k}_b(r_s;C_r))]. & \text{(EC.3)}\end{aligned}$$

Proof of Lemma EC.1: From Equation (7), $\frac{\partial \bar{k}_b}{\partial r_s} = \frac{1+\delta_r(C_r)}{(1+r_s)^2 \bar{F}(\bar{k}_b) z(\bar{k}_b)} = \frac{1}{(1+r_s)z(\bar{k}_b)}$. From Equation (4), $\frac{\partial \bar{y}_b}{\partial \bar{k}_b} = \frac{\bar{F}(\bar{k}_b)}{1+\delta_r(C_r)} = \frac{1}{1+r_s}$. Then, $\frac{\partial \bar{y}_b}{\partial r_s} = \frac{\partial \bar{y}_b}{\partial \bar{k}_b} \cdot \frac{\partial \bar{k}_b}{\partial r_s} = \frac{1}{(1+r_s)^2 z(\bar{k}_b)}$. From $\bar{k}_b = \bar{y}_b(1+\bar{r}_b)$, we have $\frac{\partial \bar{k}_b}{\partial \bar{y}_b} = 1 + \bar{r}_b + \bar{y}_b \frac{\partial \bar{r}_b}{\partial \bar{y}_b}$. Since $\frac{\partial \bar{y}_b}{\partial \bar{k}_b} = \frac{1}{1+r_s}$, we have $\frac{\partial \bar{r}_b}{\partial \bar{y}_b} = \frac{r_s - \bar{r}_b}{\bar{y}_b}$. Then, $\frac{\partial \bar{r}_b}{\partial r_s} = \frac{\partial \bar{r}_b}{\partial \bar{y}_b} \cdot \frac{\partial \bar{y}_b}{\partial r_s} = \frac{r_s - \bar{r}_b}{(1+r_s)^2 \bar{y}_b z(\bar{k}_b)}$. □

Corollary EC.1. *For $0 \leq r_s < \tilde{r}_s(y_r, C_r)$, $\partial[-\bar{y}_b(r_s;C_r)(1+r_s) + \bar{k}_b(r_s;C_r)]/\partial r_s = -\bar{y}_b(r_s;C_r)$.*

Proof of Corollary EC.1: The conclusion holds if $0 \leq r_s < \delta_r(C_r)$, since $\bar{y}_b = \bar{k}_b = 0$. For $\delta_r(C_r) \leq r_s < \tilde{r}_s$, $\frac{\partial[-\bar{y}_b(1+r_s)+\bar{k}_b]}{\partial r_s} = -(1+r_s)\frac{\partial \bar{y}_b}{\partial r_s} - \bar{y}_b + \frac{\partial \bar{k}_b}{\partial r_s} = -\bar{y}_b$, from Equations (EC.1) and (EC.2). □

Proof of Lemma 1: From Equation (4), $\frac{\partial k_b}{\partial y_b} = \frac{1+\delta_r(C_r)}{\bar{F}(k_b)} \geq 0$ and k_b increases and $\bar{F}(k_b)$ decreases in y_b. From Equations (5) and (6a), $\frac{\partial \pi}{\partial y_b} = -\bar{F}(k_s)\frac{\partial k_s}{\partial y_b} = \bar{F}(k_s)\left[(1+r_s) - \frac{1+\delta_r(C_r)}{\bar{F}(k_b)}\right]$ so that π is quasi-concave in y_b. Then, for $r_s \geq \delta_r(C_r) = \frac{\epsilon_r(C_r)+\lambda_r(C_r)}{1-\lambda_r(C_r)}$, the optimal y_b solves $\frac{\partial \pi}{\partial y_b} = 0$, i.e., $\bar{F}(k_b) = \frac{1+\delta_r(C_r)}{1+r_s}$, and for $0 \leq r_s \leq \delta_r(C_r)$, the optimal $y_b = 0$. Note that $\bar{y}_b(1+\delta_r(C_r)) = \mathbb{E}[\min(\xi, \bar{k}_b)] \geq \bar{k}_b \bar{F}(\bar{k}_b) = \bar{y}_b(1+\bar{r}_b)\frac{1+\delta_r(C_r)}{1+r_s}$. Then, $\bar{r}_b \leq r_s$. Consequently, from Lemma EC.1, $\frac{\partial \bar{r}_b}{\partial r_s} \geq 0$. □

Proof of Lemma 2: Note that $q\bar{F}(q)$ is quasi-concave in q and the largest value is achieved at $\tilde{q}$ that solves $qz(q) = 1$. From Lemma EC.1, $(y_r + \bar{y}_b)(1 + \min\{\delta_r(C_r), r_s\})$ increases in r_s and y_r. This establishes the existence of $\tilde{y}_r(C_r)$ and $\tilde{r}_s(y_r, C_r)$. Then, for a given $r_s \in [0, \tilde{r}_s)$, there are two solutions of q to each of Equations (9a)-(9c). Let $q_i^u \geq q_i^l$, for $i = 1,2,3$, be the solutions to (9a)-(9c), respectively. Then, $q_3^u \geq q_2^u \geq q_1^u \geq \tilde{q} \geq q_1^l \geq q_2^l \geq q_3^l$ since for the right hand side, $(9a) \geq (9b) \geq (9c)$.

For Equations (6a), (6b), and (6d), the optimal q satisfies $\bar{F}(q) - w(1+r_s)\bar{F}(k_s) = 0$, $\bar{F}(q) = w(1+\delta_r(C_r))$ and $\bar{F}(q) = w$, respectively. Then, the boundary of (6c) and (6d) is $wq = q\bar{F}(q) = y_r$, which is (9c). If $r_s > \delta_r(C_r)$, then since $y_b = \bar{y}_b$, $wq = y_r + \bar{y}_b$ and $k_s = \bar{k}_b$ at the boundary of (6a) and (6b), we have $wq = \frac{q\bar{F}(q)}{1+\delta_r(C_r)} = y_r + \bar{y}_b$, which is (9a). Similarly, the boundary of (6b) and (6c) is $wq = \frac{q\bar{F}(q)}{1+\delta_r(C_r)} = y_r$, which is (9b). If $r_s \leq \delta_r(C_r)$, (6b) does not exist and $\bar{y}_b = 0$. Since $wq = y_r$ and $k_s = 0$ at the boundary of (6a) and (6c), we have $wq = \frac{q\bar{F}(q)}{1+r_s} = y_r$, which is (9b). □

Lemma EC.2. *For a fixed r_s so that $\delta_r(C_r) < r_s \leq \tilde{r}_s(y_r, C_r)$, we have $q_1^l(r_s;C_r) > \bar{k}_b(r_s;C_r)$.*

Proof of Lemma EC.2: Since $r_s > \delta_r(C_r)$, $\bar{y}_b > 0$ and $q_1^l > 0$. Then, $w_1^l q_1^l(1+\delta_r(C_r)) = q_1^l \bar{F}(q_1^l) < \mathbb{E}[\min(\xi, q_1^l)]$. As $\bar{y}_b(1+\delta_r(C_r)) = \mathbb{E}[\min(\xi, \bar{k}_b)] = (w_1^l q_1^l - y_r)(1+\delta_r(C_r)) < \mathbb{E}[\min(\xi, q_1^l)]$, $q_1^l > \bar{k}_b$. □

Lemma EC.3. *Let $y_r > 0$. For $w \in [w_1^u(r_s;C_r), w_1^l(r_s;C_r)]$, or equivalently, $q^*(w, r_s;C_r) \in [q_1^l(r_s;C_r), q_1^u(r_s;C_r)]$, we have $w(1+r_s) < 1$, $k_s^*(w, r_s;C_r) < q^*(w, r_s;C_r)$, and $1 - M^*(w, r_s;C_r) > 0$.*

Proof of Lemma EC.3: Since $y_r > 0$, $q_1^l > 0$. Note that $w(1+r_s) \leq w_1^l(1+r_s) = \frac{\bar{F}(q_1^l)(1+r_s)}{1+\min\{\delta_r(C_r), r_s\}} < 1$ if $r_s \leq \delta_r(C_r)$. If $r_s > \delta_r(C_r)$, from Equation (7) and Lemma EC.2, $w(1+r_s) \leq \frac{\bar{F}(q_1^l)(1+r_s)}{1+\delta_r(C_r)} = \frac{\bar{F}(q_1^l)}{\bar{F}(\bar{k}_b)} < 1$.

Then, from Equation (10a), $k_s^* < q^*$. Finally, since $y_r > 0$ and $\bar{r}_b \leq r_s$ from Lemma 1, $k_s^* = (wq^* - y_r)(1+r_s) - \bar{y}_b(r_s - \bar{r}_b) < wq^*(1+r_s)$. Then, $q^*\bar{F}(q^*) = wq^*(1+r_s)\bar{F}(k_s^*) > wq^*(1+r_s)\bar{F}(wq^*(1+r_s))$. Since $w(1+r_s) < 1$, $wq^*(1+r_s) < \tilde{q}$. Thus, $1 - M^* > 1 - wq^*(1+r_s)z(wq^*(1+r_s)) > 0$. □

Proof of Lemma 3: The result directly follows from Lemma EC.3. □

Lemma EC.4. *For a fixed $r_s \geq 0$, $q^*(w,r_s;C_r)$ and $\pi^*(w,r_s;C_r)$ strictly decrease in w, for $w \in (0,1]$.*

Proof of Lemma EC.4: From Lemma EC.3 and Equations (10a) and (6a), $\frac{\partial q^*}{\partial w} = \frac{q^*(1-M^*)}{w[M^* - q^* z(q^*)]} < 0$, and $\frac{\partial \pi^*}{\partial w} = \frac{\partial \pi}{\partial q}|_{q=q^*} \frac{\partial q^*}{\partial w} + \frac{\partial \pi}{\partial w} = \frac{\partial \pi}{\partial w} = -(1-\rho_r(C_r))\bar{F}(k_s^*)q^*(1+r_s) < 0$. Proofs for other cases are similar. □

Lemma EC.5. *For any given $r_s \geq r_f$, the optimal order quantity to the supplier is in $[0, \tilde{q}]$.*

Proof of Lemma EC.5: Let $\hat{q}_1$ solve the first order condition of Equation (11a), i.e., $\frac{\bar{F}(\hat{q}_1)[1-\hat{q}_1 z(\hat{q}_1)]}{1-M^*} - \frac{c(1+\delta_s(C_s))}{1-\rho_r(C_r)} = 0$. From Lemma EC.3, $1 - M^* \geq 0$. Then, $\hat{q}_1 \leq \tilde{q}$. We can show (11b)-(11e) similarly. □

Proof of Lemma 4: Let $\Delta(r_s) = 1 + \min\{\delta_r(C_r), r_s\}$. Recall $q_1^l \leq \tilde{q}$ solving $q_1^l \bar{F}(q_1^l) = (y_r + \bar{y}_b)\Delta(r_s)$. Let $H(r_s) = q_y\bar{F}(q_y) - q_1^l\bar{F}(q_1^l) = q_y[\bar{F}(q_y) - c\Delta(r_s)]$. Then, for $q_y \leq \tilde{q}$, we have $q_y \geq q_1^l$ if $H(r_s) \geq 0$, and $q_y < q_1^l$ otherwise. Since $\bar{F}(q_y)$ decreases in r_s (strictly if $r_s \geq \delta_r(C_r)$) and $c\Delta(r_s)$ increases in r_s (strictly if $r_s \leq \delta_r(C_r)$), $H(r_s)$ strictly decreases in r_s and there is at most one $\bar{r}_s$ satisfying $H(\bar{r}_s) = 0$. For Part 1, as $c \leq \tilde{w}(C_r) = \frac{\bar{F}(\tilde{q})}{\Delta(\tilde{r}_s)}$, $q_1^l(\tilde{r}_s; C_r) = \tilde{q} = \frac{y_r + \bar{y}_b(\tilde{r}_s;C_r)}{\bar{F}(\tilde{q})/\Delta(\tilde{r}_s)} \leq \frac{y_r + \bar{y}_b(\tilde{r}_s;C_r)}{c} = q_y(\tilde{r}_s; C_r)$. Since q_y increases in r_s, there is an $r_s^0 \in [0, \tilde{r}_s]$ so that $q_y \geq \tilde{q} \geq q_1^l$ for $r_s \in [r_s^0, \tilde{r}_s]$ and $q_y(r_s^0; C_r) = \tilde{q}$. Then, $\bar{F}(q_y(r_s^0; C_r)) = \bar{F}(\tilde{q}) \geq c\Delta(\tilde{r}_s) \geq c\Delta(r_s^0)$. Thus, $H(r_s^0) \geq 0$ and $H(r_s) \geq 0$, i.e., $q_y \geq q_1^l$, for $r_s \in [0, r_s^0]$. For Part 2, $q_1^l(\tilde{r}_s; C_r) = \tilde{q} \geq q_y(\tilde{r}_s; C_r)$ so that $H(\tilde{r}_s) \leq 0$ and $q_y \leq \tilde{q}$ for $r_s \in [0, \tilde{r}_s]$. Suppose $H(0) < 0$ so that $\bar{F}(\frac{y_r}{c}) < c$. Then, $\frac{y_r}{c}\bar{F}(\frac{y_r}{c}) < y_r = q_3^l\bar{F}(q_3^l)$ so that $\frac{y_r}{c} < q_3^l$, i.e., $\bar{F}(\frac{y_r}{c}) > \bar{F}(q_3^l) \geq c$. Contradiction. Then, $H(0) \geq 0$ and $\bar{r}_s \in [0, \tilde{r}_s)$. Since q_y increases in y_r, $\bar{r}_s$ decreases in y_r. □

Lemma EC.6. *If $z(\cdot)$ is increasing and convex, $\frac{\bar{F}(q)[1-qz(q)]}{1-M^*(q,r_s;C_r)}$ decreases in q, for $q \leq \tilde{q}$.*

Proof of Lemma EC.6: From Equation (10a), $\frac{\partial w^*}{\partial q} = \frac{w^*[M^* - qz(q)]}{q(1-M^*)}$. As $w^*(1+r_s) < 1$ and $k_s^* < q$ from Lemma EC.3, $0 \leq \frac{1-qz(q)}{1-M^*} \leq 1$ and $0 \leq \frac{\partial k^*}{\partial q} = \frac{\partial(w^* q(1+r_s))}{\partial q} = \frac{w^*(1+r_s)[1-qz(q)]}{1-M^*} < 1$. As $z(k_s^*) \leq z(q)$ and $z'(k_s^*) \leq z'(q)$, $\partial \frac{1-qz(q)}{1-M^*}/\partial q = \frac{1}{1-M^*}\left\{\frac{1-qz(q)}{1-M^*}\left[\frac{\partial(w^* q(1+r_s))}{\partial q} z(k_s^*) + w^*(1+r_s)\frac{\partial k_s^*}{\partial q} qz'(k_s^*)\right] - z(q) - qz'(q)\right\} \leq 0$. □

For a given r_s, $\hat{q}(r_s; C_r, \alpha)$ and $w^*(\hat{q}(r_s; C_r, \alpha), r_s; C_r)$, where $\alpha = \alpha_1$ or α_2, are solutions of

$$\begin{cases} V^1(q,w) = qf(q) - \bar{F}(q) - c(1+\alpha)\left[w(1+r_s)qz(k_s) - 1\right] = 0, & \text{(EC.4a)} \\ V^2(q,w) = \bar{F}(q) - w(1+r_s)\bar{F}(k_s) = 0, & \text{(EC.4b)} \end{cases}$$

where k_s is defined in Equation (5), Equation (EC.4a) is the supplier's first order condition (12a) ($\alpha = \alpha_1$) or (12b) ($\alpha = \alpha_2$), and Equation (EC.4b) is the retailer's first order condition (10a). Then,

$$\begin{aligned} V_q^1 &= \bar{F}(q)\{q[z'(q) - (c(1+\alpha)/\bar{F}(q))(w(1+r_s))^2 z'(k_s)] + z(q)(1-qz(q)) + \\ &\qquad [z(q) - (c(1+\alpha)/\bar{F}(q))w(1+r_s)z(k_s)]\} \geq 0, && \text{(EC.5)} \\ V_w^1 &= -c(1+\alpha)(1+r_s)q\left[z(k_s) + w(1+r_s)qz'(k_s)\right] \leq 0, && \text{(EC.6)} \\ V_q^2 &= -\bar{F}(q)\left[z(q) - w(1+r_s)z(k_s)\right] \leq 0, && \text{(EC.7)} \\ V_w^2 &= -(1+r_s)\bar{F}(k_s)\left[1 - w(1+r_s)qz(k_s)\right] \leq 0, && \text{(EC.8)} \end{aligned}$$

where inequalities hold from Lemma EC.5 ($qz(q) \leq 1$), Lemma EC.3 ($k_s < q$, $w(1+r_s) < 1$, $1 - M \geq 0$), IFR with convex feature ($z(q) \geq z(k_s)$, $z'(q) \geq z'(k_s)$), and Equation (EC.4a) ($\frac{c(1+\alpha)}{\bar{F}(q)} = \frac{1-qz(q)}{1-M} \leq 1$).

LEMMA EC.7. *For a given $r_s \geq 0$, we have $\hat{q}(r_s;C_r,\alpha)$, $w^*(\hat{q}(r_s;C_r,\alpha),r_s;C_r)\hat{q}(r_s;C_r,\alpha)(1+r_s)$ and $k_s^*(\hat{q}(r_s;C_r,\alpha),r_s;C_r)$ all decrease in y_r, where α is either α_1 or α_2.*

Proof of Lemma EC.7: From Equations (EC.4a) and (EC.4b), $V_{y_r}^1 = c(1+\alpha)(1+r_s)M\frac{z'(k_s)}{z(k_s)} \geq 0$, $V_{y_r}^2 = -(1+r_s)\bar{F}(q)z(k_s) \leq 0$. Let $B = V_q^1 V_w^2 - V_w^1 V_q^2$, $B_q = V_w^1 V_{y_r}^2 - V_{y_r}^1 V_w^2$, and $B_w = V_{y_r}^1 V_q^2 - V_q^1 V_{y_r}^2$. Then, $B \leq 0$ and $B_q \geq 0$. From implicit function theorem, $\frac{\partial q}{\partial y_r} = \frac{B_q}{B} \leq 0$, $\frac{\partial w}{\partial y_r} = \frac{B_w}{B}$, $\frac{\partial (wq)}{\partial y_r} = q\frac{\partial w}{\partial y_r} + w\frac{\partial q}{\partial y_r} = \frac{(1+r_s)M}{B}\left\{c(1+\alpha)\bar{F}(q)\left[\frac{z'(k_s)}{z(k_s)}(1-qz(q)+M)+z(k_s)\right]+\bar{F}(k_s)V_q^1\right\} \leq 0$, and $\frac{\partial k_s}{\partial y_r} = (1+r_s)[\frac{\partial (wq)}{\partial y_r} - 1] \leq 0$. □

LEMMA EC.8. *We have $\hat{q}(r_s;C_r,\alpha)$, $w^*(\hat{q}(r_s;C_r,\alpha),r_s;C_r)$, $w^*(\hat{q}(r_s;C_r,\alpha),r_s;C_r)\hat{q}(r_s;C_r,\alpha)(1+r_s)$, $k_s^*(\hat{q}(r_s;C_r,\alpha),r_s;C_r)$, and $M^*(\hat{q}(r_s;C_r,\alpha),r_s;C_r)$ all decrease in r_s, where α is α_1 or α_2.*

Proof of Lemma EC.8: From Equations (EC.4a) and (EC.4b), $V_{r_s}^1 = -c(1+\alpha)wq[z(k_s)+(wq-y_r-\bar{y}_b)(1+r_s)z'(k_s)] \leq 0$. Also, $V_{r_s}^2 = -w\bar{F}(k_s)[1-(wq-y_r-\bar{y}_b)(1+r_s)z(k_s)] \leq 0$, as $1-M \geq 0$ from Lemma EC.3. Let $B = V_q^1 V_w^2 - V_w^1 V_q^2$, $B_q = V_w^1 V_{r_s}^2 - V_{r_s}^1 V_w^2$ and $B_w = V_{r_s}^1 V_q^2 - V_q^1 V_{r_s}^2$. Then, $B \leq 0$, $B_w \geq 0$ and $\frac{\partial w}{\partial r_s} = \frac{B_w}{B} \leq 0$. Substituting V_j^1 and V_j^2 for $j = q,w,r_s$ yields $B_q = (y_r+\bar{y}_b)(1+r_s)q\bar{F}(q)c(1+\alpha)(z'(k_s)+z^2(k_s)) \geq 0$ and $B = -(1+r_s)(1-M)[P-c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s))]$, where $P = \bar{F}(q)\bar{F}(k_s)[2z(q)+qz'(q)-qz^2(q)] \geq c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s))$. Then,

$$\frac{\partial q}{\partial r_s} = \frac{B_q}{B} = -\frac{(y_r+\bar{y}_b)q\bar{F}(q)c(1+\alpha)[z'(k_s)+z^2(k_s)]}{(1-M)[P-c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s))]} \leq 0. \tag{EC.9}$$

From $k_s = (wq-y_r-\bar{y}_b)(1+r_s)+\bar{y}_b(1+\bar{r}_b)$ and $\frac{\partial k_s}{\partial r_s} = q(1+r_s)\frac{\partial w}{\partial r_s} + w(1+r_s)\frac{\partial q}{\partial r_s} + (wq-y_r-\bar{y}_b)$,

$$\frac{\partial k_s}{\partial r_s} = -\frac{(y_r+\bar{y}_b)}{1-M}\cdot\frac{P-c^2(1+\alpha)^2 z(k_s)(1-M)}{P-c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s))} \leq 0, \tag{EC.10}$$

since $P-c^2(1+\alpha)^2 z(k_s)(1-M) \geq P-c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s)) \geq 0$. Also, since $\frac{1}{1-M}\cdot\frac{P-c^2(1+\alpha)^2 z(k_s)(1-M)}{P-c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s))} \geq 1$, we have $\frac{\partial k_s}{\partial r_s} \leq -(y_r+\bar{y}_b)$ and $\frac{\partial [wq(1+r_s)]}{\partial r_s} = \frac{\partial k_s}{\partial r_s} + (y_r+\bar{y}_b) \leq 0$. Finally, $\frac{\partial M}{\partial r_s} = \frac{\partial [wq(1+r_s)]}{\partial r_s}z(k_s) + wq(1+r_s)z'(k_s)\frac{\partial k_s}{\partial r_s} = \frac{(y_r+\bar{y}_b)}{1-M}\frac{P[-wq(1+r_s)z^2(k_s)-wq(1+r_s)z'(k_s)]}{P-c^2(1+\alpha)^2(z(k_s)+wq(1+r_s)z'(k_s))} \leq 0$. □

Proof of Proposition 1: $\hat{q}(r_s;C_r,\alpha_1) \leq \hat{q}(r_s;C_r,\alpha_2) \leq \tilde{q}$ from Lemma EC.5. From Lemma 4 and for $c \leq \frac{\bar{F}(\tilde{q})}{1+\min\{\delta_r(C_r),\tilde{r}_s\}}$, $q_y \geq q_1^l$ for all $r_s \in [0,\tilde{r}]$. Then, from Equations (12a) and (12b), $\hat{q}(r_s;C_r,\alpha_1)$ is realizable only if $\hat{q}(r_s;C_r,\alpha_1) \geq q_y$ and $\hat{q}(r_s;C_r,\alpha_2)$ is realizable only if $q_1^l \leq \hat{q}(r_s;C_r,\alpha_2) \leq q_y$. From Lemmas EC.8 and EC.1, $\hat{q}(r_s;C_r,\alpha_1)$ and $\hat{q}(r_s;C_r,\alpha_2)$ decrease in r_s and q_y and q_1^l increase in r_s. Then, for a given y_r, there exist thresholds $r_s^1(C_r,\alpha_1) \leq r_s^2(C_r,\alpha_2) \leq r_s^3(C_r,\alpha_2) \leq \tilde{r}_s$ so that

a) r_s^1 is defined by $\hat{q}(r_s;C_r,\alpha_1) = q_y$. For $r_s < r_s^1$, $\hat{q}(r_s;C_r,\alpha_1) > q_y$ is realizable and $\hat{q}^o = \hat{q}(r_s;C_r,\alpha_1)$.

b) r_s^2 is defined by $\hat{q}(r_s;C_r,\alpha_2) = q_y$. For $r_s^1 < r_s < r_s^2$, $\hat{q}(r_s;C_r,\alpha_1) < q_y < \hat{q}(r_s;C_r,\alpha_2)$ so that neither $\hat{q}(r_s;C_r,\alpha_1)$ nor $\hat{q}(r_s;C_r,\alpha_2)$ are realizable. Then, $\hat{q}^o = q_y$.

c) r_s^3 is defined by $\hat{q}(r_s;C_r,\alpha_2) = q_1^l$. For $r_s^2 < r_s < r_s^3$, we have $q_1^l < \hat{q}(r_s;C_r,\alpha_2) < q_y$ is realizable. Then, $\hat{q}^o = \hat{q}(r_s;C_r,\alpha_2)$. For $r_s^3 < r_s \leq \tilde{r}_s$, $\hat{q}(r_s;C_r,\alpha_2) < q_1^l$ is not realizable. Then, $\hat{q}^o = q_1^l$.

Note that r_s^1 may not exist if $\hat{q}(r_s;C_r,\alpha_1) \neq q_y(r_s;C_r)$ for any $r_s \geq 0$. This may happen if y_r is relatively large, since q_y increases in y_r and $\hat{q}(r_s;C_r,\alpha_1)$ decreases in y_r from Lemma EC.7. Then, there is a threshold $y_r^1 \geq 0$ that solves $\hat{q}(0;C_r,\alpha_1) = q_y(0;C_r)$. For $y_r < y_r^1$, $r_s^1 > 0$ and decreases in y_r, and for $y_r > y_r^1$, $\hat{q}(r_s;C_r,\alpha_1) < q_y(r_s;C_r)$ for any $r_s \geq 0$ and r_s^1 does not exist (we let $r_s^1 = 0$ for ease of exposition). Similarly, there are two thresholds y_r^2 and y_r^3 that solve $\hat{q}(0;C_r,\alpha_2) = q_y(0;C_r)$ and $\hat{q}(0;C_r,\alpha_2) = q_1^l(0;C_r)$, respectively. We have $0 \leq y_r^1 \leq y_r^2 \leq y_r^3$ since $\hat{q}(0;C_r,\alpha_1) \leq \hat{q}(0;C_r,\alpha_2)$ and $q_1^l \leq q_y$. If $y_r \in [0, y_r^1]$, $r_s^i \geq 0$ for $i = 1,2,3$, if $y_r \in (y_r^1, y_r^2]$, $r_s^i \geq 0$ for $i = 2,3$ but $r_s^1 = 0$, if $y_r \in (y_r^2, y_r^3]$, $r_s^3 \geq 0$ but $r_s^1 = r_s^2 = 0$, and if $y_r \in (y_r^3, \tilde{y}_r)$, $r_s^1 = r_s^2 = r_s^3 = 0$. Please refer to Figures 3-6. □

Proof of Proposition 2: From Equations (12a)-(12e), candidates of the global optimal ordering quantity are $\hat{q}^o(r_s;C_r,C_s)$, $\bar{q}(0)$, q_3^l, and $\bar{q}(\delta_r(C_r))$. Let $\hat{r}_s^{\#}$ and $\hat{q}^o(\hat{r}_s^{\#};C_r,C_s)$ be the optimal trade credit rate and ordering quantity, respectively, in the trade/combined financing region, where q_1^l is the region boundary (see Figure 3). For any $r_s \in [0, \tilde{r}_s]$, $q_1^l(r_s;C_r)$ is dominated by $\hat{q}^o(\hat{r}_s^{\#};C_r,C_s)$ since $\Pi(q_1^l(r_s;C_r), r_s;C_r,C_s) \leq \Pi(\hat{q}^o(r_s;C_r,C_s), r_s;C_r,C_s) \leq \Pi(\hat{q}^o(\hat{r}_s^{\#};C_r,C_s), \hat{r}_s^{\#};C_r,C_s)$.

Note that $\Pi(q, r_s;C_r,C_s)$ does not change in r_s for $q = q_3^l$, $\bar{q}(\delta_r(C_r))$, and $\bar{q}(0)$. From $q_3^l = q_1^l(0;C_r)$, q_3^l is dominated by $\hat{q}^o(\hat{r}_s^{\#};C_r,C_s)$. If $\bar{q}(\delta_r(C_r)) < q_3^l$, then $\bar{q}(\delta_r(C_r))$ is not realizable. If $q_3^l \leq \bar{q}(\delta_r(C_r)) \leq q_2^l(\delta_r(C_r);C_r)$, then $\bar{q}(\delta_r(C_r))$ is dominated by q_3^l, since for $q \in [q_3^l, q_2^l(\delta_r(C_r);C_r)]$, the retailer orders with all working capital and the supplier charges w_3^l (see Figures 1, 2 and 3). Finally, if $\bar{q}(\delta_r(C_r)) > q_2^l(\delta_r(C_r);C_r)$, there exists a r_s' so that $\bar{q}(\delta_r(C_r)) = q_1^l(r_s';C_r)$ and $\bar{q}(\delta_r(C_r))$ is dominated by $\hat{q}^o(\hat{r}_s^{\#};C_r,C_s)$. As a result, we do not need to consider $\bar{q}(\delta_r(C_r))$ explicitly.

Then, we next only consider $\bar{q}(0)$ and $\hat{q}^o(r_s;C_r,C_s)$. Since $q_1^l(r_s;C_r)$ increases in y_r and $\bar{q}(0)$ does not change in y_r, there exists a y_r^{31} so that $q_1^l(0;C_r) = \bar{q}(0)$ at $y_r = y_r^{31}$. For $y_r < y_r^{31}$, $q_3^l = q_1^l(0;C_r) < \bar{q}(0)$ so that $\bar{q}(0)$ is not realizable (see Figures 3 and 4). As a result, $\hat{q}^o(r_s;C_r,C_s)$ is the only remaining candidate, and thus is the global optimal solution. If $y_r^{31} \geq y_r^3$, since at $y_r = y_r^3$, we have $q_1^l(0;C_r) = \hat{q}(0;C_r,\alpha_2)$, then for $y_r \geq y_r^{31} > y_r^3$, $\hat{q}^o(r_s;C_r,C_s) = q_1^l(r_s;C_r)$ for $0 \leq r_s < \tilde{r}_s$. Note that $q_1^l(r_s;C_r)$ is dominated by $\bar{q}(0)$, since it is on the boundary of no financing region. Then, the global optimal is $\bar{q}(0)$. Alternatively, if $y_r^{31} < y_r^3$, then in the same logic, the global optimal is $\bar{q}(0)$ for $y_r \geq y_r^3$. For $y_r^{31} < y_r < y_r^3$, we have to compare $\Pi(\hat{q}^o(\hat{r}_s^{\#};C_r,C_s), \hat{r}_s^{\#};C_r,C_s)$ and $\Pi(\bar{q}(0), r_s;C_r,C_s)$ to determine the global optimal. Combine the two cases, the global optimal is $\hat{q}^o(\hat{r}_s^{\#};C_r,C_s)$ for $y_r \leq y_r^{31}$, is $\bar{q}(0)$ for $y_r \geq \max\{y_r^{31}, y_r^3\}$, and is either $\hat{q}^o(\hat{r}_s^{\#};C_r,C_s)$ or $\bar{q}(0)$ for $y_r^{31} \leq y_r < \max\{y_r^{31}, y_r^3\}$.

Recall that $\Pi(\bar{q}(0), r_s;C_r,C_s)$ does not change in r_s (the retailer's profit does not change in r_s as well). Then, the global optimal trade credit rate can just be determined from $\hat{r}_s^{\#}$. □

Let $w^o(r_s;C_r,C_s) = w^*(\hat{q}^o, r_s;C_r)$, $k_s^o(r_s;C_r,C_s) = k_s^*(\hat{q}^o, r_s;C_r)$, $M^o(r_s;C_r,C_s) = M^*(\hat{q}^o, r_s;C_r)$, and $\Pi^o(r_s;C_r,C_s) = \Pi(\hat{q}^o, r_s;C_r,C_s)$. Let $A(r_s;C_r,C_s) \equiv \frac{\bar{F}(\hat{q}^o)[1-\hat{q}^o z(\hat{q}^o)]}{c}$, $D(r_s;C_r,C_s) \equiv \frac{A(r_s;C_r,C_s)}{1-M^o(r_s;C_r,C_s)} - 1$, and $G(r_s;C_r,C_s) \equiv \frac{1-M^o(r_s;C_r,C_s)}{\bar{F}(k_s^o(r_s;C_r,C_s))}$. From Equation (EC.4a), $D(r_s;C_r,C_s) = \alpha_1$ for $r_s \in [0, r_s^1)$, and $D(r_s;C_r,C_s) = \alpha_2$ for $r_s \in [r_s^2, r_s^3)$. Let the indicator $\mathbb{1}_\Omega = 1$ if the condition Ω holds and 0 otherwise.

From Proposition 1, for $r_s^1(C_r,\alpha_1) \leq r_s \leq r_s^2(C_r,\alpha_2)$, $q_y(r_s;C_r)$ is the optimal quantity satisfying

$$V^3(q,w) = cq - y_r - \bar{y}_b(r_s;C_r) = 0, \tag{EC.11}$$

with $V_q^3 = c \geq 0$ and $V_w^3 = 0$. The optimal q and w are solutions of Equations (EC.11) and (EC.4b).

LEMMA EC.9. *For* $r_s^1(C_r,\alpha_1) \le r_s \le r_s^2(C_r,\alpha_2)$*, since* $\hat{q}^o(r_s;C_r,C_s) = q_y(r_s;C_r)$*, we have*

$$\frac{\partial \hat{q}^o(r_s;C_r,C_s)}{\partial r_s} = \frac{1}{c}\cdot\frac{\partial \bar{y}_b(r_s;C_r)}{\partial r_s} \ge 0, \tag{EC.12}$$

$$\frac{\partial k_s^o(r_s;C_r,C_s)}{\partial r_s} = \frac{1}{\bar{F}(k_s^o(r_s;C_r,C_s))}\Big[-\frac{y_r+\bar{y}_b(r_s;C_r)}{G(r_s;C_r,C_s)} + (D(r_s;C_r,C_s)+1)\frac{\partial \bar{y}_b(r_s;C_r)}{\partial r_s}\Big] \tag{EC.13}$$

Proof of Lemma EC.9: From Equations (EC.11) and (EC.4b), $V_{r_s}^3 = -\frac{\partial \bar{y}_b}{\partial r_s}$ and $V_{r_s}^2 = -w\bar{F}(k_s)\left[1-(wq-y_r-\bar{y}_b)(1+r_s)z(k_s)\right] \le 0$. Let $B = V_q^3V_w^2 - V_w^3V_q^2 = V_q^3V_w^2 \le 0$, $B_q = -V_{r_s}^3V_w^2 + V_w^3V_{r_s}^2 = -V_{r_s}^3V_w^2 \le 0$, and $B_w = -V_q^3V_{r_s}^2 + V_{r_s}^3V_q^2 \ge 0$. Then, $\frac{\partial q}{\partial r_s} = \frac{B_q}{B} = -\frac{V_{r_s}^3V_w^2}{V_q^3V_w^2} = \frac{1}{c}\cdot\frac{\partial \bar{y}_b}{\partial r_s} \ge 0$. Since $\frac{\partial w}{\partial r_s} = \frac{B_w}{B}$, we have $\frac{\partial k_s}{\partial r_s} = q(1+r_s)\frac{\partial w}{\partial r_s} + w(1+r_s)\frac{\partial q}{\partial r_s} + (wq-y_r-\bar{y}_b) = -\frac{y_r+\bar{y}_b}{1-M} + \frac{\bar{F}(q)[1-qz(q)]}{c\bar{F}(k_s)(1-M)}\cdot\frac{\partial \bar{y}_b}{\partial r_s}$. □

LEMMA EC.10. *In the trade/combined financing region, we have the following derivatives*

$$\partial\Pi^o(r_s;C_r,C_s)/\partial r_s = (1-\rho_s(C_s))(1-\rho_r(C_r))\cdot$$

$$\begin{cases}\Big[-\frac{y_r+\bar{y}_b(r_s;C_r)}{G(r_s;C_r,C_s)} + (D(r_s;C_r,C_s)-\delta_r(C_r))\frac{\partial \bar{y}_b(r_s;C_r)}{\partial r_s}\Big], & r_s \in [0, r_s^3(C_r,\alpha_2)), \quad \text{(EC.14a)}\\ \Big[-\frac{y_r\mathbb{1}_{r_s\le\delta_r(C_r)}}{A(r_s;C_r,C_s)} + \Big(1-\frac{1+\delta_r(C_r)}{A(r_s;C_r,C_s)}\Big)\frac{\partial \bar{y}_b(r_s;C_r)}{\partial r_s}\Big](1+\alpha_2) \le 0, & r_s \in [r_s^3(C_r,\alpha_2), \tilde{r}_s(y_r,C_r)) \quad \text{(EC.14b)}\end{cases}$$

Proof of Lemma EC.10: For notational convenience, let $\Omega = (1-\rho_s(C_s))(1-\rho_r(C_r))$. From Equations (11a) and (11b), $\frac{\partial \Pi^o}{\partial r_s} = \Omega\big[\bar{F}(k_s)\frac{\partial k_s}{\partial r_s} - c(1+\alpha)\frac{\partial q}{\partial r_s} + (\alpha-\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s}\big]$, where $\alpha = \alpha_1$ or $\alpha = \alpha_2$ and where $\frac{\partial k_s}{\partial r_s}$ and $\frac{\partial q}{\partial r_s}$ are in Equations (EC.10) and (EC.9) for $r_s \in [0, r_s^1) \cup [r_s^2, r_s^3)$. Then, $\frac{\partial \Pi^o}{\partial r_s} = \Omega\big\{-\frac{(y_r+\bar{y}_b)\bar{F}(k_s)}{1-M} - \delta_r(C_r)\frac{\partial \bar{y}_b}{\partial r_s} + \alpha\frac{\partial \bar{y}_b}{\partial r_s}\big\} = \Omega\big\{-\frac{y_r+\bar{y}_b}{G} + [D-\delta_r(C_r)]\frac{\partial \bar{y}_b}{\partial r_s}\big\}$. For $r_s \in [r_s^1, r_s^2)$, from Equation (EC.13), $\frac{\partial \Pi^o}{\partial r_s} = \Omega\big\{\bar{F}(k_s)\frac{\partial k_s}{\partial r_s} - (1+\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s}\big\} = \Omega\big\{-\frac{y_r+\bar{y}_b}{G} + [D-\delta_r(C_r)]\frac{\partial \bar{y}_b}{\partial r_s}\big\}$.

For $r_s \in [r_s^3, \tilde{r}_s)$, from Proposition 1, the optimal $q = q_1^l(r_s;C_r)$ satisfying $V^4 = q\bar{F}(q) - [y_r + \bar{y}_b(r_s;C_r)](1+\min\{\delta_r(C_r), r_s\}) = 0$. Then, $V_q^4 = \bar{F}(q)[1-qz(q)] = cA \ge 0$, $V_{r_s}^4 = -y_r\mathbb{1}_{r_s\le\delta_r(C_r)} - (1+\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s} \le 0$, and $\frac{\partial q}{\partial r_s} = -\frac{V_{r_s}^4}{V_q^4} = \frac{1}{cA}[y_r\mathbb{1}_{r_s\le\delta_r(C_r)} + (1+\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s}] \ge 0$. From $k_s = \bar{k}_b$ and Equations (7), (EC.1), and (EC.2), $\bar{F}(k_s)\frac{\partial k_s}{\partial r_s} = (1+\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s}$. From Equation (11b), $\frac{\partial \Pi^o}{\partial r_s} = \Omega\big\{\bar{F}(k_s)\frac{\partial k_s}{\partial r_s} - c(1+\alpha_2)\frac{\partial q}{\partial r_s} + (\alpha_2-\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s}\big\} = \Omega(1+\alpha_2)\big\{\frac{\partial \bar{y}_b}{\partial r_s} - c\frac{\partial q}{\partial r_s}\big\} = \Omega(1+\alpha_2)\big\{-\frac{y_r\mathbb{1}_{r_s\le\delta_r(C_r)}}{A} + \frac{\partial \bar{y}_b}{\partial r_s}[1-\frac{1+\delta_r(C_r)}{A}]\big\} \le 0$, where "$\le$" holds from $\frac{1+\delta_r(C_r)}{A} \ge \frac{c(1+\alpha_2)}{\bar{F}(q)[1-qz(q)]} \ge \frac{1}{1-M} \ge 1$ as $q = q_1^l \ge \hat{q}(r_s;C_r,\alpha_2)$. □

Proof of Proposition 3: Note that $r_s^2 = 0$ at $y_r = y_r^2$, and $r_s^2 = \delta_r(C_r) \ge 0$ at $y_r = y_r^{21}$. From the proof of Proposition 1, r_s^2 decreases in y_r. Then, $y_r^{21} \le y_r^2$. From Equations (EC.14a) and (EC.14b) and $D = \alpha_2 \le \delta_r(C_r)$ for $r_s \in [r_s^2, r_s^3)$, we have $\frac{\partial \Pi^o}{\partial r_s} \le 0$ for $r_s \in [r_s^2, \tilde{r}_s)$. Then, the optimal $r_s \in [0, r_s^2)$. For $y_r \ge y_r^{21}$, we have $r_s^2 \le \delta_r(C_r)$ so that $\frac{\partial \bar{y}_b}{\partial r_s} = 0$. From Equation (EC.14a), the supplier's profit decreases in r_s for $r_s \le \delta_r(C_r)$ so that $r_s^{\#}(C_r,C_s) = 0$. For $y_r < y_r^{21}$, we have $r_s^2 > \delta_r(C_r)$. In the same logic, the supplier's profit decreases in r_s for $r_s \le \delta_r(C_r)$. Then, $r_s^{\#} \in \{0\} \cup [\delta_r(C_r), r_s^2)$. □

LEMMA EC.11. *For* $y_r < y_r^{21}$ *so that* $r_s^2(C_r,\alpha_2) > \delta_r(C_r)$*, if* $y_r \ge y_r^{22}$ *where* y_r^{22} *is a threshold, then the supplier's profit* $\Pi^o(r_s;C_r,C_s)$ *is quasi-concave in* r_s *for* $r_s \in (\delta_r(C_r), r_s^2(C_r,\alpha_2))$*.*

Proof of Lemma EC.11: We first show $D = \frac{\bar{F}(q)(1-qz(q))}{c(1-M)} - 1$ decreases in r_s. It holds for $r_s \in [0, r_s^1) \cup [r_s^2, r_s^3)$, since D is constant. For $r_s \in [r_s^1, r_s^2)$, $\frac{\partial M}{\partial r_s} = \frac{\partial[wq(1+r_s)]}{\partial r_s}z(k_s) + wq(1+r_s)z'(k_s)\frac{\partial k_s}{\partial r_s} = [\frac{\partial k_s}{\partial r_s} + (y_r+\bar{y}_b)]z(k_s) + wq(1+r_s)z'(k_s)\frac{\partial k_s}{\partial r_s} = -(y_r+\bar{y}_b)\frac{wq(1+r_s)[z'(k_s)+z^2(k_s)]}{1-M} +$

$\frac{wq(1+r_s)z'(k_s)+z(k_s)}{c}\frac{\bar{F}(q)[1-qz(q)]}{\bar{F}(k_s)[1-M]}\frac{\partial\bar{y}_b}{\partial r_s}$, where $\frac{\partial k_s}{\partial r_s}$ is from Equation (EC.13). Also, $\frac{\partial[qz(q)]}{\partial r_s}=(z(q)+qz'(q))\frac{\partial q}{\partial r_s}=\frac{z(q)+qz'(q)}{c}\frac{\partial\bar{y}_b}{\partial r_s}$, where $\frac{\partial q}{\partial r_s}\geq 0$ from Equation (EC.12). From $0\leq\frac{\bar{F}(q)}{\bar{F}(k_s)}\leq 1$, $0\leq\frac{1-qz(q)}{1-M}\leq 1$ and $wq(1+r_s)z'(k_s)+z(k_s)\leq qz'(q)+z(q)$, we have $-\frac{\partial[qz(q)]}{\partial r_s}+\frac{1-qz(q)}{1-M}\frac{\partial M}{\partial r_s}\leq 0$. Then, $\partial[\frac{1-qz(q)}{1-M}]/\partial r_s=\frac{1}{1-M}\{-\frac{\partial[qz(q)]}{\partial r_s}+\frac{1-qz(q)}{1-M}\frac{\partial M}{\partial r_s}\}\leq 0$. As q increases in r_s so that $\bar{F}(q)$ decreases in r_s, D decreases in r_s.

Since $\frac{\partial\bar{y}_b}{\partial r_s}\geq 0$ and decreases in r_s, $[D-\delta_r(C_r)]\frac{\partial\bar{y}_b}{\partial r_s}$ decreases in r_s, as long as it is nonnegative. Then, Π^o is quasi-concave in r_s if $\frac{y_r+\bar{y}_b}{G}$ increases in r_s. We have $\partial(\frac{y_r+\bar{y}_b}{G})/\partial r_s=\frac{\bar{F}(k_s)}{(1-M)^2}\{(y_r+\bar{y}_b)\frac{\partial M}{\partial r_s}+[\frac{\partial\bar{y}_b}{\partial r_s}-(y_r+\bar{y}_b)z(k_s)\frac{\partial k_s}{\partial r_s}](1-M)\}=\frac{\bar{F}(k_s)}{1-M}\{\frac{\partial\bar{y}_b}{\partial r_s}+\frac{(y_r+\bar{y}_b)^2z(k_s)}{1-M}J\}$, where $J=1+\frac{M}{y_r+\bar{y}_b}[1+\frac{z'(k_s)}{z^2(k_s)}]\frac{\partial k_s}{\partial r_s}$.

For $r_s\in(r_s^1,r_s^2]$, from Equation (11b), $\frac{\partial\Pi^o}{\partial r_s}=(1-\rho_s(C_s))(1-\rho_r(C_r))\{\bar{F}(k_s)\frac{\partial k_s}{\partial r_s}-(1+\delta_r(C_r))\frac{\partial\bar{y}_b}{\partial r_s}\}$. As long as $\frac{\partial\Pi^o}{\partial r_s}\geq 0$, we have $\frac{\partial k_s}{\partial r_s}\geq 0$ so that $J\geq 0$ and $\partial(\frac{y_r+\bar{y}_b}{G})/\partial r_s\geq 0$. Since $r_s\geq\delta_r(C_r)$ and $\frac{\partial\Pi^o}{\partial r_s}$ is continuous in r_s, Π^o is quasi-concave in r_s. For $r_s\in[0,r_s^1)$, from $\frac{\partial k_s}{\partial r_s}$ in Equation (EC.10), $J=\frac{P[1-2M-M(z'(k_s)/z^2(k_s))]-[c(1+\alpha)(1-M)]^2z(k_s)}{(1-M)\{P-c^2(1+\alpha)^2[wq(1+r_s)z'(k_s)+z(k_s)]\}}$, where $\alpha=\alpha_1$, $P=\bar{F}(q)\bar{F}(k_s)[2z(q)+qz'(q)-qz^2(q)]$ and the denominator is nonnegative from the proof of Lemma EC.8. If the numerator is nonnegative, then $J\geq 0$ and $\frac{y_r+\bar{y}_b}{G}$ increases in r_s. Then, suppose it is negative. Thus, $\frac{\partial\bar{y}_b}{\partial r_s}+\frac{(y_r+\bar{y}_b)^2z(k_s)}{1-M}J\geq\frac{1}{(1+r_s)^2z(k_s)}[1+\frac{M^2}{1-M}J]=\frac{w^2}{G^2M}\frac{(1+q^2z'(q))H_1(r_s)-(1-qz(q))^2H_0(r_s)}{P-c^2(1+\alpha)^2[wq(1+r_s)z'(k_s)+z(k_s)]}$, where the inequality holds from $wq-(y_r+\bar{y}_b)=\frac{k_s-\bar{k}_b}{1+r_s}\geq 0$. Here $H_0(r_s)=1-M(1-M)^2+M^2(1-M)\frac{z'(k_s)}{z^2(k_s)}\leq 1+q^2z'(q)$, since $M^2(1-M)\frac{z'(k_s)}{z^2(k_s)}\leq q^2(w(1+r_s))^2z'(k_s)\leq q^2z'(q)$ from $0\leq M\leq 1$ and $w(1+r_s)<1$ (see Lemma EC.3). Also, $H_1(r_s)=(1-M)^2+M^2-2M^3-M^3\frac{z'(k_s)}{z^2(k_s)}\geq H_2(r_s)+(1-qz(q))^2$ where $H_2(r_s)\equiv[1-(\frac{c(1+\alpha)}{\bar{F}(q)})^2](1-M)^2-M^3-M(wq(1+r_s))^2z'(k_s)$. From Lemma EC.7, q, k_s, $wq(1+r_s)$ and M decrease in y_r. Then, $H_2(r_s)$ increases in y_r since $1-(\frac{c(1+\alpha)}{\bar{F}(q)})^2\geq 0$, $(1-M)^2$, and $-M^3-M(wq(1+r_s))^2z'(k_s)$ all increase in y_r. Thus, there is a $y_r^4(r_s)$ so that $H_2(r_s)\geq 0$ for $y_r\geq y_r^4(r_s)$. Similarly, from Lemma EC.8, $H_2(r_s)$ increases in r_s. Consequently, $y_r^4(r_s)$ decreases in r_s. As a result, if $y_r\geq y_r^{22}\equiv y_r^4(0)$, then $H_1(r_s)\geq(1-qz(q))^2$, $\partial\frac{y_r+\bar{y}_b}{G}/\partial r_s\geq 0$ so that Π^o is quasi-concave in r_s. □

Let $\pi^o(r_s;C_r,C_s)$ be the retailer's equilibrium profits in the combined/trade financing region for a given r_s. Let $\pi^\#(C_r,C_s)=\pi^o(r_s^\#(C_r,C_s);C_r,C_s)$. Let x^- (x^+) be the value to the left (right) of x.

LEMMA EC.12. *For a fixed $r_s\in[0,r_s^2(C_r,\alpha_2))$, the retailer's optimal profit in the trade/combined financing region (weakly) increases in the supplier's credit rating C_s.*

Proof of Lemma EC.12: For $r_s\in[0,r_s^1)$, q and w satisfy Equations (EC.4a) and (EC.4b) where $\alpha=\alpha_1=\frac{\delta_s(C_s)+\rho_r(C_r)}{1-\rho_r(C_r)}$. Then, $V_{C_s}^1=\frac{c(1-M)\delta_s'(C_s)}{1-\rho_r(C_r)}\leq 0$ and $V_{C_s}^2=0$. Let $B=V_q^1V_w^2-V_w^1V_q^2$, $B_q=V_w^1V_{C_s}^2-V_{C_s}^1V_w^2\leq 0$, and $B_w=V_{C_s}^1V_q^2-V_q^1V_{C_s}^2\geq 0$. Then, $B\leq 0$, $\frac{\partial q}{\partial C_s}=\frac{B_q}{B}\geq 0$, $\frac{\partial w}{\partial C_s}=\frac{B_w}{B}\leq 0$, and

$$\frac{\partial k_s}{\partial C_s}=q(1+r_s)\frac{\partial w}{\partial C_s}+w(1+r_s)\frac{\partial q}{\partial C_s}=\frac{1}{B}(1+r_s)V_{C_s}^1[\bar{F}(q)(1-qz(q))]\geq 0, \tag{EC.15}$$

$$\frac{\partial\pi^o}{\partial C_s}=(1-\rho_r(C_r))\left\{\bar{F}(q)\frac{\partial q}{\partial C_s}-\bar{F}(k_s)\frac{\partial k_s}{\partial C_s}\right\}=\frac{1-\rho_r(C_r)}{B}(1+r_s)\bar{F}(k_s)\bar{F}(q)V_{C_s}^1[qz(q)-M]\geq 0, \tag{EC.16}$$

where the inequality follows from $w(1+r_s)<1$ based on Lemma EC.3, $k_s\leq q$, $V_{C_s}^1\leq 0$, and $B\leq 0$.

For $r_s\in[r_s^1,r_s^2)$, q and w satisfy Equations (EC.11) and (EC.4b). Then, $V_{C_s}^3=0$ and $V_{C_s}^2=0$. Let $B=V_q^3V_w^2-V_w^3V_q^2$, $B_q=V_w^3V_{C_s}^2-V_{C_s}^3V_w^2$, and $B_w=V_{C_s}^3V_q^2-V_q^3V_{C_s}^2$. Then, $\frac{\partial q}{\partial C_s}=\frac{B_q}{B}=\frac{\partial w}{\partial C_s}=\frac{B_w}{B}=0$ and $\frac{\partial k_s}{\partial C_s}=(1+r_s)\frac{\partial(wq)}{\partial C_s}=0$. From Equation (6a), $\frac{\partial\pi^o}{\partial C_s}=(1-\rho_r(C_r))\{\bar{F}(q)\frac{\partial q}{\partial C_s}-\bar{F}(k_s)\frac{\partial k_s}{\partial C_s}\}=0$. □

Proof of Proposition 4: the result follows directly from the proof of Proposition 2. □

Proof of Proposition 5: From $\frac{\partial(wq(1+r_s))}{\partial C_s}=\frac{\partial k_s}{\partial C_s}$, we have $\partial[-\frac{(y_r+\bar{y}_b)}{G}]/\partial C_s=-\frac{(y_r+\bar{y}_b)}{(1-M)G}\{-z(k_s)(1-M)\frac{\partial k_s}{\partial C_s}+z(k_s)\frac{\partial(wq(1+r_s))}{\partial C_s}+wq(1+r_s)z'(k_s)\frac{\partial k_s}{\partial C_s}\}=-\frac{(y_r+\bar{y}_b)}{(1-M)G}\{Mz(k_s)+wq(1+r_s)z'(k_s)\}\frac{\partial k_s}{\partial C_s}$. Note that $\frac{\partial^2\bar{y}_b}{\partial r_s\partial C_s}=0$. For $r_s\in[0,r_s^1)$, from Equation (EC.15), $\frac{\partial k_s}{\partial C_s}\geq 0$, and for $r_s\in[r_s^1,r_s^2)$, from the proof of Lemma EC.12, $\frac{\partial k_s}{\partial C_s}=0$. Then, $-\frac{(y_r+\bar{y}_b)}{G}$ decreases in C_s. For $r_s\in[0,r_s^1)$, $D=\alpha_1=\frac{\delta_s(C_s)+\rho_r(C_r)}{1-\rho_r(C_r)}$ decreases in C_s. For $r_s\in[r_s^1,r_s^2)$, $\frac{\partial q}{\partial C_s}=\frac{\partial w}{\partial C_s}=\frac{\partial k_s}{\partial C_s}=0$ and $D=\frac{\bar{F}(q)(1-qz(q))}{c(1-M)}-1$ does not change in C_s.

Let $H^o(r_s;C_s)=\frac{\Pi^o(r_s;C_r,C_s)}{1-\rho_s(C_s))(1-\rho_r(C_r))}$. From Equation (EC.14a), $\frac{\partial^2H^o}{\partial r_s\partial C_s}\leq 0$ for $r_s\in[0,r_s^1)$ and $\frac{\partial^2H^o}{\partial r_s\partial C_s}=0$ for $r_s\in[r_s^1,r_s^2)$. Let $\Delta(r_s;C_s)=H^o(r_s;C_s)-H^o(0;C_s)$. For $\hat{r}_s\in[0,r_s^1)$, $\Delta(\hat{r}_s;C_s)=\int_0^{\hat{r}_s}\frac{\partial H^o}{\partial r_s}dr_s$ and $\frac{\partial\Delta(\hat{r}_s;C_s)}{\partial C_s}=\int_0^{\hat{r}_s}\frac{\partial^2H^o}{\partial r_s\partial C_s}dr_s\leq 0$. For $\hat{r}_s\in[r_s^1,r_s^2)$, $\Delta(\hat{r}_s;C_s)=\int_0^{r_s^1}\frac{\partial H^o}{\partial r_s}dr_s+\int_{r_s^1}^{\hat{r}_s}\frac{\partial H^o}{\partial r_s}dr_s$ and $\frac{\partial\Delta(\hat{r}_s;C_s)}{\partial C_s}=[\int_0^{r_s^1}\frac{\partial^2H^o}{\partial r_s\partial C_s}dr_s+\frac{\partial H^o}{\partial r_s}|_{r_s=r_s^{1-}}\cdot\frac{\partial r_s^1}{\partial C_s}]+[\int_{r_s^1}^{\hat{r}_s}\frac{\partial^2H^o}{\partial r_s\partial C_s}dr_s-\frac{\partial H^o}{\partial r_s}|_{r_s=r_s^{1+}}\cdot\frac{\partial r_s^1}{\partial C_s}]=\int_0^{r_s^1}\frac{\partial^2H^o}{\partial r_s\partial C_s}dr_s\leq 0$, where the second equality holds from $\int_{r_s^1}^{\hat{r}_s}\frac{\partial^2H^o}{\partial r_s\partial C_s}dr_s=0$ and $\frac{\partial H^o}{\partial r_s}|_{r_s=r_s^{1-}}=\frac{\partial H^o}{\partial r_s}|_{r_s=r_s^{1+}}$ since $\frac{\partial H^o}{\partial r_s}$ is continuous at r_s^1 implied from Equation (EC.14a). (As $\frac{\partial\bar{y}_b}{\partial r_s}$ is discontinuous at $r_s=\delta_r(C_r)$, $\frac{\partial H^o}{\partial r_s}$ is discontinuous at $r_s=\delta_r(C_r)$. However, if $r_s^1=\delta_r(C_r)$, then $\frac{\partial r_s^1}{\partial C_s}=0$.) Thus, $\Delta(\hat{r}_s;C_s)$ decreases in C_s.

For $C_s\geq\delta_s^{-1}(\epsilon_r(C_r))$ so that $\alpha_1\leq\delta_r(C_r)$, since $\frac{\partial H^o}{\partial r_s}\leq 0$, $\Delta(r_s;C_s)\leq 0$ for $r_s\in(0,r_s^2)$. For $r_s\in[\delta_r(C_r),r_s^2)$, it is possible that $\Delta(r_s;C_s)>0$ after C_s drops below a threshold $C_s^{11}(r_s;y_r,C_r)\in[\underline{C}_s,\delta_s^{-1}(\epsilon_r(C_r))]$. Let $C_s^1(y_r,C_r)=\sup_{r_s\in[\delta_r(C_r),r_s^2)}\{C_s^{11}(r_s;y_r,C_r)\}$. Then, $H^o(r_s^{\#};C_s)>H^o(0;C_s)$ so that $\Pi^o(r_s^{\#};C_r,C_s)>\Pi^o(0;C_r,C_s)$ and $r_s^{\#}>0$ if $C_s\in[\underline{C}_s,C_s^1(y_r,C_r))$. Otherwise, $r_s^{\#}=0$. □

Lemma EC.13. *For the same supplier, i.e., C_s is fixed, the retailer's equilibrium profit in the combined/trade region increases in the supplier's interest rate r_s.*

Proof of Lemma EC.13: From Proposition 3, let $r_s\in[0,r_s^2)$. For $r_s\in[0,r_s^1)$, from Equations (EC.9) and (EC.10), $\frac{\partial\pi^o}{\partial r_s}=(1-\rho_r(C_r))\big[\bar{F}(q)\frac{\partial q}{\partial r_s}-\bar{F}(k_s)\frac{\partial k_s}{\partial r_s}\big]=\frac{(1-\rho_r(C_r))(y_r+\bar{y}_b)\cdot J}{\{P-[c(1+\alpha)]^2[z(k_s)+w(1+r_s)qz'(k_s)]\}G}$, where $\alpha=\alpha_1$, $P=\bar{F}(q)\bar{F}(k_s)[2z(q)+qz'(q)-qz^2(q)]$ and the denominator is nonnegative from the proof of Lemma EC.8, and $J=-\bar{F}(q)c(1+\alpha)wq(1+r_s)[z'(k_s)+z^2(k_s)]+P-c^2(1+\alpha)^2z(k_s)(1-M)$ for short. Substituting P and $c(1+\alpha)(1-M)=\bar{F}(q)(1-qz(q))$ (from $V^1=0$) into J yields $J=\bar{F}(q)\big\{q[\bar{F}(k_s)z'(q)-c(1+\alpha)w(1+r_s)z'(k_s)]+[\bar{F}(k_s)z(q)-c(1+\alpha)wq(1+r_s)z^2(k_s)]+(1-qz(q))[\bar{F}(k_s)z(q)-c(1+\alpha)z(k_s)]\big\}\geq 0$, where "$\geq$" holds from $qz(q)\leq 1$, $k_s\leq q$ and $w(1+r_s)<1$ from Lemma EC.3, $z(q)\geq z(k_s)$ and $z'(q)\geq z'(k_s)$, and $\frac{\bar{F}(k_s)}{c(1+\alpha_1)}\geq\frac{\bar{F}(q)}{c(1+\alpha_1)}=\frac{1-M}{1-qz(q)}\geq 1$. Then, π^o increases in r_s. For $r_s\in[r_s^1,r_s^2)$, from Equations (EC.12) and (EC.13), π^o increases in r_s since $\frac{\partial\pi^o}{\partial r_s}=(1-\rho_r(C_r))\big[\bar{F}(q)\frac{\partial q}{\partial r_s}-\bar{F}(k_s)\frac{\partial k_s}{\partial r_s}\big]=(1-\rho_r(C_r))\big[\frac{\bar{F}(q)}{c}\frac{\partial\bar{y}_b}{\partial r_s}\big(1-\frac{1-qz(q)}{1-M}\big)+\frac{y_r+\bar{y}_b}{G}\big]\geq 0$. □

Proof of Proposition 6: For Part 1, from Proposition 3, if $y_r\geq y_r^{21}$, $r_s^{\#}(C_r,C_s)=0$. For Part 2, if $c\leq\frac{y_r}{\hat{q}(0;C_r,\alpha_2)}$, then $\hat{q}(0;C_r,\bar{C}_s)\leq q_y(0;C_r)$. Since q_y increases in r_s and $\hat{q}(r_s;C_r,\alpha_2)$ decreases in r_s from Lemma EC.8, $r_s^2=0$. From Proposition 3, $r_s^{\#}(C_r,C_s)=0$. For both parts, $C_s^1(y_r,C_r)=\underline{C}_s$. □

Proof of Proposition 7: From Proposition 3, $r_s^{\#}=0$ or $r_s^{\#}\in[\delta_r(C_r),r_s^2)$ satisfying the first order condition. Therefore, $\frac{\partial\Pi^{\#}(C_r,C_s)}{\partial C_r}=\frac{\partial\Pi^o(r_s^{\#};C_r,C_s)}{\partial C_r}+\frac{\partial\Pi^o(r_s;C_r,C_s)}{\partial r_s}|_{r_s=r_s^{\#}}\cdot\frac{\partial r_s^{\#}}{\partial C_r}=\frac{\partial\Pi^o(r_s^{\#};C_r,C_s)}{\partial C_r}$. Then, we can let r_s be fixed at $r_s^{\#}$. For $r_s > \delta_r(C_r)$, we have $\frac{\partial \bar{k}_b}{\partial C_r} = -\frac{\delta_r'(C_r)}{(1+\delta_r(C_r))z(\bar{k}_b)}$, $\frac{\partial \bar{y}_b}{\partial C_r} = -\frac{\bar{y}_b \delta_r'(C_r)}{1+\delta_r(C_r)} - \frac{\delta_r'(C_r)}{(1+\delta_r(C_r))(1+r_s)z(\bar{k}_b)} \geq 0$, and $\frac{\partial k_s}{\partial C_r} = \frac{\partial[-\bar{y}_b(1+r_s)+\bar{k}_b]}{\partial C_r} = \frac{\bar{y}_b \delta_r'(C_r)}{\bar{F}(\bar{k}_b)}$. From Equations (EC.4a), (EC.4b) and (EC.11), $V_{C_r}^1 = \Omega - c(1+\alpha_1)w(1+r_s)qz'(k_s)\frac{\bar{y}_b \delta_r'(C_r)}{\bar{F}(\bar{k}_b)} = \Omega - V_{y_r}^1 \frac{\bar{y}_b \delta_r'(C_r)}{(1+r_s)\bar{F}(\bar{k}_b)}$, where $\Omega = \frac{\rho_r'(C_r)}{1-\rho_r(C_r)}c(1+\alpha_1)(1-M) \leq 0$, $V_{C_r}^2 = w(1+r_s)\bar{F}(k_s)z(k_s)\frac{\bar{y}_b \delta_r'(C_r)}{\bar{F}(\bar{k}_b)} = -V_{y_r}^2 \frac{\bar{y}_b \delta_r'(C_r)}{(1+r_s)\bar{F}(\bar{k}_b)} \leq 0$, and $V_{C_r}^3 = -\frac{\partial \bar{y}_b}{\partial C_r} \leq 0$.

For $r_s \in [0, r_s^1)$, let $B = V_q^1 V_w^2 - V_w^1 V_q^2$, $B_q = -V_{C_r}^1 V_w^2 + V_w^1 V_{C_r}^2$ and $B_w = -V_q^1 V_{C_r}^2 + V_{C_r}^1 V_q^2$. Then, $B \leq 0$, $\frac{\partial q}{\partial C_r} = \frac{B_q}{B} = \frac{-\Omega V_w^2}{B} + L_1$, $\frac{\partial w}{\partial C_r} = \frac{B_w}{B} = \frac{\Omega V_q^2}{B} + \frac{\bar{y}_b \delta_r'(C_r)}{(1+r_s)\bar{F}(\bar{k}_b)} \frac{V_q^1 V_{y_r}^2 - V_{y_r}^1 V_q^2}{B}$, and $\frac{\partial k_s}{\partial C_r} = (1+r_s)\big[q\frac{\partial w}{\partial C_r} + w\frac{\partial q}{\partial C_r}\big] + \frac{\bar{y}_b \delta_r'(C_r)}{\bar{F}(\bar{k}_b)} = (1+r_s)\frac{q\Omega V_q^2 - w\Omega V_w^2}{B} + L_2$, where $L_1 = \frac{\bar{y}_b \delta_r'(C_r)}{(1+r_s)\bar{F}(\bar{k}_b)} \frac{V_{y_r}^1 V_w^2 - V_w^1 V_{y_r}^2}{B}$ and $L_2 = (1+r_s)\frac{\bar{y}_b \delta_r'(C_r)}{\bar{F}(\bar{k}_b)\cdot B}[-\bar{F}(k_s)V_q^1 + c(1+\alpha_1)qz(k_s)V_q^2 - c(1+\alpha_1)w^2 q(1+r_s)^2 \bar{F}(k_s)(z'(k_s)+z^2(k_s))]$. From the proof of Lemma EC.7, $L_1 \leq 0$, and since $V_q^1 \geq 0$, $V_q^2 \leq 0$ and $B \leq 0$, we have $L_2 \geq 0$.

Let $\Pi^o = (1-\rho_s(C_s))H^o$. From Equation (11a), $\frac{\partial H^o}{\partial C_r} = -\rho_r'(C_r)\mathbb{E}[\min\{\xi, k_s\}] + (1-\rho_r(C_r))[L_3 + (\alpha_1 - \delta_r(C_r))\frac{\partial \bar{y}_b}{\partial C_r}] - \bar{y}_b \epsilon_r'(C_r)$, where $L_3 = \bar{F}(k_s)\frac{\partial k_s}{\partial C_r} - c(1+\alpha_1)\frac{\partial q}{\partial C_r}$. Since $\rho_r'(C_r) \leq 0$ and $\epsilon_r'(C_r) \leq 0$, if $L_3 \geq 0$ and $(\alpha_1 - \delta_r(C_r))\frac{\partial \bar{y}_b}{\partial C_r} \geq 0$, then $\frac{\partial H^o}{\partial C_r} \geq 0$. Note that $L_3 = \frac{\Omega}{B}\big[(qV_q^2 - wV_w^2)(1+r_s)\bar{F}(k_s) + c(1+\alpha_1)V_w^2\big] + \bar{F}(k_s)L_2 - c(1+\alpha_1)L_1 = \bar{F}(k_s)L_2 - c(1+\alpha_1)L_1 \geq 0$. If $\alpha_1 \geq \delta_r(C_r)$, then $\frac{\partial H^o}{\partial C_r} \geq 0$. If $\alpha_1 < \delta_r(C_r)$, however, Proposition 5 implies $r_s^{\#} = 0 \leq \delta_r(C_r)$ and $\bar{y}_b = \frac{\partial \bar{y}_b}{\partial C_r} = 0$. Again, $\frac{\partial H^o}{\partial C_r} \geq 0$.

For $r_s \in [r_s^1, r_s^2)$, let $B = V_q^3 V_w^2 - V_w^3 V_q^2 = V_q^3 V_w^2 \leq 0$, $B_q = V_w^3 V_{C_r}^2 - V_{C_r}^3 V_w^2 = -V_{C_r}^3 V_w^2$, and $B_w = V_{C_r}^3 V_q^2 - V_q^3 V_{C_r}^2 \geq 0$. Then, $\frac{\partial k_s}{\partial C_r} = (1+r_s)\big[q\frac{\partial w}{\partial C_r} + w\frac{\partial q}{\partial C_r}\big] + \frac{\bar{y}_b \delta_r'(C_r)}{\bar{F}(\bar{k}_b)} = -\frac{\delta_r'(C_r)}{\bar{F}(\bar{k}_b)}\big\{-\frac{\bar{y}_b}{1-M} + \frac{1}{\bar{F}(k_s)}[\frac{\bar{y}_b}{1+r_s} + \frac{\partial \bar{y}_b}{\partial r_s}](D+1)\big\}$. From Equation (11a) or (11b), $\frac{\partial H^o}{\partial C_r} = -\rho_r'(C_r)\mathbb{E}[\min\{\xi, k_s\}] + (1-\rho_r(C_r))L_4 - \epsilon_r'(C_r)\bar{y}_b$, where $L_4 = \bar{F}(k_s)\frac{\partial k_s}{\partial C_r} - (1+\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial C_r} = -\frac{\delta_r'(C_r)}{\bar{F}(\bar{k}_b)}\{-\frac{\bar{y}_b}{G} + \frac{\bar{y}_b}{1+r_s}(D+1) + (D-\delta_r(C_r))\frac{\partial \bar{y}_b}{\partial r_s}\} = 0$ if $r_s^{\#} = 0 \leq \delta_r(C_r)$ so that $\bar{y}_b = \frac{\partial \bar{y}_b}{\partial C_r} = 0$. If $r_s = r_s^{\#} > 0$ so that $\frac{\partial \Pi^o(r_s; C_r, C_s)}{\partial r_s} = 0$, then $L_4 = -\frac{\delta_r'(C_r)}{\bar{F}(\bar{k}_b)}\big[\frac{\partial \Pi^o(r_s; C_r, C_s)/\partial r_s}{(1-\rho_s(C_s))(1-\rho_r(C_r))} + \frac{y_r}{G} + \frac{\bar{y}_b}{1+r_s}(D+1)\big] \geq 0$. In both cases, we have $\frac{\partial H^o}{\partial C_r} \geq 0$. □

Proof of Proposition 8: Let C_s^m be the worst credit rating of the supplier in $[\underline{C}_s, C_s^1(y_r, C_r))$ for the retailer, i.e., $\pi^{\#}(C_r, C_s^m) \leq \pi^{\#}(C_r, C_s)$ for $C_s \in [\underline{C}_s, C_s^1(y_r, C_r))$. From Lemma EC.13, $\pi^{\#}(C_r, C_s^m) > \pi^0(0; C_r, C_s^m)$, i.e., the retailer receives larger expected profit when the supplier is forced to use 0 trade credit rate. From Lemma EC.12, $\pi^0(0; C_r, C_s)$ increases in C_s. Note that $\pi^0(0; C_r, C_s) = \pi^{\#}(C_r, C_s)$ for $C_s \in [C_s^1(y_r, C_r), \bar{C}_s]$, since $r_s^{\#}(C_r, C_s) = 0$ from Proposition 5. Let $C_s^2(y_r, C_r)$ be the largest value in $(C_s^m, \bar{C}_s]$ so that $\pi^0(0; C_r, C_s^2(y_r, C_r)) \leq \pi^{\#}(C_r, C_s^m)$. Then, $(C_s^1(y_r, C_r), C_s^2(y_r, C_r)]$ is the supplier's credit rating hole, since $\pi^{\#}(C_r, C_s) > \pi^{\#}(C_r, C_s')$ for $C_s \in [\underline{C}_s, C_s^1) \cup (C_s^2, \bar{C}_s]$ and $C_s' \in (C_s^1, C_s^2]$. That is, the retailer gets better profit from suppliers outside the hole than ones within the hole. Note that $(C_s^1(y_r, C_r), C_s^2(y_r, C_r)] = \emptyset$ if $C_s^2(y_r, C_r) \leq C_s^1(y_r, C_r)$.

We now study the special case $y_r = 0$. For $C_s \in (C_s^1, \bar{C}_s]$, $r_s^{\#} = 0$ so that $\bar{y}_b = \bar{k}_b = 0$. Let $q^{\#}$, $w^{\#}$ and $k_s^{\#}$ be the equilibrium quantity, wholesale price, and trade credit bankruptcy threshold, respectively. From Equation (10a), $q^{\#}\bar{F}(q^{\#}) = w^{\#}q^{\#}\bar{F}(w^{\#}q^{\#})$. Then, $w^{\#} = 1$, since $q^{\#} \leq \tilde{q}$. Also, $k_s^{\#} = q^{\#}$. From Equation (6a), $\pi^{\#}(C_r, C_s) = 0$. Since $z(q^{\#}) - w^{\#}z(k_s^{\#}) = 0$, from Equation (EC.16), $\frac{\partial \pi^{\#}(C_r, C_s)}{\partial C_s} = 0$. Then, $\pi^{\#}(C_r, C_s) = 0$ for $C_s \in (C_s^1(0, C_r), \bar{C}_s]$ and $C_s^2(0, C_r) = \bar{C}_s$.

Combining the above general case y_r and special case $y_r = 0$ together, we can conclude that there exists a threshold $y_r^4 > 0$ so that $(C_s^1(y_r, C_r), C_s^2(y_r, C_r)] \neq \emptyset$ for $0 \leq y_r \leq y_r^4$. □