

## Online Appendix

### O.1 Technical Appendix—Lemma Proofs

*Proof of joint concavity of firm objectives.* FOCs for each firm  $i \in \mathcal{N}_d$  and firm  $j \in \mathcal{N}_f$  are

$$\frac{\partial \pi_i(n_d, n_f, l, y, \varepsilon)}{\partial x_{i,l}} = A - b(x_{i,l} + (n_d - 1)x_{-i,l} + n_f y) - bx_{i,l} - c_d(\varepsilon) = 0, \quad \forall i \in \mathcal{N}_d, \quad (\text{O.1})$$

and

$$\frac{\partial \pi_j(n_d, n_f, x_l)}{\partial y_j} = A - b(n_d x_l + (n_f - 1)y_{-j} + y_j) - by_j - \delta\gamma_f - \tau = 0, \quad \forall j \in \mathcal{N}_f. \quad (\text{O.2})$$

The joint concavity of firm objectives can be proven directly through the Hessian. Based on the FOCs in Equations (O.1) and (O.2), the second derivative of domestic and offshore objectives are

$$\frac{\partial^2 \pi_i(\cdot)}{\partial x_{i,l}^2} = -2b, \quad \forall i \in \mathcal{N}_d, \quad \text{and} \quad \frac{\partial^2 \pi_j(\cdot)}{\partial y_j^2} = -2b, \quad \forall j \in \mathcal{N}_f,$$

while the cross-partials are

$$\frac{\partial^2 \pi_i(\cdot)}{\partial x_{i,l} \partial y} = -b, \quad \text{and} \quad \frac{\partial^2 \pi_j(\cdot)}{\partial y_j \partial x_l} = -b, \quad \forall i \in \mathcal{N}_d, \quad \forall j \in \mathcal{N}_f.$$

Elements composing the main diagonal of the Hessian are equal to  $-2b$  while all other elements are equal to  $-b$ . As a consequence, all odd-ordered leading principal minors are strictly negative and all even-ordered leading principal minors are positive, thereby implying strict concavity.  $\square$

**Proof of Lemma 1.** The following equilibrium quantities are required to prove Lemma 1:

LEMMA 8. *With the number of domestic and foreign competitors fixed at  $n_d$  and  $n_f > 0$ , domestic and foreign firms, respectively, produce at equilibrium quantities*

$$x_l^* = \frac{A - c_d(l, \varepsilon)}{b(n_d + n_f + 1)} + \frac{n_f(c_f - c_d(l, \varepsilon))}{b(n_d + n_f + 1)}, \quad \text{and} \quad y^* = \frac{A - c_f}{b(n_d + n_f + 1)} - \frac{n_d(c_f - c_d(l, \varepsilon))}{b(n_d + n_f + 1)}.$$

**Proof of Lemma 8.** Since the problem is symmetric for all domestic firms and is likewise symmetric for all foreign firms, solving (O.2) for  $y^*$  yields

$$y^* = \frac{A - c_f - bn_d x_l}{b(n_f + 1)}, \quad \forall j \in \mathcal{N}_f. \quad (\text{O.3})$$

Substituting (O.3) into (O.1) and then solving for  $x_l$  yields

$$x_l^* = \frac{A - c_d(l, \varepsilon)}{b(n_d + n_f + 1)} + \frac{n_f(c_f - c_d(l, \varepsilon))}{b(n_d + n_f + 1)}, \quad \forall i \in \mathcal{N}_d, \quad (\text{O.4})$$

which, by substituting into (O.3) yields

$$y^* = \frac{A - c_f}{b(n_d + n_f + 1)} - \frac{n_d(c_f - c_d(l, \varepsilon))}{b(n_d + n_f + 1)}, \quad \forall j \in \mathcal{N}_f. \quad \blacksquare \quad (\text{O.5})$$

The number of offshore entrants follows directly from its definition,

$$\begin{aligned} \max_y \pi_f(n_d, n_f, x_l) &= \max_y [P(n_d, n_f, x_l, y)y - c_f y] = F_e, \quad \forall j \in \mathcal{N}_f. \\ &\Rightarrow [A - b(n_d x_l + n_f y)]y - c_f y = F_e, \quad \forall j \in \mathcal{N}_f. \end{aligned}$$

The result follows from the quantities in Equations (O.4) and (O.5), and the constraint that  $n_f \geq 0$ .

$$n_f^* = \max \left\{ 0, \frac{A - c_f - n_d(c_f - c_d(l, \varepsilon))}{\sqrt{F_e b}} - n_d - 1 \right\}. \quad \square \quad (\text{O.6})$$

**Proof of Lemma 2** When domestic firms produce in the regulated region  $c_d(\varepsilon) = \gamma_d + \alpha_d \varepsilon$ , and when domestic firms produce offshore  $c_d(\varepsilon) = \delta\gamma_1 + \tau$ . Domestic quantity solutions for  $\Gamma_1$  follow from (O.4) when  $c_d(\varepsilon) = \delta\gamma_1 + \tau$ ,  $n_d^* = N_d$ , and  $n_f^* = 0$ . Similarly, domestic quantity solutions for  $\Gamma_3$  follow from (O.4) when  $c_d(\varepsilon) = \delta\gamma_1 + \tau$ ,  $n_d^* = N_d$  and  $n_f^* = 0$ . For  $\Gamma_2$  and  $\Gamma_4$ ,  $n_f^* > 0$ , and therefore is equal to the righthand argument of the maximum statement given in (O.6), and  $c_f = \delta\gamma_1 + \tau$ . Domestic quantities then follow directly from (O.4) and foreign quantities follow from (O.5). For  $\Gamma_5$ , foreign quantities are determined directly from (O.5) when  $n_f^*$  is again equal to the righthand argument of (O.6) and  $n_d^* = 0$ .  $\square$

**Proof of Lemma 3** Define  $Q_o$  as the change in total output due to offshoring. Where  $F_o \leq \bar{F}_o$ ,

$$Q_o = n_d^* x_r^*(\varepsilon_o) + n_f^* y^*(\varepsilon_o) - \left[ \lim_{\varepsilon \rightarrow \varepsilon_o^-} (n_d^* x_r^*(\varepsilon) + n_f^* y^*(\varepsilon)) \right].$$

If  $F_o \leq \bar{F}_o$ , then without entry (i.e., when  $n_f^* = y^* = 0$ ), the unique solution to  $\varepsilon_o$  is

$$\varepsilon_o = \frac{c_f - \gamma_d}{\alpha_d} + \frac{A - c_f - \sqrt{(A - c_f)^2 - F_o b(N_d + 1)^2}}{\alpha_d}, \quad (\text{O.7})$$

while, with entry (i.e.,  $n_f^* > 0$ ), the unique solution is

$$\varepsilon_o = \frac{c_f - \gamma_d}{\alpha_d} + \frac{\sqrt{b}(\sqrt{F_e} - \sqrt{F_e - F_o})}{\alpha_d}. \quad (\text{O.8})$$

If  $F_o = 0$ , then  $\lim_{\varepsilon \rightarrow \varepsilon_o^-} c_d(\varepsilon) = c_f$ . Wrt  $F_o$ ,  $c'_d(r, \varepsilon) > 0$ . Therefore, if  $0 < F_o \leq \bar{F}_o$ , then  $h(\varepsilon_o) < \lim_{\varepsilon \rightarrow \varepsilon_o^-} h(\varepsilon)$  because  $c_f = c_d(\varepsilon_o) < \lim_{\varepsilon \rightarrow \varepsilon_o^-} c_d(\varepsilon)$ . Substituting the appropriate values of  $x_l^*$  and  $y^*$  from Lemma 2,  $n_f^*$  from Lemma 1, and  $\varepsilon_o$  from (O.7) and (O.8),

$$Q_o = \begin{cases} \frac{N_d(A - c_f - \sqrt{(A - c_f)^2 - F_o b(N_d + 1)^2})}{b(N_d + 1)} & \text{if } \lim_{\varepsilon \rightarrow \varepsilon_o^-} h(\varepsilon) \leq 0 \\ \frac{((N_d + 1)\sqrt{F_e b} - A + c_f)}{b} & \text{if } h(\varepsilon_o) < 0 < \lim_{\varepsilon \rightarrow \varepsilon_o^-} h(\varepsilon) \\ 0 & \text{if } h(\varepsilon_o) \geq 0. \end{cases} \quad (\text{O.9})$$

By Lemma 1,  $h(\varepsilon_o) < 0 \Leftrightarrow ((N_d + 1)\sqrt{F_e b} - A + c_f)/b > 0$ . Therefore, when  $0 < F_o \leq \bar{F}_o$ ,  $Q_o > 0$  iff  $h(\varepsilon_o) < 0$ , and  $Q_o = 0$  otherwise. If  $F_o = 0$ , then  $h(\varepsilon_o) = \lim_{\varepsilon \rightarrow \varepsilon_o^-} h(\varepsilon)$  with both equal to  $(A - c_f)/\sqrt{F_e b} - N_d - 1$  per Lemma 1. Consequently, per (O.9), if  $F_o = 0$  the interval  $(h(\varepsilon_o), \lim_{\varepsilon \rightarrow \varepsilon_o^-} h(\varepsilon))$  is empty and  $Q_o = 0$ .  $\square$

**Proof of Lemma 4** The proof for Lemma 4 is symmetric to that of Lemma 2 except that  $\hat{c}_d(\varepsilon) = \hat{\gamma}_d + \hat{\alpha}_d\varepsilon$  when domestic firms produce in the regulated region,  $\hat{c}_d(\varepsilon) = \delta\hat{\gamma}_d + \hat{\alpha}_d\varepsilon + \tau$  when they produce offshore, and  $\hat{c}_f(\varepsilon) = \delta\hat{\gamma}_f + \hat{\alpha}_f\varepsilon + \tau$ .  $\square$

**Proof of Lemma 5** The proof to Lemma 5 is symmetric to that of Lemma 3 except that the unique solution to  $\hat{\varepsilon}_o$  without entry is given by (14), while it is given by (15) with entry, and where  $\Delta = (A - \delta\gamma_2 - \tau)\alpha_1 - (A - \gamma_1)\alpha_2$ ,

$$\hat{Q}_o = \begin{cases} \frac{N_d(\Delta - \sqrt{\Delta^2 - F_o b(N_d+1)^2(\alpha_1^2 - \alpha_2^2)})}{(N_d+1)(\alpha_1 + \alpha_2)b} & \text{if } \lim_{\varepsilon \rightarrow \hat{\varepsilon}_o^-} h(\varepsilon) \leq 0 \\ \frac{((N_d+1)\alpha_1 - N_d\alpha_2)\sqrt{F_e b - \alpha_2}\sqrt{(F_e - F_o)b - \Delta}}{(\alpha_1 - \alpha_2)b} & \text{if } h(\hat{\varepsilon}_o) < 0 < \lim_{\varepsilon \rightarrow \hat{\varepsilon}_o^-} h(\varepsilon) \\ 0 & \text{if } h(\hat{\varepsilon}_o) \geq 0. \end{cases} \quad \square$$