Online Supplement

This supplement addresses the setting where there is uncertainty in both customers' valuations and agents' opportunity costs. For convenience, we replace the upper-case superscripts on K^H and K^L with lower-case superscripts: $K^h = k + \Delta$ and $K^l = k - \Delta$. For simplicity, we suppose that the correlation of any pair $\{i, j\} \in \{1, .., \bar{N}\}^2$, where $i \neq j$, of agents' opportunity costs $\text{Corr}(\hat{K}_i, \hat{K}_j) = 1$, which implies $\hat{K}_i = \hat{K}_j$. Accordingly, we drop the subscript on $\hat{K}$. For further simplicity, we suppose that in one state of the world (when the weather is "bad") all customers have valuation V^h and all agents have opportunity cost K^h, and in the other state of the world (when the weather is "good") all customers have valuation V^l and all agents have opportunity cost K^l. More precisely, $(\hat{V}, \hat{K}) = (V^h, K^h)$ and $(\hat{V}, \hat{K}) = (V^l, K^l)$ with equal probability. Let $\pi^a = r^a - \bar{N}k$ and $\pi^j = [r^j - \bar{N}K^j]/2$, where $r^a = \max_{p \geq 0}\{p\Sigma_{j\in\{h,l\}}\lambda(V^j, p, \bar{N})/2\}$ and $r^j = \max_{p\geq 0}\{p\lambda(V^j, p, \bar{N})\}$ for $j \in \{h, l\}$. The definitions of p^a and p^h provided prior to Lemma A2 imply that $p^a \in \arg\max_{p\geq 0}\{p\Sigma_{j\in\{h,l\}}\lambda(V^j, p, \bar{N})/2\}$ and $p^h \in \arg\max_{p\geq 0}\{p\lambda(V^h, p, \bar{N})\}$. We abuse notation by defining $p^l \in \arg\max_{p\geq 0}\{p\lambda(V^l, p, \bar{N})\}$. Let $\delta^L = \sup\{\delta : \pi^l \geq \pi^h,\ \delta \in [0, v]\}$ and $\Delta^a = (2r^a - r^l)/\bar{N} - k$. Note $\lim_{\delta\to 0} \Delta^a = r/\bar{N} - k$, where $r = \max_{p\geq 0}\{p\lambda(v, p, \bar{N})\}$.

Lemma B1 *In the firm-employee business model, the firm's optimal price $p_I^* = p^a$ and the firm activates $\bar{N}$ employee-agents. The firm activates at least one agent for any realized customers' valuation and agents' opportunity cost (V, K) only if $K^h < p^a\lambda(V^h, p^a, \bar{N})/\bar{N}$. Further, $\delta^L > 0$. Suppose $\delta < \delta^L$; then $\pi^l > \pi^h$; and further, the firm activates at least one agent for any realized (V, K) if and only if $\Delta < \Delta^a$.*

Proof of Lemma B1: After setting price p and observing realized valuation and opportunity cost (V^j, K^j) where $j \in \{h, l\}$, the firm's employee-agent activation problem is $\max_{n\in\{0,1,..,\bar{N}\}}\{p\lambda(V^j, p, n) - nK^j\}$. Because $\lambda(V^j, p, n)/n$ strictly increases with n (by Lemma A1a),

$$\arg\max_{n\in\{0,1,..,\bar{N}\}}\{p\lambda(V^j, p, n) - nK^j\} = \begin{cases} \bar{N} \text{ if } K^j < p\lambda(V^j, p, \bar{N})/\bar{N} \\ 0 \text{ otherwise.} \end{cases}$$

If

$$K^j \geq p\lambda(V^j, p, \bar{N})/\bar{N}, \tag{41}$$

then it is optimal for the firm to activate zero agents. Because, by assumption, the firm activates at least one agent for any realized (V, K), this implies that there exists $p \geq 0$ such that

$$K^i < p\lambda(V^i, p, \bar{N})/\bar{N} \tag{42}$$

and that the optimal p satisfies (42) for $i = h$ and $i = l$. If p satisfies (42) for $i = h$ and $i = l$, then the firm's objective function simplifies to $\pi^a(p) = p\Sigma_{j\in\{h,l\}}\lambda(V^j, p, \bar{N})/2 - \bar{N}k$, which is maximized at $p = p^a$, yielding expected profit rate π^a. If p satisfies (41) for $j \in \{h, l\}$ and (42) for $i \in \{h, l\}$

where $i \neq j$, then the firm's objective function simplifies to $\pi^i(p) = [p\lambda(V^i, p, \bar{N}) - \bar{N}K^i]/2$, which is maximized at $p = p^i$, yielding expected profit rate π^i. Thus, our assumption that the firm activates at least one agent for any realized (V, K) is satisfied if and only if $\pi^a > \max\{\pi^j\}_{j \in \{h,l\}}$. We next show that $\delta^L > 0$ and that for $\delta \in [0, \delta^L)$, $\pi^l > \pi^h$. As $\delta \to 0$, $\pi^j \to [r - \bar{N}K^j]/2$ for $j \in \{h, l\}$ and $\pi^h - \pi^l \to -\bar{N}\Delta < 0$. Therefore that $\delta^L > 0$ follows from π^h and π^l being continuous in δ. For the remainder of the proof, suppose $\delta < \delta^L$. Because π^h strictly increases and π^l strictly decreases with δ, this implies $\pi^l > \pi^h$. Further, $\pi^a > \pi^l$ if and only if $\Delta < \Delta^a$. Therefore, $\pi^a > \max\{\pi^j\}_{j \in \{h,l\}}$ if and only if $\Delta < \Delta^a$.■

Lemma B2 *There exists* $\Delta^l \in [0, k]$ *such that if* $\delta < \delta^L$ *and* $\Delta > \Delta^l$, *then the platform's optimal price* $p^* = p^l$. *Further,* $\lim_{\delta \to 0} \Delta^l = \min(k, (r/\bar{N} - k)/3)$.

Proof of Lemma B2: We first characterize how many agents participate in equilibrium under various wages, prices and realized valuations and opportunity costs. Let $(\underline{j}(p), \bar{j}(p)) \in \{h, l\}^2$ denote the solution to $K^{\underline{j}(p)}/\lambda(V^{\underline{j}(p)}, p, \bar{N}) \leq K^{\bar{j}(p)}/\lambda(V^{\bar{j}(p)}, p, \bar{N})$ and $\underline{j}(p) \neq \bar{j}(p)$. If price $p < V^l$ and wage $\omega \geq \bar{N}K^{\bar{j}(p)}/\lambda(V^{\bar{j}(p)}, p, \bar{N})$, then $\bar{N}$ agents participate. If $p < V^{\underline{j}(p)}$ and $K^{\underline{j}(p)}/\lambda(V^{\underline{j}(p)}, p, \bar{N}) \leq \omega < K^{\bar{j}(p)}/\lambda(V^{\bar{j}(p)}, p, \bar{N})$, then $\bar{N}$ agents participate if $(V, K) = (V^{\underline{j}(p)}, K^{\underline{j}(p)})$, and no agents participate otherwise. Otherwise, no agents participate. Consequently, we can restrict attention to two wages $\omega \in \{K^j/\lambda(V^j, p, \bar{N})\}_{j \in \{\underline{j}(p), \bar{j}(p)\}}$. Under $\omega = K^{\underline{j}(p)}/\lambda(V^{\underline{j}(p)}, p, \bar{N})$, the platform's objective function simplifies to $\pi^{\underline{j}(p)}(p) = [p\lambda(V^{\underline{j}(p)}, p, \bar{N}) - \bar{N}K^{\underline{j}(p)}]/2$, which is maximized at $p = p^{\underline{j}(p)}$, yielding expected profit rate $\pi^{\underline{j}(p)}$. Under $\omega = K^{\bar{j}(p)}/\lambda(V^{\bar{j}(p)}, p, \bar{N})$, the platform's objective function simplifies to $\pi^m(p) = [p - \bar{N}K^{\bar{j}(p)}/\lambda(V^{\bar{j}(p)}, p, \bar{N})]\Sigma_{j \in \{h,l\}}\lambda(V^j, p, \bar{N})/2$. Let $p^m \in \arg\max_{p \geq 0} \pi^m(p)$ and $\pi^m = \pi^m(p^m)$. Therefore, if $\pi^l > \max\{\pi^j\}_{j \in \{h,m\}}$, then the platform's optimal price $p^* = p^l$. Let $\overline{\pi}^m(p) = [p - \bar{N}K^h/\lambda(V^h, p, \bar{N})]\Sigma_{j \in \{h,l\}}\lambda(V^j, p, \bar{N})/2$, $\overline{p}^m \in \arg\max_{p \geq 0} \overline{\pi}^m(p)$ and $\overline{\pi}^m = \overline{\pi}^m(\overline{p}^m)$. Because $K^{\bar{j}(p)}/\lambda(V^{\bar{j}(p)}, p, \bar{N}) \geq K^h/\lambda(V^h, p, \bar{N})$, $\overline{\pi}^m \geq \pi^m$. Because $\overline{\pi}^m$ strictly decreases and π^l strictly increases with Δ, there exists $\Delta^l \in [0, k]$ such that $\pi^l > \overline{\pi}^m$ if and only if $\Delta > \Delta^l$. As $\delta \to 0$, $\overline{\pi}^m \to r - \bar{N}K^h$ and $\pi^l \to (r - \bar{N}K^l)/2$, so $\Delta^l \to \min(k, (r/\bar{N} - k)/3)$. Because $\pi^l > \pi^h$ for $\delta < \delta^L$ (by Lemma B1), if $\delta < \delta^L$ and $\Delta > \Delta^l$, then $\pi^l > \max\{\pi^j\}_{j \in \{h,m\}}$.■

Proposition B1 *There exists* $\delta^l > 0$ and $\Delta^l \in [0, k]$ *such that: if* $\delta < \delta^l$ *and* $\Delta > \Delta^l$, *then agent independence decreases the optimal price* $p^* < p^*_I$; *if* $\delta < \delta^l$, *then* $\Delta^l < \Delta^a$, *and* $\Delta^l < k$ *if* $k > 5r/(9\bar{N})$.

Proof of Proposition B1: The proof proceeds in four steps. First, by argument parallel to that in the proof of Lemma A2b, $p^l < p^a$. Second, let $\delta^l = \min(\delta^L, \delta^l_2, \delta^l_3, \delta^l_4)$, where $\delta^l_j > 0$ for $j \in \{2, 3, 4\}$ will be defined subsequently. Because $\delta^l \leq \delta^L$, if $\delta < \delta^l$ and $\Delta > \Delta^l$, then $p^* = p^l$ (from Lemma B2). Further $p^l < p^a = p^*_I$, where the inequality follows from step one and the equality from Lemma B1. Third, suppose $r/\bar{N} \leq k$. This implies $r/\bar{N} < K^h$. Because $p^a\lambda(V^h, p^a, \bar{N})$ is continuous in δ and $\lim_{\delta\to 0} p^a\lambda(V^h, p^a, \bar{N}) = r$, $r/\bar{N} < K^h$ implies that there exists $\delta_1^l > 0$ such that $\delta < \delta_1^l$ implies $p^a\lambda(V^h, p^a, \bar{N}) < K^h$, which contradicts our assumption that the firm activates at least one agent under any realized (V, K) (from Lemma B1). We conclude that there exists $\delta_2^l > 0$ such that $\delta < \delta_2^l$ implies $r/\bar{N} > k$. Fourth, because Δ^a and Δ^l are continuous in δ and $\lim_{\delta\to 0}[\Delta^a - \Delta^l] = \max(r/\bar{N} - 2k, 2(r/\bar{N} - k)/3) > 0$ (from Lemma B2), there exists $\delta_3^l > 0$ such that $\delta < \delta_3^l$ implies $\Delta^a > \Delta^l$. Because Δ^l is continuous in δ and $\lim_{\delta\to 0}\Delta^l = \min(k, (r/\bar{N} - k)/3) > 0$ (from Lemma B2), there exists $\delta_4^l > 0$ such that $\delta < \delta_4^l$ and $k > 5r/(9\bar{N})$ imply $\Delta^l < k$.■