Online Supplement
"Shared Mobility for Last-Mile Delivery: Design, Operational Prescriptions and Environmental Impact"

Appendix A: Notation

Table 1 Notation

Symbol	Description
x	location coordinate, or the index of the service zone with its terminal at x
$n(x)$	mean density of demand destinations at x
$g(x)$	mean package weight at destination x
$\bar{m}(x)$	mean density of cars subscribed for shared use at x
$m(x)$	mean density of available cars at x
$\tilde{m}(x)$	endogenous mean density of available cars at x
$A(x)$	area of the service zone that has a terminal at x
$m_z(x)$	average number of cars required to deliver packages in the service zone that has a terminal at x
v_c, v_t	maximum weight of packages loaded to a car and a truck, respectively
k	maximum number of destinations visited by a car, $= v_c/g$ on average
λ	detour coefficient
α, β, γ	line-haul coefficients
$w(d_o)$	wage payment to a car that travels an outbound trip of distance d_o
w_b,w_m	base fare and per-minute fare of a ride-share service, respectively
$w_p(\tau)$	expected car driver's income out of ride-share services that are requested over a time window $(0, \tau)$
s	normal cruise speed of a car
s_o	speed of a car in delivery services
$\bar{\mu}(x)$	rate of total ride-share demand per unit area at x
$\mu(x)$	rate of ride-share service requests that a driver observes at x
$\tilde{\mu}(x)$	endogenous rate of ride-share requests that a driver observes at x
$\nu(x)$	1/(mean duration of a shared ride)
η	discount factor accounting for faster vehicle depreciation and more physical effort during delivery services
$F(\cdot)$	Proportion of the registered car drivers that are available for shared use, as a function of the expected earning rate
$W_c(x, A(x)), W_t(x, A(x))$	cost density of hiring shared mobility and bulk trucking, respectively, for the service zone that has a terminal at x and area $A(x)$
$d_{t,l}, d_{t,d}$	total truck line-haul trip length, total truck detour trip length
$r_d(x)$	distance from the depot to destination x
r_c	terminal-bound trip distance, cars within which are sufficient for deliveries.
c_t, c_t'	per-km truck operating cost in line-haul trips and last-mile trips, respectively
C	total operating cost
C_b	total operating cost of the truck-only system

Appendix B: Continuous Approximation of OVRP Routes

A car's outbound trip consists of the following two parts:

$$d_{OVRP} = d_l(n, \frac{v_c}{g}) m_z + \lambda A \sqrt{n}, \tag{E1}$$

where the two terms on the right-hand side represent the total line-haul and detour trip lengths, respectively. In particular, $d_l(n, \frac{v_c}{g})$ is the average of individual line-haul trip lengths. d_l primarily depends on the demand density n as well as the number of destinations that each car visits, as measured by $\frac{v_c}{g}$. The detour portion is asymptotically structurally similar to a traveling salesman problem (TSP) tour as n goes to infinity. Hence, the total detour length is approximated to be proportional to $\sqrt{A(An)} = A\sqrt{n}$ with a coefficient λ.

The appropriate function form of $d_l(n, \frac{v_c}{g})$ and the value of λ should fit with the optimal routes, which are solutions to the open vehicle routing problems (OVRPs). In this work, a record-to-record heuristic embedded in the open-source library VRPH (Groër et al. (2010)) is applied to solve $96,000$ randomly generated OVRP instances with different values of n, $\frac{v_c}{g}$, variation coefficient of package weights and random realizations of destinations (Li et al. (2007) demonstrate the high accuracy of this type of heuristic). Based on these numerical results, the following function form is proposed to approximate the average individual line-haul trip length:

$$d_l(n, \frac{v_c}{g}) = (\frac{\sqrt{2}}{3} - \alpha(\frac{v_c}{g})^{\beta}(1 - \frac{1}{\gamma\sqrt{An}}))\sqrt{A}, \tag{E2}$$

where $\alpha = 0.055$, $\beta = 0.470$ and $\gamma = 0.374$ are parameters that minimize the sum of squared approximation errors of the problem instances. Despite the certain degree of misestimation (up to 10.7% when $n = 70$ and $\frac{v_c}{g} = 23$), the model and the estimates capture the one-way routing structural properties and are effective for a wide range of parameter settings. Further validation with real out-of-sample data is beyond the scope of this paper. Similarly, the minimum-squared-error estimate of λ is 1.005, which is close to 0.97 as Jaillet (1988) estimates for TSP tour with the Manhattan metric. Substituting $m_z = \frac{gAn}{v_c}$ and (E2) into formula (E1) yields the routing CA model in Result 1.

Appendix C: Structure Properties of OVRP

Proof for Proposition 1 The proof is constructive. A heuristic procedure is used to construct a feasible solution to VRP_k starting from an optimal OVRP tour with vehicles making most use of their capacities. Given such an optimal OVRP tour, there are $\lceil n/k \rceil$ routes originating from the center. To make the open-loop routes closed, endpoint of each route is connected to the center by a line segment. As a result, $\lceil n/k \rceil$ line segments are added and a feasible solution to $\text{VRP}_k(X^n)$ is derived with the length that equals to $\sum_{i \in \{\text{End Points}\}} r_i + \text{OVRP}_k(X^n)$.

Clearly, the optimal solution to VRP_k has tour length less than or equal to the above heuristic solution. Each r_i is bounded by $r^n_{\max}$. Hence, $\text{VRP}_k(X^n) \leq \sum_{i \in \{\text{End Points}\}} r_i + \text{OVRP}_k(X^n) \leq \left\lceil \frac{n}{k} \right\rceil r^n_{\max} + \text{VRP}_k(X^n)$. By rearranging terms the second inequality does hold and the first inequality follows naturally from the fact that the optimal TSP solution is no greater than the optimal VRP solution. □

Proof for Proposition 2 The proof is constructive. A heuristic procedure is used to construct a feasible solution to VRP_{2k} in which each vehicle can carry at most $2k$ packages. Given an optimal OVRP tour with vehicles fully loaded, there are $\lceil n/k \rceil$ routes originating from the center. Denote the set of endpoints of routes in $\text{OVRP}_k(X^n)$ as $X^{\lceil \frac{n}{k} \rceil}$. Consider two cases in terms of $\lceil n/k \rceil$:

(1) If $\lceil n/k \rceil$ is even, an optimal TSP tour that connects points in $X^{\lceil \frac{n}{k} \rceil}$ is first constructed. The number of edges in this TSP tour is also even. Then select $\frac{1}{2}\lceil n/k \rceil$ edges from the TSP tour such that each point is connected with exactly one other point in the tour. It should be noted that there are two different selections and they are compliment to each other in the TSP tour. Compare the two sets of edges and choose the one with smaller total length. Hence the chosen $\frac{1}{2}\lceil n/k \rceil$ edges have a total length less than $\frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil})$. Next, add these edges to the OVRP tour as such each route is connected to one other route in the OVRP tour. So every pairs of routes in the OVRP solution plus added edges constitute closed tours with each traversing at most $2k$ demand points. As a result, a feasible solution to $\text{VRP}_{2k}(X^n)$ is generated with the length of $\big(\text{Total Length of Selected Edges from } \text{TSP}(X^{\lceil \frac{n}{k} \rceil}) + \text{OVRP}_k(X^n)\big)$, which is less than $\frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil}) + \text{OVRP}_k(X^n)$. It thus follows $\text{VRP}_{2k}(X^n) \ \leq\ \frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil}) + \text{OVRP}_k(X^n)$.

(2) If $\lceil n/k \rceil$ is odd, first randomly take out one end point in $X^{\lceil \frac{n}{k} \rceil}$ and construct an optimal TSP tour for the remaining end points, i.e. $X^{\lceil \frac{n}{k} \rceil - 1}$. Then select $\frac{1}{2}(\lceil n/k \rceil - 1)$ from the TSP tour in a manner similar to (1). The total length of selected edges should be less than $\frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil - 1})$. Add these edges to the OVRP tour and every pairs of routes except for one route (that contains the taken-out point) will make up closed tours with each traversing at most $2k$ demand points. To transform the remaining route into a closed tour, a line segment that links the center and the end point is added afterward. With these extra edges and line segment, a feasible VRP tour with capacity of $2k$ is constructed selected from the OVRP tour. The length of the feasible VRP tour consists of three parts: length of edges from the TSP tour, length of the line segment and $\text{OVRP}_k(X^n)$. The first part is less than $\frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil - 1}) \leq \frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil})$ and the second part is less than $r^n_{\max}$. So the total length of the feasible solution is less than $\frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil}) + r^n_{\max} + \text{OVRP}_k(X^n)$. Obviously, the optimal VRP tour is a lower bound to the feasible solution, so $\text{VRP}_{2k}(X^n) \ \leq\ \frac{1}{2}\text{TSP}(X^{\lceil \frac{n}{k} \rceil}) + r^n_{\max} + \text{OVRP}_k(X^n)$.

Combine the two inequalities in (1) and (2) together and the desired result follows. □

Proof for Proposition 3 To prove this proposition, the following lemma is introduced first.

LEMMA 1. *$\text{OVRP}(X^n) \ \leq\ \left\lceil \frac{n}{k} \right\rceil \overline{r} + (1 - \frac{\lceil n/k \rceil}{n})\text{TSP}(X^n)$, where $\overline{r}$ is the mean distance from X^n to its center.*

The proof of this lemma follows the well known heuristic, Optimal Tour Partition (OTP), as used in Haimovich and Kan (1985). The basic idea is to construct a feasible OVRP tour by breaking a TSP tour into several segments and repeat the construction by moving the endpoints in original routes in a certain orientation.

By Lemma 1 and Proposition 2,

$$\frac{\text{VRP}_{2k}(X^n)}{n} - \frac{\text{TSP}(X^{\lceil \frac{n}{k} \rceil})}{2n} - \frac{r^n_{\max}}{n} \ \leq\ \frac{\text{OVRP}_k(X^n)}{n} \ \leq\ \frac{1}{n}\lceil \frac{n}{k} \rceil \overline{r} + (1 - \frac{\lceil n/k \rceil}{n})\frac{\text{TSP}(X^n)}{n}.$$

Utilize the asymptotic results for TSP and VRP derived in Haimovich and Kan (1985) that

$$\lim_{n\to\infty}\frac{\mathrm{TSP}(X^n)}{n}=0 \quad \text{and} \quad \lim_{n\to\infty}\frac{\mathrm{VRP}_k(X^n)}{n}=\frac{2E(r)}{k}.$$

Also note that $\bar{r}\to E(r)$, $\frac{1}{n}\lceil n/k\rceil\to\frac{1}{k}$ and $\frac{1}{n}r^n_{\max}\to 0$ as n goes to infinity (the last one follows from the Borel-Cantelli Lemma mentioned by Haimovich and Kan (1985)). Combining these facts gives

$$\lim_{n\to\infty}\frac{\mathrm{VRP}_{2k}(X^n)}{n}-\frac{\mathrm{TSP}(X^{\lceil \frac{n}{k}\rceil})}{2n}-\frac{r^n_{\max}}{n}=\frac{E(r)}{k} \quad \text{and} \quad \lim_{n\to\infty}\frac{1}{n}\lceil\frac{n}{k}\rceil\bar{r}+(1-\frac{\lceil n/k\rceil}{n})\frac{\mathrm{TSP}(X^n)}{n}=\frac{E(r)}{k}.$$

Hence, $\lim_{n\to\infty}\frac{1}{n}\mathrm{OVRP}_k(X^n)=\frac{E(r)}{k}$ $(a.s.)$. □

Appendix D: Model Derivations and Analysis

Result 1 is derived in Appendix B. Results 2, 4 and 5 are directly derived in the main text preceding them. Result 6 is from page 284, Appendix A of Daganzo (2005).

Proof of Result 3 With justifications for omitting the small exponential term, the wage model (4) in Result 4 becomes: $w^*(d_o)=\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)}\sqrt{\frac{gAn}{2mv_c}}\frac{1}{s}+\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)}\frac{d_o}{s_o}+\eta\frac{\mu^2(w_b\nu+w_m)}{\nu(\mu+\nu)^2}$.

Given identities (8a) and (8b) and the d_{OVRP} formula in Result 1, the cost density of shared mobility at x becomes:

$$\begin{aligned}
W_c(x,A(x)) &= \frac{\int_{A(x)} w^*(d_o,x)n_o(d_o,x)dd_o}{A(x)}\\
&=\frac{(\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)}\sqrt{\frac{gAn}{2mv_c}}\frac{1}{s}+\eta\frac{\mu^2(w_b\nu+w_m)}{\nu(\mu+\nu)^2})}{A}\int_{A(x)}n_o(d_o,x)dd_o+\frac{\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)s_o}}{A}\int_{A(x)}d_on_o(d_o,x)dd_o\\
&=\frac{(\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)}\sqrt{\frac{gAn}{2mv_c}}\frac{1}{s}+\eta\frac{\mu^2(w_b\nu+w_m)}{\nu(\mu+\nu)^2})}{A}\frac{gAn}{v_c}+\frac{\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)s_o}}{A}d_{OVRP}\\
&=\frac{(\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)}\sqrt{\frac{gAn}{2mv_c}}\frac{1}{s}+\eta\frac{\mu^2(w_b\nu+w_m)}{\nu(\mu+\nu)^2})}{A}\frac{aAn}{v_c}+\frac{\eta\frac{\mu(w_b\nu+w_m)}{(\mu+\nu)s_o}}{A}((\frac{\sqrt{2}}{3}-\alpha(\frac{v_c}{g})^\beta(1-\frac{1}{\gamma\sqrt{An}}))\frac{gA^{1.5}n}{v_c}+\lambda A\sqrt{n})\\
&=\eta(\frac{g\mu}{v_c}\sqrt{\frac{gn}{2mv_c}}\frac{w_b\nu+w_m}{s(\mu+\nu)}+\frac{\mu(w_b\nu+w_m)}{s_o(\mu+\nu)}(\frac{\sqrt{2}}{3}-\alpha(\frac{v_c}{g})^\beta)\frac{g}{v_c})n\sqrt{A}\\
&\quad+\frac{\eta g\mu^2(w_b\nu+w_m)n}{v_c\nu(\mu+\nu)^2}+\frac{\eta\mu(w_b\nu+w_m)}{s_o(\mu+\nu)}\big(\alpha(\frac{v_c}{g})^\beta\frac{\sqrt{n}g}{\gamma v_c}+\lambda\sqrt{n}\big)\\
&=\Phi_c(x)\sqrt{A(x)}+\Psi_c(x).
\end{aligned}$$

Proof of formula (3) Model the passenger riding process as an alternate renewal process, in which the *on* state is waiting and the *off* state is riding. The driver is initially on for a time Z_1 and then remains off for a time Y_1; then he/she goes on for a time Z_2; then off for a time Y_2; then on, and so forth. The sequence of two random variables $\{Z_i\}$ and $\{Y_i\}$, $i\geq 1$, are independent and identically distributed as exponentials with μ and ν. A renewal cycle $\{V_i=Z_i+Y_i\}$ consists of an on period and an off period. The process starts over again when the driver picks up a new passenger. Let $N(x)$ denote the number of complete renewal cycle up to time x and S_n be the completing time of nth renewal. They have a relationship that $N(x)=\max\{n:\ S_n\leq x\}$. $m(t)$ is the mean of $N(x)$, i.e. $m(x)=E(N(x))$.

During renewal cycle i, the driver receives a reward $R_i=w_b+w_mY_i$. It follows that $E(R_i)=w_b+w_mE(Y_i)=w_b+w_m/\nu=E(R)$. The objective is to calculate the expected total rewards during $(0,x)$. In the stochastic setting, the total rewards is differentiated in two cases: (1) When the driver is on at time x, i.e., the driver has finished $N(x)$ rides and the expected total rewards is $E(\sum_{i=1}^{N(x)} R_i)$; (2) When the driver is off at time x, the expected total rewards has two components: the reward from the $N(x)$ complete rides and the reward from the last incomplete ride. Since in practice the ride is always completed, the reward for the last ride is assumed to be complete, that is, the driver gets full rewards. Then the expected total reward is $E(\sum_{i=1}^{N(x)+1} R_i)$ in this case. Hence, combining these two cases, the expected total rewards is given by

$$\begin{aligned} w_p(x) &= E(\sum_{i=1}^{N(x)} R_i | \text{ on at } x)\mathbb{P}(\text{on at } x) + E(\sum_{i=1}^{N(x)+1} R_i | \text{ off at } x)\mathbb{P}(\text{off at } x) \\ &= E(\sum_{i=1}^{N(x)+1} R_i) - E(R_{N(x)+1} | \text{ on at } x)\mathbb{P}(\text{on at } x) = (m(x)+1)E(R) - E(R_{N(x)+1} | \text{ on at } x)\mathbb{P}(\text{on at } x), \end{aligned}$$

where the first two equalities use the law of total probability and the last equality utilizes the Wald's equation by observing that $N(x)+1$ is a stopping time for $\{R_i\}$, $i \geq 1$ (Ross (1996)). First, $m(x)$ is known as $m(x) = \frac{\mu\nu}{\mu+\nu}\Big(x - \frac{1}{\mu+\nu}(1-\exp(-(\mu+\nu)x)\Big)$ (Wolstenholme (1999)). Second, $\mathbb{P}(\text{on at } x)$ is given by $\mathbb{P}(\text{on at } x) = \frac{\nu}{\mu+\nu} + \frac{\mu}{\mu+\nu}\exp(-(\mu+\nu)x)$ (Tijms (2003)). And

$$\begin{aligned} E(R_{N(x)+1} | \text{ on at } x) &= \int_0^\infty E(R_{N(x)+1} | \; S_{N(x)} = y, \text{on at } x) dF_{S_{N(x)} | \text{ on at } x} \\ &= \int_0^\infty E(R_{N(x)+1} | \; Z_{N(x)+1} > x - y) dF_{S_{N(x)} | \text{ on at } x} = \int_0^\infty E(R) dF_{S_{N(x)} | \text{ on at } x} = E(R). \end{aligned}$$

Combing the above equations, it follows that $w_p(x) = \frac{\mu(w_b + w_m/\nu)}{\mu+\nu}\Big(\nu x + \frac{\mu}{\mu+\nu}(1-\exp(-(\mu+\nu)x))\Big)$. $\square$

Proof of Proposition 4 To show $\frac{d\tilde{m}}{d\bar{\mu}} > 0$, let $y = \frac{\tilde{\mu}(\nu w_b + w_m)}{\tilde{\mu}+\nu}$, which is the car drivers' expected earning rate. Substituting $\tilde{\mu} = \frac{\bar{\mu}}{\tilde{m} - \frac{ng}{v_c}}$ from (6) yields $y = \frac{\bar{\mu}(\nu w_b + w_m)}{\bar{\mu}+\nu(\tilde{m}-ng/v_c)}$. From the model (5), $\tilde{m} = \bar{m}F(y)$. Let $G(\tilde{m}, y) = \bar{m}F(y) - \tilde{m} = 0$. By the implicit function theorem, $\frac{d\tilde{m}}{d\bar{\mu}} = -\frac{\partial G/\partial\bar{\mu}}{\partial G/\partial\tilde{m}} = -\frac{\frac{\partial F}{\partial y}\frac{dy}{d\bar{\mu}}\bar{m}}{\frac{\partial F}{\partial y}\frac{dy}{d\tilde{m}}\bar{m}-1}$, in which $\frac{dy}{d\bar{\mu}} = \frac{\nu(\tilde{m}-ng/v_c)}{(\bar{\mu}+\nu(\tilde{m}-ng/v_c))^2} > 0$, $\frac{dy}{d\tilde{m}} = \frac{-\nu}{(\bar{\mu}+\nu(\tilde{m}-ng/v_c))^2} < 0$, since $\nu > 0$ and $\tilde{m} - ng/v_c > 0$ in reality. In addition, $\frac{\partial F}{\partial y} > 0$ by definition. Hence, $\frac{d\tilde{m}}{d\bar{\mu}} > 0$.

To show $\frac{d\tilde{\mu}}{d\bar{\mu}} > 0$, first notice that $\tilde{m}$ is a concave function of $\bar{\mu}$. This is true because, from the above equations, $\frac{d\tilde{m}}{d\bar{\mu}} = -\frac{\frac{\partial F}{\partial y}\frac{dy}{d\bar{\mu}}\bar{m}}{\frac{\partial F}{\partial y}\frac{dy}{d\tilde{m}}\bar{m}-1} = \frac{-\frac{dy}{d\bar{\mu}}}{\frac{dy}{d\tilde{m}} - \frac{1}{\frac{\partial F}{\partial y}\bar{m}}} = \frac{-\frac{\nu(\tilde{m}-ng/v_c)}{(\bar{\mu}+\nu(\tilde{m}-ng/v_c))^2}}{\frac{-\nu}{(\bar{\mu}+\nu(\tilde{m}-ng/v_c))^2} - \frac{1}{\frac{\partial F}{\partial y}\bar{m}}} = \frac{\tilde{m}-ng/v_c}{1+\frac{1}{\frac{\partial F}{\partial y}\bar{m}\nu}(\bar{\mu}+\nu(\tilde{m}-ng/v_c))^2}$. Then, $\frac{d^2\tilde{m}}{d\bar{\mu}^2} = \frac{-\frac{2}{\frac{\partial F}{\partial y}\bar{m}\nu}(\bar{\mu}+\nu(\tilde{m}-ng/v_c))(\tilde{m}-ng/v_c)}{(1+\frac{1}{\frac{\partial F}{\partial y}\bar{m}\nu}(\bar{\mu}+\nu(\tilde{m}-ng/v_c))^2)^2} < 0$. Therefore, $\tilde{m}$ is a strongly concave function of $\bar{\mu}$. Immediately, the density of cars for ride-share services, denoted by $H(\bar{\mu}) \equiv \tilde{m} - ng/v_c$, is a strongly concave function of $\bar{\mu}$, too. In addition, $H(0) = 0$, which represents the extreme case with neither supply nor demand for ride-share. By the definition of the differentiable and strongly concave function, $H(\bar{\mu})$ satisfies $\frac{dH(\bar{\mu})}{d\bar{\mu}}(0 - \bar{\mu}) + H(\bar{\mu}) > H(0)$. Plugging $H(\bar{\mu}) = \tilde{m} - ng/v_c$, $\frac{dH(\bar{\mu})}{d\bar{\mu}} = \frac{d\tilde{m}}{d\bar{\mu}}$ and $H(0) = 0$ into the above inequality yields $\tilde{m} - ng/v_c - \frac{d\tilde{m}}{d\bar{\mu}}\bar{\mu} > 0$. Finally, $\frac{d\tilde{\mu}}{d\bar{\mu}} = \frac{d(\frac{\bar{\mu}}{\tilde{m}-ng/v_c})}{d\bar{\mu}} = \frac{\tilde{m}-ng/v_c - \frac{d\tilde{m}}{d\bar{\mu}}\bar{\mu}}{(\tilde{m}-ng/v_c)^2} > 0$. $\square$

Numerical test of the synergy and the competition effects Proposition 4 identifies the synergy and the competition effects that the ride-share service market exerts on the delivery service market. The overall effect of increasing ride-share demand on the total operating cost cannot be conclusively stated, given the involved cost expression (14b). However, numerical experiments with wide ranges of parameter values reveal that the competition effect dominates the overall effect. This is expected, since the operating cost is significantly more sensitive to the ride-share request rate $\tilde{\mu}$ than to the mobility supply $\tilde{m}$. In fact, $\tilde{m}$ only affects cars' inbound trip distances (as the first term on the righthand-side of (10a) captures), which are on average much less than the outbound trips for delivery services.

Proof of Proposition 5 ρ_c and ρ_t shares a common part Ψ_t, so the comparison between those two cost densities boils down to comparing $2\sqrt{\Phi_c\Phi_t}+\Psi_c$ and $0.82c_t'\sqrt{n}$. First, note that $\tilde{m}<\bar{m}$, so $\tilde{\mu}=\frac{\bar{\mu}}{\tilde{m}-\frac{ng}{v_c}}\geq\frac{\bar{\mu}}{\bar{m}-\frac{ng}{v_c}}$. It follows that $\frac{\tilde{\mu}}{\tilde{\mu}+\nu}\geq\frac{\bar{\mu}}{\bar{\mu}+\nu(\bar{m}-\frac{ng}{v_c})}\geq\frac{\bar{\mu}}{\bar{\mu}+\nu\bar{m}}$. Then

$$\begin{aligned}\Phi_c&=\eta\Big(\frac{g\tilde{\mu}}{v_c}\sqrt{\frac{gn}{2\tilde{m}v_c}}\frac{w_b\nu+w_m}{s(\tilde{\mu}+\nu)}+\frac{\tilde{\mu}(w_b\nu+w_m)}{s_o(\tilde{\mu}+\nu)}\Big(\frac{\sqrt{2}}{3}-\alpha\Big(\frac{v_c}{g}\Big)^{\beta}\Big)\frac{g}{v_c}\Big)n\\&\geq\eta\Big(\frac{g}{v_c}\sqrt{\frac{gn}{2\bar{m}v_c}}\frac{(w_b\nu+w_m)\bar{\mu}}{s(\bar{\mu}+\nu\bar{m})}+\frac{\bar{\mu}(w_b\nu+w_m)}{s_o(\bar{\mu}+\nu\bar{m})}\Big(\frac{\sqrt{2}}{3}-\alpha\Big(\frac{v_c}{g}\Big)^{\beta}\Big)\frac{g}{v_c}\Big)n\\&\geq\frac{\bar{\mu}(w_b\nu+w_m)}{s_o(\bar{\mu}+\nu\bar{m})}\Big(\frac{\sqrt{2}}{3}-\alpha\Big(\frac{v_c}{g}\Big)^{\beta}\Big)\frac{g}{v_c}\Big)n=\underline{f}_{\Phi_c}n.\end{aligned}$$

Similarly,

$$\begin{aligned}\Psi_c&=\frac{\eta g\tilde{\mu}^2(w_b\nu+w_m)n}{v_c\nu(\tilde{\mu}+\nu)^2}+\frac{\eta\tilde{\mu}(w_b\nu+w_m)}{s_o(\tilde{\mu}+\nu)}\Big(\alpha\Big(\frac{v_c}{g}\Big)^{\beta}\frac{\sqrt{n}g}{\gamma v_c}+\lambda\sqrt{n}\Big)\\&\geq\frac{\eta g\bar{\mu}^2(w_b\nu+w_m)n}{v_c\nu(\bar{\mu}+\nu\bar{m})^2}+\frac{\eta\bar{\mu}(w_b\nu+w_m)}{s_o(\bar{\mu}+\nu\bar{m})}\Big(\alpha\Big(\frac{v_c}{g}\Big)^{\beta}\frac{\sqrt{n}g}{\gamma v_c}+\lambda\sqrt{n}\Big)=\underline{f}_{\Psi_c,1}n+\underline{f}_{\Psi_c,2}\sqrt{n}.\end{aligned}$$

As a result, $2\sqrt{\Phi_c\Phi_t}+\Psi_c\geq\underline{f}_{\Psi_c,1}n+\Big(\sqrt{\frac{\underline{f}_{\Phi_c}c_t}{\sqrt{2}}}+\underline{f}_{\Psi_c,2}\Big)\sqrt{n}$. In the case when $\sqrt{\frac{\underline{f}_{\Phi_c}c_t}{\sqrt{2}}}+\underline{f}_{\Psi_c,2}\geq 0.82c_t'$, we have $2\sqrt{\Phi_c\Phi_t}+\Psi_c\geq 0.82c_t'\sqrt{n}$ for all $n\geq 0$. Then $\rho_s\geq\rho_t$ for all $n\geq 0$ (the threshold $\bar{n}<0$). Otherwise, $2\sqrt{\Phi_c\Phi_t}+\Psi_c$ may be smaller than $0.82c_t'\sqrt{n}$ for some $n\geq 0$, but it can be easily shown that when $n\geq\Big[\frac{0.82c_t'-\sqrt{\frac{\underline{f}_{\Phi_c}c_t}{\sqrt{2}}}-\underline{f}_{\Psi_c,2}}{\underline{f}_{\Psi_c,1}}\Big]^2$, $\underline{f}_{\Psi_c,1}n+\Big(\sqrt{\frac{\underline{f}_{\Phi_c}c_t}{\sqrt{2}}}+\underline{f}_{\Psi_c,2}\Big)\sqrt{n}\geq 0.82c_t'\sqrt{n}$ and so $\rho_s\geq\rho_t$, in which the threshold $\bar{n}\geq 0$. □

Numerical test of the short-run and the long-rum scalability Figure 1 illustrates the short-run and the long-rum scalability of the business model of using shared mobility for last-mile delivery for Zip-Code Area 9 in the case study. Specifically, Figure 1 (a) shows that, as the delivery demand n scales up (with the baseline pool size of shared mobility $\bar{m}$), the truck-only system is more economically scalable than the shared-mobility system. The shared mobility system is only preferable for small demand densities, which corresponds to the scenario 2 discussed above. Figure 1 (b) identifies the pool sizes of shared mobility that keeps the operating cost of the shared-mobility system smaller than that of the truck-only system as the delivery demand scales up. However, increasing the pool size of shared mobility will reduce the expected earning rate of individual car drivers. The on-demand platforms of shared-mobility services should carefully control this pool size to ensure that the car drivers' expected earning rate is above a certain level. For example, Figure 1 (b) also shows the value ranges of n and $\bar{m}$ that keep the car drivers' long-run expected earning rate to be above 16.23\$/hr, which is the low end of the range of median hourly earnings of Uber driver-partners in six major cities of the U.S. (Hall and Krueger (2016)). For the business model of using shared mobility for last-mile delivery to be both achievable and cost-advantageous, $(n,\bar{m})$ should fall within the area above the dashed line and below the solid line, which is small in this case.

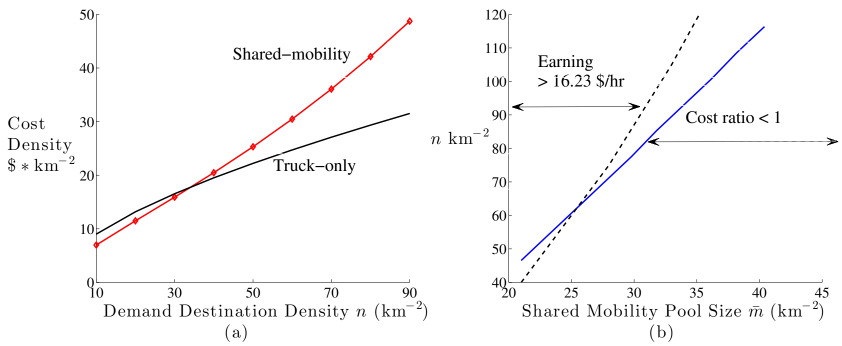

Figure 1 (a) Operating costs per unit area of the shared-mobility logistics system and the truck-only system as the delivery demand scales up. (b) Value ranges of delivery demand and shared mobility pool size that 1) keep the shared-mobility logistics system more cost-efficient than the truck-only system (right of the solid line), and 2) keep the car drivers' earning rate higher than 16.23$/hr (left of the dashed line).

Appendix E: Parameter Settings and Estimates

E.1. Service Region Setting and Baseline Results

E.2. Vehicle-Related Parameter Estimates

E.3. Shared Mobility Supply and Wages

See the External Supplement for E.1 - E.3 from `http://www.optimization-online.org/DB_FILE/2017/10/6264.pdf`

E.4. Justification for Linearizing the Wage Response Function

To justify dropping the exponential part in the third term of the wage response function (4), one needs to show that the value of $exp(-(\tilde{\mu}+\nu)\tau)$ is far less than 1, where τ is the time a car driver takes to finish the terminal-bound trip and the outbound delivery trip. In the baseline scenario of the case study, $\tilde{\mu} = 3.43$ hr^{-1}, $\nu = 4.07$ hr^{-1}. Given the assumptions that the car capacity $v_c = 15g$ and the average duration of a car stop at a destination is 48.5s, and simply assuming that the time between car stops is 48.5s, too, τ can be conservatively estimated as $15 \times 48.5 \times 2 = 1,455\text{s} = 0.40\text{hr}$. With these values of $\tilde{\mu}$, ν and τ, $exp(-(\tilde{\mu}+\nu)\tau) = 0.049$, which is far less than 1. As an even more conservative estimate of $(\tilde{\mu}+\nu)$, choose $\nu = 3.69$ hr^{-1} (which is the lowest among its values in ten major cities in the U.S. in 2015 (SherpaShare (2016)) and $\tilde{\mu} = 2$ hr^{-1} (which is likely to be too low to be realistic according to anecdotal evidence (https://www.quora.com/What-do-Uber-drivers-do-between-rides)). In this case, $exp(-(\tilde{\mu}+\nu)\tau) = 0.1101$. Moreover, this linearization has insignificant impact on the wage payment cost density $W_c(x, A(x))$ in (9). The linearized wage term is factored into the first term of $\Psi_c(x)$ in (10b). Of all the 15 zip-code areas in the baseline scenario, this term accounts for $17.9\% - 51.0\%$ of the cost density $W_c(x, A(x))$, with the mean 39.6%. Therefore, the overall error that this linearization causes to the wage payment estimation is less than 4.4%.

When the demand for delivery services scales up (i.e., n increases), this approximation error is still bounded. In fact, as n increases, the outbound delivery trip time reduces, but the time of doorstep delivery remains unchanged as the product of the number of car stops and the duration of each stop. Put in the above numerical example, τ is bounded from below by $15 \times 48.5 = 727.5\text{s} = 0.20\text{hr}$. The resulting values of $exp(-(\tilde{\mu}+\nu)\tau)$ is bounded from above by 0.220 in the baseline scenario and by 0.317 in the more conservative scenario. The actual values of $exp(-(\tilde{\mu}+\nu)\tau)$ should be smaller in these scenarios, since increasing n intensifies the competition for shared mobility with the ride-share market and thus increases $\tilde{\mu}$. The overestimation of the operating cost can be corrected by approximating $1-exp(-(\tilde{\mu}+\nu)\tau)$ by a factor that is smaller than 1. Since the error is bounded as n increases, the finding that using shared mobility for last-mile delivery is not as economically scalable as the truck-only system is still valid.

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