Online Appendix to the "Optimizing Local Content Requirements Under Technology Gaps" paper co-authored by Shiliang Cui and Lauren X. Lu

Proof of Lemma 1:

Omitted. See pp. 282 of Munson and Rosenblatt (1997).

Proof of Proposition 1:

When $l < l_0$, we have $s = c_1T_1\alpha_1 + c_1(1-\alpha_1) + c_2 = c_1T_1\alpha_1 + c_1 + c_2 - c_1\alpha_1$ and $s \cdot l = c_1T_1\alpha_1$. Therefore, $\alpha_1 = \frac{s \cdot l}{c_1T_1} = \frac{s-c_1-c_2}{c_1(T_1-1)} \Rightarrow s \cdot l \cdot T_1 - s \cdot l = s \cdot T_1 - c_1T_1 - c_2T_1 \Rightarrow c_1T_1 + c_2T_1 = s(T_1 - l \cdot T_1 + l) \Rightarrow s = \frac{(c_1+c_2)T_1}{T_1 - l \cdot T_1 + l} = \frac{c_1+c_2}{1-l+l/T_1}$. In particular, when $l = 0$, $s = c_1 + c_2$. Plugging $s = \frac{c_1+c_2}{1-l+l/T_1}$ into $\alpha_1 = \frac{s \cdot l}{c_1T_1}$, we get $\alpha_1(l) = \frac{(c_1+c_2)l}{c_1(l+T_1-lT_1)}$ for $l < l_0$. In follows that $\alpha_1(0) = 0$ and $\alpha_1(l_1) = \alpha_1(\frac{c_1T_1}{c_1T_1+c_2}) = 1$. Furthermore, $\frac{\partial \alpha_1(l)}{\partial l} = \frac{(c_1+c_2)T_1}{c_1(l+T_1-lT_1)^2} > 0$. When $l = l_0$, $\alpha_1 = 1$ and $\alpha_2 = 0$. And $s = c_1T_1 + c_2$. When $l > l_0$, we have $s = c_1T_1 + c_2T_2\alpha_2 + c_2(1-\alpha_2) = c_1T_1 + c_2T_2\alpha_2 + c_2 - c_2\alpha_2$ and $s \cdot l = c_1T_1 + c_2T_2\alpha_2$. Therefore, $\alpha_2 = \frac{s \cdot l - c_1T_1}{c_2T_2} = \frac{s - c_1T_1 - c_2}{c_2(T_2-1)} \Rightarrow s \cdot l \cdot T_2 - s \cdot l + c_1T_1 = s \cdot T_2 - c_2T_2 \Rightarrow c_1T_1 + c_2T_2 = s[(1-l)T_2 + l] \Rightarrow s = \frac{c_1T_1+c_2T_2}{(1-l)T_2+l}$. In particular, when $l = 1$, $s = c_1T_1 + c_2T_2$. Plugging $s = \frac{c_1T_1+c_2T_2}{(1-l)T_2+l}$ into $\alpha_2 = \frac{s \cdot l - c_1T_1}{c_2T_2}$, we get $\alpha_2(l) = \frac{c_2l - c_1T_1(1-l)}{c_2(l+T_2-lT_2)}$. It follows that $\alpha_2(l_1) = \alpha_2(\frac{c_1T_1}{c_1T_1+c_2}) = 0$ and $\alpha_2(1) = 1$. Furthermore, $\frac{\partial \alpha_2(l)}{\partial l} = \frac{c_1T_1+c_2T_2}{c_2(l+T_2-lT_2)^2} > 0$. Finally, s is continuous at $l = l_0$ (and anywhere else between 0 and 1), because $\frac{c_1+c_2}{1-l+l/T_1}|_{l=l_0} = \frac{c_1T_1+c_2T_2}{(1-l)T_2+l}|_{l=l_0} = c_1T_1 + c_2$ for $l_0 = \frac{c_1T_1}{c_1T_1+c_2}$.

Proof of Proposition 2:

It follows from $p^*(s) = \frac{A+s}{2}$ and $q^* = \frac{A-s}{2}$ that $\frac{1}{2}(A-p^*) \cdot q^* = \frac{1}{2}[p^* - s(l)] \cdot q^*$. Therefore,

$$\begin{aligned} W &= \gamma[p^* - s(l)] \cdot q^* + l \cdot s(l) \cdot q^* + \frac{1}{2}(A - p^*) \cdot q^* \text{ from Eq. (5)} \\ &= (\gamma + \frac{1}{2})[p^* - s(l)] \cdot q^* + l \cdot s(l) \cdot q^* = \{(\gamma + \frac{1}{2})[p^* - s(l)] + l \cdot s(l)\} \cdot q^* \qquad (13) \\ &= \frac{[A + (Kl-1)s]\,(A-s)}{2K} \text{ where } K \triangleq \frac{2}{\gamma + \frac{1}{2}} \in [2, 4]. \qquad (14) \end{aligned}$$

The last step is again due to $p^*(s) = \frac{A+s}{2}$ and $q^* = \frac{A-s}{2}$. We proceed by investigating the partial derivative of W w.r.t. l to identify the optimal LCR level l^*.

When $l \le l_0$, $s = \frac{c_1+c_2}{1-l+l/T_1}$ and $W = \frac{1}{2K}[A + \frac{(c_1+c_2)(Kl-1)}{1-l+l/T_1}](A-s)$. By letting $X_1 \triangleq 1 - l + l/T_1$ and $Y_1 \triangleq c_1 + c_2$, we have

$$\begin{aligned} \frac{\partial(2K \cdot W)}{\partial l} &= \left[A + \frac{(c_1+c_2)(Kl-1)}{1-l+l/T_1}\right]\left[-\frac{(c_1+c_2)(1-1/T_1)}{(1-l+l/T_1)^2}\right] + \left[A - \frac{c_1+c_2}{1-l+l/T_1}\right]\left[\frac{(c_1+c_2)(K-1+1/T_1)}{(1-l+l/T_1)^2}\right] \\ &= \left[A + \frac{Y_1(Kl-1)}{X_1}\right]\left[-\frac{Y_1(1-1/T_1)}{X_1^2}\right] + \left[A - \frac{Y_1}{X_1}\right]\left[\frac{Y_1(K-1+1/T_1)}{X_1^2}\right] \\ &= -A\frac{Y_1}{X_1^2}(1 - \frac{1}{T_1}) - \frac{Y_1^2}{X_1^3}(Kl-1)(1-\frac{1}{T_1}) + A\frac{Y_1}{X_1^2}(K - 1 + \frac{1}{T_1}) - \frac{Y_1^2}{X_1^3}(K-1+\frac{1}{T_1}) \\ &= A\frac{Y_1}{X_1^2}(K - 2 + \frac{2}{T_1}) - \frac{Y_1^2}{X_1^3}\left[(Kl-1)(1-\frac{1}{T_1}) + (K-1+\frac{1}{T_1})\right] \\ &= A\frac{Y_1}{X_1^2}(K-2+\frac{2}{T_1}) - \frac{Y_1^2}{X_1^3}\frac{2(1-T_1) + K[T_1 - l(1-T_1)]}{T_1} \end{aligned}$$

Since $A, X_1, Y_1 > 0, T_1 > 1, K > 2$, W increases in l if and only if $A\frac{Y_1}{X_1^2}(K-2+\frac{2}{T_1}) \ge \frac{Y_1^2}{X_1^3}\frac{2(1-T_1)+K[T_1-l(1-T_1)]}{T_1} \Leftrightarrow A \ge \frac{Y_1}{X_1}\frac{2(1-T_1)+K[T_1-l(1-T_1)]}{(K-2)T_1+2} = (c_1+c_2)T_1\frac{2(1-T_1)+K[T_1-l(1-T_1)]}{[(K-2)T_1+2][T_1+l(1-T_1)]}$ which increases in l because the numerator $\uparrow$ and the denominator $\downarrow$ in l. Moreover, for $l \in [0, l_0 = \frac{c_1T_1}{c_1T_1+c_2}]$, $(c_1+c_2)T_1\frac{2(1-T_1)+K[T_1-l(1-T_1)]}{[(K-2)T_1+2][T_1+l(1-T_1)]} \in [c_1 + c_2, \frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1 + c_2]$. Define $f_1(l) \triangleq (c_1+c_2)T_1\frac{2(1-T_1)+K[T_1-l(1-T_1)]}{[(K-2)T_1+2][T_1+l(1-T_1)]}$. Let l_1^* be such that $f_1(l_1^*) = A$. It follows that $l_1^* = \frac{[A-(c_1+c_2)][(K-2)T_1+2]T_1}{(T_1-1)[A[(K-2)T_1+2]+KT_1(c_1+c_2)]} (\le l_0)$ and $s(l_1^*) = \frac{c_1+c_2}{1-l+l/T_1}|_{l=l_1^*} = \frac{A[(K-2)T_1+2]+KT_1(c_1+c_2)}{2(K-1)T_1+2}$.

When $l \ge l_0$, $s = \frac{c_1T_1+c_2T_2}{(1-l)T_2+l}$ and $W = \frac{1}{2K}[A + \frac{(c_1T_1+c_2T_2)(Kl-1)}{(1-l)T_2+l}](A-s)$. By letting $X_2 \triangleq (1-l)T_2 + l = T_2 - l(T_2-1)$ and $Y_2 \triangleq c_1T_1 + c_2T_2$, this time we have

$$\frac{\partial(2K \cdot W)}{\partial l} = \left[A + \frac{(c_1T_1+c_2T_2)(Kl-1)}{(1-l)T_2+l}\right]\left[-\frac{(c_1T_1+c_2T_2)(T_2-1)}{[(1-l)T_2+l]^2}\right] + \left[A - \frac{c_1T_1+c_2T_2}{(1-l)T_2+l}\right]\left[\frac{(c_1T_1+c_2T_2)[(K-1)T_2+1]}{[(1-l)T_2+l]^2}\right]$$

$$
\begin{aligned}
&= \left[A + \frac{Y_2(Kl-1)}{X_2}\right]\left[-\frac{Y_2(T_2-1)}{X_2^2}\right] + \left[A - \frac{Y_2}{X_2}\right]\left[\frac{Y_2[(K-1)T_2+1]}{X_2^2}\right] \\
&= -A\frac{Y_2}{X_2^2}(T_2-1) - \frac{Y_2^2}{X_2^3}(Kl-1)(T_2-1) + A\frac{Y_2}{X_2^2}[(K-1)T_2+1] - \frac{Y_2^2}{X_2^3}[(K-1)T_2+1] \\
&= A\frac{Y_2}{X_2^2}[(K-2)T_2+2] - \frac{Y_2^2}{X_2^3}\left[K[l(T_2-1)+T_2]+2(1-T_2)\right]
\end{aligned}
$$

Since $A, X_2, Y_2 > 0, T_2 > 1, K > 2$, W increases in l iff $A\frac{Y_2}{X_2^2}[(K-2)T_2+2] \geq \frac{Y_2^2}{X_2^3}\left[K[l(T_2-1)+T_2]+2(1-T_2)\right]$ $\Leftrightarrow A \geq \frac{Y}{X}\frac{K[l(T_2-1)+T_2]+2(1-T_2)}{(K-2)T_2+2} = (c_1T_1+c_2T_2)\frac{K[l(T_2-1)+T_2]+2(1-T_2)}{[(K-2)T_2+2][T_2-l(T_2-1)]}$ which increases in l because the numerator $\uparrow$ and the denominator $\downarrow$ in l. For $l \in [l_0 = \frac{c_1T_1}{c_1T_1+c_2}, 1]$, $(c_1T_1+c_2T_2)\frac{K[l(T_2-1)+T_2]+2(1-T_2)}{[(K-2)T_2+2][T_2-l(T_2-1)]} \in [\frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1 + c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)]$. Define $f_2(l) \triangleq (c_1T_1+c_2T_2)\frac{K[l(T_2-1)+T_2]+2(1-T_2)}{[(K-2)T_2+2][T_2-l(T_2-1)]}$. Let l_2^* be such that $f_2(l_2^*) = A$. It follows that $l_2^* = \frac{[AT_2-(c_1T_1+c_2T_2)][(K-2)T_2+2]}{(T_2-1)[A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)]} (\geq l_0)$ and $s(l_2^*) = \frac{c_1T_1+c_2T_2}{(1-l)T_2+l}|_{l=l_2^*} = \frac{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)}{2(K-1)T_2+2}$.

Now we discuss the five cases listed in the proposition. Case 0: If $A \leq c_1+c_2$, W decreases for $l \in [0, l_0]$ and for $l \in [l_0, 1]$, so $l^* = 0$. It follows that $s(l^*) = s(0) = c_1+c_2 \geq A$, so there is no positive demand outcome in this case because $q^* = \frac{A-s}{2}$. Case I: If $A \in \left(c_1+c_2, \frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2\right)$, W increases for $l \in [0, l_1^*]$ but decreases for $l \in [l_1^*, l_0]$ and for $l \in [l_0, 1]$, so $l^* = l_1^* = \frac{A[(K-2)T_1+2]+KT_1(c_1+c_2)}{2(K-1)T_1+2}$. Also $A > c_1+c_2 \Rightarrow A \cdot KT_1 > KT_1(c_1+c_2) \Rightarrow A[2(K-1)T_1+2] > A[(K-2)T_1+2]+KT_1(c_1+c_2) \Rightarrow A > \frac{c_1+c_2}{1-l+l/T_1}|_{l=l_1^*} = \frac{A[(K-2)T_1+2]+KT_1(c_1+c_2)}{2(K-1)T_1+2} = s(l_1^*) = s(l^*)$ so there is positive demand outcome in this case. Case II: If $A \in \left[\frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2\right]$, W increases for $l \in [0, l_0]$ and decreases for $l \in [l_0, 1]$, so $l^* = l_0 = \frac{c_1T_1}{c_1T_1+c_2}$. Also $A \geq \frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2 > c_1T_1+c_2 = s(l_0) = s(l^*)$ so there is positive demand outcome in this case. Case III: If $A \in \left(\frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)\right)$, W increases for $l \in [0, l_0]$ and for $l \in [l_0, l_2^*]$ but decreases for $l \in [l_2^*, 1]$, so $l^* = l_2^* = \frac{[AT_2-(c_1T_1+c_2T_2)][(K-2)T_2+2]}{(T_2-1)[A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)]}$. Furthermore, $A > c_1T_1+c_2 \Rightarrow AKT_2 > K(c_1T_1+c_2T_2) \Rightarrow A > \frac{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)}{2(K-1)T_2+2} = s(l_2^*) = s(l^*)$ so there is positive demand outcome in this case. Case IV: If $A \geq \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)$, W increases for $l \in [0, l_0]$ and for $l \in [l_0, 1]$, so $l^* = 1$. And $A > c_1T_1+c_2T_2 = s(1) = s(l^*)$ so there is again positive demand outcome. In summary,

Cases	Scenarios (increase in A)	l^* (increases in A)
0.	$A \leq c_1+c_2$	0
I.	$A \in \left(c_1+c_2, \frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2\right)$	$\frac{[A-(c_1+c_2)][(K-2)T_1+2]T_1}{(T_1-1)\{A[(K-2)T_1+2]+KT_1(c_1+c_2)\}} (\leq l_0)$
II.	$A \in \left[\frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2\right]$	$\frac{c_1T_1}{(c_1T_1+c_2)} (= l_0)$
III.	$A \in \left(\frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)\right)$	$\frac{[A\cdot T_2-(c_1T_1+c_2T_2)][(K-2)T_2+2]}{(T_2-1)\{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)\}} (\geq l_0)$
IV.	$A \geq \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)$	1

and results of the proposition follow by defining $F_{K,T} \triangleq \frac{2(K-1)T-(K-2)}{(K-2)T+2} > 1$ and $G_{A,K,T} \triangleq \frac{[AT-(c_1T_1+c_2T)][(K-2)T+2]}{(T-1)[A[(K-2)T+2]+K(c_1T_1+c_2T)]}$, except that we still need to show continuity of l^* in A and that it is concave and increasing on the intervals of Cases I and III. First, l^* is continuous in A because

$$
\begin{aligned}
\frac{[A-(c_1+c_2)][(K-2)T_1+2]T_1}{(T_1-1)\{A[(K-2)T_1+2]+KT_1(c_1+c_2)\}}|_{A\to c_1+c_2} &= 0, \\
\frac{[A-(c_1+c_2)][(K-2)T_1+2]T_1}{(T_1-1)\{A[(K-2)T_1+2]+KT_1(c_1+c_2)\}}|_{A\to \frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2} &= \frac{c_1T_1}{c_1T_1+c_2}, \\
\frac{[A\cdot T_2-(c_1T_1+c_2T_2)][(K-2)T_2+2]}{(T_2-1)\{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)\}}|_{A\to \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2} &= \frac{c_1T_1}{c_1T_1+c_2}, \\
\frac{[A\cdot T_2-(c_1T_1+c_2T_2)][(K-2)T_2+2]}{(T_2-1)\{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)\}}|_{A\to \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)} &= 1.
\end{aligned}
$$

Second, for Case I, when $A \in \left(\frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2\right)$, l^* is concave and increasing in A because given $K > 2$ and $T_1 > 1$,

$$
\partial l^*/\partial(A) = \frac{2(c_1+c_2)T_1[(K-2)T_1+2][(K-1)T_1+1]}{(T_1-1)\{A[(K-2)T_1+2]+K(c_1+c_2)T_1\}^2} > 0,
$$

$$\partial^2 l^*/\partial(A)^2 = -\frac{4(c_1+c_2)T_1[(K-2)T_1+2]^2[(K-1)T_1+1]}{(T_1-1)\{A[(K-2)T_1+2]+K(c_1+c_2)T_1\}^3} < 0.$$

Third, for Case III, when $A \in \left(\frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2, \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)\right)$, l^* is concave and increasing in A because given $K > 2$ and $T_1, T_2 > 1$,

$$\partial l^*/\partial(A) = \frac{(c_1T_1+c_2T_2)[(K-2)T_2+2][(K-2)T_2+K+2]}{(T_2-1)\{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)\}^2} > 0,$$
$$\partial^2 l^*/\partial(A)^2 = -\frac{2(c_1T_1+c_2T_2)[(K-2)T_2+2]^2[(K-2)T_2+K+2]}{(T_2-1)\{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)\}^3} < 0.$$

Proof of Corollary 1:

$s(l^*)$ can be obtained by plugging l^* from Proposition 2 into Proposition 1. $s(l^*)$ is continuous in A because

$$\frac{A[(K-2)T_1+2]+KT_1(c_1+c_2)}{2(K-1)T_1+2}\Big|_{A\to\frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2} = c_1T_1+c_2,$$
$$\frac{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)}{2(K-1)T_2+2}\Big|_{A\to\frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2} = c_1T_1+c_2,$$
$$\frac{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)}{2(K-1)T_2+2}\Big|_{A\to\frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)} = c_1T_1+c_2T_2$$

Furthermore, $s(l^*)$ is clearly piecewise-linear and increasing in A.

Proof of Corollary 2:

Expressions of $W(l^*)$ can be obtained by plugging l^* from Proposition 2 into (5). $W(l^*)$ is continuous in A because when $A \to \frac{2(K-1)T_1-(K-2)}{(K-2)T_1+2}c_1T_1+c_2$,

$$\frac{K[A-(c_1+c_2)]^2T_1^2}{8(T_1-1)[(K-1)T_1+1]} = \frac{1}{2K}(A-c_1T_1-c_2)[A+(K-1)c_1T_1-c_2] = \frac{Kc_1^2T_1^2(T_1-1)[(K-1)T_1+1]}{2[(K-2)T_1+2]^2};$$

When $A \to \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}c_1T_1+c_2$,

$$\frac{1}{2K}(A-c_1T_1-c_2)[A+(K-1)c_1T_1-c_2] = \frac{K[AT_2-(c_1T_1+c_2T_2)]^2}{8(T_2-1)[(K-1)T_2+1]} = \frac{Kc_1^2T_1^2(T_2-1)[(K-1)T_2+1]}{2[(K-2)T_2+2]^2};$$

And when $A \to \frac{2(K-1)T_2-(K-2)}{(K-2)T_2+2}(c_1T_1+c_2T_2)$,

$$\frac{K[AT_2-(c_1T_1+c_2T_2)]^2}{8(T_2-1)[(K-1)T_2+1]} = \frac{1}{2K}(A-c_1T_1-c_2T_2)[A+(K-1)(c_1T_1+c_2T_2)] = \frac{K(T_2-1)[(K-1)T_2+1](c_1T_1+c_2T_2)^2}{2[(K-2)T_2+2]^2}.$$

Furthermore, $W(l^*)$ is increasing and piecewise-convex in A because

Cases	$\frac{\partial W(l^*)}{\partial A}$	$\frac{\partial^2 W(l^*)}{\partial A^2}$
I.	$\frac{K[A-(c_1+c_2)]T_1^2}{4(T_1-1)[(K-1)T_1+1]} > 0$	$\frac{KT_1^2}{4(T_1-1)[(K-1)T_1+1]} > 0$
II.	$\frac{2[A-(c_1+c_2)]+Kc_1T_1}{2K} > 0$	$\frac{1}{K} > 0$
III.	$\frac{KT_2[AT_2-(c_1T_1+c_2T_2)]}{4(T_2-1)[(K-1)T_2+1]} > 0$	$\frac{KT_2^2}{4(T_2-1)[(K-1)T_2+1]} > 0$
IV.	$\frac{2[A-(c_1T_1+c_2T_2)]+K(c_1T_1+c_2T_2)}{2K} > 0$	$\frac{1}{K} > 0$

Proof of Lemma 2:

Recall from (9) that the government's goal is to maximize

$$\begin{aligned} W(\alpha_1,\alpha_2) &= \gamma(p^*-s)q^* + (c_1T_1\alpha_1+c_2T_2\alpha_2)q^* + \frac{1}{2}(A-p^*)q^* \\ &= [(\gamma+\frac{1}{2})\frac{A-s}{2}+c_1T_1\alpha_1+c_2T_2\alpha_2]\frac{A-s}{2} = \frac{1}{2K}(A-s+Kc_1T_1\alpha_1+Kc_2T_2\alpha_2)(A-s) \quad (15) \end{aligned}$$

where $s = c_1T_1\alpha_1 + c_1(1-\alpha_1) + c_1T_1\alpha_2 + c_1(1-\alpha_2)$ and K is still equal to $\frac{2}{\gamma+\frac{1}{2}} \in [2,4]$.

We show below that (α_1^*, α_2^*) can only be in one of the two forms: i) $0 \le \alpha_1^* \le 1$ and $\alpha_2^* = 0$, or ii) $\alpha_1^* = 1$ and $0 \le \alpha_2^* \le 1$. That is, both of these two forms can be achieved by using a product-level LCR studied in Section 3.

From (15), it is clear that $(\alpha_1^*, \alpha_2^*) = \underset{\alpha_1,\alpha_2\in[0,1]}{\text{argmax}} \{(A - s + Kc_1T_1\alpha_1 + Kc_2T_2\alpha_2)(A - s)\}$. Let $F(\alpha_1, \alpha_2) \triangleq (A-s+K\alpha_1c_1T_1+K\alpha_2c_2T_2)(A-s) = \{A-[c_1T_1\alpha_1+c_1(1-\alpha_1)+c_1T_1\alpha_2+c_1(1-\alpha_2)]\}+K\alpha_1c_1T_1+K\alpha_2c_2T_2][A-(c_1T_1\alpha_1+c_1(1-\alpha_1)+c_1T_1\alpha_2+c_1(1-\alpha_2))]$. We have

$$\frac{\partial^2 F}{\partial \alpha_1^2} = -2c_1^2(T_1-1)[(K'-1)T_1+1] < 0,$$

$$\frac{\partial^2 F}{\partial \alpha_2^2} = -2c_2^2(T_2-1)[(K-1)T_2+1] < 0,$$

$$\frac{\partial^2 F}{\partial \alpha_1^2}\cdot\frac{\partial^2 F}{\partial \alpha_2^2} - (\frac{\partial^2 F}{\partial \alpha_1 \partial \alpha_2})^2 = -c_1^2c_2^2(K)^2(T_2-T_1)^2 < 0.$$

So F is neither concave nor convex with respect to α_1 and α_2. In other words, the maximum value of F occurs on the boundary or at the first-order conditions.

From the first-order conditions, we have $\alpha_1 = \frac{T_2(A-c_1-c_2)}{c_1(T_1-T_2)} < 0$ and $\alpha_2 = \frac{T_1(A-c_1-c_2)}{c_2(T_2-T_1)} > 0$ which is not feasible. Therefore, the maximum value of F can only occur at the boundary conditions, i.e., (α_1^*, α_2^*) can only be i) $0 \le \alpha_1^* \le 1$ and $\alpha_2^* = 0$, ii) $\alpha_1^* = 1$ and $0 \le \alpha_2^* \le 1$, iii) $0 \le \alpha_1 < 1$ and $\alpha_2 = 1$, or (iv) $\alpha_1 = 0, 0 < \alpha_2 \le 1$.

To eliminate (iii) and (iv), it suffices to show that for any given parameters A, c_1, c_2, T_1, T_2, K and at any point (α_1, α_2), we have $\frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_1} > 0$ *or* $\frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0$, i.e., (α_1, α_2) is not a maximizer unless at least $\alpha_1 = 1$ or $\alpha_2 = 0$. It is then sufficient to show that $\frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_1} \le 0 \Rightarrow \frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0$, i.e., if $\frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_1} > 0$ does not hold then we must have $\frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0$.

For $i = 1, 2$: $\frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_i} = [A - s + K(c_1T_1\alpha_1 + c_2T_2\alpha_2)](c_i - c_iT_i) + (A - s)[c_i + (K - 1)c_iT_i]$.

$$\text{We have } \frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_1} \le 0 \Leftrightarrow \frac{K(\alpha_1c_1T_1+\alpha_2c_2T_2)}{A-s} \ge \frac{2+(K-2)T_1}{T_1-1} \Rightarrow \frac{K(\alpha_1c_1T_1+\alpha_2c_2T_2)}{A-s} > \frac{2+(K-2)T_2}{T_2-1}$$

$$\text{since } K \ge 2 \text{ and RHS} \downarrow \text{(because } \frac{\partial \text{RHS}}{\partial T} = -\frac{K}{(T-1)^2} < 0) \Leftrightarrow \frac{\partial F(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0.$$

So we have successfully argued that (α_1^*, α_2^*) can only be i) $0 \le \alpha_1^* \le 1$ and $\alpha_2^* = 0$, or ii) $\alpha_1^* = 1$ and $0 \le \alpha_2^* \le 1$. As a result, for any (α_1^*, α_2^*), it is equivalent for the government to impose a product-level LCR at

$$l \triangleq \frac{c_1T_1\alpha_1^* + c_2T_2\alpha_2^*}{s} = \frac{c_1T_1\alpha_1^* + c_2T_2\alpha_2^*}{c_1T_1\alpha_1^* + c_1(1-\alpha_1^*) + c_2T_2\alpha_2^* + c_2(1-\alpha_2^*)} \tag{16}$$

and l must be l^* found in Proposition 2 since (α_1^*, α_2^*) maximizes government's welfare objective and so does l^*. Given the one-to-one correspondence between l and (α_1^*, α_2^*) shown in Eq. (16) (one-to-one because $\alpha_2^* > 0$ if and only if $\alpha_1^* = 1$), we can use the l^* information in Proposition 2 to derive closed-form solutions for (α_1^*, α_2^*) which is presented in Proposition 2.

Proof of Proposition 3:

The expressions of (l_1^*, l_2^*) follow directly by plugging Eq. (8) into Lemma 2. Furthermore, in Case 0, $l_1^* = l^* = l_2^* = 0$. In Case I, $l_1^* = \frac{c_1T_1\alpha_1}{c_1T_1\alpha_1+c_1(1-\alpha_1)} > \frac{c_1T_1\alpha_1}{c_1T_1\alpha_1+c_1(1-\alpha_1)+c_2} = l^* > 0 = l_2^*$. In Case II, $l_1^* = 1 > \frac{c_1T_1}{c_1T_1+c_2} = l^* > 0 = l_2^*$. In Case III, $l_1^* = 1 > \frac{c_1T_1+c_2T_2\alpha_2}{c_1T_1+c_2T_2\alpha_2+c_2T_2(1-\alpha_2)} > \frac{c_2T_2\alpha_2}{c_2T_2\alpha_2+c_2T_2(1-\alpha_2)} = l_2^*$. In Case IV, $l_1^* = l^* = l_2^* = 1$. Therefore, we conclude that $l_1^* \ge l^* \ge l_2^*$.

Proof of Proposition 4:

Recall from (14) that $W(s) = \frac{[A+(Kl-1)s](A-s)}{2K}$ where $K = \frac{2}{\gamma+\frac{1}{2}} \in [2, 4]$ for $s \in [s(l), A)$ and $s(l)$ is given in Proposition 1. Case 1: When $Kl^c \in [0, 1)$, $W(s)$ is a quadratic function opening up with roots A and $\frac{A}{1-Kl^c} > A$. Therefore, $W(s)$ is decreasing in s on $s \in (0, A)$. Thus, $W(s)$ is maximized at $s^c = s(l^c)$. Case 2: When $Kl^c = 1$, $W(s) = \frac{A(A-s)}{2K}$ which is clearly decreasing on $s \in (0, A)$ thus $s^c = s(l^c)$. Case 3: When $Kl^c \in (1, 2]$, $W(s)$ is a quadratic function opening down with roots $\frac{A}{1-Kl^c} < 0$ and A. Moreover, the axis of symmetry is $-\frac{2-Kl^c}{2(Kl^c-1)} \le 0$ Therefore, $W(s)$ is again decreasing in s on $s \in (0, A)$ thus $s^c = s(l^c)$. Case 4: When $Kl^c \in (2, 4] \Rightarrow K \in (2, 4]$, $W(s)$ is a quadratic function opening down with roots $\frac{A}{1-Kl^c} < 0$ and A. This time, the axis of symmetry is $\frac{Kl^c-2}{2(Kl^c-1)} \in (0, A)$.

Therefore, $W(s)$ is maximized at $s = \frac{Kl^c-2}{2(Kl^c-1)}$ with a value of $\frac{A^2K(l^c)^2}{8(Kl^c-1)}$. Note that $\partial[\frac{A^2K(l^c)^2}{8(Kl^c-1)}]/\partial l = \frac{A^2Kl^c(Kl^c-2)}{8(Kl^c-1)^2} > 0$ in this case, so l^c must be equal to 1 because by definition it is the maximizer, and it is feasible because $K \in (2,4]$. But when $l^c = 1$, the sourcing cost for the foreign OEM can only be $s = c_1T_1 + c_2T_2$ with everything sourced locally, i.e., $s^c = s(l^c)$ again.

Proof of Proposition 5:

The government's goal in the oligopoly game with n foreign OEMs and product-level LCR l is given in (11):

$$W(l) = \gamma \cdot \sum_{j=1}^{n} \pi_j^* + l \cdot s(l) \cdot \sum_{j=1}^{n} q_j^* + \frac{1}{2}(A - p^*) \sum_{j=1}^{n} q_j^*.$$

By plugging in $q_j^* = \frac{A-s}{n+1}$, $p^* = \frac{A+n\cdot s}{n+1}$ and $\pi_j^* = \frac{(A-s)^2}{(n+1)^2}$ for all j, we can reduce (11) to

$$\frac{n}{(n+1)K_n}[A + (K_n l - 1)s](A - s) \text{ where } K_n \triangleq \frac{n+1}{\gamma + n/2} \tag{17}$$

which reduces to the monopoly case in (14) when $n = 1$. In particular $K_1 \equiv K$.

Part (i): Comparing (17) to (14), it is easy to see that the optimal LCR level in the monopoly setting is $l_M^*(A) \triangleq \underset{l}{\operatorname{argmax}}\{[A + (Kl-1)s](A-s)\}$ while the optimal LCR level in the n-firm-oligopoly setting is $l_n^*(A) \triangleq \underset{l}{\operatorname{argmax}}\{[A + (K_n l-1)s](A-s)\}$. Therefore, $l_n^*(A)$ follows the exact same structure as $l_M^*(A)$ provided in Proposition 2. Furthermore, we can obtain the closed-form solution of $l_n^*(A)$ by replacing K with $K_n = \frac{n+1}{\gamma+n/2}$ in Proposition 2.

Part (ii): For any given $\gamma \in [0\%, 50\%]$ and any $n \geq 1$, $n^2 + 2n + 1 \geq n^2 + 2n + 2\gamma \Leftrightarrow (n+1)^2 + 2\gamma(n+1) \geq n(n+2) + 2\gamma + 2\gamma(n+1) \Leftrightarrow \frac{n+1}{2\gamma+n} \geq \frac{n+2}{2\gamma+(n+1)} \Leftrightarrow K_n \geq K_{n+1}$ with equalities being held only when $\gamma = 50\%$. It follows that the thresholds between Cases I-IV, i.e., $F_{K_n,T_1} \cdot c_1T_1 + c_2$, $F_{K_n,T_2} \cdot c_1T_1 + c_2$, and $F_{K_n,T_2} \cdot (c_1T_1 + c_2T_2)$ are increasing in n because $\frac{\partial F_{K,T}}{\partial K} = -\frac{2(T-1)^2}{[(K-2)T+2]^2} < 0$ for $T \in \{T_1, T_2\}$. It is clear that $\frac{\partial l^*}{\partial K} = 0$ when A are in Cases II and IV. Since l^* is an increasing function in A (Proposition 2), the thresholds are increasing in n, and K_n is decreasing in n, it suffices to show that $\frac{\partial l^*}{\partial K} > 0$ when A are in Cases I and III, i.e., l^* decreases when K is reduced in Case I and Case III. When $A \in (c_1 + c_2, F_{K,T_1} \cdot c_1T_1 + c_2)$ (Case I), $\frac{\partial l^*}{\partial K} = \frac{2(c_1+c_2)T_1^2[A-(c_1+c_2)]}{\{A[(K-2)T_1+2]+(c_1+c_2)KT_1\}^2} > 0$. And when $A \in (F_{K,T_2} \cdot c_1T_1 + c_2, F_{K,T_2} \cdot (c_1T_1 + c_2T_2))$ (Case III), $\frac{\partial l^*}{\partial K} = \frac{2(c_1T_1+c_2T_2)[AT_2-(c_1T_1+c_2T_2)]}{\{A[(K-2)T_2+2]+K(c_1T_1+c_2T_2)\}^2} > 0$.

Part (iii): As $n \uparrow \infty$, $K_n = \frac{n+1}{\gamma+n/2} \downarrow 2$. Therefore, $\lim_{n\to\infty} l_n^*(A) = \underset{l}{\operatorname{argmax}}\{[A + (2l-1)s](A-s)\}$ which is the same case as $n = 1$ and $\gamma = 50\%$. Therefore, $l_n^*(A)$ converges a non-zero lower bound when $A > c_1 + c_2$.

Part (iv) Let (α_1, α_2) continue to denote the sourcing percentages that we assume hypothetically that the government can determine. Then the government's objective function can be written as

$$\begin{aligned} W(\alpha_1, \alpha_2) &= \gamma \cdot \sum_{j=1}^{n} \pi_j^* + (c_1T_1\alpha_1 + c_2T_2\alpha_2) \cdot \sum_{j=1}^{n} q_j^* + \frac{1}{2}(A - p^*) \sum_{j=1}^{n} q_j^* \\ &= \gamma \cdot \frac{n(A-s)^2}{(n+1)^2} + (c_1T_1\alpha_1 + c_2T_2\alpha_2)\frac{n(A-s)}{n+1} + \frac{1}{2}\frac{n^2(A-s)^2}{(n+1)^2} \\ &= \frac{n}{(n+1)K_n}(A - s + K_n c_1T_1\alpha_1 + K_n c_2T_2T_2)(A - s) \end{aligned} \tag{18}$$

Note that (19) is identical to (15) with K simply replaced by K_n. Because $K_n = \frac{n+1}{\gamma+n/2} \in [2,4]$ for $\gamma \in [0\%, 50\%]$, it can then be easily verified that Lemma 2 and thus the equivalence result continues to hold for the oligopoly model of n foreign OEMs.

Proof of Proposition 6:

To show each individual firm's profit $\frac{(A-s)^2}{(n+1)^2}$ decreases as $n \uparrow$ for any A, it suffices to show that $\frac{A-s}{n+1}$ decreases in n. From Corollary 1, for fixed n, we have

$$A - s = \begin{cases} A \cdot \frac{K_nT_1}{2(K_n-1)T_1+2} - \frac{K_nT_1(c_1+c_2)}{2(K_n-1)T_1+2} & \text{Case 1: } A \in (c_1 + c_2, F_{K_n,T_1} \cdot c_1T_1 + c_2) \\ A - (c_1T_1 + c_2) & \text{Case 2: } A \in [F_{K_n,T_1} \cdot c_1T_1 + c_2, F_{K_n,T_2} \cdot c_1T_1 + c_2] \\ A \cdot \frac{K_nT_2}{2(K_n-1)T_2+2} - \frac{K_n(c_1T_1+c_2T_2)}{2(K_n-1)T_2+2} & \text{Case 3: } A \in (F_{K_n,T_2} \cdot c_1T_1 + c_2, F_{K_n,T_2} \cdot (c_1T_1 + c_2T_2)) \\ A - (c_1T_1 + c_2T_2) & \text{Case 4: } A \geq F_{K_n,T_2} \cdot (c_1T_1 + c_2T_2) \end{cases}$$

which is continuous, increasing and piece-wise linear in A. Therefore, for fixed n, $\frac{A-s}{n+1}$ is also continuous, increasing and piece-wise linear in A. Next we note that,

Cases	Scenarios (increase in A)	$\frac{\partial[\frac{A-s}{n+1}]}{\partial n}$
I.	$A \in (c_1+c_2, F_{K_n,T_1}\cdot c_1T_1+c_2)$	$-\frac{[A-(c_1+c_2)]T_1(T_1+1)}{[n(T_1+1)+2T_1-2\gamma(T_1-1)]^2}<0$
II.	$A \in [F_{K_n,T_1}\cdot c_1T_1+c_2, F_{K_n,T_2}\cdot c_1T_1+c_2]$	$-\frac{A-(c_1T_1+c_2)}{(n+1)^2}<0$
III.	$A \in (F_{K_n,T_2}\cdot c_1T_1+c_2, F_{K_n,T_2}\cdot(c_1T_1+c_2T_2))$	$-\frac{[AT_2-(c_1T_1+c_2T_2)](T_2+1)}{[n(T_2+1)+2T_2-2\gamma(T_2-1)]^2}<0$
IV.	$A \geq F_{K_n,T_2}\cdot(c_1T_1+c_2T_2)$	$-\frac{A-(c_1T_1+c_2T_2)}{(n+1)^2}<0$

That is, $\frac{A-s}{n+1}$ decreases in n for any given A within each case of Cases I-IV. Finally, the thresholds between Cases I-IV, i.e., $F_{K_n,T_1}\cdot c_1T_1+c_2$, $F_{K_n,T_2}\cdot c_1T_1+c_2$, and $F_{K_n,T_2}\cdot(c_1T_1+c_2T_2)$ are increasing in n because K_n decreases in n and $\frac{\partial F_{K,T}}{\partial K}=-\frac{2(T-1)^2}{[(K-2)T+2]^2}<0$ for $T\in\{T_1,T_2\}$. With these pieces, we can conclude that each individual firm's profit decreases as $n\uparrow$ for any fixed A. Moreover, since $A-s\in(0,A)$, the profit $\frac{(A-s)^2}{(n+1)^2}$ converges to 0 as $n\uparrow\infty$. Firms will stop entering the market when potential profit becomes less than the fixed cost of entry. Since profit increases in A for any given n, an increase in A (or a decrease in the fixed entry cost R) induces more entries if any.

Proof of Proposition 7:

Part (i): The proof is the same as that of Proposition 5/(i) because $\gamma\uparrow\Rightarrow K_n\downarrow$, which is the same effect as $n\uparrow$.

Part (ii): Consider the case γ increases to $\gamma+\Delta\gamma$, and let l^*_γ and $l^*_{\gamma+\Delta\gamma}$ (and W_γ and $W_{\gamma+\Delta\gamma}$) denote the optimal LCR levels (and welfare functions) in the two cases. Then $W_{\gamma+\Delta\gamma}(l^*_{\gamma+\Delta\gamma})\geq W_\gamma(l^*_\gamma)$ because $W_{\gamma+\Delta\gamma}(l^*_{\gamma+\Delta\gamma})\geq W_{\gamma+\Delta\gamma}(l^*_\gamma)$ by the definition of $l^*_{\gamma+\Delta\gamma}$, and $W_{\gamma+\Delta\gamma}(l^*_\gamma)\geq W_\gamma(l^*_\gamma)$ by the definition of W in (11), i.e., as the tax rate increases, the government already improves welfare without changing the LCR requirement, and will do even better if it changes the LCR requirement.

As a consequence of $\gamma\uparrow$, $l^*\downarrow\Rightarrow s\downarrow\Rightarrow(A-s)\uparrow$. Therefore, the first and third components of the welfare function in (11), $\frac{\gamma n(A-s)^2}{(n+1)^2}$ and $\frac{n^2(A-s)^2}{2(n+1)^2}$ both go up. For the second component, $\frac{n}{n+1}\cdot l^*\cdot s\cdot(A-s)$, we first note that it is a piecewise increasing function in A because

Cases	Scenarios (increase in A)	$\frac{\partial[l^*\cdot s\cdot(A-s)]}{\partial A}$
I.	$A \in (c_1+c_2, F_{K_n,T_1}\cdot c_1T_1+c_2)$	$\frac{K_nT_1^2[(K_n-2)T_1+2][A-(c_1+c_2)]}{2(T_1-1)[(K_n-1)T_1+1]^2}>0$
II.	$A \in [F_{K_n,T_1}\cdot c_1T_1+c_2, F_{K_n,T_2}\cdot c_1T_1+c_2]$	$c_1T_1>0$
III.	$A \in (F_{K_n,T_2}\cdot c_1T_1+c_2, F_{K_n,T_2}\cdot(c_1T_1+c_2T_2))$	$\frac{K_nT_2[(K_n-2)T_2+2][AT_2-(c_1T_1+c_2T_2)]}{2(T_2-1)[(K_n-1)T_2+1]^2}>0$
IV.	$A \geq F_{K_n,T_2}\cdot(c_1T_1+c_2T_2)$	$c_1T_1+c_2T_2>0$

Second, in each of Cases I-IV, $\frac{n}{n+1}\cdot l^*\cdot s\cdot(A-s)$ weakly decreases as $\gamma\uparrow$ ($K_n\downarrow$) because

Cases	Scenarios (increase in A)	$\frac{\partial[l^*\cdot s\cdot(A-s)]}{\partial K_n}$
I.	$A \in (c_1+c_2, F_{K_n,T_1}\cdot c_1T_1+c_2)$	$\frac{(T_1-1)T_1^2[A-(c_1+c_2)^2]}{2[(K_n-1)T_1+1]^3}>0$
II.	$A \in [F_{K_n,T_1}\cdot c_1T_1+c_2, F_{K_n,T_2}\cdot c_1T_1+c_2]$	0
III.	$A \in (F_{K_n,T_2}\cdot c_1T_1+c_2, F_{K_n,T_2}\cdot(c_1T_1+c_2T_2))$	$\frac{(T_2-1)[AT_2-(c_1T_1+c_2T_2)]^2}{2[(K-1)T_2+1]^3}>0$
IV.	$A \geq F_{K_n,T_2}\cdot(c_1T_1+c_2T_2)$	0

On the other hand, $\gamma\uparrow$ increases the thresholds between Cases I-IV, i.e., $F_{K_n,T_1}\cdot c_1T_1+c_2$, $F_{K_n,T_2}\cdot c_1T_1+c_2$, and $F_{K_n,T_2}\cdot(c_1T_1+c_2T_2)$, thus shifts the domain if any from Case 4 $\rightarrow$ Case 3 $\rightarrow$ Case 2 $\rightarrow$ Case 1 reducing the value of $\frac{n}{n+1}\cdot l^*\cdot s\cdot(A-s)$ because it is an increasing function in A. Therefore, value of $\frac{n}{n+1}\cdot l^*\cdot s\cdot(A-s)$ decreases as γ increases.

Part (iii): Since $A-s$ increases as $\gamma\uparrow$, the before-tax profit for an OEM, $\pi_j(l^*)=\frac{(A-s)^2}{(n+1)^2}$, also increases. On the other hand, the after-tax profit for the OEM is $\pi_e(l^*)=(1-\gamma)\pi_j(l^*)$, and it can go up or down because $(1-\gamma)$ decreases in γ. Through some algebra manipulations, we have

Cases	Scenarios (increase in A)	$\frac{\partial[\pi_e(l^*)]}{\partial\gamma}$
I.	$A \in (c_1 + c_2, F_{K_n,T_1} \cdot c_1T_1 + c_2)$	$\frac{[T_1-2\gamma(T_1-1)-5][A-(c_1+c_2)]^2T_1^2}{[3T_1-2\gamma(T_1-1)+1]^3}$
II.	$A \in [F_{K_n,T_1} \cdot c_1T_1 + c_2, F_{K_n,T_2} \cdot c_1T_1 + c_2]$	$-\frac{[A-(c_1T_1+c_2)]^2}{4} < 0$
III.	$A \in (F_{K_n,T_2} \cdot c_1T_1 + c_2, F_{K_n,T_2} \cdot (c_1T_1 + c_2T_2))$	$\frac{[T_2-2\gamma(T_2-1)-5][AT_2-(c_1T_1+c_2T_2)]^2}{[3T_2-2\gamma(T_2-1)+1]^3}$
IV.	$A \geq F_{K_n,T_2} \cdot (c_1T_1 + c_2T_2)$	$-\frac{[A-(c_1T_1+c_2T_2)]^2}{4} < 0$

When the tax rate is sufficiently large (i.e., $\gamma \to 50\%$), $\frac{[T_1-2\gamma(T_1-1)-5][A-(c_1+c_2)]^2T_1^2}{[3T_1-2\gamma(T_1-1)+1]^3} \to -\frac{2[A-(c_1+c_2)]^2T_1^2}{[T_1+1]^3} < 0$ and $\frac{[T_2-2\gamma(T_2-1)-5][AT_2-(c_1T_1+c_2T_2)]^2}{[3T_2-2\gamma(T_2-1)+1]^3} \to -\frac{2[AT_2-(c_1T_1+c_2T_2)]^2}{[T_2+1]^3} < 0$. With the fact that $\pi_e(l^*)$ is a continuous and increasing function in A (see the proof of Proposition 6), and the thresholds on A are increasing with a reduced K_n, we can conclude that $\pi_e(l^*)$ decreases with a reduced K (or a bigger γ) when $\gamma \to 50\%$.

Proof of Proposition 8:

Part (i) follows directly from Proposition 5/(i). For part (ii) and part (iii), we know from the proof of Proposition 6, that $A - s(l^*)$ is a continuous, increasing and piece-wise linear function in A. Therefore, the first and third components of the welfare function, $\frac{\gamma n(A-s)^2}{(n+1)^2}$ and $\frac{n^2(A-s)^2}{2(n+1)^2}$ increase as $A \uparrow$. Furthermore, the second term, $\frac{n}{n+1} \cdot l \cdot s \cdot (A - s)$, also increases in A as shown in the proof of Proposition 7/(ii). As a result, welfare which is a sum of the three components increases as $A \uparrow$. Similarly, both the before-tax and the after-tax profit for any OEM, $\pi_j = \frac{(A-s)^2}{(n+1)^2}$ and $\pi_e = (1 - \gamma)\pi_j$, would increase.

Proof of Propositions 9 and 10:

We first present $\frac{\partial\pi_j(l^*)}{\partial T_i}$ for $i \in \{1, 2\}$:

Cases	$\frac{\partial\pi_j(l^*)}{\partial T_1}$	$\frac{\partial\pi_j(l^*)}{\partial T_2}$
I.	$\frac{K_n^2T_1[A-(c_1+c_2)]^2}{2(n+1)^2[(K_n-1)T_1+1]^3} > 0$	0
II.	$-\frac{2c_1[A-(c_1T_1+c_2)]}{(n+1)^2} < 0$	0
III.	$-\frac{c_1K_n^2[AT_2-(c_1T_1+c_2T_2)]}{2(n+1)^2[(K_n-1)T_2+1]^2} < 0$	$\frac{K_n^2[AT_2-(c_1T_1+c_2T_2)][A+(K_n-1)c_1T_1-c_2]}{2(n+1)^2[(K_n-1)T_2+1]^3} > 0$
IV.	$-\frac{2c_1[A-(c_1T_1+c_2T_2)]}{(n+1)^2} < 0$	$-\frac{2c_2[A-(c_1T_1+c_2T_2)]}{(n+1)^2} < 0$

It is clear that $\frac{\partial\pi_j(l^*)}{\partial T_1} > 0$ in Case I and $\frac{\partial\pi_j(l^*)}{\partial T_2} > 0$ in Case III. For the purpose of completeness, we also state $\frac{\partial s(l^*)}{\partial T_1}$ and $\frac{\partial s(l^*)}{\partial T_2}$ below which would have the opposite signs as $\frac{\partial\pi_j(l^*)}{\partial T_1}$ and $\frac{\partial\pi_j(l^*)}{\partial T_2}$ given that an OEM's profit decreases in the sourcing cost s.

Cases	$\frac{\partial s(l^*)}{\partial T_1}$	$\frac{\partial s(l^*)}{\partial T_2}$
I.	$-\frac{[A-(c_1+c_2)]K_n}{2[(K-1)T_1+1]^2} < 0$	0
II.	$c_1 > 0$	0
III.	$\frac{c_1K_n}{2(K_n-1)T_2+2} > 0$	$-\frac{A-(c_1T_1+c_2)+K_nc_1T_1}{2[(K_n-1)T_2+1]^2} < 0$
IV.	$c_1 > 0$	$c_2 > 0$

Next, we present $\frac{\partial W(l^*)}{\partial T_i}$ for $i \in \{1, 2\}$:

Cases	$\frac{\partial W(l^*)}{\partial T_1}$	$\frac{\partial W(l^*)}{\partial T_2}$
I.	$-\frac{nK_nT_1[A-(c_1+c_2)]^2[(K_n-2)T_1+2]}{4(n+1)(T_1-1)^2[(K_n-1)T_1+1]^2} < 0$	0
II.	$\frac{nc_1}{(n+1)K_n}(K_n - 2)[A - \frac{2(K_n-1)}{K_n-2}c_1T_1 - c_2] < 0$	0
III.	$-\frac{nK_nc_1[AT_2-(c_1T_1+c_2T_2)]}{2(n+1)(T_2-1)[(K_n-1)T_2+1]} < 0$	$-\frac{nK_nc_1[AT_2-(c_1T_1+c_2T_2)]\{[(K_n-2)T_2+2]A-[2(K_n-1)T_2-(K_n-2)]c_1T_1-[(K_n-2)T_2+2]c_2\}}{4(n+1)(T_2-1)^2[(K_n-1)T_2+1]^2} < 0$
IV.	$\frac{nc_1}{(n+1)K_n}(K_n - 2)[A - \frac{2(K_n-1)}{K_n-2}(c_1T_1 + c_2T_2)]$	$\frac{nc_2}{(n+1)K_n}(K_n - 2)[A - \frac{2(K_n-1)}{K_n-2}(c_1T_1 + c_2T_2)]$

In Case III, when $c_1 > c_2$, we always have $\frac{\partial W(l^*)}{\partial T_1} < \frac{\partial W(l^*)}{\partial T_2} < 0$. When $c_1 \le c_2$, we have $\frac{\partial W(l^*)}{\partial T_1} < \frac{\partial W(l^*)}{\partial T_2} < 0$ iff $A \in \left(\frac{2(K_n-1)T_2-(K_n-2)}{(K_n-2)T_2+2}c_1T_1 + c_2, \frac{[2(K_n-1)T_2-(K_n-2)]T_1+2(T_2-1)[(K_n-1)T_2+1]}{(K_n-2)T_2+2}c_1 + c_2\right)$. That is, when $c_1 \le c_2$ and $A \in (\frac{[2(K_n-1)T_2-(K_n-2)]T_1+2(T_2-1)[(K_n-1)T_2+1]}{(K_n-2)T_2+2}c_1+c_2, \frac{2(K_n-1)T_2-(K_n-2)}{(K_n-2)T_2+2}\times(c_1T_1+c_2T_2))$, we have $\frac{\partial W(l^*)}{\partial T_2} < \frac{\partial W(l^*)}{\partial T_1} < 0$. In Case IV, when $c_1 > c_2$, $\frac{\partial W(l^*)}{\partial T_1} < (>)\frac{\partial W(l^*)}{\partial T_2} < (>)0$ if $A < (>)\frac{2(K_n-1)}{K_n-2}(c_1T_1 + c_2T_2)$. When $c_1 \le c_2$, $\frac{\partial W(l^*)}{\partial T_2} < (>)\frac{\partial W(l^*)}{\partial T_1} < (>)0$ if $A < (>)\frac{2(K_n-1)}{K_n-2}(c_1T_1 + c_2T_2)$.

It then becomes clear that $\frac{\partial W(l^*)}{\partial T_i} > 0$ for $i = 1, 2$ when $A > \frac{2(K_n-1)}{(K_n-2)}(c_1T_1 + c_2T_2)$. On the other hand, suppose $A < \frac{2(K_n-1)}{(K_n-2)}(c_1T_1 + c_2T_2)$. Then when $c_1 > c_2$, $\frac{\partial W(l^*)}{\partial T_1} < \frac{\partial W(l^*)}{\partial T_2} < 0$. But When $c_1 \le c_2$, $\frac{\partial W(l^*)}{\partial T_1} < \frac{\partial W(l^*)}{\partial T_2} < 0$ if and only if $A < \frac{[2(K_n-1)T_2-(K_n-2)]T_1+2(T_2-1)[(K_n-1)T_2+1]}{(K_n-2)T_2+2}c_1 + c_2 = F_{K_n,T_2}\cdot c_1T_1 + c_2 + (T_2-1)\left[\frac{K_n}{(K_n-2)T_2+2}+1\right]c_1$ which is an inner point of Case III because $F_{K_n,T_2}\cdot c_1T_1 + c_2 + (T_2-1)\left[\frac{K_n}{(K_n-2)T_2+2}+1\right]c_1 - (F_{K_n,T_2}\cdot c_1T_1 + c_2) = (T_2-1)\left[\frac{K_n}{(K_n-2)T_2+2}+1\right]c_1 > 0$ and $F_{K_n,T_2}\cdot(c_1T_1 + c_2T_2) - (F_{K_n,T_2}\cdot c_1T_1 + c_2) = \frac{2\{(c_1+c_2)(T_2-1)[(K_n-1)T_2+1]+c_1T_1[2(K_n-1)T_2+2-K_n]\}}{(K_n-2)T_2+2} > 0$ where $2(K_n-1)T_2 + 2 - K_n > 2(K_n-1) + 2 - K_n = K_n$.

Finally, we examine the impact of reduced technology gap T_1 or T_2 on the three individual components that make up the welfare. Breaking down the welfare function into three components as given in (4), we find that the foreign OEMs' tax (FOT), the local suppliers' revenue (LSR), and the consumer surplus (CS) change as follows:

	When T_1 is reduced			When T_2 is reduced		
	FOT	LSR	CS	FOT	LSR	CS
Case I.	$\downarrow$	$\uparrow$	$\downarrow$	no change	no change	no change
Case II.	$\uparrow$	$\uparrow$ iff $A < 2c_1T_1 + c_2$	$\uparrow$	no change	no change	no change
Case III.	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Case IV.	$\uparrow$	$\uparrow$ iff $A < 2(c_1T_1 + c_2T_2)$	$\uparrow$	$\uparrow$	$\uparrow$ iff $A < 2(c_1T_1 + c_2T_2)$	$\uparrow$

The proof is as follows. Given that the foreign OEMs' tax (i.e., the first term) is $\frac{\gamma n(A-s)^2}{n+1}$ and consumer surplus (i.e., the third term) is $\frac{1}{2}\frac{n^2(A-s)^2}{n+1}$, their derivatives with respect to $T_i, i \in \{1,2\}$ would have the opposite sign of $\frac{\partial s(l^*)}{\partial T_i}$ which was given earlier in this proof. For the local suppliers' revenue (the second term), it is equal to $\frac{n}{n+1}l \cdot s \cdot (A-s)$ and we have

Cases	$\frac{\partial[l\cdot s\cdot(A-s)]}{\partial T_1}$	$\frac{\partial[l\cdot s\cdot(A-s)]}{\partial T_2}$
I.	$-\frac{[A-(c_1+c_2)]^2K_nT_1}{4(T_1-1)^2[(K_n-1)T_1+1]^3}\cdot Pos.Factor_1 < 0$	0
II.	$c_1[A-(2c_1T_1+c_2)]$	0
III.	$-\frac{c_1K_n[AT_2-(c_1T_1+c_2T_2)][(K_n-2)T_2+2]}{2(T_2-1)[(K_n-1)T_2+1]^2} < 0$	$-\frac{K_n[AT_2-(c_1T_1+c_2T_2)]}{4(T_2-1)^2[(K_n-1)T_2+1]^3}\cdot Pos.Factor_2 < 0$
IV.	$c_1[A-2(c_1T_1+c_2T_2)]$	$c_2[A-2(c_1T_1+c_2T_2)]$

where $Pos.Factor_1 = \{[4+(K_n-3)K_n]T_1 - (8-3K_n)\}T_1 + 4 \ge K_n^2 \ge 4 > 0$ and $Pos.Factor_2 = 4(A-c_2) + (A-c_2)T_2\{-8+3K_n+[4+(K_n-3)K_n]T_2\} - c_1T_1\{4-3K_n+(K_n-1)[8-K_n+(K_n-2)T_2]T_2\} > 4[A-(c_1T_1+c_2)] + (A-c_2)T_2\{-8+3K_n+[4+(K_n-3)K_n]T_2\} - c_1T_1T_2\{-3K_n+(K_n-1)[8-K_n+(K_n-2)T_2]\} \ge (A-c_2)T_2\{-8+3K_n+[4+(K_n-3)K_n]T_2\} - c_1T_1T_2\{-3K_n+(K_n-1)[8-K_n+(K_n-2)T_2]\} \ge 0$ and the last step is true because $A-c_2 \ge c_1T_1$ under Case III and $-8+3K_n+[4+(K_n-3)K_n]T_2 \ge -3K_n+(K_n-1)[8-K_n+(K_n-2)T_2] \ge 0$ since $T_2 > 1$ and $K_n \ge 2$. This completes the proof.