Online Supplement to "Is Servicization a Win-Win Strategy? Profitability and Environmental Implications of Servicization"

In this online supplement, we study social surplus (SS) in Appendix A, provide characterization of thresholds that appear in Section 6.1 in Appendix B and provide the proofs of the analytical results in Appendix C in the order their corresponding results appear in the paper.

A. Social Surplus

In this section, we investigate the impact of servicization on SS. The SS is the sum of the firm's profit and consumer surplus (CS), and it is important in analyzing the overall efficiency of servicization.

Proposition 13. *(Social Surplus) When servicization is more profitable than the selling strategy, servicization increases SS except when $r < k_1(\alpha, \beta)$ in R1. Furthermore, $k_1(\alpha, \beta) > 1$ if and only if $1 - \sigma(\beta) > 1 - \alpha$. The expressions for k_1 and σ are explicitly characterized in the proof of the proposition.*

Intuition suggests that whenever the firm's operating cost is smaller than those of consumers ($r > 1$), the servicization should improve the SS because shifting the operating cost burden to the firm should improve overall efficiency in the system. However, contrary to this intuition, Proposition 13 shows that servicization may decrease the SS even when the firm has a better operating efficiency than the consumers. This happens when the firm serves both segments under both selling and servicization and the valuation gap between the segments is low, i.e., $1 - \sigma(\beta) > 1 - \alpha$. This result indicates that the social planner may prefer selling over servicization even when consumers have an inferior operating efficiency. This outcome is due to two factors: product durability choice and consumer use durations under selling and servicization strategies. To the extent that the product design, i.e., product durability, and the consumer use durations are close to the *socially optimum*[1], servicization would improve the social surplus. On one hand, when servicization is chosen, the firm's product durability choice in $R1$ is always closer to the socially optimum compared to firm's product durability choice under selling in the same region. On the other hand, the servicizing firm distorts the use duration offered to the low-end segment away from the socially optimum level to make this option less attractive for high segment, so it can charge a higher price to the high-end segment. In some cases, this distortion may be too high and make low end segment's offered use duration socially worse than the segment's use duration under selling strategy. These observations are summarized in the following lemma.

Lemma 2. *When servicization is more profitable than the selling strategy, in $R1$,*

(i) *product durability choice under servicization is always closer to the socially optimum level relative to the product durability choice under selling strategy.*

(ii) *for a given product durability, low end segment's use duration under selling strategy is always closer to*

[1] The firm's contracting problem leads to socially optimum outcome when there is no information asymmetry, i.e., consumer types are common knowledge (Netessine and Taylor 2007, cf. Laffont and Martimort 2009 Ch. 2) and the firm can charge different prices to different consumer segments. The characterization of this case is given in Orsdemir et al. (2017).

the socially optimum use duration relative to the low end segment's use duration under servicization if and only if $r < \frac{\alpha^2(1-\beta)}{\alpha^2+\beta-2\alpha\beta}$. *Furthermore,* $\frac{\alpha^2(1-\beta)}{\alpha^2+\beta-2\alpha\beta} > 1$.

Because the firm determines product durability based on low-end segment's valuation in $R1$ in the case of selling (as explained in §4.3), as the gap between segment valuations widens, i.e., $1-\alpha$ gets larger, product durability moves further away from the social optimum, and servicization becomes more socially preferable. On the other hand, when the gap between the segments is low, i.e., $1-\alpha < 1-\sigma(\beta)$, product durability under selling is not too far from the social optimum, and the inefficient use duration of low end segment under servicization dominates. Therefore, servicization may lead to a lower social surplus even when the firm has a better operating efficiency than the consumers.

In region $R2$, a servicizing firm serves more consumer segments than a selling firm, that is, the low-end segment is served only by the servicizing firm. The high end segment can capture positive consumer surplus. Thus, servicization always results in a higher social surplus in $R2$ as long as it is more profitable. In region $R3$, only the high end segment is served in both cases. Therefore, the consumer surplus is always zero, and the social surplus is equal to firm's profit. In this region, servicization increases the social surplus, when it is more profitable.

B. Characterization of thresholds in Section 6.1:

The next proposition characterizes when servicization can be a win-win strategy in $R1_a^{pl}$ and $R1_b^{pl}$. Recall that $R1^{pl} = R1_a^{pl} \cup R1_b^{pl}$.

Proposition 14. *In $R1$, servicization is a win-win strategy only if* $r > \frac{(\beta-1)^2\left(\alpha^4-2\alpha^2\beta+\beta\right)}{\alpha^4-4\alpha^3\beta+3(2\alpha^2-1)\beta^2+(3-4\alpha)\beta^3+\beta}$, *and*
(i) when $U_f^{pl} < U_c^{pl}$ *and* $D_f^{pl} > D_c^{pl}$, $\frac{e_u}{e_p+e_d} > \Delta_{R1}^{pl}(\alpha,\beta,m_c,m_f)$.
(ii) when $U_f^{pl} > U_c^{pl}$ *and* $D_f^{pl} < D_c^{pl}$, $\frac{e_u}{e_p+e_d} < \Delta_{R1}^{pl}(\alpha,\beta,m_c,m_f)$.
(iii) when $U_f^{pl} < U_c^{pl}$ *and* $D_f^{pl} < D_c^{pl}$.
Furthermore, $U_f^{pl}, U_c^{pl}, D_f^{pl}$ *and* D_c^{pl} *are defined in the proof of the propositon.*

C. Proofs

Proof of Proposition 1: We first solve the equilibrium for selling strategy, then we solve it for servicization strategy. In selling strategy, if type-θ_i purchases the product, it uses the product for $\tau_i = \frac{\delta\theta}{1+m_c}$. Then, $U_c(\theta_i) = \frac{\delta\theta_i^2}{2(1+m_c)} - p$. If the firm sells to both segments, $p = \frac{\delta\theta_L^2}{2(1+m_c)}$; otherwise, $p = \frac{\delta\theta_H^2}{2(1+m_c)}$. It is easy to see that when these prices are plugged into firm's profit function, the function becomes concave in product durability δ. Thus, from the first order conditions: If the firm serves both segments, product durability is $\delta^* = \frac{\alpha^2\theta^2}{4k(1+m_c)}$; otherwise, $\delta^* = \frac{\theta^2}{4k(1+m_c)}$. The firm profits are $\pi_{c,B} = \frac{\alpha^4 M\theta^4}{16k(1+m_c)^2}$, $\pi_{c,H} = \frac{\beta^4 M\theta^4}{16k(1+m_c)^2}$, respectively. Therefore, the firm serves both segments if and only if $\alpha > \sqrt[4]{\beta}$.

In servicization, if the firm serves both segments, one can show that at the equilibrium, IR_H and IC_L constraints do not bind, but IR_L and IC_H bind. Hence, $F_L = \tau_L\left(\theta_L - \frac{\tau_L}{2\delta}\right)$ and $F_H = \tau_H\left(\theta_H - \frac{\tau_H}{2\delta}\right) - \tau_L\left(\theta_H - \frac{\tau_L}{2\delta}\right) + F_L$. When we plugged in these to the profit function, we have $\pi_{f,B}(.|F_H, F_L) = M\left(-k\delta^2 + b\theta_H(\tau_H - \tau_L) + \theta_L\tau_L\right) - \frac{(1+m)M\left(b\tau_H^2-(-1+b)\tau_L^2\right)}{2\delta}$. This function is concave in τ_H, then from FOC, $\tau_H = \frac{\delta\theta_H}{1+m_f}$. After incorporating this expression to the profit function, we obtain $\pi_{f,B}(.|F_L, F_H, \tau_H) =$

$-M\left(k\delta^2+\beta\theta_H\tau_L-\theta_L\tau_L\right)+\frac{M\beta\delta\theta_H^2}{2+2m_f}+\frac{M(-1+\beta)\left(1+m_f\right)\tau_L^2}{2\delta}$. This is concave in τ_L; hence, τ_L can be found as $\frac{\delta(\beta\theta_H-\theta_L)}{(-1+\beta)\left(1+m_f\right)}$. Note that $\tau_L>0$ if and only if $\alpha>\beta$. After plugging this in, δ can be found as $\frac{\left(\alpha^2+\beta-2\alpha\beta\right)\theta^2}{4k\alpha^2(-1+\beta)\left(1+m_f\right)}$, similarly. When the firm serves only the high-end segment, only IR_H binds and the equilibrium can be obtained similar to the selling model.

Under servicization, the firm serves both segments if and only if $\pi_{f,B}\geq\pi_{f,H}$. One can show that $\lim_{\alpha\to1}\frac{\pi_{f,B}}{\pi_{f,H}}>1$, $\lim_{\alpha\to\beta}\frac{\pi_{f,B}}{\pi_{f,H}}<1$, and $\frac{\pi_{f,B}}{\pi_{f,H}}$ is increasing in α when $\alpha\in(\beta,1)$. Therefore, there is only one threshold α_t where $\frac{\pi_{f,B}(\alpha_t(\beta))}{\pi_{f,H}(\alpha_t(\beta))}=1$. Define $\gamma(\alpha,\beta)=\frac{\alpha_t(\beta)}{\beta}=\gamma(\alpha,\beta)$. It can be shown that at $\alpha=\sqrt[4]{\beta}$, serving to both segment is more profitable. Hence, $\gamma(\alpha,\beta)<\sqrt[4]{\beta}$. □

Proof of Proposition 2: In R1, $\frac{\pi_{f,B}}{\pi_{c,B}}>1$ if and only if $\frac{M\left(\alpha^2+\beta-2\alpha\beta\right)^2\theta^4}{16k(-1+\beta)^2\left(1+m_f\right)^2}>\frac{M\alpha^4\theta^4}{16k(1+m_c)^2}$. This can be rearranged to show that $\frac{\pi_{f,B}}{\pi_{c,B}}>1$ if and only if $r>\frac{\alpha^2(1-\beta)}{\alpha^2+\beta-2\alpha\beta}\triangleq f_1$. Simple algebra shows that $f_1<1$. The the other parts can be shown similarly. □

Proof of Proposition 3: We only show the proof for region $R2$. The rest can be shown similarly. In R2, $\delta_f^*=\frac{(\alpha^2+\beta-2\alpha\beta)\theta^2}{4k(1-\beta)(1+m_f)}$ and $\delta_c^*=\frac{\theta^2}{4k(1+m_c)}$. We can rearrange the terms and find that product durability is higher under servicization if and only if $r>\frac{1-\beta}{\alpha^2+\beta-2\alpha\beta}\triangleq r_\delta$. We compare this with the minimum operating efficiency threshold above which servicization is more profitable than selling, i.e., $f_1=\frac{\alpha^2(1-\beta)}{\alpha^2+\beta-2\alpha\beta}$. $r_\delta>f_1$ if and only if $1>\alpha^2$ which is true by assumption. $\frac{\partial r_\delta}{\partial\alpha}<0$ for $\alpha\in(\beta,1)$ and $\lim_{\alpha\to1}r_\delta>1$. Hence $r_\delta>1$. □

Proof of Lemma 1: In $R1$, $\tau_{H,f}>\tau_{H,c}$ if and only if $r>\sqrt{\frac{\alpha^2(1-\beta)}{\alpha^2-2\alpha\beta+\beta}}$. Similarly, $\tau_{L,f}>\tau_{L,c}$ if and only if $r>\sqrt{\frac{\alpha^3(\beta-1)^2}{(\alpha-\beta)(\alpha^2-2\alpha\beta+\beta)}}$. One can show that $\sqrt{\frac{\alpha^3(\beta-1)^2}{(\alpha-\beta)(\alpha^2-2\alpha\beta+\beta)}}>\sqrt{\frac{\alpha^2(1-\beta)}{\alpha^2-2\alpha\beta+\beta}}>f_1$, where f_1 is the thresholds above which servicization is more profitable and it is defined in the proof of Proposition 2. This proves the first part of the proposition. The other parts can be proved similarly. □

Proof of Proposition 4: First define total use duration (total disposal per unit of time) under selling when the firm serves both segments and only the high end segment as U_c^B (D_c^B) and U_c^H (D_c^H), respectively. U_f^B (D_f^B) and U_f^H (D_f^H) are defined similarly for servicization. In what follows, we characterize when servicization decreases the environmental impact for each region, $R1$, $R2$ and $R3$. To do so, we first analyze how the use impact and the production and disposal impact changes as a function of relative operating efficiency in each region. This allows us to find the critical relative operating efficiency values that determine whether the use, and the production and disposal impact increase/decrease under servicization.

In $R1$, product use impact under servicization is $e_u\frac{M\alpha\left(\alpha^2+\beta-2\alpha\beta\right)\theta^3}{4k(1-\beta)\left(1+m_f\right)^2}=e_uU_f^B$ and under selling is $e_u\frac{M\alpha^2(\alpha+\beta-\alpha\beta)\theta^3}{4k(1+m_c)^2}=e_uU_c^B$. Then, use impact under servicization is smaller than under selling strategy if and only if $r<\sqrt{\frac{\alpha(1-\beta)(\alpha+\beta-\alpha\beta)}{\alpha^2+\beta-2\alpha\beta}}\triangleq g_1$. Product disposal and production impact under servicization is $(e_d+e_p)\frac{4kM(1-\beta)(1+(-2+\alpha)\beta)\left(1+m_f\right)^2}{(\alpha-\beta)(\alpha^2+\beta-2\alpha\beta)\theta^3}=(e_p+e_d)D_f^B$ and under selling is $(e_d+e_p)\frac{4kM(1+(-1+\alpha)\beta)(1+m_c)^2}{\alpha^3\theta^3}=(e_p+e_d)D_c^B$. Hence, production and disposal impact is lower under servicization if and only if $r>\sqrt{\frac{\alpha^3(1-\beta)(1+(-2+\alpha)\beta)}{(\alpha-\beta)(1+(-1+\alpha)\beta)(\alpha^2+\beta-2\alpha\beta)}}\triangleq g_2$.

We now show that $g_2>g_1$. $\frac{g_2}{g_1}>1$ if and only if $\frac{\alpha^2(1+(-2+\alpha)\beta)}{(\alpha-\beta)(1+(-1+\alpha)\beta)(\alpha+\beta-\alpha\beta)}>1$. This can be rewritten as $(-1+\alpha)^2(1+\alpha-\beta)\beta^2>0$. Because in this region $\alpha>\beta$, the result follows. We now show that $g_1>f_1$. Recall that f_1 is the threshold that determines when servicization is more profitable than selling (see Proposition 2). First, define $j_1\triangleq\left(\alpha-2\alpha^2-5\alpha^3+11\alpha^4-\alpha^5-\alpha^6\right)\beta^2+\left(1-6\alpha+12\alpha^2-8\alpha^3\right)\beta^3$ and $j_2\triangleq3\alpha^5-2\alpha^6+$

$(3\alpha^3 - 6\alpha^4 - 3\alpha^5 + 3\alpha^6)\beta$. $\frac{g_1}{f_1} > 1$ if and only if $j \triangleq j_1 + j_2 > 0$. $\frac{\partial^3 j}{\partial \beta^3} = -6(-1+2\alpha)^3$ and it is greater than 0 if and only if $\alpha < 1/2$. Hence, $\frac{\partial^2 j}{\partial \beta^2}$ has its minimum at $\alpha = \frac{1}{2}$, and it is $\frac{1}{32} > 0$. This proves that $\frac{\partial^2 (j)}{\partial \beta^2}$ is always positive. Furthermore, $j(\beta = \alpha) = -(-1+\alpha)^3 \alpha^3 (2 + \alpha + \alpha^2) > 0$ and $j(\beta = 0) = (3 - 2\alpha)\alpha^5 > 0$, which proves that $j > 0$. Therefore, $g_2 > g_1 > f_1$. This shows that the following regions exist: 1) A region in which servicization decreases the total use impact but increases the production and disposal impact, i.e., $f_1 < r < g_1$. 2) A region in which servicization increases the total use impact but decreases the production and disposal impact, i.e., $r > g_2$.

Let $\Delta_1 = \frac{D_c^B - D_f^B}{U_f^B - U_c^B}$. We now show that in $R1$, for low relative use impact products characterized by $\frac{e_u}{e_p + e_d} < \Delta_1$, the necessary and sufficient condition for servicization to decrease the environmental impact and increase the profit is $r > g_2$. We know that $r > g_2$ if and only if $D_c^B > D_f^B$ and $U_c^B < U_f^B$. Therefore, $\frac{e_u}{e_p + e_d} < \Delta_1$ if and only if $e_u U_f^B + (e_p + e_d) D_f^B < e_u U_c^B + (e_p + e_d) D_c^B$. In addition, $\frac{e_u}{e_p + e_d} < \Delta_1$ defines a non-empty set since $\Delta_1 > 0$ in this region, and $r > g_2$ implies that $r > f_1$. Thus, the result follows.

We now show that in $R1$, for high relative use impact products characterized by $\frac{e_u}{e_p + e_d} > \Delta_1$, the necessary and sufficient condition for servicization to decrease the environmental impact and increase the profit is $f_1 < r < g_1$. We know that $r < g_1$ if and only if $D_c^B < D_f^B$ and $U_c^B > U_f^B$. Therefore, $\frac{e_u}{e_p + e_d} > \Delta_1$ if and only if $e_u U_f^B + (e_p + e_d) D_f^B < e_u U_c^B + (e_p + e_d) D_c^B$. In addition, $\frac{e_u}{e_p + e_d} < \Delta_1$ defines a non-empty set since $\Delta_1 > 0$ in this region, and $f_1 < g_1$. Thus, the result follows.

For $R1$, finally, note that when $g_1 \le r \le g_2$, $U_f^B \ge U_c^B$ and $D_f^B \ge D_c^B$, and hence, servicization can never improve the environmental impact regardless of the product type.

In R2, under servicization, product use impact, and production and disposal impact are same as in $R1$. Under selling, product use impact is $e_u \frac{M\beta\theta^3}{4k(1+m_c)^2} = e_u U_c^H$. Use impact under servicization is lower if and only if $e_u \frac{M\beta\theta^3}{4k(1+m_c)^2} > e_u \frac{M\alpha\left(\alpha^2+\beta-2\alpha\beta\right)\theta^3}{4k(1-\beta)\left(1+m_f\right)^2}$. This can be reorganized as $r < \sqrt{\frac{(1-\beta)\beta}{\alpha(\alpha^2+\beta-2\alpha\beta)}} \triangleq g_3$. Under selling, product production and disposal impact is given by $(e_d + e_p)\frac{4kM\beta(1+m_c)^2}{\theta^3}$. Then, production and disposal impact under servicization is lower than under selling if and only if $r > \sqrt{\frac{(1-\beta)(1+(-2+\alpha)\beta)}{(\alpha-\beta)\beta(\alpha^2+\beta-2\alpha\beta)}} \triangleq g_4$.

We now show that $g_4 > f_2 > g_3$. $g_4 > f_2$ (see Proposition 2 for f_2) if and only if $\frac{(1-\beta)(1+(-2+\alpha)\beta)}{(\alpha-\beta)\beta(\alpha^2+\beta-2\alpha\beta)} > \sqrt{\frac{(1-\beta)^2\beta}{(\alpha^2+\beta-2\alpha\beta)^2}}$. By taking the square of both sides, the expression can be rewritten as $\alpha^2(1-\beta)^3\beta^2 - 2\alpha(1-\beta)^3\beta(-1+\beta+\beta^2) + (1-\beta)^3(1-3\beta+\beta^2+\beta^3+\beta^4) > 0$. The expression is strictly convex in α and has its minimum at $\alpha = 1 - \frac{1}{\beta} + \beta < \beta$. Therefore, if the expression is positive at $\alpha = \beta$, it is always positive in $R2$. The value of the expression at $\alpha = \beta$ is $(-1+\beta)^6 > 0$. $f_2 > g_3$ if and only if $f_2^2 - g_3^2 = \frac{\left(\alpha^2-\beta\right)(-1+\beta)^2\beta}{\alpha^2(\alpha^2+\beta-2\alpha\beta)^2} > 0$. Thus, it is enough to show that in $R2$, $\alpha > \sqrt{\beta}$. In order to show this, we will prove that at $\frac{\pi_{f,B}}{\pi_{f,H}}|_{\beta=\alpha^2} < 1$. $\frac{\pi_{f,B}}{\pi_{f,H}}|_{\beta=\alpha^2} = \frac{\left(2\alpha^2-2\alpha^3\right)^2}{\alpha^2(-1+\alpha^2)^2} < 1$ if and only if $(-1+\alpha)^3\alpha^2(1+3\alpha) < 0$, which indeed holds. Thus, $\gamma > \sqrt{\beta}$, and we have $\alpha > \sqrt{\beta}$ in $R2$. Therefore, $g_4 > f_2 > g_3$. This shows that a region in which servicization increases the total use impact but decreases the production and disposal impact exists, i.e., $r > g_4$.

Let $\Delta_2 = \frac{D_c^H - D_f^B}{U_f^B - U_c^H}$. We now show that for low relative use impact products characterized by $\frac{e_u}{e_p + e_d} < \Delta_2$, the necessary and sufficient condition for servicization to decrease the environmental impact and increase the profit is $r > g_4$. We know that $r > g_4$ if and only if $D_c^H > D_f^B$ and $U_c^H < U_f^B$. Therefore $\frac{e_u}{e_p + e_d} < \Delta_2$ if and only if $e_u U_f^B + (e_p + e_d) D_f^B < e_u U_c^H + (e_p + e_d) D_c^H$. In addition, $\frac{e_u}{e_p + e_d} < \Delta_2$ defines a non-empty set since $\Delta_2 > 0$ in this region, and $r > g_4$ implies that $r > f_2$. Thus, the result follows.

For $R2$, finally, note that when $f_2 < r \leq g_4$, $U_f^B \geq U_c^H$ and $D_f^H \geq D_c^B$, and hence, servicization can never improve the environmental impact regardless of the product type.

In R3, under selling, product use impact, and production and disposal impact are same as in $R2$, i.e., use impact is $e_u \frac{M\beta\theta^3}{4k(1+m_c)^2} = e_u U_c^H$ and production and disposal impact is $(e_d + e_p)\frac{4kM\beta(1+m_c)^2}{\theta^3}$. Under servicization product use impact is $e_u \frac{M\beta\theta^3}{4k\left(1+m_f\right)^2} = e_u U_f^H$. Comparison of the use impacts shows that the use impact under servicization is lower if and only if $r<1$. Under servicization, product production and disposal impact is given by $(e_d + e_p)\frac{4kM\beta\left(1+m_f\right)^2}{\theta^3}$. Comparison of the production and disposal impacts shows that production and disposal impact under servicization is lower than under selling if and only if $r>1$.

Let $\Delta_3 = \frac{D_c^H - D_f^H}{U_f^H - U_c^H}$. We now show that in $R3$, for low relative use impact products characterized by $\frac{e_u}{e_p+e_d} < \Delta_3$, the necessary and sufficient condition for servicization to decrease the environmental impact and increase the profit is $r>1$. We know that $r>1$ if and only if $D_c^H > D_f^H$ and $U_c^H < U_f^H$. Therefore $\frac{e_u}{e_p+e_d} < \Delta_3$ if and only if $e_u U_f^H + (e_p+e_d)D_f^H < e_u U_c^H + (e_p+e_d)D_c^H$. In addition, $\frac{e_u}{e_p+e_d} < \Delta_3$ defines a feasible set since $\Delta_3 > 0$ in this region, and the profit is higher under servicization if and only if $r>1$. Thus, the result follows.

Finally we define,

$$\Delta(\alpha,\beta,m_c,m_f) = \begin{cases} \Delta_1 & : \text{in R1} \\ \Delta_2 & : \text{in R2} \\ \Delta_3 & : \text{in R3} \end{cases} \quad \overline{r}(\alpha,\beta) = \begin{cases} g_1 & : \text{in R1} \\ 0 & : \text{in R2} \\ 0 & : \text{in R3} \end{cases} \quad \underline{r}(\alpha,\beta) = \begin{cases} g_2 & : \text{in R1} \\ g_4 & : \text{in R2} \\ 1 & : \text{in R3} \end{cases}.$$

□

Proof of Corollary 1 In the proof of Proposition 4, we showed that $g_2 > g_1$, which proves the result. □

Proof of Proposition 5: $CS_f^{R1} = \sum_{i=L,H} \int_0^{\tau_{i,f}^*} (\theta_i - \frac{t}{\delta})dt - F_i = \frac{M(-1+\alpha)\beta\theta^4}{8c}\left(\frac{\alpha^2(1+\alpha)}{(1+m_c)^2} - \frac{2(\alpha-\beta)\left(\alpha^2+\beta-2\alpha\beta\right)}{(-1+\beta)^2\left(1+m_f\right)^2}\right)$ and $CS_c^{R1} = \sum_{i=L,H} \int_0^{\tau_{i,c}^*} (\theta_i - \frac{t}{\delta})dt - \frac{m_c}{2\delta}\tau_{i,c}^{*2} - p = -\frac{M\alpha^2\left(-1+\alpha^2\right)\beta\theta^4}{8c(1+m_c)^2}$. $CS_f^{R1} > CS_c^{R1}$ if and only if $-\left(\frac{\alpha^2(1+\alpha)}{(1+m_c)^2} - \frac{2(\alpha-\beta)\left(\alpha^2+\beta-2\alpha\beta\right)}{(-1+\beta)^2\left(1+m_f\right)^2}\right) > 0$. Then, $CS_f^{R1} > CS_c^{R1}$ if and only if $r^2 > \frac{(1-\beta)^2\alpha^2(1+\alpha)}{2(\alpha-\beta)(\alpha^2+\beta-2\alpha\beta)} = h^2$, and the result follows. $h>1$ if and only if $(-1+\alpha)\left(-\alpha^2+2\alpha\beta-2\alpha^2\beta-2\beta^2+2\alpha\beta^2+\alpha^2\beta^2\right)>0$. $\alpha<1$ by assumption; hence, if we show that second expression is negative, the result follows. Second derivative of the expression is $2(-1+(-2+\beta)\beta)<0$; hence, it is concave in α. At $\alpha=1$, it is $-1+\beta^2<0$, and at $\alpha=\beta$, it is $\beta^2\left(-1+\beta^2\right)<0$. Therefore, the expression is negative in $R1$.

In R2, the firm has to leave positive informational rent to high end segment under servicization. However, the firm extracts the entire surplus under selling. Hence, the CS under servicization is always higher. In R3, since the firm only serves high-end segment, the CS is zero for both selling and servicization strategies. □

Proof of Proposition 6 This can be shown easily by comparing Proposition 4 and 5. Hence, it is omitted. □

Proof of Proposition 7: The analysis are very similar to Proposition 1 except that we need to account for different product durabilities. Under servicization, if the firm serves only the high end segment, it solves the problem in (7). On the other hand, if it serves both segments, it solves the following:

$$\max_{F_i,\tau_i,\delta_i,i=H,L} \sum_{i=H,L} (F_i - \frac{m_f\tau_i^2}{2\delta_i} - k\delta_i^2)Q_i, \tag{11}$$

$$\begin{aligned} s.t. \quad & IR_i^{pl}: \quad \int_0^{\tau_i} (\theta_i - \frac{t}{\delta_i})dt - F_i \geq 0, \quad i=H,L \\ & IC_i^{pl}: \quad \int_0^{\tau_i} (\theta_i - \frac{t}{\delta_i})dt - F_i \geq \int_0^{\tau_j} (\theta_i - \frac{t}{\delta_j})dt - F_j, \quad i \neq j \text{ and } i,j=H,L. \end{aligned}$$

Similarly, under selling, if the firm serves only the high end segment, it solves the problem in (2). On the other hand, if it serves both segments, it solves the following:

$$\max_{F_i,\delta_i,i=H,L} \sum_{i=H,L}(F_i - k\delta_i^2)Q_i, \qquad (12)$$
$$s.t. \quad U_c(\theta_L) \geq 0 \quad .$$

Finally, comparison of these two strategies will give the optimal strategy and the decisions as stated. □

Proof of Proposition 8: This can be easily shown by computing the average product durabilities using Table 3 and then comparing these average product durabilities. Hence, it is omitted. □

Proof of Proposition 9: For simplicity, we drop the superscript *pl* when we refer to the equilibrium regions. It can be shown that in $R2$ servicization increases use impact if and only if $r^2 > \frac{(\beta-1)^2\beta}{\alpha^3-3\alpha^2\beta+(3\alpha-2)\beta^2+\beta} \triangleq u_{R2}$ and the disposal impact if and only if $r^2 < -\frac{\beta\left(\alpha^3-3\left(\alpha^2-2\right)\beta+(3\alpha-4)\beta^2-4\right)+1}{\beta(\beta-\alpha)^3} \triangleq d_{R2}$. We will prove that $u_{R2} < \overline{f}_2^2 < d_{R2}$. $u_{R2} < \overline{f}_2^2$ if and only if $-(\alpha-1)(\alpha-\beta)^3 > 0$, which is always true for $R2$.

$$d_{R2} - \overline{f}_2^2 = \frac{\nu_1}{\beta(\alpha-\beta)^3\left(\alpha^4 - 4\alpha^3\beta + 3\left(2\alpha^2-1\right)\beta^2 + (3-4\alpha)\beta^3 + \beta\right)} > 0.$$

By some algebra, it can be shown that the denominator is always positive in $R2$. Thus, we can only work with ν_2. Starting with $i=5$, it can be shown that $\partial^i\nu_1/\partial\alpha^i > 0$ for all $i \in \{1,2,3,4,5\}$. In addition, $\lim_{\beta\to 0}\nu_1(\alpha) = 0$. Thus, $\nu > 0$ in $R2$. Define $\Delta_{R2}^{pl} \triangleq \frac{\beta/\tau_{H,c}-\beta/\tau_{H,f}-(1-\beta)/\tau_{L,f}}{\beta\tau_{H,f}+(1-\beta)\tau_{L,f}-\beta\tau_{H,c}}$. Similarly for $R3$, we can define $\Delta_{R3}^{pl} \triangleq \frac{\beta/\tau_{H,c}-\beta/\tau_{H,f}}{\beta\tau_{H,f}-\beta\tau_{H,c}}$. Finally, define

$$\Delta^{pl}(\alpha,\beta,m_c,m_f) = \begin{cases} \Delta_{R2}^{pl} & \text{: in R2} \\ \Delta_{R3}^{pl} & \text{: in R3} \end{cases} \qquad \underline{r}^{pl} = \begin{cases} \sqrt{d_{R2}} & \text{: in R2} \\ 1 & \text{: in R3} \end{cases} \quad .$$

Then, the result follows. □

Proof of Proposition 10: To find the optimal strategy we need to compare six different strategies: serve both segments by selling (servicization), serve only high end segment by selling (servicization), serve high end segment by selling (servicization) and low end segment by servicization (selling). From Proposition 1, we already know the comparison of the first four strategies. Denote the last two strategies as $(sell, serv.)$ and $(serv., sell)$, respectively. The optimal durability, prices and the use durations for these strategies can be obtained using standard contract design tools similar to Proposition 1 as follows. If the firm uses $(sell, serv.)$ strategy, it solves

$$\max_{\delta,p_H,F_L,\tau_L} (p_H - c\delta^2)Q_H + (F_L - \frac{m_f\tau_L^2}{2\delta} - c\delta^2)Q_L \qquad (13)$$
$$s.t. \quad U_c(\theta_H|\theta_H) \geq U_f(\theta_L|\theta_H), \quad U_f(\theta_L|\theta_L) \geq U_c(\theta_H|\theta_L), \quad U_f(\theta_L|\theta_L) \geq 0, \quad U_c(\theta_H|\theta_H) \geq 0.$$

where $U_c(\theta_j|\theta_i)$ and $U_f(\theta_j|\theta_i)$ are utilities of type-i if he takes the contract intended for type-j under selling and servicization models, respectively. Thus,

$$U_c(\theta_j|\theta_i) = \max_{\tau_c}\int_0^{\tau_c}\left(\theta_i - \frac{t}{\delta}\right)dt - \frac{m_c\tau_c^2}{2\delta} - p_j \text{ and } U_f(\theta_j|\theta_i) = \theta_i\tau_j - \frac{\tau_j^2}{2\delta} - F_j$$

Similarly, when the firm uses $(serv, sell)$ strategy, it solves

$$\max_{\delta,p_L,F_H,\tau_H} (p_L - c\delta^2)Q_L + (F_H - \frac{m_f\tau_H^2}{2\delta} - c\delta^2)Q_H \qquad (14)$$
$$s.t. \quad U_c(\theta_L|\theta_L) \geq U_f(\theta_H|\theta_L), \quad U_f(\theta_H|\theta_H) \geq U_c(\theta_L|\theta_H), \quad U_f(\theta_H|\theta_H) \geq 0, \quad U_c(\theta_L|\theta_L) \geq 0.$$

Strategy	δ^*	τ^*	π^*
$(Sell, Serv.)$	$-\frac{\theta^2\left(\alpha^2-2\alpha\beta+\beta+(\alpha-\beta)^2 m_c-(\beta-1)\beta m_f\right)}{4(\beta-1)k(m_c+1)\left(m_f+1\right)}$	$\tau_H^*=\frac{\theta\delta^*}{1+m_c},\ \tau_L^*=\frac{\theta(\alpha-\beta)\delta^*}{(1-\beta)\left(1+m_f\right)}$	$kM\delta^{*2}$
$(Serv., Sell)$	$\frac{\theta^2\left(\alpha^2+\beta m_c+\left(\alpha^2-\beta\right)m_f\right)}{4k(1+m_c)\left(1+m_f\right)}$	$\tau_H^*=\frac{\theta\delta^*}{1+m_f},\ \tau_L^*=\frac{\alpha\theta\delta^*}{1+m_c}$	$kM\delta^{*2}$

Table 4 Equilibrium under hybrid strategy.

Table 4 gives the equilibriums. Comparison of all six strategies yields the result, where

$$m_1(\alpha,\beta,m_c)=\begin{cases}-\frac{(\alpha-1)^2\beta+(\alpha-\beta)^2 m_c}{(\beta-1)(\alpha^2-\beta)} & : \text{in R1}\\ \frac{\left(\sqrt{\beta}+1\right)\left(\alpha^2-2\alpha\beta+\beta^{3/2}+\beta-\sqrt{\beta}+(\alpha-\beta)^2 m_c\right)}{(\beta-1)^2\sqrt{\beta}} & : \text{in R2}\end{cases}$$

□

Proof of Proposition 11: The result can be found by comparing the product durabilities in Tables 1 and 4. Hence, omitted. □

Proof of Proposition 12: This can be shown similar to Proposition 4 using the use durations in Table 4. Hence, omitted. In the proposition, $m_3=\max\left(m_c,m_4\right)$, and

$$\beta_1=\min\left(\alpha^4,\frac{-\alpha^3+4\alpha^2-1}{2(2\alpha-1)}-\frac{1}{2}\sqrt{\frac{\alpha^6-8\alpha^5+16\alpha^4-14\alpha^3+8\alpha^2-4\alpha+1}{(2\alpha-1)^2}}\right),\quad \Delta_h\ =\frac{D_c^B-D_{(sell,serv.)}}{U_{(sell,serv.)}-U_c^B}.$$

In the above expressions $D_{(sell,serv.)}$ ($U_{(sell,serv.)}$) denotes total production and disposal impacts (use impact) under the hybrid strategy, and D_r^B (U_c^B) are given in the proof of Proposition 4. m_4 can be found as the larger root of $U_{(sell,serv.)}-U_c^B$. $U_{(sell,serv.)}-U_c^B$ can be shown to have to two roots with respect to m_f. Finally, m_2 can be found as the larger root of $D_{(sell,serv.)}-D_c^B$. $D_{(sell,serv.)}-D_c^B$ can be shown to have to two roots with respect to m_f. Due to page count limitations, we do not state these roots in the paper explicitly; however, the details are available from the authors. □

Proof of Proposition 13: In R1, $SS_{f,B}=-\frac{M\left(-\alpha^2-\beta+2\alpha\beta\right)\left(\alpha^2+\beta+2(1-2\alpha)\alpha\beta+4(-1+\alpha)\beta^2\right)\theta^4}{16k(-1+\beta)^2\left(1+m_f\right)^2}$ and $SS_{c,B}=\frac{M\alpha^2\left(\alpha^2(1-2\beta)+2\beta\right)\theta^4}{16k(1+m_c)^2}$. By rearranging the terms, $SS_{f,B}>SS_{c,B}$ if and only if $r^2>\frac{\alpha^2(-1+\beta)^2\left(-2\beta+\alpha^2(-1+2\beta)\right)}{(\alpha^2+\beta-2\alpha\beta)(-2\alpha\beta(1+2\beta)+\alpha^2(-1+4\beta)+\beta(-1+4\beta))}\triangleq k_1^2$. We need to show that $k_1>f_1$. This is true if and only if $\frac{\alpha^2(-1+\beta)^2\left(-2\beta+\alpha^2(-1+2\beta)\right)}{(\alpha^2+\beta-2\alpha\beta)(-2\alpha\beta(1+2\beta)+\alpha^2(-1+4\beta)+\beta(-1+4\beta))}>\frac{\alpha^4(-1+\beta)^2}{(\alpha^2+\beta-2\alpha\beta)^2}$. This inequality can be written as $-\frac{2(-1+\alpha)^2\alpha^2(-1+\beta)^2\beta\left(\alpha^2+\beta\right)}{(\alpha^2+\beta-2\alpha\beta)^2(-2\alpha\beta(1+2\beta)+\alpha^2(-1+4\beta)+\beta(-1+4\beta))}>0$. Note that numerator is always positive and $\left(\alpha^2+\beta-2\alpha\beta\right)^2>0$. Hence, we need to show that $e_1\triangleq-(-2\alpha\beta(1+2\beta)+\alpha^2(-1+4\beta)+\beta(-1+4\beta))>0$. $\frac{\partial^2 e_1}{\partial\beta^2}=8(-1+\alpha)<0$, and hence, e_1 is concave in β. Then it is sufficient to show that e_1 is positive at $\beta=0$ and $\beta=\alpha$. $e_1(\beta=0)=\alpha^2$ and $e_1(\beta=\alpha)=\alpha(1-\alpha)$. This proves that when servicization is more profitable than selling strategy in R1, $SS_{f,B}>SS_{c,B}$ if and only if $r>k_1$. To complete the proof for region $R1$, we need to find when $k_1>1$ is true. Observe that this is true if and only if $l=-1-2\alpha+5\alpha^2+(4-2\alpha(2+\alpha))\beta>0$. When $\beta<\frac{1}{2}$, l has two roots $\alpha_1^l=\frac{-1-2\beta-\sqrt{6}\sqrt{1-3\beta+2\beta^2}}{-5+2\beta}$ and $\alpha_2^l=\frac{-1-2\beta+\sqrt{6}\sqrt{1-3\beta+2\beta^2}}{-5+2\beta}$. Furthermore, $\alpha_1^l>\alpha_2^l$ and $\alpha_2^l<\beta$. Since, l is convex in this region, $l>0$ if and only if $\alpha>\min\{\alpha_1^l,\sqrt[4]{\beta}\}$. $\frac{\partial\alpha_l^1}{\partial\beta}<0$ for $\beta\in(0,0.5)$. $\alpha_l^1(\beta=0)\approx 0.69$ and $\alpha_l^1(\beta=1)=0.5$. Therefore, there exist a $\beta_c\in(0,0.5)$ such that $\alpha_1^l>\sqrt[4]{\beta}$ if and if $0<\beta<\beta_c$. When $\beta\geq\frac{1}{2}$, l does not have any roots. Hence it is either always positive or always negative. It is easy to see that it is always positive. Then, define

$$\sigma(\beta)=\begin{cases}\sqrt[4]{\beta} & :\beta\geq\beta_c\\ \alpha_1^l & :\beta<\beta_c\end{cases}$$

In R2, $SS_{c,H} = \frac{M\beta\theta^4}{16k\left(1+m_f\right)^2}$. By rearranging the terms we can show that $SS_{f,B} > SS_{c,H}$ if and only if $r > \sqrt{-\frac{(-1+\beta)^2\beta}{(-\alpha^2-\beta+2\alpha\beta)(\alpha^2+\beta+2\alpha\beta-4\alpha^2\beta-4\beta^2+4\alpha\beta^2)}} \triangleq k_2$. $f_2 > k_2$ if and only if $\frac{(1-\beta)^2\beta}{(\alpha^2+\beta-2\alpha\beta)^2} > -\frac{(-1+\beta)^2\beta}{(-\alpha^2-\beta+2\alpha\beta)(\alpha^2+\beta+2\alpha\beta-4\alpha^2\beta-4\beta^2+4\alpha\beta^2)}$. The inequality can be written as $(\alpha^2+\beta-2\alpha\beta)(-1-\alpha^2-\beta-2\alpha\beta+4\alpha^2\beta+4\beta^2-4\alpha\beta^2) < 0$. First expression in the equality is convex in α and takes its minimum value at $\alpha=\beta$ which is $\beta(1-\beta)>0$. Second expression is convex in β, and hence, it is sufficient to show that it is negative at $\beta=0$ and $\beta=\alpha$. These can be shown by simple algebra.

In R3, since the SS is equivalent to firm's profit under both selling and servicization. The proof is same as the comparison of the profits in the proof of Proposition 2. □

Proof of Lemma 2: These can be shown easily by comparing the product durabilities and consumer segments' use durations in Proposition 1 in this paper and Proposition 17 in Orsdemir et al. (2017). □

Proof of Proposition 14 Define $U_i^{pl} = \beta M\tau_{i,H} + (1-\beta)M\tau_{i,L}$ and $D_i^{pl} = \beta M/\tau_{i,H} + (1-\beta)M/\tau_{i,L}$ where $\tau_{i,j}$'s are given in Table 3. In addition, $\Delta_{R1}^{pl} = (D_f^{pl} - D_c^{pl})/(U_c^{pl} - U_f^{pl})$. Finally, it can be shown that $\pi_f > \pi_c$ if and only if $r > \frac{(\beta-1)^2\left(\alpha^4-2\alpha^2\beta+\beta\right)}{\alpha^4-4\alpha^3\beta+3(2\alpha^2-1)\beta^2+(3-4\alpha)\beta^3+\beta}$. Then, the proposition follows. □