

Online Appendix to “Buyback Pricing of Durable Goods in Dual Distribution Channels”

We derive the inverse demand functions in Appendix A, and analyze three extensions to our base model: (1) In Appendix B, we assume that the dealer can lease new cars to consumers; (2) In Appendix C, we consider competition between two dealers; (3) In Appendix D, we analyze the case where program cars depreciate faster than used cars. The proofs of the results in Appendices C-D are similar to proofs for the base model and thus omitted for brevity (available upon request).

A. Derivation of the Inverse Demand Functions

Sales Market. A consumer purchases a car, if any, that gives him/her the highest positive utility. Given the consumer valuations specified in Section 3, a new car is valued higher than a program car, which is valued higher than a used car. Assume, in period 2, the consumer who is indifferent between buying a used car and not buying a car is located at ϕ_1 . Therefore, $\phi_1 = 1 - q_{2n} - q_{2p} - q_{2u}$ and $(1 - \theta_u)\phi_1 - p_{2u} = (1 - \theta_u)(1 - q_{2n} - q_{2p} - q_{2u}) - p_{2u} = 0$. Solving for p_{2u} , we get Eq. (4): $p_{2u} = (1 - \theta_u)(1 - q_{2n} - q_{2p} - q_{2u})$. Now, assume the consumer who is indifferent between purchasing a used car and a program car is located at ϕ_2 . Thus, $\phi_2 = 1 - q_{2n} - q_{2p}$ and $(1 - \theta_p)\phi_2 - p_{2p} = (1 - \theta_p)\phi_2 - p_{2p}$. Plugging in ϕ_2 , we get $(1 - \theta_u)(1 - q_{2n} - q_{2p}) - p_{2u} = (1 - \theta_p)(1 - q_{2n} - q_{2p}) - p_{2p}$. Solving for p_{2p} , we get Eq. (3): $p_{2p} = p_{2u} + (\theta_u - \theta_p)(1 - q_{2n} - q_{2p})$. Finally, assume the consumer who is indifferent between purchasing a program car and a new car is located at ϕ_3 . Therefore, $\phi_3 = 1 - q_{2n}$ and $(1 - \theta_p)\phi_3 - p_{2p} = \phi_3 - p_{2n}$. Plugging in ϕ_3 , we get $(1 - \theta_p)(1 - q_{2n}) - p_{2p} = (1 - q_{2n}) - p_{2n}$. Solving for p_{2n} , we get Eq. (2): $p_{2n} = p_{2p} + \theta_p(1 - q_{2n})$. In period 1, only new cars exist in the sales market. A consumer can purchase a car and use it for two periods or do not purchase a car in period 1 and buy a used car in period 2. Assume the consumer who is indifferent between the two alternatives is located at ϕ_4 . Therefore, $\phi_4 = 1 - q_{1n}$ and $\phi_4 - p_{1n} + (1 - \theta_u)\phi_4 = (1 - \theta_u)\phi_4 - p_{2u}$. Plugging in ϕ_4 and then solving for p_{1n} , we get Eq. (1): $p_{1n} = p_{2u} + (1 - q_{1n})$.

Rental Market. In the rental market only new and program cars exist in period 2. Assume a consumer located at ϕ_5 is indifferent between renting a program car and not renting at all. Therefore, $\phi_5 = 1 - \bar{q}_{2n} - \bar{q}_{2p}$ and $(1 - \theta_p)\phi_5 - \bar{p}_{2p} = 0$. Plugging in ϕ_5 and solving for \bar{p}_{2p} , we get Eq. (7): $\bar{p}_{2p} = (1 - \theta_p)(1 - \bar{q}_{2n} - \bar{q}_{2p})$. Now, assume the consumer located at ϕ_6 is indifferent between renting a new and a program car. Therefore, $\phi_6 = 1 - \bar{q}_{2n}$ and $(1 - \theta_p)\phi_6 - \bar{p}_{2p} = \phi_6 - \bar{p}_{2n}$. Plugging in ϕ_6 and solving for \bar{p}_{2n} , we get Eq. (6): $\bar{p}_{2n} = \bar{p}_{2p} + \theta_p(1 - \bar{q}_{2n})$. Finally, in period 1 only new cars exist in the rental market and therefore we have Eq. (5): $\bar{p}_{1n} = 1 - \bar{q}_{1n}$.

B. Leasing

We assume that the dealer can sell and lease new cars to consumers. (In reality, manufacturers may lease to consumers directly, but for simplicity we do not consider it in this model.)

B.1 Leasing: The Model

Following Bhaskaran and Gilbert (2009), we assume that the lease contract is for one period, and at the end of one period leased cars are returned to and owned by the dealer. Furthermore, leased cars depreciate at the same rate as used cars. Therefore, (similar to Bhaskaran and Gilbert) there is no difference between selling and leasing in period 2. Let q_L denote the quantity of leased cars and q_S denote the quantity of sold cars by the dealer in period 1. At the beginning of period 2, the leased cars are returned to the dealer. The dealer decides how many new cars to buy (q_{2n}) at wholesale price w_2 and how many program cars to buy (q_{2p}) at price v from the manufacturer, maximizing the total profit from new and program car sales:

$$(PL1) \quad \max_{q_{2n}, q_{2p}} \pi_2 = q_{2n}(p_{2n} - w_2) + q_{2p}(p_{2p} - v) + q_L p_{2u}. \quad (B.1)$$

Note the dealer's profit includes the revenue from the sales of previously leased cars (q_L) in the used car market at p_{2u} . Also, we make the reasonable assumption that the dealer does not withhold leased cars once they are returned. The rental agency's second-period problem (P2) (see Eq. (9)) and the manufacturer's second-period problem (P3) (see Eq. (10)) remain the same.

In period 1, the dealer's problem is as follows:

$$(PL4) \quad \max_{q_S \geq 0, q_L \geq 0} \pi_1 = \pi_2^* + p_{1n}q_S + p_Lq_L - w_1(q_S + q_L), \quad (B.2)$$

where $q_S + q_L = q_{1n}$ and $p_L = 1 - q_{1n}$. Here, we assume that the manufacturer sets a single wholesale price for the dealer (Bhaskaran and Gilbert, 2009). The rental agency's and manufacturer's first-period problems are given in (P4) and (P6) (see Eq. (13) and (14)), respectively.

Consumer demand. In period 2, the total used-car quantity is $q_{2u} = q_L + q_S = q_{1n}$ (since leased cars depreciate as much as sold cars). Thus, same as our base model, used-car quantity in period 2 is equal to the quantity of new cars the dealer purchases in period 1, i.e., $q_{2u} = q_{1n}$. Thus, the inverse demand functions for the sales market given in Eq. (2)-(4) (i.e., p_{2n} , p_{2p} , and p_{2u}) remain the same. As leasing does not affect the underlying utility model in the rental market, the inverse demand functions for the rental market given in Eq. (5)-(7) (i.e., \bar{p}_{1n} , \bar{p}_{2p} , and \bar{p}_{2n}) remain the same too.

In period 1, to derive the inverse demand functions in the sales market, we consider the indifferent customer between buying and leasing located at ϕ_1 . This customer can buy a new car and use it for two periods or lease a car in period 1 and buy a used car in period 2. Hence, we have $(\phi_1 - p_{1n}) + \phi_1(1 - \theta_u) = (\phi_1 - p_L) + \phi_1(1 - \theta_u) - p_{2u}$. Solving for p_{1n} we find the following inverse demand function which is similar to those in Desai and Purohit (1999) and Bhaskaran and Gilbert (2009):

$$p_{1n} = p_L + p_{2u} \quad (B.3)$$

Now, consider the customer located at ϕ_2 , who is indifferent between leasing and not leasing. This customer can lease in period 1 and buy a used car in period 2 or do nothing in period 1 and buy a used car in period 2. Hence, $(\phi_2 - p_L) + (\phi_2(1 - \theta_u) - p_{2u}) = \phi_2(1 - \theta_u) - p_{2u}$. Solving for p_L , we find:

$$p_L = \phi_2 = 1 - q_{1n}. \quad (B.4)$$

From Eq. (B.3) and (B.4) we find $p_{1n} = (1 - q_{1n}) + p_{2u}$, i.e., the same as in the base case (see Eq. (1)).

B.2 Leasing: Equilibrium Analysis

We characterize the subgame perfect equilibrium under the two buyback pricing schemes in §B.2.1 and §B.2.2 and then find the manufacturer's preference in §B.2.3. As in the base model, the intermediaries' second-period problems do not depend on the buyback pricing scheme, thus the second-period equilibria between the intermediaries are identical for the two schemes:

$$q_{2n}^* = \frac{\gamma\theta_u - w_2 + v}{2\gamma\theta_u} \quad (B.5)$$

$$\bar{q}_{2n}^* = \frac{1 - \bar{w}_2}{2} - (1 - \gamma\theta_u)(\bar{q}_{1n} - \bar{q}_v^*) \quad (B.6)$$

$$q_{2p}^* = \frac{(1 - \gamma\theta_u)w_2 - \gamma\theta_u(1 - \theta_u)(2q_L + q_S) - v}{2\gamma\theta_u(1 - \gamma\theta_u)} \quad (B.7)$$

$$\bar{q}_v^* = \max\left\{\frac{2\gamma\theta_u\bar{q}_{1n} - 2\gamma^2\theta_u^2\bar{q}_{1n} + \gamma\theta_u\bar{w}_2 - \bar{w}_2 + \bar{v}}{2\gamma\theta_u(1 - \gamma\theta_u)}, 0\right\} \quad (B.8)$$

As in the base model, we find that there are two possible scenarios: *i*) positive buyback quantity, i.e., $\bar{q}_v^* = q_{2p}^* > 0$, and *ii*) zero buyback quantity, i.e., $\bar{q}_v^* = q_{2p}^* = 0$. Next, for each of the two possible scenarios, we characterize each player's optimal decisions under the two pricing schemes.

B.2.1 Leasing: Postponed Buyback Pricing

We first consider the case of positive buyback quantity and analyze the players' optimal decisions through backward induction.

Second Period. In period 2, the manufacturer maximizes her profit $\Pi_2 = q_{2n}^* w_2 + \bar{q}_{2n}^* \bar{w}_2 + q_{2p}^* v^* - \bar{q}_v^* \bar{v}$ by choosing wholesale prices (w_2, \bar{w}_2) as well as the buyback price \bar{v} . From the optimality conditions, we find that the wholesale prices and the buyback price as follows:

$$w_2^* = \frac{1 - (1 - \theta_u)(2q_L + q_S)}{2}, \quad (\text{B.9})$$

$$\bar{w}_2^* = \frac{1 - 2(1 - \gamma\theta_u)\bar{q}_{1n}}{2}, \quad (\text{B.10})$$

$$\bar{v}^* = \frac{2 - 2\theta_u\gamma - \theta_u\gamma(1 - \theta_u)(2q_L + q_S) - 2(2 + \gamma\theta_u)(1 - \gamma\theta_u)\bar{q}_{1n}}{4}. \quad (\text{B.11})$$

As in the base model, the wholesale price offered to each intermediary *inversely* depends on his own first-period order quantity but is independent of the other's (see Eq. (B.9) and (B.10)); and the buyback price \bar{v}^* decreases as first-period dealer- and/or rental-car sales increase (see Eq. (B.11)).

First Period. In period 1, the intermediaries maximize their profits by optimizing their ordering decisions. We find that $q_S^* = 0$ (see Appendix B.3). Recall that $q_{1n}^* = q_L^* + q_S^*$. Next result identifies how these quantities depend on the wholesale prices.

Proposition B.1 (Leasing:Cross-Channel Interactions: Postponed Scheme) *When $\bar{q}_v^* > 0$,*

- i. the dealer's order quantity is increasing in the rental-car wholesale price (i.e., $\frac{\partial q_{1n}^*}{\partial \bar{w}_1} > 0$), and*
- ii. the rental agency's order quantity is increasing in the dealer-car wholesale price (i.e., $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} > 0$).*

Comparing Proposition B.1 with Proposition 1, note that the sign of partial derivatives remain the same as those in the base model. Finally, the manufacturer maximizes her profit $\Pi_1 = \bar{q}_{1n}^*(\bar{w}_1, w_1)\bar{w}_1 + q_{1n}^*(\bar{w}_1, w_1)w_1 + \Pi_2^*$ by setting the wholesale prices (\bar{w}_1, w_1) . We omit the details for brevity. In addition, we solve the manufacturer's problem when the buyback quantity is zero (see Appendix B.3). By comparing the manufacturer's profits under the two scenarios, we show that the equilibrium outcome is the same as in the base model:

Proposition B.2 (Leasing) *Under postponed buyback pricing, a positive buyback quantity always arises in equilibrium, i.e., $\bar{q}_v^* > 0$, which maximizes the manufacturer's profit.*

B.2.2 Leasing: Precommitted Buyback Pricing

Second Period. If the manufacturer precommits the buyback price in period 1, she sets wholesale prices (w_2, \bar{w}_2) to maximize her profit in period 2, i.e., solving problem (P3). Her dealer-car wholesale price w_2^* remains the same as under the postponed scheme (see Eq. (B.9)) while her rental-car wholesale price becomes

$$\bar{w}_2^* = \frac{\gamma\theta_u(1 - \theta_u)(2q_L + q_S) + 4\gamma\theta_u(1 - \gamma\theta_u)\bar{q}_{1n} + \gamma\theta_u + 4\bar{v}}{2(2 - \gamma\theta_u)}. \quad (\text{B.12})$$

Comparing Eq. (B.10) with (B.12), observe that under the postponed scheme, \bar{w}_2^* decreases as the rental agency's first-period order quantity \bar{q}_{1n} increases. Under the precommitted scheme, however, \bar{w}_2^* increases as \bar{q}_{1n} and/or q_{1n}^d increases. This difference in the direction of change in \bar{q}_{1n} is the same as in base model.

First Period. We solve the dealer's problem (PL4) and the rental agency's problem (P5) simultaneously. With the following proposition, we show how each intermediary's ordering decision depends on the manufacturer's prices offered to the other intermediary.

Proposition B.3 (Leasing:Cross-Channel Interactions: Precommitted Scheme) *When $\bar{q}_v^* > 0$*

- i) *the dealer's order quantity is increasing in the rental-car wholesale price and decreases in the buyback price (i.e., $\frac{\partial q_{1n}^*}{\partial \bar{w}_1} > 0$ and $\frac{\partial q_{1n}^*}{\partial \bar{v}} < 0$),*
- ii) *the rental agency's order quantity is decreasing in the dealer-car wholesale price and increases in the buyback price (i.e., $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} < 0$ and $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{v}} > 0$).*

Comparing Proposition B.3 with Proposition 3, we see that the direction of change in the intermediaries' ordering decisions w.r.t. wholesale prices remain the same as in the base model. Finally, solving the manufacturer's first-period problem (P6), as in the base model, we find that a sufficiently high (low) buyback price \bar{v} yields a positive (zero) buyback quantity. The next proposition shows, as in the base model, the manufacturer would always prefer zero buyback quantity in equilibrium.

Proposition B.4 (Leasing) *Under precommitted buyback pricing, zero buyback quantity always arises in equilibrium, i.e., $\bar{q}_v^* = 0$, which maximizes the manufacturer's profit.*

When the equilibrium buyback quantity is positive, next we show that how buyback prices and the rental car wholesale prices compare between the buyback pricing schemes remain the same as in base model. Furthermore, the buyback price the manufacturer pays to the rental agency is always higher than the resale price charged to the dealer. This result is also the same as before.

Corollary B.1 (Leasing) *When $\bar{q}_v^* > 0$, the buyback price is higher but the first-period rental-car wholesale price is lower under the precommitted scheme than those under the postponed scheme. Furthermore, under the precommitted (postponed) scheme, the buyback price paid to the rental agency is higher (lower) than the resale price charged to the dealer, i.e., $\bar{v} > v$ ($\bar{v} < v$).*

B.2.3 Leasing: The Manufacturer's Preferred Buyback Pricing scheme

Next we show that our finding in the base model regarding a manufacturer's preference over the two buyback pricing schemes continues to hold when leasing is considered.

Proposition B.5 (Leasing) *The manufacturer's total two-period profit Π_1^* is higher under the postponed scheme than under the precommitted scheme.*

B.3 Leasing: Characterization of Solution

In this section, we derive the equilibrium solution for the leasing model. First, we write the Lagrangian of the rental agency's problem as $L_R = \bar{\pi}_2 + b_R \bar{q}_v$ where b_R is the Lagrangian multiplier. Then the first order conditions (FOCs) $\frac{\partial L_R}{\partial q_{2n}} = 1 - 2(\bar{q}_{1n} - \bar{q}_v)(1 - \gamma\theta_u) - \bar{w}_2 - 2\bar{q}_{2n} = 0$, $\frac{\partial L_R}{\partial \bar{q}_v} = \bar{v} - (1 - \gamma\theta_u)(1 - 2\bar{q}_{2n} - 2\bar{q}_{1n} + 2\bar{q}_v) + b_R = 0$, $\frac{\partial \pi_2}{\partial q_{2n}} = 1 - (2q_L + q_S)(1 - \theta_u) - 2q_{2n} - w_2 - 2q_{2p}(1 - \gamma\theta_u) = 0$, and $\frac{\partial \pi_2}{\partial q_{2p}} = (1 - \gamma\theta_u)(1 - 2q_{2p} - 2q_{2n}) - (2q_L + q_S)(1 - \theta_u) - v = 0$, the complementary slackness condition $b_R \bar{q}_v = 0$ as well as feasibility conditions $\bar{q}_v \geq 0$ and $b_R \geq 0$ should hold. Solving all optimality conditions simultaneously, we find two possible equilibrium solutions:

$$\begin{aligned} \text{-Soln 1: } & b_R^* = (\bar{w}_2 - 2\gamma\theta_u \bar{q}_{1n})(1 - \gamma\theta_u) - \bar{v}, q_{2n}^* = \frac{\gamma\theta_u - w_2 + v}{2\gamma\theta_u}, \bar{q}_{2n}^* = \frac{1 - \bar{w}_2 - 2(1 - \gamma\theta_u)\bar{q}_{1n}}{2}, q_{2p}^* = \frac{(1 - \gamma\theta_u)w_2 - (1 - \theta_u)\gamma\theta_u(2q_L + q_S) - v}{2\gamma\theta_u(1 - \gamma\theta_u)}, \bar{q}_v^* = 0, \\ \text{-Soln 2: } & b_R^* = 0, q_{2n}^* = \frac{\gamma\theta_u - w_2 + v}{\gamma\theta_u}, \bar{q}_{2n}^* = \frac{\gamma\theta_u - \bar{w}_2 + \bar{v}}{2\gamma\theta_u}, q_{2p}^* = \frac{-(1 - \theta_u)\gamma\theta_u(2q_L + q_S) + (1 - \gamma\theta_u)w_2 - v}{2\gamma\theta_u(1 - \gamma\theta_u)}, \bar{q}_v^* = \frac{(2\gamma\theta_u \bar{q}_{1n} - \bar{w}_2)(1 - \gamma\theta_u) + \bar{v}}{2\gamma\theta_u(1 - \gamma\theta_u)}. \end{aligned}$$

The resale price v_p^* comes from the equilibrium $\bar{q}_v^* = q_{2p}^*$. From here, under soln 1 we get $v_p^* = (1 - \gamma\theta_u)w_2 - \gamma\theta_u(1 - \theta_u)(2q_L + q_S)$; and under soln 2 we get $v_p^* = (1 - \gamma\theta_u)(w_2 + \bar{w}_2) - \gamma\theta_u(1 - \theta_u)(2q_L + q_S) - 2\gamma\theta_u(1 - \gamma\theta_u)\bar{q}_{1n} - \bar{v}$. Accordingly, soln 1 (soln 2) results in zero (positive) buyback

quantity, i.e., $\bar{q}_v^* = q_{2p}^* = 0$ (i.e., $\bar{q}_v^* = q_{2p}^* > 0$). For each solution, we continue with standard application of backward induction. For brevity we do not show details of each intermediate step (available upon request) but present the final solutions. We verify that all optimal values of the decision variables and objective functions are nonnegative in the parameter space (i.e., when $\gamma \geq 0$ and $\theta_u \geq 0$).

B.3.1 Zero Buyback Quantity (For precommitted and postponed buyback pricing):

Recall that buyback pricing scheme becomes irrelevant under zero buyback quantity. Therefore, using soln 1 ($\bar{q}_v^* = q_{2p}^* = 0$) in backward induction for both precommitted and postponed schemes

gives the following: $\bar{w}_1^* = \frac{11\gamma^3\theta_u^3 - 25\gamma^2\theta_u^2 - 19\gamma\theta_u + 49}{16(3 - \gamma^2\theta_u^2)}$, $w_1^* = \frac{11\theta_u^3 - 25\theta_u^2 - 19\theta_u + 49}{16(3 - \theta_u^2)}$, $\bar{q}_{1n}^* = \frac{5 - \gamma\theta_u}{8(3 - \gamma^2\theta_u^2)}$, $q_L^* = q_{1n}^* = \frac{5 - \theta_u}{8(3 - \theta_u^2)}$, $q_S^* = 0$, $\bar{w}_2^* = \frac{7 - 5\gamma^2\theta_u^2 + 6\gamma\theta_u}{8(3 - \gamma^2\theta_u^2)}$, $w_2^* = \frac{7 - 5\theta_u^2 + 6\theta_u}{8(3 - \theta_u^2)}$, $\bar{q}_{2n}^* = \frac{7 - 5\gamma^2\theta_u^2 + 6\gamma\theta_u}{16(3 - \gamma^2\theta_u^2)}$, $q_{2n}^* = \frac{7 - 5\theta_u^2 + 6\theta_u}{16(3 - \theta_u^2)}$, $\bar{\pi}_1^* = \frac{15\gamma^4\theta_u^4 + 8\gamma^3\theta_u^3 - 94\gamma^2\theta_u^2 - 120\gamma\theta_u + 319}{256(3 - \gamma^2\theta_u^2)^2}$, $\pi_1^* = \frac{15\theta_u^4 + 8\theta_u^3 - 94\theta_u^2 - 120\theta_u + 319}{256(\theta_u^2 - 3)^2}$. And the manufacturer's profit is:

$$\Pi_1^* = \frac{7\gamma^2\theta_u^4 + 5\gamma^2\theta_u^3 - 35\gamma^2\theta_u^2 + 5\gamma\theta_u^3 - 15\gamma\theta_u - 35\theta_u^2 - 15\theta_u + 147}{32(3 - \theta_u^2)(3 - \gamma^2\theta_u^2)} \quad (\text{B.13})$$

We find $b_R^* = \frac{(3\gamma^2\theta_u^2 + \gamma\theta_u - 11)\gamma\theta_u + (8\gamma^2\theta_u^2 - 24)\bar{v} + 7}{8(3 - \gamma^2\theta_u^2)}$ by substituting the variables' optimal values in b_R^* of soln 1. Note that $b_R^* \geq 0$ can be written as $\bar{v} \leq \bar{v}_T$ where b_T is as in Eq. (A.2) (i.e., same as base model). Finally, $\frac{\partial \bar{v}_T}{\partial \gamma} < 0$ and $\frac{\partial \bar{v}_T}{\partial \theta_u} < 0$. This concludes the solution for $\bar{q}_v^* = 0$. Positive-buyback quantity, however, needs to be analyzed separately under each pricing scheme. We present our findings next. In the remainder of this appendix, we provide only the expressions of variables and profit functions which are necessary for the derivation of results. The expressions of the remaining variables and profit functions are omitted for brevity; but available upon request.

B.3.2 Precommitted Buyback Pricing with Positive Buyback Quantity:

Using soln 2 ($\bar{q}_v^* = q_{2p}^* > 0$) in backward induction for precommitted buyback pricing, we find:

$$\bar{v}^* = \frac{1 - \gamma\theta_u}{2A} \left(\frac{768 - 2\gamma^6\theta_u^8 - \gamma^5(4\gamma - 19)\theta_u^7 + \gamma^4(10\gamma^2 + 46\gamma - 91)\theta_u^6 - \gamma^3(117\gamma^2 + 286\gamma - 316)\theta_u^5}{+\gamma^2(717\gamma^2 + 792\gamma - 652)\theta_u^4 - 8\gamma(280\gamma^2 + 121\gamma - 83)\theta_u^3 + (3548\gamma^2 + 432\gamma - 256)\theta_u^2 - 2696\gamma\theta_u} \right) \quad (\text{B.14})$$

$$v^* = \frac{1 - \gamma\theta_u}{2A} \left(\frac{448 - 5\gamma^6\theta_u^8 - 2\gamma^5(6\gamma - 23)\theta_u^7 + \gamma^4(29\gamma^2 + 94\gamma - 197)\theta_u^6 - \gamma^3(248\gamma^2 + 360\gamma - 553)\theta_u^5}{+\gamma^2(1065\gamma^2 + 640\gamma - 959)\theta_u^4 - \gamma(2589\gamma^2 + 302\gamma - 880)\theta_u^3 + (3397\gamma^2 - 416\gamma - 320)\theta_u^2 + (-2128\gamma + 384)\theta_u} \right) \quad (\text{B.15})$$

$$\bar{w}_1^* = \frac{1}{2A} \left(\frac{1536 - 8\gamma^6(2\gamma^2 + 12\gamma - 1)\theta_u^8 + \gamma^5(188\gamma^2 + 438\gamma - 15)\theta_u^7 - \gamma^4(870\gamma^2 + 956\gamma + 187)\theta_u^6 + \gamma^3(1859\gamma^2 + 932\gamma + 921)\theta_u^5}{-\gamma^2(1121\gamma^2 + 40\gamma + 1731)\theta_u^4 - \gamma(2885\gamma^2 + 614\gamma - 1512)\theta_u^3 + (6545\gamma^2 + 336\gamma - 512)\theta_u^2 - 5240\gamma\theta_u + 8\gamma^8\theta_u^9} \right) \quad (\text{B.16})$$

$$\Pi_1^* = \frac{1}{8(2 - \gamma\theta_u)A} \left(\frac{6208 - \gamma^8\theta_u^{10} - 2\gamma^7(\gamma - 8)\theta_u^9 - \gamma^6(\gamma^2 - 32\gamma + 110)\theta_u^8 - 4\gamma^5(2\gamma^2 + 66\gamma - 121)\theta_u^7}{+2\gamma^4(147\gamma^2 + 698\gamma - 802)\theta_u^6 - 4\gamma^3(652\gamma^2 + 1110\gamma - 945)\theta_u^5 + \gamma^2(11072\gamma^2 + 8140\gamma - 5569)\theta_u^4}{-2\gamma(12832\gamma^2 + 4099\gamma - 2232)\theta_u^3 + (33239\gamma^2 + 4000\gamma - 1472)\theta_u^2 + (-22544\gamma - 640)\theta_u} \right) \quad (\text{B.17})$$

where $A = 768 - (4\gamma + 1)\gamma^6\theta_u^8 + 2(4\gamma^2 + 21\gamma + 2)\gamma^5\theta_u^7 - (81\gamma^2 + 160\gamma + 27)\gamma^4\theta_u^6 + 2(154\gamma^2 + 115\gamma + 93)\gamma^3\theta_u^5 - (379\gamma^2 - 8\gamma + 518)\gamma^2\theta_u^4 - 2(287\gamma^2 + 150\gamma - 304)\gamma\theta_u^3 + (2154\gamma^2 + 192\gamma - 256)\theta_u^2 - 2208\gamma\theta_u$. Note that, for the optimal buyback price \bar{v}^* (given in Eq. (B.14)), the following holds: $\bar{v}^* \geq \bar{v}_T$.

B.3.3 Postponed Buyback Pricing with Positive Buyback Quantity:

Using soln 2 ($\bar{q}_v^* = q_{2p}^* > 0$) in backward induction for postponed buyback pricing, we find:

$$\bar{v}^* = \frac{(1 - \gamma\theta_u)(84 - 3\gamma^3\theta_u^5 - 2\gamma^2(6\gamma - 1)\theta_u^4 + \gamma(31\gamma^2 - 12\gamma + 25)\theta_u^3 + (18\gamma^2 + 48\gamma - 28)\theta_u^2 - 153\theta_u\gamma)}{8A} \quad (\text{B.18})$$

$$v^* = \frac{(1 - \gamma\theta_u)(84 - 3\gamma^3\theta_u^5 - 2\gamma^2(6\gamma - 1)\theta_u^4 + \gamma(31\gamma^2 - 36\gamma + 49)\theta_u^3 + (50\gamma^2 + 48\gamma - 60)\theta_u^2 + (-225\gamma + 72)\theta_u)}{8A} \quad (\text{B.19})$$

$$\bar{w}_1^* = \frac{588 + 11\gamma^4\theta_u^6 + 3\gamma^3(10\gamma - 11)\theta_u^5 - \gamma^2(95\gamma^2 + 90\gamma + 49)\theta_u^4 + \gamma(269\gamma^2 + 6\gamma + 243)\theta_u^3 + (177\gamma^2 + 102\gamma - 196)\theta_u^2 - 963\theta_u\gamma}{16A} \quad (\text{B.20})$$

$$\Pi_1^* = \frac{588 - 7\gamma^3\theta_u^5 - 2\gamma^2(13\gamma - 1)\theta_u^4 + \gamma(89\gamma^2 - 64\gamma + 109)\theta_u^3 + (82\gamma^2 + 222\gamma - 140)\theta_u^2 + (-795\gamma - 60)\theta_u}{32A} \quad (\text{B.21})$$

where $A = 36 - \gamma^3\theta_u^5 - \gamma^2(3\gamma - 1)\theta_u^4 + 9\gamma(\gamma^2 + 1)\theta_u^3 + (-3\gamma^2 + 9\gamma - 12)\theta_u^2 - 45\theta_u\gamma$. Finally, for the optimal buyback price \bar{v}^* (given in Eq. (B.18)), the following holds: $\bar{v}^* \geq \bar{v}_T$.

B.4 Leasing: Proofs of Analytical Results

Proof of Proposition B.1. Under postponed buyback pricing, consider soln 1 where buyback quantity is positive ($\bar{q}_v^* = q_{2p}^* > 0$). Using backward induction, we substitute the second-period quantities (Eq. (B.5)-(B.8)) and prices (Eq. (B.10)-(B.9)) as well as the buyback price \bar{v}^* (Eq. (B.11)) in first-period problems (PL4) and (P5) for the dealer and rental agency, respectively. Then we solve the intermediaries' problems simultaneously and find the first-period order quantities in terms of wholesale prices: $q_L^* = q_{1n}^* = \frac{(1-\gamma\theta_u)((6\gamma^2\theta_u^2+18\gamma\theta_u-56)w_1+2\theta_u\gamma(1-\theta_u)\bar{w}_1+3\theta_u^3\gamma^2-9\gamma^2\theta_u^2+17\gamma\theta_u^2-35\theta_u\gamma-42\theta_u+98)}{2A}$ and $\bar{q}_{1n}^* = \frac{\theta_u\gamma(1-\theta_u)(1-\gamma\theta_u)w_1+(-6\gamma\theta_u^3-20\gamma\theta_u^2+58\gamma\theta_u+8\theta_u^2+16\theta_u-56)\bar{w}_1-3\theta_u^4\gamma^2-20\theta_u^3\gamma^2+47\gamma^2\theta_u^2+15\gamma\theta_u^3+52\gamma\theta_u^2-147\theta_u\gamma-14\theta_u^2-28\theta_u+98}{2A}$

where $A = 98 - \gamma^3\theta_u^5 - 2\gamma^2(2\gamma + 1)\theta_u^4 + \gamma(11\gamma^2 - 8\gamma + 15)\theta_u^3 + (22\gamma^2 + 44\gamma - 14)\theta_u^2 - (133\gamma + 28)\theta_u \geq 0$. Taking the derivatives of expression for \bar{q}_{1n}^* and q_{1n}^* above we find *i.* $\frac{\partial q_{1n}^*}{\partial w_1} > 0$ and *ii.* $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} > 0$. \square

Proof of Proposition B.2. For postponed buyback pricing, the optimal profits under zero- and positive- buyback quantities are in Eq. (B.13) and (B.21), respectively. Let Δ denote the difference between the two, i.e., $\Delta = (\text{B.21}) - (\text{B.13})$. Minimizing Δ s.t. $0 \leq \gamma \leq 1$, $0 \leq \theta_u \leq 1$ gives $\Delta^* = 0$. Thus $\Delta \geq 0$, i.e., $(\text{B.21}) \geq (\text{B.13})$. \square

Proof of Proposition B.3. Under precommitted buyback pricing, consider soln 1 where buyback quantity is positive ($\bar{q}_v^* = q_{2p}^* > 0$). Using backward induction, we substitute the second-period quantities and prices (i.e., Eq. (B.5)-(B.8), (B.9) and (B.12)) in first-period problems (PL4) (see Eq. (B.2)) and (P5) (see §3.2) for the dealer and rental agency, respectively. Then we solve their problems simultaneously and find the first-period order quantities in terms of the first-period wholesale prices: $\bar{q}_{1n}^* = \frac{1}{2A}(2\gamma\theta_u(1-\theta_u)(1-\gamma\theta_u)(\gamma\theta_u-2)w_1-(2-\gamma\theta_u)(28-4\gamma^2\theta_u^3+8\gamma^2\theta_u^2+3\gamma\theta_u^3+10\gamma\theta_u^2-29\gamma\theta_u-4\theta_u^2-8\theta_u)\bar{w}_1+(56-\gamma^2\theta_u^4-10\gamma^2\theta_u^3+23\gamma^2\theta_u^2+8\gamma\theta_u^3+24\gamma\theta_u^2-72\gamma\theta_u-8\theta_u^2-16\theta_u)\bar{v}+56+4\gamma^3(1-\gamma)\theta_u^5+\gamma^2(8\gamma^2+12\gamma-13)\theta_u^4-2\gamma(19\gamma^2+9\gamma-8)\theta_u^3+(81\gamma^2+24\gamma-8)\theta_u^2+(-100\gamma-16)\theta_u)$ and $q_L^* = q_{1n}^* = \frac{1}{2A}((8-4\gamma\theta_u)(\gamma^3\theta_u^3-3\gamma^2\theta_u^2+5\gamma\theta_u-4)w_1+2\gamma\theta_u(1-\theta_u)(1-\gamma\theta_u)(2-\gamma\theta_u)\bar{w}_1-2\gamma\theta_u(1-\theta_u)(2-\gamma\theta_u)^2\bar{v}+56-4\gamma^4\theta_u^5+2\gamma^3(4\gamma+9)\theta_u^4-\gamma^2(38\gamma+37)\theta_u^3+\gamma(81\gamma+44)\theta_u^2-(100\gamma+24)\theta_u)$ where $A = 56 - 2\gamma^3(2\gamma - 1)\theta_u^5 + \gamma^2(8\gamma^2 + 16\gamma - 7)\theta_u^4 - 2\gamma(19\gamma^2 + 15\gamma - 6)\theta_u^3 + (81\gamma^2 + 32\gamma - 8)\theta_u^2 - (100\gamma + 16)\theta_u > 0$. Taking derivatives of the expressions above we find *i.* $\frac{\partial q_{1n}^*}{\partial w_1} > 0$, $\frac{\partial q_{1n}^*}{\partial \bar{v}} < 0$, and *ii.* $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} < 0$, $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{v}} > 0$. \square

Proof of Proposition B.4. Under precommitted buyback pricing, the optimal profits under zero- and positive- buyback quantities are given in Eq. (B.13) and (B.17), respectively (see Appendix B.3.1 and B.3.2). Let Δ denote the difference between the two, i.e., $\Delta = (\text{B.17}) - (\text{B.13})$. Maximizing Δ s.t. $0 \leq \gamma \leq 1$, $0 \leq \theta_u \leq 1$ gives $\Delta^* = 0$ at $\gamma = 1$, $\theta_u = 1$. Therefore $\Delta \leq 0$ and thus $(\text{B.17}) \leq (\text{B.13})$ and the manufacturer is better off under zero-buyback quantity (i.e., $\bar{q}_v^* = 0$). \square

Proof of Corollary B.1. Consider $\bar{q}_v^* > 0$. Under the precommitted scheme (see §B.3.2), the first-period rental-car wholesale $\bar{w}_1^*(precommitted)$ is given in Eq. (B.16), the buyback price $\bar{v}^*(precommitted)$ is given Eq. (B.14), and the resale price $v^*(precommitted)$ is given in Eq. (B.15). The difference is $\bar{v}^*(precommitted) - v^*(precommitted) > 0$. Under postponed scheme (see §B.3.3), the first-period rental-car wholesale price $\bar{w}_1^*(postponed)$ is given in Eq. (B.20) and the buyback price $\bar{v}^*(postponed)$ is given in Eq. (B.18). Taking the differences, we find that $\bar{w}_1^*(precommitted) < \bar{w}_1^*(postponed)$ and $\bar{v}^*(precommitted) > \bar{v}^*(postponed)$. Also, the resale price $v^*(postponed)$ is given in Eq. (B.19). Here, $\bar{v}^*(postponed) - v^*(postponed) < 0$. \square

Proof of Proposition B.5. From Proposition B.2 (Proposition B.4), we know that postponed (pre-

committed) buyback pricing yields positive (zero) buyback quantity, i.e., $q_{2p}^* > 0$ (i.e., $q_{2p}^* = 0$). To compare the profits under two strategies, let $\Delta = \Pi_1^*(\text{postponed}) - \Pi_1^*(\text{precommitted})$. Minimizing Δ s.t. $0 \leq \gamma \leq 1$, $0 \leq \theta_u \leq 1$ gives $\Delta^* = 0$. Therefore, $\Delta \geq 0$ and $\Pi_1^*(\text{postponed}) \geq \Pi_1^*(\text{precommitted})$. \square

C. Dealer Competition

We consider two symmetric dealers competing in the sales market in the base model. They are denoted by superscript d for $d = 1, 2$. Therefore, the first- and second-period quantities in the sales market are: $q_{2n} = q_{2n}^1 + q_{2n}^2$, $q_{2u} = q_{2u}^1 + q_{2u}^2$, $q_{2p} = q_{2p}^1 + q_{2p}^2$, $q_{1n} = q_{1n}^1 + q_{1n}^2$. Next we present the model:

Second Period. Each dealer decides how many new cars to buy (q_{2n}^d) at wholesale price w_2^d and how many program cars to buy (q_{2p}^d) at price v from the manufacturer, maximizing the total profit:

$$\max_{q_{2n}^d, q_{2p}^d} \pi_2^d = q_{2n}^d(p_{2n} - w_2^d) + q_{2p}^d(p_{2p} - v) \text{ for } d = 1, 2.$$

The program cars are generally sold to dealers through auctions. Therefore, we assume there is a single resale price v for both dealers, which comes from the equilibrium where $q_{2p}^{1*} + q_{2p}^{2*} = \bar{q}_v^*$. The rental agency's problem remains the same as in (P2) (see Eq. (9)). Given the optimal quantity decisions of intermediaries (i.e., $q_{2n}^{d*}, q_{2p}^{d*}, \bar{q}_{2n}^{d*}, \bar{q}_v^*$), the manufacturer chooses new- and rental-car wholesale prices: $\max_{\bar{w}_2, w_2^1, w_2^2} \Pi_2 = \sum_{d=1}^2 q_{2n}^{d*} w_2^d + \bar{q}_{2n}^{d*} \bar{w}_2 + \sum_{d=1}^2 q_{2p}^{d*} v^* - \bar{q}_v^* \bar{v} = \sum_{d=1}^2 q_{2n}^{d*} w_2^d + \bar{q}_{2n}^{d*} \bar{w}_2 + \bar{q}_v^* (v^* - \bar{v})$.

First period. In period 1, each dealer d decides how many new cars to buy (q_{1n}^d) at price w_1^d , whereas the rental agency decides how many to buy (\bar{q}_{1n}) at price \bar{w}_1 . Therefore, dealer d 's problem is: $\max_{q_{1n}^d} \pi_1^d = \pi_2^{d*} + q_{1n}^d(p_{1n} - w_1^d)$, while the rental agency's problem remains the same as in (P5)

(see Eq. (13)). Finally, the manufacturer chooses new- and rental-car wholesale prices (\bar{w}_1, w_1^1, w_1^2) as well as the buyback price \bar{v} to maximize her total profit in the two periods: $\max_{\bar{w}_1, w_1^1, w_1^2, \bar{v}} \Pi_1 = \Pi_2^* + \sum_{d=1}^2 q_{1n}^{d*} w_1^d + \bar{q}_{1n}^* \bar{w}_1$.

We again analyze the players' optimal decisions through backward induction. We first characterize the subgame perfect equilibrium under the postponed and precommitted buyback pricing schemes. Then, we characterize the manufacturer's preference on the pricing scheme. We find that Propositions 1-5 and Corollary 1 continue to hold as are. We also have studied the manufacturer competition in the rental market under this extension (i.e., two manufacturers competing through the same rental agency and each having two dealers competing in their own sales markets, resulting in a total of seven players) and conducted thorough numerical analyses for the resulting model. We observe that Observations 1-3 continue to hold (see Figure C.1).

D. Program Cars Depreciate Faster Than Used Cars

We now assume program cars depreciate faster than used cars, which is an opposite assumption to the one used in the base model. Accordingly, $0 \leq \theta_u \leq \theta_p \leq 1$. We let $\theta_u = \lambda \theta_p$ where $\lambda \in [0, 1]$ denotes the ratio of the depreciation rates of used cars versus program cars. Having program cars depreciate more than used cars requires deriving new inverse demand functions for the sales market. That for the first period remains the same, i.e., $p_{1n} = p_{2u} + (1 - q_{1n})$, whereas

$$p_{2n} = p_{2u} + \theta_u(1 - q_{2n}), \tag{D.1}$$

$$p_{2p} = (1 - \theta_p)(1 - q_{2u} - q_{2p} - q_{2n}), \tag{D.2}$$

$$p_{2u} = p_{2p} + (\theta_p - \theta_u)(1 - q_{2n} - q_{2u}). \tag{D.3}$$

Note that Eq. (D.1)-(D.3) are different from Eq. (2)-(4). The inverse demand functions in the rental market remain the same as in Eq. (5)-(7). We again analyze the players' optimal decisions through backward induction. We first characterize the subgame perfect equilibrium under each buyback pricing schemes. Then, we characterize the manufacturer's preference. We find that

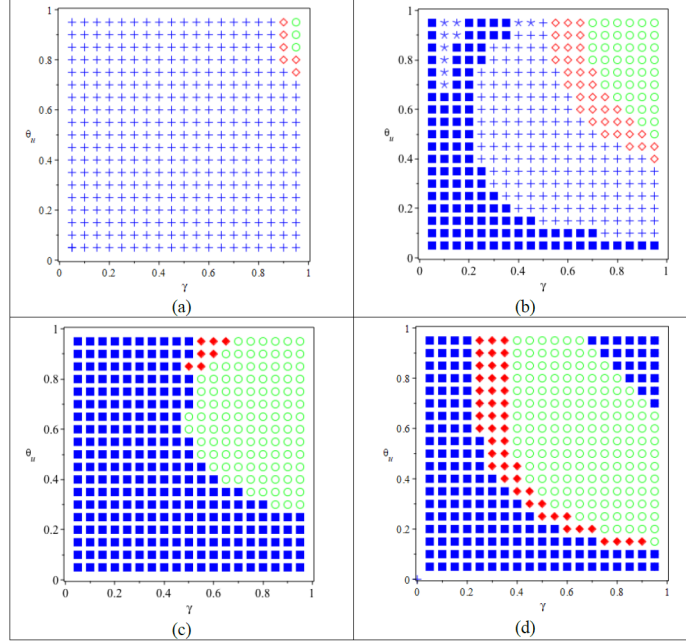


Figure C.1: As labeled in Figure 4 in the paper

Propositions 1-5 and Corollary 1 continue to hold as are. We also studied manufacturer competition in the rental market under this extension and conducted thorough numerical analyses for the resulting model. We find Observations 1-3 continue to hold (see Figure D.1). Here, different than those in the previous figures depicting equilibrium under manufacturer competition, these plots are with respect to the depreciation of program cars θ_p and the ratio of the depreciation rates of used cars versus program cars λ .

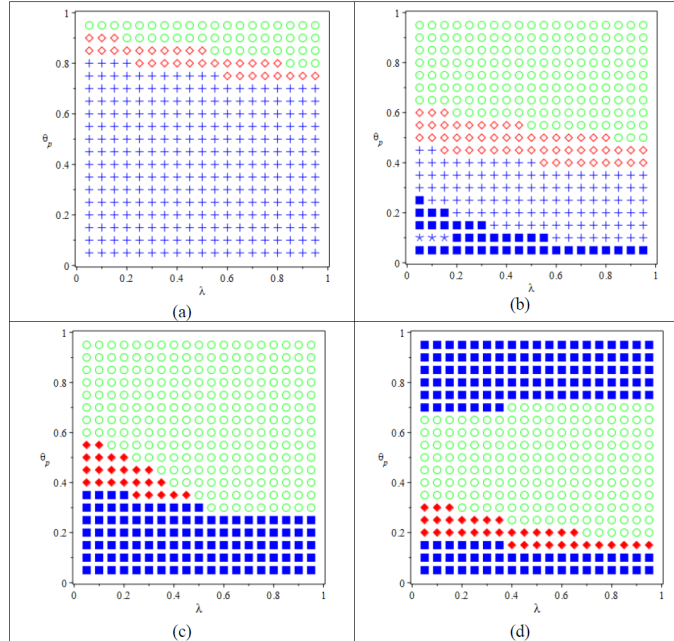


Figure D.1: As labeled in Figure 4 in the paper.