Designing and Evaluating Dynamic Pricing Policies for Major League Baseball Tickets: Online Supplement

S1. Additional Model Validations

S1.1. Model Fit for the Game Demand Model

We examine the model fit for the game demand model. We first compare the negative binomial model against alternative models. We consider an OLS model (which can result in a negative demand estimate) and a Poisson regression model. The negative binomial model is superior to these models in terms of both appropriateness and model fit. These findings are summarized in Table S1. Among the three candidate models, the NBD model has the lowest AIC using the training data and the highest forecast accuracy using the testing data with actual prices.

Table S1 Comparison of NBD, Poisson, and OLS Game Demand Models

Model	AIC	Forecast	Error
NBD	23,488	50,106	-3.5%
Poisson	45,764	47,430	-8.7%
OLS	36,976	98,427	89.5%

Figure S1 is the boxplot of forecast error using the testing data in terms of the daily order count. Figure S2 is the density plot of the same forecast errors, focused around zero. These figures show that errors are roughly normally distributed, which is indicative of model appropriateness.

Figure S1 Boxplot of Order Count Forecast Errors

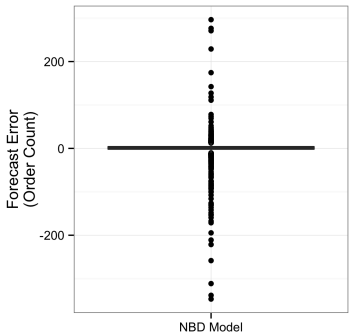


Figure S3 shows the order count forecast error using the testing data, aggregated by the number of remaining selling periods. Figures S4-S6 show the average order count forecast error aggregated by home team performance (wins above .500), opponent performance (wins above .500), and home

Figure S2 Density Plot of Order Count Forecast Errors (Near Zero)

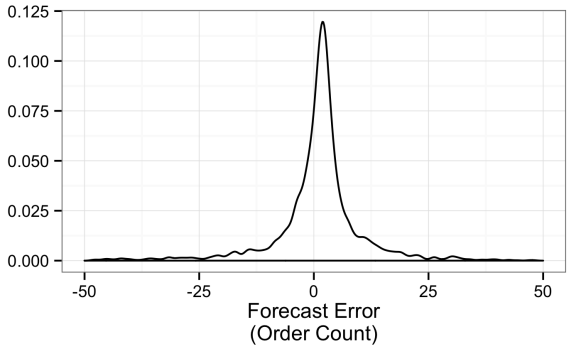


Figure S3 Average Order Count Forecast Error by Remaining Selling Periods

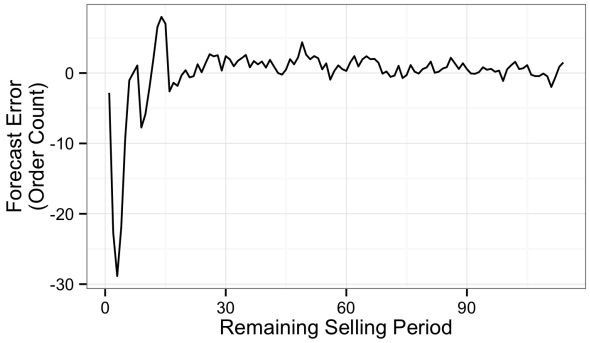
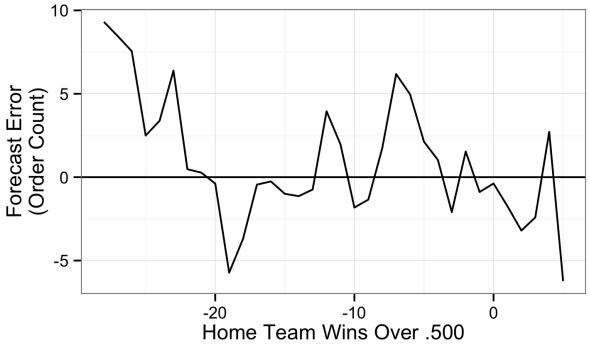


Figure S4 Average Order Count Forecast Error by Home Team Performance



team playoff odds on the game day, respectively. We find that there is no major systematic forecast error along these dimensions.

Figures S7-S9 show the revenue forecast error using the testing data at the game level with respect to the home game number (i.e., n-th home game of the year), home team performance on game day, and the opposing team's performance on game day, respectively. We do not observe systematic errors in the revenue forecast based on these factors.

Figure S5 Average Order Count Forecast Error by Opponent Performance

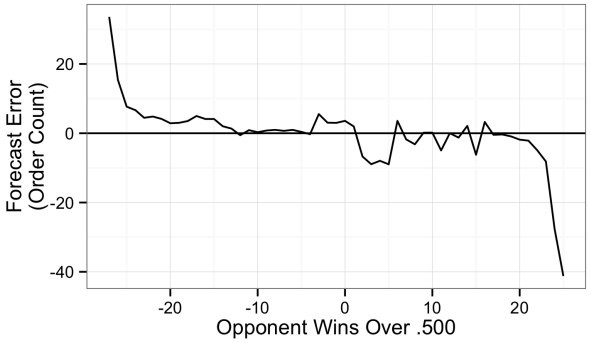


Figure S6 Average Order Count Forecast Error by Home Team Playoff Odds

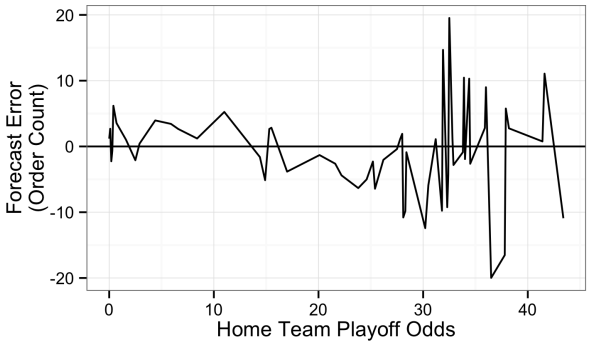
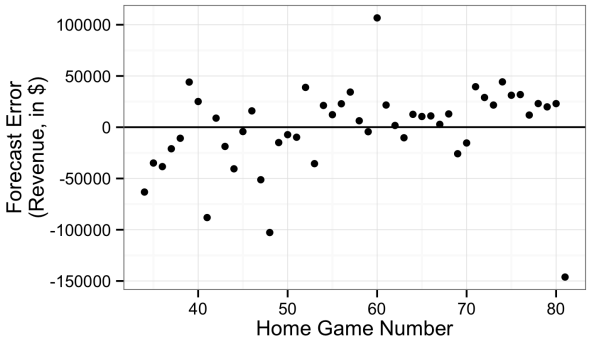


Figure S7 Game-Level Revenue Forecast Error by Home Game Number



Finally, Figure S10 shows the order count forecast plotted against the actual order count at the game level. The mean absolute percentage error (MAPE) of the order count metric at the game level is 36.9% (25.3% for the first 40 games in the testing period). A similar scatterplot for the revenue metric is shown in Figure S11. The MAPE of the revenue metric at the game level is 50.1% (28.6% for the first 40 games in the testing period).

Figure S8 Game-Level Revenue Forecast Error by Home Team Performance

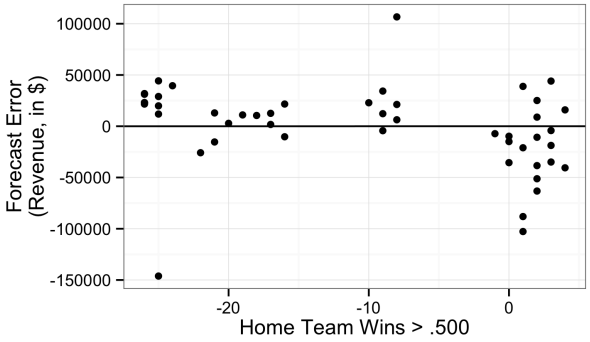


Figure S9 Game-Level Revenue Forecast Error by Opponent Performance

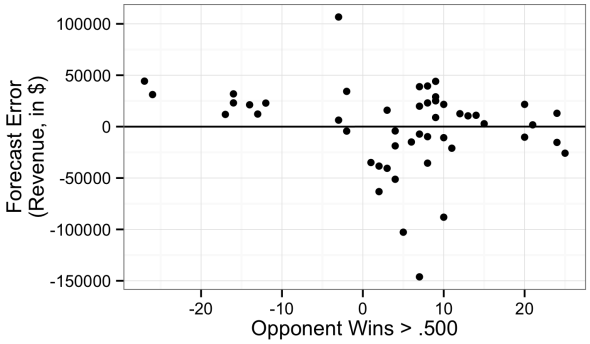
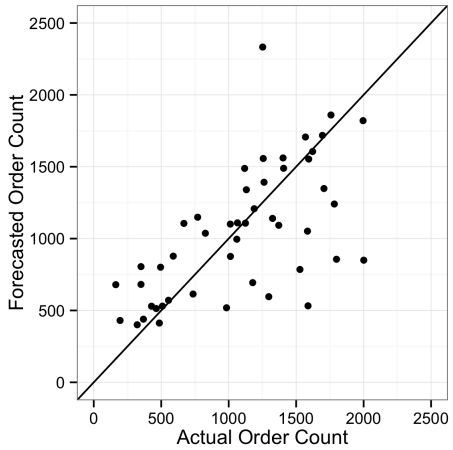


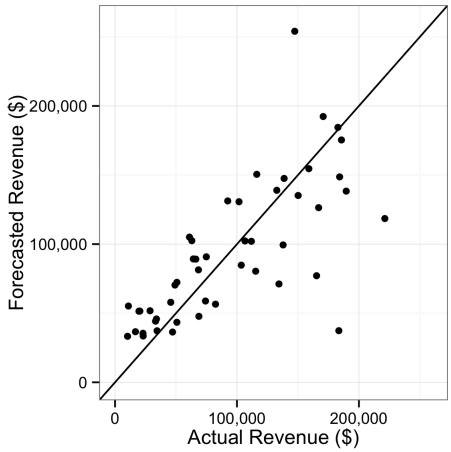
Figure S10 Forecast vs. Actual Number of Orders by Each Home Game



S1.2. Forecast Accuracy Comparison with Naive Forecasting Approaches

We benchmark our forecasts relative to two naive forecasting approaches: the average sales per game in the training data and the average daily sales with respect to the remaining selling period. In the first approach, we compute the average sales per game in the training set and use this single

Figure S11 Forecast vs. Actual Revenue by Each Home Game



value as the sales forecast in the testing data for all games (naive forecast 1). For example, this naive forecast would use 601 orders as the forecast for every game in the second half. In the second approach, we aggregate the observations in the training data by the remaining selling period to compute the average sales, given the amount of the selling period remaining. We then use these values as the forecast in the testing data for every game, given the remaining selling period (naive forecast 2).

Table S2 summarizes the game-level MAPE for various sales metrics. We find that the first naive forecast leads to 56.5% MAPE in terms of order count and 79% MAPE in terms of revenue. The second naive forecast performs worse, with over 100% MAPE across all sales metrics. Our model forecasts represent significant improvement over these naive approaches.

Table S2 Forecast Accuracy of Naive Forecasting Methods

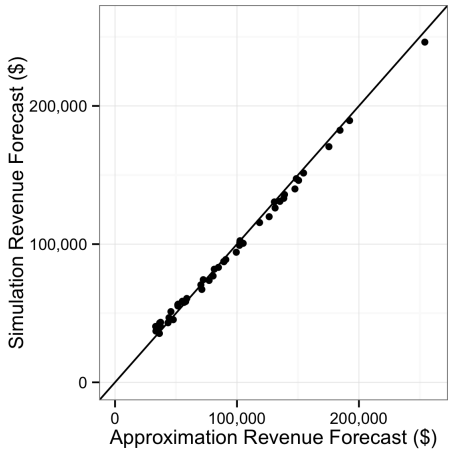
	Game-Level MAPE	
	Naive Forecast 1	Naive Forecast 2
Total Order Count	56.5%	103.7%
Total Ticket Count	75.1%	140.4%
Total Revenue ($k)	79.0%	148.9%

S1.3. Comparison of Revenue Approximation and Simulation

We evaluate the quality of our revenue approximation by comparing the game-level revenue forecasts against Monte Carlo simulation results. We generate 1000 sample paths of demand realization for each game using the full demand model. The average demand across the 1000 sample paths is used as the revenue forecast under the Monte Carlo simulation.

Figure S12 shows the revenue forecasts from the approximation and revenue forecasts from Monte Carlo simulation. We note that our revenue approximation results in a minimal sacrifice of accuracy compared to the Monte Carlo simulation.

Figure S12 Comparison of Revenue Forecasts by Approximation and by Simulation



S2. Seat-Section-Level Forecast Accuracy

We report the forecast accuracy for each seat section from the full model. Table S3 presents the aggregate forecast accuracy and game-level MAPE in terms of the number of tickets sold by each seat section. Table S4 presents the aggregate forecast accuracy and game-level MAPE in terms of dollar revenue by each seat section. As expected, the errors are larger when disaggregated at the seat section level. We are unable to compute game-level MAPE for the Home Plate sections, since there were many games with zero sales. We do not find obvious bias in the distribution of errors across seat sections.

S3. Optimized Clairvoyant Fixed Policy

Figure S13 shows the optimized price path for several seat sections (as well as the stadium average price) for the clairvoyant fixed price. We note that these are the sample price paths for the same home game shown in Figure 1, with the dashed lines representing the actual dynamic prices adopted by our partner franchise. The actual dynamic price for the Bleacher section is not visible since it is the same as the optimized price, which is constrained to be \$10.

We summarize the model-recommended price changes for the daily re-optimized pricing policy and the clairvoyant fixed pricing policy in Table S5. This table is equivalent to Table 11 presented in the main text. For the pricing policies under consideration, we show the average price change on the first day of the testing period aggregated over remaining games (Δp Start), the average price

Table S3 Model Sales Forecast vs. Actual Sales by Seat Section (Ticket Count)

				Game-Level MAPE		
	Actual	Forecast	Error (%)	All	First 40	Last 8
Premium 1	2,951	2,863	-3.0%	96.8%	39.4%	383.8%
Premium 2	1,818	1,459	-19.7%	68.2%	62.0%	99.4%
Premium 3	1,854	2,391	29.0%	119.3%	106.6%	183.0%
Home Plate 1	82	20	-75.7%	n.a.	n.a.	94.8%
Home Plate 2	763	436	-42.8%	n.a.	n.a.	50.0%
Home Plate 3	713	381	-46.6%	n.a.	n.a.	n.a.
Dugout	13,536	14,630	8.1%	91.8%	46.1%	319.8%
1st/3rd Base Line	16,583	17,855	7.7%	74.4%	47.2%	210.7%
Outfield 1	19,373	25,970	34.1%	118.0%	110.0%	157.8%
Outfield 2	10,615	13,114	23.5%	238.5%	187.9%	491.7%
Upper Deck 1	7,905	5,714	-27.7%	59.4%	47.4%	119.5%
Upper Deck 2	24,433	18,227	-25.4%	71.9%	48.7%	118.0%
Upper Deck 3	12,720	6,185	-51.4%	166.9%	144.6%	278.0%
Bleacher	15,305	17,244	12.7%	79.9%	50.5%	227.1%

Table S4 Model Sales Forecast vs. Actual Sales by Seat Section (Total Revenue)

				Game-Level MAPE		
	Actual	Forecast	Error (%)	All	First 40	Last 8
Premium 1	266,240	245,676	-7.7%	94.5%	36.9%	382.6%
Premium 2	143,747	111,526	-22.4%	65.3%	58.6%	98.7%
Premium 3	147,974	166,522	12.5%	114.9%	101.3%	182.6%
Home Plate 1	5,890	1,586	-73.1%	n.a.	n.a.	94.2%
Home Plate 2	63,881	36,318	-43.1%	n.a.	n.a.	48.6%
Home Plate 3	36,711	19,174	-47.8%	n.a.	n.a.	n.a.
Dugout	932,224	1,025,001	10.0%	99.2%	47.7%	356.7%
1st/3rd Base Line	727,030	852,339	17.2%	112.1%	59.8%	373.8%
Outfield 1	538,542	871,673	61.9%	171.9%	155.6%	253.4%
Outfield 2	266,892	304,205	14.0%	121.9%	121.9%	121.5%
Upper Deck 1	253,511	203,992	-19.5%	73.4%	49.6%	192.0%
Upper Deck 2	697,241	356,885	-48.8%	48.0%	39.5%	90.8%
Upper Deck 3	259,769	92,212	-64.5%	74.0%	68.7%	100.8%
Bleacher	178,251	172,437	-3.3%	59.3%	34.5%	183.5%

change between the first day of the testing period and the game day aggregated over remaining games (Δp G.D.), and the % of tickets sold in each seat section according to forecasts. We note that the actual dynamic pricing policy does not involve any price change on day 1 of the testing period, and the clairvoyant fixed pricing policy does not involve any price change until the game day.

Figure S13 Optimized Prices for Home Game #68 (Clairvoyant Fixed)

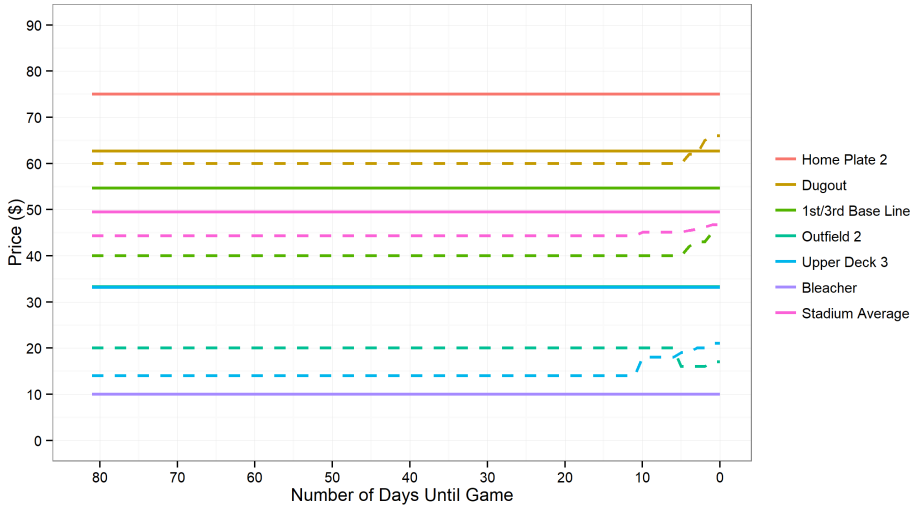


Table S5 Summary of Model-Recommended Price Changes

	Actual Dynamic		Daily Re-optimized			Clairvoyant Fixed	
	Δp G.D.	% Tickets	Δp Start	Δp G.D.	% Tickets	Δp Start	% Tickets
Bleacher	\$0	13.6%	\$0	\$0	16.7%	\$0	16.4%
Premium 1	\$2.44	2.3%	\$0	\$0	2.9%	\$0	2.8%
Premium 2	\$2.53	1.2%	\$0	\$0	1.5%	\$0	1.5%
Premium 3	\$2.35	1.9%	\$0	\$0	2.4%	\$0	2.4%
Home Plate 1	\$0	0.02%	\$0	\$0	0.02%	\$0	0.02%
Home Plate 2	\$0.93	0.3%	-\$0.07	\$0	0.4%	-\$0.07	0.4%
Home Plate 3	\$2.14	0.3%	\$1.45	-\$0.15	0.4%	\$0.57	0.4%
Dugout	\$3.09	11.6%	\$6.49	-\$6.49	13.7%	\$1.07	14.0%
1st/3rd Base Line	\$2.65	14.1%	\$17.77	-\$10.10	13.3%	\$10.67	13.8%
Outfield 1	\$2.56	20.5%	\$18.45	-\$0.22	16.3%	\$17.44	16.2%
Outfield 2	\$2.26	10.4%	\$13.93	-\$0.42	9.7%	\$12.60	9.5%
Upper Deck 1	\$1.16	4.5%	\$2.41	-\$0.89	5.6%	\$1.14	5.6%
Upper Deck 2	\$1.49	14.4%	\$16.74	-\$0.56	11.9%	\$15.83	11.8%
Upper Deck 3	\$0.49	4.9%	\$4.96	\$2.53	5.0%	\$3.96	5.2%