Electronic Appendix to: "Is Diversity (Un)Biased? Project Selection Decisions in Executive Committees"

Illustrative example of the pivotal contingency Consider a committee of three members that decides based on a unanimous acceptance rule, e.g., a compound gets approved for funding during the Phase IIb, only if all three members support it. Focal member 1 recommends approval only if $EU_1(YES) > EU_1(NO)$ where the expectation accounts for the possible recommendations of the other two members, and the fact that the member 1's payoff depends on these recommendations. With a slight abuse of notation, let $Pr(a_2 a_3)$ denote the probability that members 2 and 3 recommend $a_2 = \{YES, NO\}$ and $a_3 = \{YES, NO\}$, respectively. Then, we can write member 1's expected utilities from voting YES and NO, respectively, as follows:

$$EU_1(YES) = Pr(YES, YES)U_1(YES|YES, YES) + Pr(YES, NO)U_1(YES|YES, NO) +$$

$$Pr(NO, YES)U_1(YES|NO, YES) + Pr(NO, NO)U_1(YES|NO, NO)$$

and

$$EU_1(NO) = Pr(YES, YES)U_1(NO|YES, YES) + Pr(YES, NO)U_1(NO|YES, NO) +$$

$$Pr(NO, YES)U_1(NO|NO, YES) + Pr(NO, NO)U_1(NO|NO, NO).$$

However, since unanimous acceptance is required for the project to be approved, $U_1(YES|YES, NO) = U_1(YES|NO, YES) = U_1(YES|NO, NO) = U_1(NO|YES, YES) = U_1(NO|YES, NO) = U_1(NO|NO, NO) = 0$, as in all of the above contingencies the project is rejected irrespective of the focal member's decision. In short, the condition $EU_1(YES) > EU_1(NO)$ is equivalent to $U_1(YES|YES, YES) > U_1(NO|YES, YES)$ which represents the pivotal contingency: the other two members express support for the project. Proposition 1 below shows more formally how a utility maximization strategy results in the pivotal contingency.

Proof of Proposition 1. The proof follows the same logic as in Lemmas 1 and 2 in Gerardi (2000). Similarly, we focus on symmetric Bayesian Nash equilibria in which players do not use weakly dominated strategies. Let $\Psi(z,\omega)$ be the probability that exactly z members (out of the remaining $N-1$) recommend approval when the state is $\omega \in \{G,B\}$. Then, the expected utility of member i, if she recommends approval is:

$$EU_i(a_i = YES) = \Pr(G|s_i)\sum_{z=r-1}^{N-1}\Psi(z,G)(V-c-t_i) + \Pr(B|s_i)\sum_{z=r-1}^{N-1}\Psi(z,B)(-c-t_i),$$

as the project gets approved when more than $r-1$ peer members have supported it, while if she recommends rejection is:

$$EU_i(a_i = NO) = \Pr(G|s_i)\sum_{z=r}^{N-1}\Psi(z,G)(V-c-t_i) + \Pr(B|s_i)\sum_{z=r}^{N-1}\Psi(z,B)(-c-t_i),$$

as the project gets approved when more than r peer members have supported it.

Member i recommends approval, i.e., $a_i = YES$, if and only if :

$$EU_i(a_i = YES) \geq EU_i(a_i = NO), \text{ or equivalently,}$$

$$\Pr(G|s_i)\Psi(r-1,G)(V-c-t_i) + \Pr(B|s_i)\Psi(r-1,B)(-c-t_i) \geq 0, \text{ or equivalently,}$$

$$t_i \leq \tfrac{\Pr(G|s_i)\Psi(r-1,G)}{\Pr(G|s_i)\Psi(r-1,G)+\Pr(B|s_i)\Psi(r-1,B)}V - c, \text{ or equivalently,}$$

$$t_i \leq \Pr(G|s_i,piv)V - c$$

where $\Pr(G|s_i,piv)$ denotes the posterior belief about the project being good ($\omega = G$) of a member that receives a signal s_i and conditions on the event of being pivotal ($r-1$ members recommend approval). Therefore, an expected utility maximization strategy is equivalent to a member accounting for the pivotal contingency. Also note that, given that we are looking at equilibria in which players do not use weakly dominated strategies (i.e., before observing her signal or her type a member has a positive probability to approve or reject the project), the probability that a member's vote is pivotal is strictly positive. Moreover, the posterior belief of member i upon receiving signal s_i can be written as:

$$\Pr(G|s_i,piv) = \frac{\Pr(s_i|G)\Pr(piv|G)\Pr(G)}{\Pr(s_i|G)\Pr(piv|G)\Pr(G) + \Pr(s_i|B)\Pr(piv|B)\Pr(B)}$$

which satisfies $0 < \Pr(G|s_i,piv) < 1$, given that $\Pr(piv|G) > 0$ and $\Pr(piv|B) > 0$. Thus, there exist threshold values t_g and t_b such that a member recommends approval when the opportunity cost she assigns to the project, t_i, is below the cutoff determined by the posterior values for a good and bad signal, respectively, that it, $t_i \leq t_g = \Pr(G|s_i = g, piv)V - c$ and $t_i \leq t_b = \Pr(G|s_i = b, piv)V - c$.

Proof of Corollary 1. In Proposition 1 we showed that a symmetric Bayesian Nash equilibrium is completely characterized by the pair (t_g, t_b) which denote the cutoff thresholds associated with a good and a bad signal, respectively. We can now formally derive those thresholds by solving the following equations: $\Pr(G|s_i = g, piv)V - c = t_g$ and $\Pr(G|s_i = b, piv)V - c = t_b$, or equivalently,

$$t_g = \frac{1}{1+\frac{(1-\pi)}{\pi}\frac{(1-q)}{q}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}}V - c \text{ and } t_b = \frac{1}{1+\frac{(1-\pi)}{\pi}\frac{q}{(1-q)}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}}V - c.$$

From the last two equations, we get $t_b = k(t_g) \equiv \frac{(1-q)^2(c+t_g)V}{(1-q)^2(c+t_g)+(V-(c+t_g))q^2}$.

Moreover, under strategic voting, the posterior belief of member i upon receiving a good signal is:

$$\Pr(G|s_i = g, piv) = \frac{1}{1+\frac{(1-\pi)}{\pi}\frac{(1-q)}{q}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}}$$

while under non-strategic voting, the corresponding belief is:

$$\Pr(G|s_i = g) = \frac{1}{1+\frac{(1-\pi)}{\pi}\frac{(1-q)}{q}}.$$

It is straightforward to show (by substitution) that for neutral projects ($\mu_t = \pi V - c$), the threshold values are $t_g = qV - c$ and $t_b = (1-q)V - c$, in which case, again by simple substitution, we get $\gamma_G + \gamma_B = 1$. As such, $\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)} = 1$ and both strategic and non-strategic voting lead to the same posterior belief. We now show that $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G})$ decreases in μ_t. First note that,

$$\frac{d(\frac{\gamma_B}{\gamma_G})}{d\mu_t} = -\frac{4(t_g+c)(V-t_g-c)((1-2q)(t_g+c)+q^2V)(q-0.5)^2}{[(-2q^2+q)(t_g+c)^2+(3q^2V+(-3V-2\varepsilon_t+2\mu_t)q+V+\varepsilon_t-\mu_t)(t_g+c)+Vq^2(\varepsilon_t-\mu_t)]^2}$$

which is negative, because $V - t_g - c > 0$ and $q^2 > 2q - 1$. Thus $\frac{\gamma_B}{\gamma_G}$ decreases in μ_t. Similarly,

$$\frac{d(\frac{1-\gamma_B}{1-\gamma_G})}{d\mu_t} = -\frac{4(t_g+c)(V-t_g-c)((1-2q)(t_g+c)+q^2V)(q-0.5)^2}{[(2q^2-q)(t_g+c)^2+(-3q^2V+(3V-2\varepsilon_t-2\mu_t)q-V+\varepsilon_t+\mu_t)(t_g+c)+Vq^2(\varepsilon_t+\mu_t)]^2}$$

is negative, and therefore, $\frac{1-\gamma_B}{1-\gamma_G}$ also decreases in μ_t. Thus, $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G})$ decreases in μ_t.

Lastly, recall that for an unattractive project, $\mu_t > \pi V - c$, therefore, $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G}) < 1$, and as such, the posterior belief is higher than the case of non-strategic voting where $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G}) = 1$.

Similarly, for an attractive project, $\mu_t < \pi V - c$, therefore, $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G}) > 1$, and the posterior belief is lower than the case of non-strategic voting.

Proof of Proposition 2. From Corollary 1 we know that $t_b = k(t_g) \equiv \frac{(1-q)^2(c+t_g)V}{(1-q)^2(c+t_g)+(V-(c+t_g))q^2}$. Since t_b monotonically increases in t_g, it is sufficient to focus only on t_g. Let the function $V(t) = \Pr(G|s_i = g, piv)V - c - t$ denote the total value that a member with threshold t anticipates from the project. Applying the Implicit Function Theorem (IFT) at t_g, we get $\frac{dt_g}{d\varepsilon_t} = -\frac{\frac{\partial V(t_g)}{\partial \varepsilon_t}}{\frac{\partial V(t_g)}{\partial t}}$.

We begin by showing that $\frac{\partial V(t_g)}{\partial t} < 0$. To show the latter, it is sufficient to show that $\Pr(G|s_i = g, piv)$ decreases in t. Note that, $\Pr(G|s_i = g, piv) = \frac{1}{1+\frac{(1-\pi)}{\pi}\frac{(1-q)}{q}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}}$. Thus, it it is sufficient to show that $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G})$ increases in t. To show the latter, we start by showing that $\frac{\gamma_B}{\gamma_G}$ increases in t. Taking the derivative w.r.t. t we have $(\frac{\gamma_B}{\gamma_G})' = (\frac{(1-q)F(t)+qF(k(t))}{qF(t)+(1-q)F(k(t))})' = \frac{(1-2q)[F'(t)F(k(t))-F(t)F'(k(t))]}{[qF(t)+(1-q)F(k(t))]^2}$. We only need to examine the sign of the numerator. After substituting and some algebraic manipulation: $F'(t)F(k(t)) - F(t)F'(k(t)) = (1-2q)\Phi(t)$ where $\Phi(t) = (Vq^2 + (V + \varepsilon_t - \mu_t)(1-2q)(c+t)^2 + 2Vq^2(\varepsilon_t - \mu_t)(c+t) - V^2q^2(\varepsilon_t - \mu_t)$. But, $\Phi''(t) = (Vq^2 + (V + \varepsilon_t - \mu_t)(1-2q) > 0$, so $\Phi(t)$ is convex in t with minimum value $\Phi\min = \frac{V^2q^2(1-q)^2(\mu_t-\varepsilon_t)(V-\mu_t+\varepsilon_t)}{(1-q)^2V+(2q-1)(\mu_t-\varepsilon_t)} > 0$, and thus, $\Phi(t) > 0$ for every t. Therefore, $F'(t)F(t_b(t)) - F(t)F'(t_b(t)) < 0$, and $(\frac{\gamma_B}{\gamma_G})'$, so $\frac{\gamma_B}{\gamma_G}$ increases in t. We now show that $\frac{1-\gamma_B}{1-\gamma_G}$ also increases in t. Note that $\left(\frac{1-\gamma_B}{1-\gamma_G}\right)' = \frac{-\gamma'_B(1-\gamma_G)+(1-\gamma_B)\gamma'_G}{(1-\gamma_G)^2} = \frac{\gamma'_G\gamma_B+\gamma'_B\gamma_G+\gamma'_G-\gamma'_B}{(1-\gamma_G)^2}\frac{(1-2p)^2\Phi_1(t)}{[\Phi_2(t)]^2}$ where $\Phi_1(t) = (q^2V + (\mu_t + \varepsilon_t - V)(2q-1)(c+t)^2 - 2q^2V(\mu_t + \varepsilon_t)(c+t) + q^2V^2(\mu_t + \varepsilon_t)$ and $\Phi_2(t) = q(1-2q)(c+t) + (3q^2V + (2\mu_t + 2\varepsilon_t - 3V)q - (\mu_t + \varepsilon_t))(c+t) - Vq^2(\mu_t + \varepsilon_t)$. By differentiating twice it is easy to show that $\Phi_1(t)$ is convex in t with minimum value $\Phi_1\min = \frac{V^2q^2(1-q)^2(\mu_t+\varepsilon_t)(V-\mu_t-\varepsilon_t)}{(1-q)^2V+(2q-1)(\mu_t-\varepsilon_t)} > 0$, so $\left(\frac{1-\gamma_B}{1-\gamma_G}\right)' > 0$. Since both $\frac{\gamma_B}{\gamma_G}$ and $\frac{1-\gamma_B}{1-\gamma_G}$ increase in t, $(\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G})$ also increases in t. Thus, $\frac{\partial V(t)}{\partial t} < 0$.

We now examine the sign of $\frac{\partial V}{\partial \varepsilon_t}$. Let $t_0 \equiv \pi V - c$. Note that when $\mu_t = t_0$, we can derive a closed form solution for the system defined by the equations $\Pr(G|s_i = g, piv)V - c = t_g$ and $\Pr(G|s_i = b, piv)V - c = t_b$. In particular, the only feasible solution is $t_g = qV - c$. Let $\lambda = (\frac{\gamma_B}{\gamma_G})(\frac{1-\gamma_B}{1-\gamma_G})$. After some algebraic manipulation, we get $\frac{\partial\lambda}{\partial\varepsilon_t} = \frac{16(q-\frac{1}{2})^2[(1-2q)(c+t)+q^2V]^2(V-c-t)\varepsilon_t t\Phi_3(\mu_t)}{[\Phi_4(\mu_t)]^2}$ where $\Phi_3(\mu_t) = 2\mu_t q^2V + 2V(c+t)[q(1-q) - V - 4\mu_t q + 2q(c+t) + 2\mu_t - (c+t)]$ and $\Phi_4(\mu_t) = [(q(2q-1)(c+$

$t)^2+(-3q^2V+(-2\mu_t+3V-2\varepsilon_t)q+\mu_t-V+\varepsilon_t)(c+t)+Vq^2(\mu_t+\varepsilon_t)]][(q(1-2q)(c+t)^2+(3q^2V+(2\mu_t-3V-2\varepsilon_t)q-\mu_t+V+\varepsilon_t)(c+t)+Vq^2(\varepsilon_t-\mu_t)]$. Note that $\frac{\partial\Phi_3}{\partial\mu_t}=2q^2V-2(c+t)(2q-1)>0$ because $V>c+t$ and $q^2>2q-1$. So, $\Phi_3(\mu_t)$ increases in μ_t. Also note, that $\Phi_3(\mu_t=t_0)=0$, so $\Phi_3(\mu_t)<0$, for $\mu_t<t_0$, and $\Phi_3(\mu_t)>0$ for $\mu_t>t_0$. Thus, for $\mu_t<t_0$, $\frac{\partial\lambda}{\partial\varepsilon_t}<0$, and $\frac{\partial V}{\partial\varepsilon_t}>0$. Similarly, for $\mu_t>t_0$, $\frac{\partial V}{\partial\varepsilon_t}<0$. To conclude, by applying the IFT, $\frac{dt_g}{d\varepsilon_t}>0$ for $\mu_t<t_0\equiv\pi V-c$, and $\frac{dt_g}{d\varepsilon_t}<0$ when $\mu_t>t_0\equiv\pi V-c$. Thus, t_g (and t_b) increase in ε_t when $\mu_t>t_0\equiv\pi V-c$, and decrease in ε_t when $\mu_t<t_0\equiv\pi V-c$.

Proof of Proposition 3. We are interested in the sign of $\frac{\partial\gamma_G}{\partial\varepsilon_t}=q\frac{\partial F(t_b)}{\partial\varepsilon_t}+(1-q)\frac{\partial F(t_g)}{\partial\varepsilon_t}$ where $\frac{\partial F(t_g)}{\partial\varepsilon_t}=-(t_g-\mu_t)\frac{1}{2\varepsilon_t^2}$ and $\frac{\partial F(t_b)}{\partial\varepsilon_t}=-(t_b-\mu_t)\frac{1}{2\varepsilon_t^2}$. Also, for $\mu_t=\pi V-c$, the threshold values are $t_g=qV-c$ and $t_b=(1-q)V-c$. After substitution and some algebraic manipulation: $\frac{\partial\gamma_G}{\partial e_t}=-\frac{(2q-1)^2V}{4\varepsilon_t^2}<0$ which implies that membes who receive good signals become less likely to approve a good project. Similarly, we derive $\frac{\partial\gamma_B}{\partial e_t}=\frac{(2q-1)^2V}{4\varepsilon_t^2}>0$ which implies that members who receive a bad signal become more likely to approve a bad project. The error probabilities are given by: $\Pr(NO|G)=\sum_{j=r*N}^{N}\begin{pmatrix}N\\j\end{pmatrix}(1-\gamma_G)^j{\gamma_G}^{N-j}$ and $\Pr(YES|B)=\sum_{j=r*N}^{N}\begin{pmatrix}N\\j\end{pmatrix}{\gamma_B}^j(1-\gamma_B)^{N-j}$ and because $\gamma_G>\frac{1}{2}$ and $\gamma_B<\frac{1}{2}$ they both increase in ε_t.

Proof of Proposition 4. Similarly to the case of preference diversity, which was discussed in Proposition 1, we focus our attention on symmetric Bayesian Nash equilibria in which players do not use weakly dominated strategies. Again, a member votes in favor of the project if $\Pr(G|s_i,q_i,piv)V-c\geq t$, but in this case the type of a member determines the value q_i that he assigns to the signal s_i. If $s_i=g$, $\Pr(G|s_i=g,q_i,piv)$ increases in q_i, and thus, there is a unique threshold q_g such that $\Pr(G|s_i=g,q_g,piv)V-c=t$. If $s_i=b$, $\Pr(G|s_i=b,q_i,piv)$ decreases in q_i, and thus, there is a unique threshold q_b such that $\Pr(G|s_i=b,q_b,piv)V-c=t$.

To determine the above thresholds q_g and q_b we need to solve the following system: $\Pr(G|s_i=g,q_g,piv)V-c=t$ and $\Pr(G|s_i=b,q_b,piv)V-c=t$ Note, however, that for a given t and a distribution $q_i\sim U(\mu_q-e_q,\mu_q+e_q)$, $\Pr(G|s_i=g,q_i,piv)>\Pr(G|s_i=b,q_i,piv)$, and hence, at most one of the above equations can be satisfied.

In particular, a member that receives a good signal has a minimum ex-post valuation of $\underline{V}_g$, and therefore when the threshold t falls into Region I, he always approves the project. The decision of a member that receives a bad signal, however, depends on his interpretive type as in that range members with relative high q_i will reject the project ($\Pr(G|s_i=b,q_H,piv)V-c<t$), but members with relatively low q_i will approve the project ($\Pr(G|s_i=b,q_L,piv)V-c>t$) as they still see sufficient value in it. From the last two inequalities, and given that the ex-post project valuation is continuous in q_i, there exists q_b in Region I, such that $\Pr(G|s_i=g,q_b,piv)V-c=t$. When t falls into Region II, however, $\Pr(G|s_i=g,q_i,piv)V-c>t$ and $\Pr(G|s_i=b,q_i,piv)V-c<t$ for every q_i, so members decide based on their signals alone, regardless of their interpretive type. Lastly, when the threshold t falls into Region III, a member that receives a bad signal has a maximum ex-post valuation of $\overline{V}_b$, and therefore always rejects the project. On the other hand, the decision of a member who receives a good signal depends on his interpretive type: for relatively high q_i he approves the project (i.e., $\Pr(G|s_i=g,q_H,piv)V-c>t$), while for relatively low q_i he recommends rejection (i.e., $\Pr(G|s_i=g,q_L,piv)V-c<t$). From the last two inequalities, and the continuity of the ex-post project valuation in q_i, there exists q_g in Region III, such that $\Pr(G|s_i=g,q_g,piv)V-c=t$.

Proof of Corollary 2.

In region I, under strategic voting, the belief of member i upon receiving a bad signal is:

$$\Pr(G|s_i=b,piv)=\frac{1}{1+\frac{(1-\pi)}{\pi}\frac{q}{(1-q)}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}}$$

while under non-strategic voting, the corresponding belief is:

$$\Pr(G|s_i=g)=\frac{1}{1+\frac{(1-\pi)}{\pi}\frac{q}{(1-q)}}.$$

Thus, all we need to show is that $\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}>1$. Note that in region I a member that receives a good signal always accepts a project, while a member i that receives a bad signal approves the project if $q_i<q_b$. Thus, $\gamma_G=q+(1-q)F(q_b)$ and $\gamma_B=(1-q)+qF(q_b)$. So, $\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}=$

$\frac{q(q+(1-q)F(q_b))}{(1-q)(1-q+qF(q_b))} = \frac{q^2+(1-q)qF(q_b)}{(1-q)^2+(1-q)qF(q_b)} > 1$ because $q > 1-q$. The proof for region III follows exactly the same steps.

Proof of Proposition 5. Recall that q_b denotes the threshold value for which

$\Pr(G|s_i = b, q_b, piv)V - c = t$, that is, $\frac{1}{1+\frac{(1-\pi)}{\pi}\frac{q_b}{(1-q_b)}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}} = \frac{c+t}{V}$. The proof follows steps with that of proposition 2: we first define the appropriate function, and then apply the implicit function theorem at $q = q_b$. Let $G(q) \equiv \frac{1}{1+\frac{(1-\pi)}{\pi}\frac{q}{(1-q)}\left(\frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}\right)^{\frac{N-1}{2}}} - \frac{c+t}{V}$. We first show that $\frac{\partial G}{\partial q} < 0$, and then we derive the sign of $\frac{\partial G}{\partial \varepsilon_q}$. To show that $\frac{\partial G}{\partial q} < 0$, it is sufficient to show that $\delta \equiv \frac{\gamma_B(1-\gamma_B)}{\gamma_G(1-\gamma_G)}$ increases in q. But $\frac{\partial \delta}{\partial q} = -\frac{2}{(1-q)^2}\frac{q^4+(2(\varepsilon_q-\mu_q)-1)q^3+(\mu_q^2-\varepsilon_q^2-3\varepsilon_q+2\mu_q)q^2+(\varepsilon_q^2-\mu_q^2)q+\varepsilon_q(\mu_q-\varepsilon_q)}{(-q^2+\mu_q q-\mu_q+\varepsilon_q+q(\varepsilon_q-c))^2}$ so we need to show that $\Phi_5(q) \equiv q^4 + (2(\varepsilon_q - \mu_q) - 1)q^3 + (\mu_q^2 - \varepsilon_q^2 - 3\varepsilon_q + 2\mu_q)q_b^2 + (\varepsilon_q^2 - \mu_q^2)q + \varepsilon_q(\mu_q - \varepsilon_q) < 0$. First note that, $\frac{\partial \Phi_5}{\partial \mu_q} = 2q(1-q)(q-\mu_q) + \varepsilon_q$. If $q > \mu_q$ then clearly $\frac{\partial \Phi_5}{\partial \mu_q} > 0$. If $q < \mu_q$, then since $q > \mu_q - \varepsilon_q$, or equivalently $\varepsilon_q > -(q-\mu_q) > 0$ and $1 > 2q(1-q) > 0$, by multiplying the last two inequalities, we get $\varepsilon_q > -2q(1-q)(q-\mu_q)$, or equivalently, $2q(1-q)(q-\mu_q) + \varepsilon_q > 0$. Thus, $\frac{\partial \Phi_5}{\partial \mu_q} > 0$ for every q. Also, for $\mu_q = 1$, $\Phi_5(q) \equiv q^4 + (2\varepsilon_q - 3)q^3 + (3 - \varepsilon_q^2 - 3\varepsilon_q)q^2 + (\varepsilon_q^2 - 1)q + \varepsilon_q(1-\varepsilon_q)$. It is trivial to show that the latter function decreases in ε_q, and is negative at $\varepsilon_q = 0$ for every $q \in (\frac{1}{2}, 1)$. Thus, $\Phi_5(q) < 0$, $\frac{\partial \delta}{\partial q} > 0$, and $\frac{\partial G}{\partial q} < 0$.

We now study the sign of $\frac{\partial G}{\partial \varepsilon_q}$. As before it is easier to examine the sign of $\frac{\partial \delta}{\partial \varepsilon_q} = \frac{2q(1-2q)(\mu_q-q)}{[(1-q)(q(\varepsilon_q-\mu_q)+\varepsilon_q-q^2+q)]^2}$, so the sign of $\frac{\partial \delta}{\partial \varepsilon_q}$ is determined by the sign of $\mu_q - q$. Define $\widehat{V}_b$ such that when $t = \widehat{V}_b$, then $q_b = \mu_q$. Intuitively, for that specific value of the opportunity cost, a member who observes a bad signal is equally likely to accept or reject the project. We are interested in cases, where members who receive a bad signal are more likely to recommend rejection rather than approval, such that $t > \widehat{V}_b$. Thus, for $t > \widehat{V}_b$, $\mu_q > q_b$, and therefore $\frac{\partial \delta}{\partial \varepsilon_q} > 0$. Given that $\frac{\partial G}{\partial \varepsilon_q} < 0$, from the IFT, q_b decreases in ε_q. The proof for q_g follows identical steps as Regions I and III are symmetrical.

Extensions: Model Formulations

Preference Diversity

Hierarchy: Recall that the posterior probability of member i is:

$$\Pr(G|s_i,piv)=\frac{\Pr(s_i,piv|G)\Pr(G)}{\Pr(s_i,piv|G)\Pr(G)+\Pr(s_i,piv|B)\Pr(B)}.$$

Therefore, in a hierarchy, the posterior beliefs of member i upon receiving a good and a bad signal are respectively:

$$\Pr(G|s_i=g,piv)=\frac{q\gamma_G^1\gamma_G^2...\gamma_G^{i-1}}{q\gamma_G^1\gamma_G^2...\gamma_G^{i-1}+(1-q)\gamma_B^1\gamma_B^2...\gamma_B^{i-1}}$$

$$\Pr(G|s_i=b,piv)=\frac{(1-q)\gamma_G^1\gamma_G^2...\gamma_G^{i-1}}{(1-q)\gamma_G^1\gamma_G^2...\gamma_G^{i-1}+q\gamma_B^1\gamma_B^2...\gamma_B^{i-1}}$$

where for $i=1$ the expressions simplifies to $\Pr(G|s_1=g)=q$ and $\Pr(G|s_1=b)=1-q$.

Polyarchy

In a polyarchy, the posterior beliefs of member $i=2,...,N$ upon receiving a good and bad signal are respectively:

$$\Pr(G|s_i=g,piv)=\frac{q(1-\gamma_G^1)(1-\gamma_G^2)...(1-\gamma_G^{i-1})}{q(1-\gamma_G^1)(1-\gamma_G^2)...(1-\gamma_G^{i-1})+(1-q)(1-\gamma_B^1)(1-\gamma_B^2)...(1-\gamma_B^{i-1})}$$

and

$$\Pr(G|s_i=b,piv)=\frac{(1-q)(1-\gamma_G^1)(1-\gamma_G^2)...(1-\gamma_G^{i-1})}{(1-q)(1-\gamma_G^1)(1-\gamma_G^2)...(1-\gamma_G^{i-1})+q(1-\gamma_B^1)(1-\gamma_B^2)...(1-\gamma_B^{i-1})}.$$

Interpretive diversity Note that the formulas for the posterior belief of member i, with $i=1,...,N$ remain the same as in the case of preference diversity. The only difference with the analysis of the previous subsection, is that now the decision of each member i is determined by how her type q_i compares to her thresholds q_g^i and q_b^i. These thresholds are determined by solving recursively the system of equations $V\Pr(G|s_i=g,q_g^i,piv)-c=t$ and $V\Pr(G|s_i=b,q_b^i,piv)-c=t$ for every $i=1,2,...N$. Once the thresholds q_g^i and q_b^i are derived, the probabilities of member i recommending approval for a good and a bad project are respectively, $\gamma_G^i=\bar{q}(1-F(q_g^i)+(1-\bar{q})F(q_b^i)$ and $\gamma_B^i=(1-\bar{q})(1-F(t_g^i))+\bar{q}F(q_b^i)$ where $\bar{q}$ is the actual probability of member i receiving the correct signal. Note that unlike the committee case where only one threshold existed at a time, in this case it is possible for both thresholds to exist. Finally, the error probabilities for the hierarchy and polyarchy are calculated using the same formulas as in the case of preference diversity.