Online Appendix

Due to space limitations, the Online Appendix provides the proofs of Proposition 1-3 only. Complete Online Appendix can be downloaded from SSRN at `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3113797`.

Preliminaries:

The optimal effort for S: Due to being interior solutions to (1) and (2), $\underline{e}$ and $\bar{e}$ satisfy:

$$d\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)-d(\underline{w}-c)k=r'(\underline{e}), \tag{23}$$

$$\begin{aligned}&(2-d)d\left((p-c)(2E[X]-k)+2(w-c)(k-E[X])\right)\\&-(1-d)d\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)-d(\underline{w}-c)k=r'(\bar{e}).\end{aligned} \tag{24}$$

By $r''(e)>0$, these conditions are also sufficient. The left-hand side (LHS) of (24) is strictly greater than that of (23) because $d\in(0,1)$, $p>w>\underline{w}$, $k<2E[X]$ (because $X_1+X_2>k$ with probability 1), and $k>E[X]$. Therefore, conditions $r'(0)<d((w-c)E[X]+(\underline{w}-c)(k-E[X]))-d(\underline{w}-c)k$ and $r'(1)>(2-d)d[(p-c)(2E[X]-k)+2(w-c)(k-E[X])]-(1-d)d[(w-c)E[X]+(\underline{w}-c)(k-E[X])]-d(\underline{w}-c)k$ guarantee that solutions to (23) and (24) are interior; i.e., $\{\underline{e},\bar{e}\}\in(0,1)$.

Under the alternative assumption that S chooses responsibility effort e under uncertainty (with prior probability v of having a responsibility violation) and detects and eliminates the violation (if one exists) with probability e, the expressions for the equilibrium effort levels are as follows:

$$vd\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)-vd(\underline{w}-c)k=r'(\underline{e}), \tag{25}$$

$$\begin{aligned}&v(2-d)d\left((p-c)(2E[X]-k)+2(w-c)(k-E[X])\right)\\&-v(1-d)d\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)-vd(\underline{w}-c)k=r'(\bar{e}).\end{aligned} \tag{26}$$

Expressions for Thresholds: Proposition 1 characterizes the region in which a transparency equilibrium exists. For that purpose, define

$$\underline{\tau}\equiv\frac{\left(1-\left(1-(1-d)^2\right)(1-\bar{e})v\right)\left(1-vd(1-\underline{e})\right)(p-w)(2E[X]-k)-d(\bar{e}-\underline{e})(a+s)v}{(1-d)(\bar{e}-\underline{e})v}\vee 0, \tag{27}$$

and

$$\underline{\underline{\tau}}_2=\left(-\frac{ad}{1-d}+\frac{\left(1-\left(1-(1-d)^2\right)(1-\bar{e})v\right)\left(-s+\left(1-vd(1-\underline{e})\right)(p-w)(2E[X]-k)\right)}{(1-d)v\left(\bar{e}-\underline{e}+d(1-\bar{e})\left(1-(1-\underline{e})v\right)\right)}\right)\vee 0. \tag{28}$$

Proof of the Core Outcome When Both B1 and B2 Source from the Same Supplier: If both B1 and B2 source from the same supplier S, negotiation occurs by which S allocates its capacity between the buying firms, B1 has contribution of $(p-w)(k-x_2)$, B2 has contribution of $(p-w)(k-x_1)$, and S has contribution of $(p-c)k-(p-w)(k-x_2)-(p-w)(k-x_1)$. In terms of the contribution for each of the 3 firms, this is the unique core outcome. The rationale is simple. B1 must obtain contribution of $(p-w)(k-x_2)$, at least, because if B1 refuses to renegotiate the contract that specifies the unit price w , S will choose to sell quantity $k-x_2$, at least, to B1 at price w per unit. That quantity $k-x_2$ is the supplier's residual capacity after meeting all the demand from B2. B1 cannot obtain strictly greater contribution than $(p-w)(k-x_2)$. If B1 were to obtain contribution $\varepsilon+(p-w)(k-x_2)$ with $\varepsilon>0$ then, because the total contribution is at most $(p-c)k$, B2 and S together would earn contribution of less than $(p-c)k-[\varepsilon+(p-w)(k-x_2)]<(p-c)x_2-\varepsilon+(w-c)(k-x_2)$. That could not occur, because B2 and S could each achieve strictly higher contribution by cooperating to allocate x_2 units of S's capacity to B2 while S sells $k-x_2$ to B1 at the contracted unit price w and B2 and S together would generate contribution of $(p-c)x_2+(w-c)(k-x_2)$. Hence B1 obtains contribution $(p-w)(k-x_2)$. By the same logic, B2 obtains contribution $(p-w)(k-x_1)$ and S obtains contribution $(p-c)k-(p-w)(k-x_2)-(p-w)(k-x_1)$.

Conditions for Ex Post Optimality of Responsibility-Auditing a Candidate Supplier and Sourcing from that Supplier if and only if the Supplier Passes the Audit: In our analysis below, we break ties between sourcing from a current candidate supplier and continuing the supplier search and qualification process in favor of sourcing from the current supplier.

Recall that two types of perfect Bayesian equilibria exist for the game between B1, B2, and candidate suppliers. In the first type of equilibrium, a candidate supplier has potential to supply either B1 or B2, but not both. We now verify in this equilibrium that responsibility-auditing a candidate supplier and sourcing from that supplier if and only if the supplier passes the audit is ex post optimal for B1 and for B2 if (5) holds. Undertaking supplier search and qualification is ex post optimal (than not sourcing) if the buying firm's expected profit from this process is strictly greater than 0, $\pi>0$. $\tau>(p-w)E[X]$ and $\pi>0$ guarantee that it is optimal ex post for the buying firm to continue the supplier search process and not source from a candidate supplier if that supplier failed the audit. For a buying firm to find it optimal ex post to source from a candidate supplier that passed the audit, the buying firm's expected profit from that supplier (excluding sunk costs) must be greater than the expected profit from continuing the supplier search process; i.e., $(p-w)E[X]-v(1-\underline{e})(1-d)\tau/(1-vd(1-\underline{e}))\geq\pi$. This inequality holds by $a>0$, $s\geq 0$, $(v,d,\underline{e})\in(0,1)$. In equilibrium, responsibility effort by any candidate supplier is $\underline{e}$ and the expected profit of a buying firm (B1 or B2) is π. If a buying firm deviates by not auditing, the buying firm's expected profit is $(p-w)E[X]-v(1-\underline{e})\tau-s$. Auditing a candidate supplier is therefore ex post optimal for a buying firm if and only if $\pi>(p-w)E[X]-v(1-\underline{e})\tau-s$, which is equivalent to (5).

We now focus our attention to the transparency equilibrium and prove that responsibility-auditing a candidate supplier and sourcing from that supplier if and only if the supplier passes the audit is ex post optimal for B1 if (6) holds. In a transparency equilibrium, B1 chooses to reveal its supplier's identity (over not making a transparency commitment, which would lead to an expected profit of π) and obtains an expected profit of π_{t1} by doing so. This implies that $\pi_{t1} \geq \pi$. Then, $\pi > 0$ leads to $\pi_{t1} > 0$, which ensures that undertaking supplier search and qualification is ex post optimal for B1 (than not sourcing). $\pi_{t1} > 0$ together with $\tau > (p-w)E[X]$ and $2E[X] > k$ ensure that continuing the supplier search process and not sourcing from a candidate supplier is ex post optimal for B1 if the supplier failed the audit. Sourcing from a candidate supplier that passes the audit is ex post optimal for B1 if B1's expected profit from that supplier (excluding sunk costs) must be greater than the expected profit from continuing the supplier search process; i.e., $(p-w)((1-d(1-q))(k-E[X]) + d(1-q)E[X]) - (1-q)\tau \geq \pi_{t1}$ wherein q is the likelihood that the supplier does not have a violation given that it has passed the audit which satisfies $q = (1-v(1-\bar{e}))/(1-vd(1-\bar{e}))$. By simple algebra, this inequality can be verified to hold, due to $a > 0$, $s \geq 0$ and $(v, d, \bar{e}) \in (0, 1)$. In the transparency equilibrium, responsibility effort by a candidate supplier to B1 is $\bar{e}$ and B1's expected profit is π_{t1}. If B1 deviates by not auditing, B1's expected profit is $vd(1-\bar{e})(p-w)E[X] + (1-vd(1-\bar{e}))(p-w)(k-E[X]) - v(1-\bar{e})\tau - s$. Therefore, auditing is ex post optimal for B1 if and only if $\pi_{t1} > vd(1-\bar{e})(p-w)E[X] + (1-vd(1-\bar{e}))(p-w)(k-E[X]) - v(1-\bar{e})\tau - s$, or equivalently, (6).

Finally, we prove that, in a transparency equilibrium, responsibility-auditing S and sourcing from S if and only if S passes the audit is ex post optimal for B2 if (7) holds. In a transparency equilibrium, B2 chooses to take B1's supplier S as its first candidate and not search for and qualify alternative suppliers, which implies that $\pi_{t2} \geq \pi$. Then, $\pi > 0$ leads to $\pi_{t2} > 0$, which ensures that undertaking supplier search and qualification is ex post optimal for B2 (than not sourcing). For sourcing from S to be ex post optimal for B2 if S has passed B2's audit, B2's expected profit from S (excluding sunk costs) must be greater than continuing the supplier search process: i.e., $(p-w)(k-E[X]) - v(1-\bar{e})(1-d)^2\tau/(1-v(1-(1-d)^2)(1-\bar{e})) \geq \pi$. This condition can be verified to hold due to $\pi_{t2} \geq \pi$, $a > 0$ and $(v, d, \bar{e}) \in (0, 1)$. $\tau > (p-w)E[X]$, $\pi > 0$ and $2E[X] > k$ (by $X_1 + X_2 > k$ with probability 1) lead to $\pi > (p-w)(k-E[X]) - \tau$, which implies that it is ex post optimal for B2 to continue the supplier search process and not source from S if S fails B2's audit. In a transparency equilibrium, B1's supplier S chooses the responsibility effort $\bar{e}$ and B2's expected profit is π_{t2}. If B2 deviates by not auditing, B2's expected profit is $(p-w)(k-E[X]) - (1-q)\tau$, where q is the likelihood that S does not have a violation after observing that it is revealed as B1's supplier; by rational expectations, $q = (1-v(1-\bar{e}))/(1-vd(1-\bar{e}))$. Then, auditing is ex post optimal for B2 if and only if (7) holds. In the event that S fails B2's audit, ex post optimality conditions for B2 to responsibility-audit an alternative candidate supplier and source from that supplier if and only if the supplier passes the audit are the same as in the first type of equilibrium, which are derived above.

Proofs of the Analytical Results in Section 3

The following Lemma compares the buying firms' expected profits in a transparency equilibrium (if it exists), which will be instrumental in proving Proposition 1.

LEMMA 1. *B2's expected profit in a transparency equilibrium is strictly greater than B1's expected profit;*

$$\pi_{t2} > \pi_{t1} \tag{29}$$

Proof of Lemma 1: Note that $\pi_{t2} = (q + (1-q)(1-d))(p-w)(k - E[X]) - (1-q)(1-d)\tau - a + d(1-q)\pi$ with $q = (1 - v(1-\bar{e}))/(1 - vd(1-\bar{e}))$. By simple algebra, the expected contribution of B1 and B2 can be shown to be equivalent. B1 incurs greater auditing, search, and responsibility-related costs in expectation because $(d, \underline{e}, v) \in (0,1)$ implies $1 - vd(1-\underline{e}) > 1 - d$, which together with $\bar{e} \in (0,1)$ leads to the coefficients of a, s and τ in the expression for π being strictly greater than the respective coefficients in the expression for π_{t2}.

Proof of Proposition 1: (a.) The LHS of (24) is strictly greater than that of (23) because $p > w > \underline{w}$, $d \in (0,1)$, $2E[X] > k$ (due to $X_1 + X_2 > k$ with probability 1), and $k > E[X]$. The result then follows from the strict convexity of $r(\cdot)$.

(b.) With B1's commitment to publish its supplier's identity, B2 chooses S as its first candidate supplier if and only if

$$\big(q + (1-q)(1-d)\big)(p-w)(k - E[X]) - (1-q)(1-d)\tau - a + d(1-q)\pi \geq \pi, \tag{30}$$

where q is the likelihood B2 assigns to S not having a violation after S is revealed as B1's supplier, the LHS of (30) represents B2's expected profit by taking S as its first candidate supplier, and the right-hand side (RHS) is B2's expected profit from searching for a different candidate supplier and never sourcing from S. In a transparency equilibrium, by rational expectations,

$$q = \frac{1 - v(1-\bar{e})}{1 - vd(1-\bar{e})}, \tag{31}$$

so (30) is equivalent to $\pi_{t2} \geq \pi$. This condition is satisfied if and only if $\tau \geq \underline{\tau}_2$, with $\underline{\tau}_2$ defined in (28).

The first type of equilibrium in which B2 does not source from S and a candidate supplier to B1 exerts $\underline{e}$ exists if and only if the LHS of (30) evaluated at $q = (1-v(1-\underline{e}))/(1-vd(1-\underline{e}))$ is less than or equal to π, or equivalently if and only if $\tau \le \underline{\tau}_2$ with

$$\underline{\tau}_2 = \left(-\frac{ad}{1-d} + \frac{\left(1-\left(1-(1-d)^2\right)(1-\underline{e})v\right)\left(-s+\left(1-vd(1-\underline{e})\right)(p-w)(2E[X]-k)\right)}{vd(1-d)(1-\underline{e})\left(1-(1-\underline{e})v\right)}\right) \vee 0.$$

By simple algebra, it is straightforward to verify that $\underline{\tau}_2 \ge \underline{\underline{\tau}}_2$. When B1 is committed to publish its supplier's identity, in the region where both types of equilibria exist ($\tau \in [\underline{\underline{\tau}}_2, \underline{\tau}_2]$), the equilibrium in which B2 chooses S as its first candidate supplier and S exerts greater effort $\bar{e}$ leads to higher expected profit for B2 and S each and is more likely to be preferred. As a result, we proceed by focusing our attention to the region $\tau \ge \underline{\underline{\tau}}_2$ (in which B2 is attracted to take S as its first candidate and S exerts responsibility effort $\bar{e}$) in order to characterize the conditions for the transparency equilibrium to occur in the remainder of the proof.

For $\tau \ge \underline{\underline{\tau}}_2$, B1 obtains expected profit π_{t1} by committing to publish its supplier's identity. Without such commitment, B1 obtains π. B1 commits to publish supplier identity if and only if $\pi_{t1} \ge \pi$, which is equivalent to $\tau \ge \underline{\tau}$, with $\underline{\tau}$ defined in (27). Therefore, a transparency equilibrium exists if and only if $\tau \ge \max(\underline{\tau}, \underline{\underline{\tau}}_2)$. By Lemma 1, B2 is strictly better offthan B1 in a transparency equilibrium, $\pi_{t2} > \pi_{t1}$, which implies that

$$\underline{\tau} > \underline{\underline{\tau}}_2. \tag{32}$$

(c.) If $\underline{\tau} > 0$, the derivatives of $\underline{\tau}$ (defined in (27)) with respect to a and with respect to s are both $-d/(1-d)$, which is negative due to $d \in (0,1)$. The derivative of $\underline{\tau}$ with respect to v is $-(1-(1-(1-d)^2)dv^2(1-\bar{e})(1-\underline{e}))(p-w)(2E[X]-k)/((1-d)(\bar{e}-\underline{e})v^2)$, which is negative due to $p > w$, $k < 2E[X]$, $\bar{e} > \underline{e}$ (due to Proposition 1a), and $(v, d, \underline{e}, \bar{e}) \in (0,1)$. Therefore, $\underline{\tau}$ is strictly decreasing with s, a or v if $\underline{\tau} > 0$.

$\underline{\tau} = 0$ if and only if the numerator of the first expression on the RHS of (27) is nonpositive; i.e.,

$$\left(1-\left(1-(1-d)^2\right)(1-\bar{e})v\right)\left(1-vd(1-\underline{e})\right)(p-w)(2E[X]-k) - d(\bar{e}-\underline{e})(a+s)v \le 0. \tag{33}$$

By (24), $d \in (0,1)$, $k < 2E[X]$ and strict convexity of $r(\cdot)$, $\bar{e}$ is strictly increasing with p, while by (23) $\underline{e}$ is independent of p. Let $\epsilon = \inf\{\bar{e} - \underline{e} : p \ge w\}$. $\epsilon > 0$ by (23) and (24), strict convexity of $r(\cdot)$, $\bar{e}$ strictly increasing with p, $d \in (0,1)$, $w > \underline{w}$, and $k > E[X]$. Then, noting that $(v, d, \bar{e}) \in (0,1)$, (33) holds for any $p \le \bar{p}$ with

$$\bar{p} \equiv w + \epsilon d(a+s)v / \left(\left(1-vd(1-\underline{e})\right)(2E[X]-k)\right). \tag{34}$$

By simple algebra, (33) also holds for $s \geq \underline{s}$ or $a \geq \underline{a}$ where $\underline{s}$ and $\underline{a}$ are defined as

$$\underline{s} \equiv \frac{\left(1-\left(1-(1-d)^2\right)(1-\bar{e})v\right)\left(1-vd(1-\underline{e})\right)(p-w)(2E[X]-k)}{d(\bar{e}-\underline{e})v} - a, \tag{35}$$

$$\underline{a} \equiv \frac{\left(1-\left(1-(1-d)^2\right)(1-\bar{e})v\right)\left(1-vd(1-\underline{e})\right)(p-w)(2E[X]-k)}{d(\bar{e}-\underline{e})v} - s. \tag{36}$$

Proof of Proposition 2: S's expected profit in a transparency equilibrium satisfies

$$\begin{aligned}
&\left(\bar{e}+(1-\bar{e})(1-d)^2\right)\left((p-c)(2E[X]-k)+2(w-c)(k-E[X])\right)\\
&+(1-\bar{e})(1-d)d\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)+(1-\bar{e})d(\underline{w}-c)k-r(\bar{e})\\
&>\left(\underline{e}+(1-\underline{e})(1-d)^2\right)\left((p-c)(2E[X]-k)+2(w-c)(k-E[X])\right)\\
&+(1-\underline{e})(1-d)d\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)+(1-\underline{e})d(\underline{w}-c)k-r(\underline{e})\\
&>\left(\underline{e}+(1-\underline{e})(1-d)\right)\left((w-c)E[X]+(\underline{w}-c)(k-E[X])\right)+(1-\underline{e})d(\underline{w}-c)k-r(\underline{e}),
\end{aligned} \tag{37}$$

where the first inequality is by the optimality of $\bar{e}$ and strict convexity of $r(\cdot)$, and the second inequality is due to $p > w > \underline{w}$, $E[X] < k < 2E[X]$, and $\underline{e} \in (0,1)$. Since the quantity in the last line of above is S's expected profit in an equilibrium in which S has potential to supply B1 but not B2, (37) establishes the result.

Proof of Proposition 3: (a.) By the implicit differentiation of (24),

$$\frac{d\bar{e}}{dk} = \frac{\left(1-(1-d)^2\right)(2w-p-\underline{w})}{r''(\bar{e})}, \tag{38}$$

which is positive if and only if $(w-\underline{w}) > (p-\underline{w})/2$ due to $d \in (0,1)$ and strict convexity of $r(\cdot)$. The LHS of (23) equals $d(w-\underline{w})E[X]$, which implies that k has no effect on $\underline{e}$. To show the third part of (a.), we suppose that $\underline{\tau}$ equals the first expression in the RHS of (27) and show that $\underline{\tau}$ strictly decreases with k if (8) holds. By differentiating (27), we obtain $\partial\underline{\tau}/\partial k = (-1+(1-(1-d)^2)(1-\bar{e})v)(1-vd(1-\underline{e}))(p-w)/(v(1-d)(\bar{e}-\underline{e}))$ and $\partial\underline{\tau}/\partial\bar{e} = -((2E[X]-k)(1-vd(1-\underline{e}))(1-(1-(1-d)^2)(1-\underline{e})v)(p-w))/(v(1-d)(\bar{e}-\underline{e})^2)$, which are both negative due to $k < 2E[X]$, $(v,d,\underline{e},\bar{e}) \in (0,1)$, $p > w$ and $\bar{e} > \underline{e}$ (by Proposition 1a). Therefore, $\partial\underline{\tau}/\partial k < 0$ and $\partial\underline{\tau}/\partial\bar{e} < 0$ together with (38) lead to $d\underline{\tau}/dk < 0$ if $(w-\underline{w}) > (p-\underline{w})/2$. In the region where k is sufficiently high that $\underline{\tau} = 0$, the result holds weakly.

(b.) By implicit differentiation of (24) and noting that $\underline{w} = c$,

$$\frac{d\bar{e}}{dc} = -\frac{(2-d)dk-(1-d)dE[X]}{r''(\bar{e})} < 0, \tag{39}$$

where the inequality is due to $E[X] < k < 2E[X]$, $d \in (0,1)$ and strict convexity of $r(\cdot)$. By implicit differentiation of (23) and noting that $\underline{w} = c$,

$$\frac{d\underline{e}}{dc} = -\frac{dE[X]}{r''(\underline{e})} < 0, \tag{40}$$

where the inequality is due to $E[X]>0$ (which is implied by $x_1+x_2>k$ with probability 1 and $k>0$), $d\in(0,1)$, and strict convexity of $r(\cdot)$. The result that decreasing the production cost c strictly increases $\bar{e}-\underline{e}$ if and only if (9) follows from (39) and (40).

It remains to show that, for $\underline{w}=c$, a decrease in production cost c strictly reduces $\underline{\tau}$ if and only if (10) and $\underline{\tau}>0$. In the region where the first expression in the RHS of (27) is negative, $\underline{\tau}=0$ and hence $\underline{\tau}$ is invariant with respect to c. In the region where $\underline{\tau}$ equals the first expression in the RHS of (27), $\underline{\tau}$ is affected indirectly by c through $\bar{e}$ and $\underline{e}$. The derivatives of $\underline{\tau}$ with respect to $\underline{e}$ and $\bar{e}$ are

$$\frac{\partial\underline{\tau}}{d\bar{e}}=-\frac{(2E[X]-k)(1-vd(1-\underline{e}))(1-(1-(1-d)^2)(1-\underline{e})v)(p-w)}{v(1-d)(\bar{e}-\underline{e})^2},$$
$$\frac{\partial\underline{\tau}}{d\underline{e}}=\frac{(2E[X]-k)(1-vd(1-\bar{e}))(1-(1-(1-d)^2)(1-\bar{e})v)(p-w)}{v(1-d)(\bar{e}-\underline{e})^2},$$

which together with (39) and (40) lead to

$$\begin{aligned}\frac{d\underline{\tau}}{dc}=&\frac{d(2E[X]-k)(p-w)}{v(1-d)(\bar{e}-\underline{e})^2}\\ &\left[-\frac{(1-vd(1-\bar{e}))(1-(1-(1-d)^2)(1-\bar{e})v)E[X]}{r''(\underline{e})}\right.\\ &\left.+\frac{(1-vd(1-\underline{e}))(1-(1-(1-d)^2)(1-\underline{e})v)((2-d)k-(1-d)E[X])}{r''(\bar{e})}\right].\end{aligned}$$

For $\underline{\tau}>0$, $d\underline{\tau}/dc>0$ if and only if the term in squared brackets above is positive because $(v,d)\in(0,1)$, $E[X]<k<2E[X]$, $\bar{e}>\underline{e}$ (by Proposition 1a), and $p>w$. The term in squared brackets is positive if and only if (10) is satisfied, which establishes the result.

If suppliers are not demand-constrained, each supplier's total production cost equals ck (regardless of B1 or B2's decisions) and is independent of the supplier's responsibility effort. As a result, $\bar{e}$ and $\underline{e}$ are invariant with respect to c, which leads $\underline{\tau}$ to be invariant with respect to c as well.