Does crowdfunding benefit entrepreneurs and venture capital investors?—Online Appendix

Volodymyr Babich, Simone Marinesi, Gerry Tsoukalas[1]

B Auxiliary Tables

Table 4 presents the definitions of the cases in Proposition 2.

Table 4: Notation for cases of Proposition 2. Conditions are for a given signal s and demand for external capital L.

Legend: & is the logical 'and' operator, ¬ is the logical 'not' operator. Abbreviations: VB stands for VC financing in the presence of bank competition, V stands for VC financing, B stands for Bank financing, ∅ stands for No financing.

Abbreviations	Cases of Proposition 2	Conditions
VB	1a	$V(\bar{e}, 0, s) \geq 0$ & (14) & (11)
B	1b	$V(\bar{e}, 0, s) \geq 0$ & (14) & ¬ (11)
V	2a	¬ [$V(\bar{e}, 0, s) \geq 0$ & (14)] & $V(\bar{e}, \bar{v}, s) \geq 0$ & (12)
∅	2b	¬ [$V(\bar{e}, 0, s) \geq 0$ & (14)] & ¬ [$V(\bar{e}, \bar{v}, s) \geq 0$ & (12)]

Table 5 presents the change in the VC's and the entrepreneur's values due to a successful campaign, for the regions described by Proposition 3.

Table 5: Change in the VC's and the entrepreneur's values of the project due to a successful campaign

Region	Effect on VC's value:	Effect on Entrepreneur's value:
(i)	0	0
(ii)	0	$V(\bar{e}, 0, 1)$
(iii)	$(1-\theta)V(\bar{e}, \bar{v}, 1)$	$\theta V(\bar{e}, \bar{v}, 1)$
(iv)	$(1-\theta)(V(\bar{e}, \bar{v}, 1) - V(\bar{e}, 0, 1))$	$\theta V(\bar{e}, \bar{v}, 1) + (1-\theta)V(\bar{e}, 0, 1)$
(v)	0	$V(\bar{e}, 0, 1) - V(\bar{e}, 0, 0)$
(vi)	$(1-\theta)(V(\bar{e}, \bar{v}, 1) - V(\bar{e}, 0, 1))$	$\theta V(\bar{e}, \bar{v}, 1) + (1-\theta)V(\bar{e}, 0, 1) - V(\bar{e}, 0, 0)$
(vii)	$-(1-\theta)V(\bar{e}, \bar{v}, 0)$	$-\theta V(\bar{e}, \bar{v}, 0)$
(viii)	$-(1-\theta)V(\bar{e}, \bar{v}, 0)$	$V(\bar{e}, 0, 1) - \theta V(\bar{e}, \bar{v}, 0)$
(ix)	$(1-\theta)[V(\bar{e}, \bar{v}, 1) - V(\bar{e}, \bar{v}, 0)]$	$\theta [V(\bar{e}, \bar{v}, 1) - V(\bar{e}, \bar{v}, 0)]$
(x)	$(1-\theta)[V(\bar{e}, \bar{v}, 1) - V(\bar{e}, 0, 1) - V(\bar{e}, \bar{v}, 0)]$	$\theta [V(\bar{e}, \bar{v}, 1) - V(\bar{e}, \bar{v}, 0)] + (1-\theta)V(\bar{e}, 0, 1)$
(xi)	$-(1-\theta)(V(\bar{e}, \bar{v}, 0) - V(\bar{e}, 0, 0))$	$V(\bar{e}, 0, 1) - \theta V(\bar{e}, \bar{v}, 0) - (1-\theta)V(\bar{e}, 0, 0)$
(xii)	$(1-\theta)[V(\bar{e}, \bar{v}, 1) - V(\bar{e}, 0, 1) - V(\bar{e}, \bar{v}, 0) + V(\bar{e}, 0, 0)]$	$\theta [V(\bar{e}, \bar{v}, 1) - V(\bar{e}, \bar{v}, 0)] + (1-\theta)[V(\bar{e}, 0, 1) - V(\bar{e}, 0, 0)]$

[1]Volodymyr Babich: McDonough School of Business, Georgetown University, vob2@georgetown.edu; Simone Marinesi: The Wharton School, University of Pennsylvania, marinesi@wharton.upenn.edu; Gerry Tsoukalas: The Wharton School, University of Pennsylvania, gtsouk@wharton.upenn.edu.

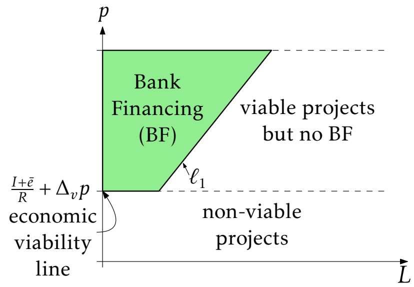


Figure 9: Illustration of bank financing feasibility in Proposition 8.

Legend: Axes are $L =$ external capital required, $p = p(\bar{e}, \bar{v}, s)$ probability of project success with both the entrepreneur's and VC's efforts. Shaded region represents bank financing feasibility conditions. Table 6 presents equations of line ℓ_1 and the economic viability line $V(\bar{e}, 0, s) = 0$. Recall, $\Delta_v p = p(e, \bar{v}, s) - p(e, 0, s)$.

C Financing Models

C.1 Model of the entrepreneur financing projects with bank investors in the economy without the VC

Proposition 8. *Bank financing is feasible if and only if the project is economically viable, that is* $V(\bar{e}, 0, s) \geq 0$*, where* V *is defined in (1), and*

$$L/p(\bar{e}, 0, s) \leq R - \bar{e}/\Delta_e p. \tag{14}$$

This is a known result (Tirole 2010, Chapter 3), but it is convenient to reproduce it in our notation. Due to perfect competition among bank investors, the entrepreneur can offer the repayment at the lower boundary of the financing feasible region (5). Then $r = L/p(\bar{e}, 0, s)$ and the entrepreneur extracts the entire value of the project, i.e., $V(\bar{e}, 0, s)$.

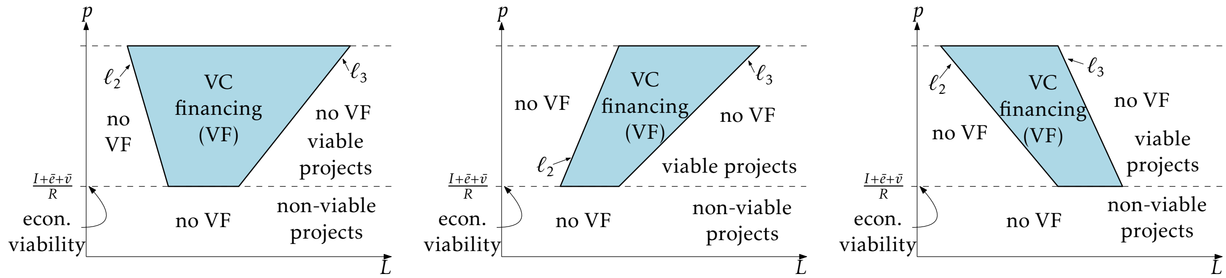
Figure 9 illustrates bank financing feasibility from Proposition 8. On the horizontal axis of this figure, we measure the amount of external capital needed L to invest in the project. On the vertical axis, we measure the probability of a successful outcome $p = p(\bar{e}, \bar{v}, s)$ for the project, when both the entrepreneur and VC's efforts are contributed. Even though there are no VC in the model in this subsection, we use variable p to facilitate the comparisons with other models in this paper. From definitions (2), there is a connection between $p = p(\bar{e}, \bar{v}, s)$ and $p(\bar{e}, 0, s)$: $p(\bar{e}, 0, s) = p - \Delta_v p$.

From Proposition 8, projects must be economically viable, i.e., $V(\bar{e}, 0, s) = p(\bar{e}, 0, s)R - I - \bar{e} \geq 0$. This is equivalent to condition $p \geq (I + \bar{e})/R + \Delta_v p$. From Proposition 8, projects must satisfy (14). This is equivalent to condition $p \geq L/\left(R - \bar{e}/\Delta_e p\right) + \Delta_v p$. We define line ℓ_1 as the set of points (L, p), where this inequality is binding. Table 6 in Online Appendix D presents equations of lines.

In the shaded region of Figure 9 bank financing is feasible. In the non-shaded region bank financing is not feasible, even for projects that are economically viable. This illustrates the notorious underinvestment inefficiency due to moral hazard (Holmström and Tirole 1998).

C.2 Equilibrium in the negotiations between the entrepreneur and the VC in the economy without bank investors

This section describes a special case of Proposition 1 for the economy without bank investors. In this case, the entrepreneur does not have alternatives to VC financing, which means that disagreement values are $d^e = d^v = 0$ in equations (8) and Proposition 1. Consequently, we have the following



(a) $\theta R \geq \bar{e}/\Delta_e p$, $(1-\theta)R \geq \bar{v}/\Delta_v p$ (b) $\theta R \geq \bar{e}/\Delta_e p$, $(1-\theta)R < \bar{v}/\Delta_v p$ (c) $\theta R < \bar{e}/\Delta_e p$, $(1-\theta)R \geq \bar{v}/\Delta_v p$

Figure 10: VC financing feasibility when only VC financing is available.

Legend: Axes are as in Figure 9. Shaded regions represent VC financing feasibility conditions. Table 6 presents equations of lines ℓ_2, ℓ_3, and the economic viability $V(\bar{e}, \bar{v}, s) = 0$.

corollary to Proposition 1.

Proposition 9. *Suppose $d^e = d^v = 0$. VC financing is feasible if and only if the project is economically viable ($V(\bar{e}, \bar{v}, s) \geq 0$) and*

$$\bar{v}/\Delta_v p \leq \tfrac{(1-\theta)V(\bar{e},\bar{v},s)+\bar{v}+L}{p(\bar{e},\bar{v},s)} \leq R - \bar{e}/\Delta_e p. \tag{15}$$

If VC financing is feasible, then the shares of the entrepreneur and the VC are $S^e = \theta V(\bar{e}, \bar{v}, s)$ and $S^v = (1-\theta)V(\bar{e}, \bar{v}, s)$, respectively. If VC financing is not feasible, then the shares are zero.

Figure 10 illustrates financing feasibility conditions in Proposition 9. We use the same variables (L, p) for axes as we did in Figure 9. Depending on the choice of the parameter values, the feasible region takes different forms, as discussed in Lemma 2 (Online Appendix §D) and shown in panels of Figure 10. The economic viability line, $V(\bar{e}, \bar{v}, s) = 0$, is $p = (I + \bar{e} + \bar{v})/R$. Line ℓ_2 corresponds to the first inequality in (12) becoming equality. This inequality controls the VC's moral hazard incentives. Line ℓ_3 corresponds to the second inequality in (12) becoming equality. This inequality controls the entrepreneur's moral hazard incentives. Table 6 (online appendix) presents equations of these lines. Figure 10 shows representative graphs, though the collection of feasible region shapes is more diverse.

As in the case of bank financing (Online Appendix §C.1), moral hazard-based frictions can lead to credit rationing. Projects that are above the economic viability line are financed only if they fall into a shaded region in Figure 10. Because of double-sided moral hazard, credit rationing happens for projects to the left and to the right of VC financing feasibility region.

D Construction of Figures

The figures in the paper are constructed analytically using the lines (representing problem constraints) listed in Table 6 below.

We collect useful properties of these lines in Lemma 2.

Lemma 2. *Properties of lines from Table 6 are as follows:*

(i) *Lines ℓ_1 and ℓ_5 are parallel.*
(ii) *Line ℓ_1 runs below line ℓ_5 if and only if $\theta\,(R - \bar{v}/\Delta_v p) < \bar{e}/\Delta_e p$.*
(iii) *Lines ℓ_2 and ℓ_3 intersect at $(L, p) = ([(1-\theta) - \bar{v}/(I + \bar{e} + \bar{v})]\,(I + \bar{e} + \bar{v}), 0)$.*

Table 6: Lines used in figures.

First use	Label	Equation in (L,p)-space
§4.2	$V(\bar{e},0,0)=0$	$p=\frac{I+\bar{e}}{R}+\Delta_v p$
	ℓ_1	$p=\frac{L}{R-\bar{e}/\Delta_e p}+\Delta_v p$
	$V(\bar{e},\bar{v},0)=0$	$p=\frac{I+\bar{e}+\bar{v}}{R}$
	ℓ_2	$p=\frac{L+[\bar{v}/(I+\bar{e}+\bar{v})-(1-\theta)](I+\bar{e}+\bar{v})}{\bar{v}/\Delta_v p-(1-\theta)R}$
	ℓ_3	$p=\frac{L+[\bar{v}/(I+\bar{e}+\bar{v})-(1-\theta)](I+\bar{e}+\bar{v})}{\theta R-\bar{e}/\Delta_e p}$
	ℓ_4	$p=\frac{L+\Delta_v p(1-\theta)R+\theta\bar{v}}{\bar{v}/\Delta_v p}$
	ℓ_5	$p=\frac{L+\Delta_v p(1-\theta)R+\theta\bar{v}}{R-\bar{e}/\Delta_e p}$
§A.1	ℓ_6	$p=\frac{L+\bar{v}-(1-\theta)\{(I+\bar{e})(1-\pi)+\bar{v}+\pi[A+R(\Delta_s p-\Delta_v p)]-k\}}{\bar{v}/\Delta_v p-(1-\theta)(1-\pi)R}$
	ℓ_7	$p=\frac{L+\bar{v}-(1-\theta)\{(I+\bar{e})(1-\pi)+\bar{v}+\pi[A+R(\Delta_s p-\Delta_v p)]-k\}}{R-\bar{e}/\Delta_e p-(1-\theta)(1-\pi)R}$
	ℓ_8	$p=\frac{I+\bar{e}+\bar{v}-k+\pi[A+R(\Delta_s p-\Delta_v p)-I-\bar{e}]}{R(1-\pi)}$
	$V(\bar{e},0,1)=0$	$p=\frac{I+\bar{e}}{R}+\Delta_v p-\Delta_s p$
	$V(\bar{e},\bar{v},1)=0$	$p=\frac{I+\bar{e}+\bar{v}}{R}-\Delta_s p$
	ℓ_1'	$p=\frac{L}{R-\bar{e}/\Delta_e p}+\Delta_v p-\Delta_s p+\frac{\Delta L}{R-\bar{e}/\Delta_e p}$
	ℓ_2'	$p=\frac{L+[\bar{v}/(I+\bar{e}+\bar{v})-(1-\theta)](I+\bar{e}+\bar{v})}{\bar{v}/\Delta_v p-(1-\theta)R}-\Delta_s p+\frac{\Delta L}{\bar{v}/\Delta_v p-(1-\theta)R}$
	ℓ_3'	$p=\frac{L+[\bar{v}/(I+\bar{e}+\bar{v})-(1-\theta)](I+\bar{e}+\bar{v})}{\theta R-\bar{e}/\Delta_e p}-\Delta_s p+\frac{\Delta L}{\theta R-\bar{e}/\Delta_e p}$
	ℓ_4'	$p=\frac{L+\Delta_v p(1-\theta)R+\theta\bar{v}}{\bar{v}/\Delta_v p}-\Delta_s p+\frac{\Delta L}{\bar{v}/\Delta_v p}$
	ℓ_5'	$p=\frac{L+\Delta_v p(1-\theta)R+\theta\bar{v}}{R-\bar{e}/\Delta_e p}-\Delta_s p+\frac{\Delta L}{R-\bar{e}/\Delta_e p}$

(iv) *Slopes of lines ℓ_4 and ℓ_5 are positive and the slope of line ℓ_4 is greater than that of line ℓ_5.*
(v) *Lines ℓ_4 and ℓ_5 intersect at point $(-\Delta_v p\,[R(1-\theta)+\theta\bar{v}/\Delta_v p]\,,0)$.*
(vi) *Lines ℓ_2, ℓ_4, and $p=(I+\bar{e})/R+\Delta_v p$ intersect.*
(vii) *Lines ℓ_3, ℓ_5, and $p=(I+\bar{e})/R+\Delta_v p$ intersect.*
(viii) *The properties of lines ℓ_i', preserve the above properties of lines ℓ_i, for $i=1,\ldots,5$.*
(ix) *The slope of ℓ_6 is steeper than that of ℓ_4'.*
(x) *Line ℓ_6 runs above line ℓ_4' if and only if $R(1-\theta)-\bar{v}/\Delta_v p\le\frac{A[1-\pi]+\theta[A\pi-k]}{\Delta_s p}$.*
(xi) *Line ℓ_2' is parallel to line ℓ_2.*
(xii) *Line ℓ_2' runs below line ℓ_2 if and only if $\Delta L=A-k\ge\frac{\Delta_s p[R\Delta_v p(1-\theta)-\bar{v}]}{\Delta_v p}$.*
(xiii) *Line ℓ_8 is above the economic viability lines and below line $1-\Delta_s p$ if and only if $\frac{\bar{e}+\bar{v}+I-(\bar{e}+I)\pi+[A\pi-k]}{1-\Delta_s p-\pi(1-\Delta_v p)}\le R\le\frac{\bar{v}+[A\pi-k]}{(\Delta_v p-\Delta_s p)}$.*

Construction of Figure 1: The economic viability line for the project financed by bank investors is $V(\bar{e},0,s)=(p-\Delta_v p)R-I-\bar{e}=0$ (from (2), $p(\bar{e},0,s)=p-\Delta_v p$). The economic viability line for project financed by the VC is $V(\bar{e},\bar{v},s)=pR-I-\bar{e}-\bar{v}=0$. Line ℓ_1 corresponds to binding condition (14), representing feasibility of bank financing (Online Appendix §C.1). Line ℓ_2 corresponds to the first inequality in (12) binding. This inequality controls the VC's moral hazard incentives, when bank financing is not feasible (Online Appendix §C.2). Line ℓ_3 corresponds to the second inequality in (12) binding. This inequality controls the entrepreneur's moral hazard incentives (Online Appendix §C.2). Lines ℓ_4 and ℓ_5 are defined by the first inequalities in (11) binding. These correspond to the VC's and the entrepreneur's moral hazard incentives, when bank financing is feasible. Table 6 (online appendix) presents equations of these lines and Lemma 2 describes their properties.

Construction of Figure 2: Figure 2, similar to Figure 1, is constructed in the (L,p) space, where L is the amount of capital required, and p is the project success probability. However, Figure 1 was based on a generic signal s and a generic capital requirement L. Now we specialize them to the no-crowdfunding economy and henceforth interpret (L, p) values relative to $s = 0$, thus $L = I - a$ and $p = p(\bar{e}, \bar{v}, 0)$. Line equations remain as given in Table 6 (online appendix) and we continue using labels ℓ_1, ℓ_2, etc.

Figure 2(a) presents equilibria in the no-crowdfunding economy. Figure 2(b) presents equilibria in the economy after a successful crowdfunding. In this economy, the equilibrium is expressed in variables $L' = I - a - (A - k)$ and $p' = p(\bar{e}, \bar{v}, 1)$. But to facilitate the comparison between two economies, we convert all lines to variables (L, p) using a transformation of variables: $L' = L - \Delta L$ and $p' = p + \Delta_s p$. Quantities $\Delta L = (A - k) \geq 0$ and $\Delta_s p = p(\bar{e}, \bar{v}, 1) - p(\bar{e}, \bar{v}, 0) \geq 0$ capture the positive effect of crowdfunding on the capital and the success probability. Lines ℓ'_1 through ℓ'_5 in Figure 2(b) are defined in variables (L, p) of the no-crowdfunding economy. Their algebraic expressions are also collected in Table 6.

Construction of Figure 3(a). Follows by overlaying Figures 1 and 2.

Construction of Figure 7. Compared to Figure 2, there are three new lines in Figure 7: ℓ_6, ℓ_7, and ℓ_8, all three coming from case (3) of Proposition 6. Lines ℓ_6 and ℓ_7 correspond to two inequalities in expression (10) binding. Line ℓ_8 corresponds to inequality $V(\bar{e}, \bar{v}, 0) \geq d^e$ binding.

Let's compare the equilibrium for the economy with a crowdfunding platform (Proposition 6) with the equilibrium for the economy after a successful campaign (Corollary 1, part (b)). Conditions in column $t = 2$ of Table 2, come from the solution of the subgame representing the economy after a successful campaign. Therefore, there is significant commonality between cases in Corollary 1, part (b) and cases in Proposition 6. In particular, regions VB and V in Corollary 1, part (b) are the same as cases (1) and (2), respectively, in Proposition 6. Region B in Corollary 1, part (b) is divided into two cases: (3) and (4), in Proposition 6. Region $\emptyset$ is divided into two cases: (5) and (6), in Proposition 6.

E Proofs

Proof of Lemma 1. From (2), we have $p(\bar{e}, \bar{v}, s) = p(\bar{e}, 0, s) + \Delta_v p$. Using this, equation (4) can be rewritten as $F(\bar{v}) = F(0) - (1 - \pi)(\Delta_v p\, R - \bar{v})$. □

Proof of Proposition 1. Condition $V(\bar{e}, \bar{v}, s) \geq 0$ ensures there is a non-negative NPV to be split. Condition $V(\bar{e}, \bar{v}, s) \geq d^e + d^v$ follow from $S^i \geq d^i, i \in \{e, v\}$, i.e., a player's equilibrium share must be greater than or equal to her/his disagreement value (outside option).

The players' shares are computed from the following optimization problem (Nash 1950):

$$\max_{S^v, S^e} (S^v - d^v)^{1-\theta} (S^e - d^e)^{\theta} \tag{16a}$$

$$\text{subject to } S^e + S^v = V(\bar{e}, \bar{v}, s). \tag{16b}$$

The solution of the Nash bargaining problem in (16) is $S^e = (V(\bar{e}, \bar{v}, s) - d^v - d^e)\, \theta + d^e$, for the entrepreneur and $S^v = (V(\bar{e}, \bar{v}, s) - d^v - d^e)(1 - \theta) + d^v$, for the VC. The bargaining outcome is then obtained by matching the expected player's cash flows to the equilibrium shares. Letting $\hat{\rho}(L, s)$ be the equilibrium transfer payment, we derive

$$p(\bar{e}, \bar{v}, s)(R - \hat{\rho}(L, s)) - \bar{e} - (I - L) = S^e \tag{17a}$$

$$p(\bar{e}, \bar{v}, s)\hat{\rho}(L, s) - \bar{v} - L = S^v. \tag{17b}$$

From (17b), the payment is $\hat{\rho}(L, s) = \frac{S^v + \bar{v} + L}{p(\bar{e}, \bar{v}, s)}$. Replacing this in (7) yields (10). □

Proof of Proposition 2. The different cases follows from Proposition 1, by assigning the correct values for d^e, d^v. Specifically:
Case 1(a): Bank financing is feasible. Thus, $d^e = V(\bar{e}, 0, s), d^v = 0$. Replacing these in (8b) and (10) gives the desired result. Case 1(b): Negotiations with the VC break down. Hence, the entrepreneur follows his outside option of bank financing with $S^e = V(\bar{e}, 0, s)$. Case 2(a): Bank financing is not feasible, implying $d^e = 0, d^v = 0$. Replacing these in (8b) and (10) gives the desired result. Case 2(b): Negotiations with the VC break down. Hence, the players are left with zero value. □

Proof of Corollary 1. Figures are constructed by setting the correct parameters in the general inequalities describing bank financing (14) and VC financing (10) and taking advantage of the properties in Lemma 2.

Figure 2(a) represents the no-crowdfunding economy. Therefore, demand for external capital is $L = I - a$, and the signal $s = 0$. The bank financing feasibility area "B" is defined by the incentive compatibility inequality (14), which yields line ℓ_1, and the project economic viability without the VC's effort inequality $V(\bar{e}, 0, 0) \geq 0$, which yields line $p = (I + \bar{e})/R + \Delta_v p$. The VC's financing feasibility area depends on whether bank financing is feasible or not:

If bank financing is not feasible, then players' disagreement values are set to $d^e = d^v = 0$ in (10), and the resulting VC financing feasibility area, "V", is defined by the incentive compatibility inequalities (10), which yield lines ℓ_2 and ℓ_3 if they are binding, and by the project's economic viability constraint $V(\bar{e}, \bar{v}, 0) \geq 0$, which yields line $p = (I + \bar{e} + \bar{v})/R$.

If bank financing is feasible, then players' disagreement values are set to $d^e = V(\bar{e}, 0, 0), d^v = 0$, which reflects the entrepreneur's outside option to pursue bank financing. The resulting VC financing feasibility area, "VB", is defined by the incentive compatibility inequalities (10), which yield lines ℓ_4 and ℓ_5 if they are binding.

Figure 2(b) represents the economy after a successful crowdfunding campaign at $t = 2$ with $s = 1$ and $L = I - a - (A - k)$. We first derive regions similar to those in Figure 2(a) using variables (L', p'), with $L' = I - a - (A - k)$ and $p' = p(\bar{e}, \bar{v}, 1)$. Then we express definitions of those regions in variables (L, p) using the following transformations: $L' = L - \Delta L$, where $\Delta L \stackrel{\text{def}}{=} (A - k)$, and $p' = p + \Delta_s p$, where $\Delta_s p = p(e, v, 1) - p(e, v, 0)$. The formal expressions of all lines are given in Table 6 (online appendix) and their properties are given in Lemma 2 (online appendix). □

Proof of Proposition 3. Regions in Table 1 follow from overlaying Figures 2(a) and 2(b). In each region, using region definitions, and the expressions for the entrepreneur's and the VC's values from Proposition 2, we derive the change in the project's valuations due to a successful campaign, as given in Table 5. Analysing these expressions, we determine whether the change has been positive or negative. The proofs for negative effects are given in the subsequent propositions. □

Proof of Proposition 4. Regions (vii) and (viii): From Table 5, the VC is worse off if $(1-\theta)V(\bar{e}, \bar{v}, 0) > 0$, which holds because $V(\bar{e}, \bar{v}, 0) > 0$ and $\theta \in (0, 1)$.

Region (x): From Table 5, the VC is worse off if $V(\bar{e}, \bar{v}, 1) - V(\bar{e}, 0, 1) - V(\bar{e}, \bar{v}, 0) \leq 0$. Using $p(\bar{e}, 0, 1) = p(\bar{e}, \bar{v}, 1) - \Delta_v p$ and $p(\bar{e}, 0, 0) = p(\bar{e}, \bar{v}, 0) - \Delta_v p$, we show that this is equivalent to $V(\bar{e}, 0, 0) \geq 0$, which holds by assumption.

Region (xi): From Table 5, the VC is worse off if $V(\bar{e}, \bar{v}, 0) - V(\bar{e}, 0, 0) \geq 0$, which holds because the effort is efficient (see equation (3)). □

Table 7: Disagreement values at $t = 0$ in the presence of a crowdfunding option.

Disagreement values ($t = 0$)	Entrepreneur d_0^e	Venture Capitalist d_0^v
Conditions at $t = 2$:		
$\emptyset$	0	0
B	$\pi\big[V(\bar{e},0,1)+A\big]-k$	0
V	$\pi\big[\theta V(\bar{e},\bar{v},1)+A\big]-k$	$\pi(1-\theta)V(\bar{e},\bar{v},1)$
VB	$\pi\big[\theta V(\bar{e},\bar{v},1)+(1-\theta)V(\bar{e},0,1)+A\big]-k$	$\pi(1-\theta)\big[V(\bar{e},\bar{v},1)-V(\bar{e},0,1)\big]$

Proof of Proposition 5. Region (vii): from Table 5, the entrepreneur is worse off if $\theta V(\bar{e},\bar{v},0) > 0$, which holds by assumption.

Region (viii): from Table 5, the entrepreneur is worse off if $\theta V(\bar{e},\bar{v},0) > V(\bar{e},0,1)$. Using $V(\bar{e},0,1) = V(\bar{e},0,0) + \Delta_s p R$ and $p(\bar{e},0,0) = p(\bar{e},\bar{v},0) - \Delta_v p$ this condition is equivalent to $\Delta_v p R - \bar{v} \geq (1-\theta)(p(\bar{e},\bar{v},0)R - I - \bar{e} - \bar{v}) + \Delta_s p R = (1-\theta)V(\bar{e},\bar{v},0) + \Delta_s p R$.

Region (xi): from Table 5, the entrepreneur is worse off if $\theta V(\bar{e},\bar{v},0)+(1-\theta)V(\bar{e},0,0) \geq V(\bar{e},0,1)$. Similar to the derivation in the previous paragraph, we find that this condition is equivalent to $\Delta_v p R - \bar{v} \geq (1-\theta)[V(\bar{e},\bar{v},0) - V(\bar{e},0,0)] + \Delta_s p R$. □

Proof of Proposition 6. We start by deriving the cases and equilibrium financing in Table 2. In an economy with a crowdfunding platform, there exist four scenarios after a successful campaign at $t = 2$: No financing ($\emptyset$), Bank Financing only (B), VC financing only (V), and VC financing under bank competition (VB).

In order to characterize the $t = 0$ negotiation outcome, we first compute the values that crowdfunding brings to each player at $t = 0$, using the $t = 2$ subgame equilibrium values derived in Corollary 1. These values are presented in Table 7. Values in Table 7 serve as disagreement values (d_0^e and d_0^v) in $t = 0$ negotiations, because they dominate other outside options for the entrepreneur. Specifically, at $t = 0$, in addition to running a campaign, the entrepreneur may also have an outside option to finance the project with a bank at $t = 0$, which generates $V(\bar{e},0,0)$ for the entrepreneur. By Assumption 3, if bank financing is available at $t = 0$, it is also available at $t = 2$ after a successful campaign. But $\pi[V(\bar{e},0,1)+A]-k > V(\bar{e},0,0)$ because this condition is equivalent to $F(0) \geq 0$, which is true by Lemma 1 and Assumption 2. Similarly, $\pi\big[\theta V(\bar{e},\bar{v},1)+(1-\theta)V(\bar{e},0,1)+A\big]-k \geq \pi[V(\bar{e},0,1)+A]-k \geq V(\bar{e},0,0)$. Thus, we proved that values in Table 7 are disagreement values.

We can now solve the $t = 0$ negotiations. From Proposition 1, these negotiations will succeed and lead to the immediate project investment if

$$V(\bar{e},\bar{v},0) \geq d_0^e + d_0^v, \quad \text{and} \quad \bar{v}/\Delta_v p \leq \tfrac{S_0^v+\bar{v}+(I-a)}{p(\bar{e},\bar{v},0)} \leq R - \bar{e}/\Delta_e p, \tag{18}$$

where S_0^v is in (8b), and the disagreement values d_0^e, d_0^v are in Table 7. Under these conditions, the entrepreneur and VC obtain value S_0^e and S_0^v, given by (8a) and (8b), respectively. Otherwise, the entrepreneur crowdfunds.

Consider cases VB and V in Table 7. Condition $V(\bar{e},\bar{v},0) \geq d_0^e + d_0^v$ is equivalent to $F(\bar{v}) < 0$, which is false by Assumption 2. Therefore, in cases VB and V of Table 7, $t = 0$ negotiations always fail and crowdfunding is used. This produces cases (1) and (2) in Table 2.

Consider case B in Table 7. Condition $V(\bar{e},\bar{v},0) \geq d_0^e + d_0^v$ is equivalent to $F(0) \leq R\Delta_v p - \bar{v}$. When this condition holds, we obtain case (3) in Table 2. Otherwise, we obtain case (4).

Consider case $\emptyset$ in Table 7. Condition $V(\bar{e},\bar{v},0) \geq d_0^e + d_0^v \Leftrightarrow V(\bar{e},\bar{v},0) \geq 0$, which holds. If the second set of conditions in (18) holds, we obtain case (5) in Table 2. Otherwise, case (6).

The last two columns of Table 2 contain the equilibrium values of the entrepreneur and the VC. These values are given by (8a) and (8b), after substituting the disagreement values. □

Proof of Proposition 7. Regions in Table 3 are obtained by comparing the equilibrium of the full game, presented in Proposition 6, with that in no-crowdfunding economy, presented in Corollary 1, part (a). Because four out of six regions in Proposition 6 (specifically regions (1), (2), (4), and (6)) coincide with regions in Corollary 1 part (b), describing the economy after a successful campaign, nine regions in Table 3 coincide with regions in Table 1. We use identical labels in both tables for these regions: (i)-(vi), (ix), (x), and (xii). In these regions, the effect of adding a platform to the economy makes the entrepreneur better off, and (except for region (x)) makes the VC better off.

Next, we study regions (3) and (5) of Proposition 6. Overlaying them with regions Corollary 1, part (a), we generate regions (vii), (viii), (xi-a), and (xi-b) in Table 3.

Region (vii) in Table 3 has the same definition as region (vii) in Table 1. But, because both entrepreneur and the VC are worse off in this region after a successful campaign, in the full game, the entrepreneur would not exercise her crowdfunding option. Therefore, both players behave as if a crowdfunding platform did not exist and their values are not affected by the addition of the platform to the economy.

Region (viii) definition is the same in Tables 3 and 1 as well. But in the full game solution we need to account for the cash benefits of crowdfunding that subgame solution ignores. Specifically, the condition for the entrepreneur to be worse off when a platform is added to the economy is $\theta V(\bar{e},\bar{v},0) \geq d^e_0$. The LHS is the entrepreneur's value in the economy without crowdfunding. The RHS is the entrepreneur's value in the economy with crowdfunding. Using $d^e_0 = \pi(V(\bar{e},0,1)+A)-k$ and if $\pi \geq \theta$, this condition is equivalent to $p \leq \underline{p}$, with $\underline{p} \stackrel{\text{def}}{=} \frac{\pi[A+(\Delta_v p-\Delta_s p)R+\bar{e}+I]+k-\theta(\bar{e}+\bar{v}+I)}{R[\pi-\theta]}$. If $\pi < \theta$, the entrepreneur is worse off if and only if $p \geq \underline{p}_2$ with $\underline{p}_2 \stackrel{\text{def}}{=} \frac{(\pi A-k)+\theta(\bar{e}+\bar{v}+I)-\pi[(\Delta_v p-\Delta_s p)R+\bar{e}+I]}{R[\theta-\pi]}$.

In region (xi-a) in Table 3 (which is a subset of region (xi) in Table 1), $t = 0$ negotiations succeed and there is no campaign. Thus, we do not proceed to the subgame analysis that yielded in Table 1. But $t = 0$ negotiations are affected by the threat of $t = 2$ competition from the banks. This forces the VC to concede some value to the entrepreneur. Thus, the entrepreneur is better off and the VC is worse off if a platform is added to the economy.

Finally, in region (xi-b) in Table 3, a crowdfunding campaign is run, but we need to adjust its effect on the entrepreneur to account for cash that a successful campaign generates. Similar to the analysis above, the entrepreneur is worse off if $\theta V(\bar{e},\bar{v},0)+(1-\theta)V(\bar{e},0,0) \geq d^e_0 = \pi(V(\bar{e},0,1)+A)-k$. This is equivalent to $p \geq \bar{p}$, with $\bar{p} \stackrel{\text{def}}{=} \frac{\pi[A+(\Delta_s p-\Delta_v p)R-\bar{e}-I]-k+(1-\theta)\Delta_v p\, R+\theta\bar{v}+\bar{e}+I}{R[1-\pi]}$. □

Proof or Corollary 2. See the proof of Proposition 7. □

Proof of Lemma 2. Algebraic transformation of line definitions. Details are available on request. □

Proof of Proposition 8. Only economically viable projects ($V(\bar{e},0,s) \geq 0$) will be financed. Of those, only those for which there is a solution to (14) can be financed with bank. □

Proof of Proposition 9. Follows from Proposition 1, when $d^e = d^v = 0$ in (10). □