Online Appendix for "Cooperative Approaches to Managing Social Responsibility in a Market with Externalities"

A1 Proofs of Analytical Results

Proof of Lemma 1: We show in the following that $\theta \in (0,1)$ and $e_i > 0$ in the equilibrium. To see that $\theta = 0$ cannot be an equilibrium, one can observe from (3) that $e_i^{(0)}(\theta) = 0$ when $\theta = 0$, whereas $\theta^{(0)}(0) = 1$ from (1). Similarly, $\theta = 1$ cannot be an equilibrium because $e_i^{(0)}(\theta) = 1 - \left\{\frac{\beta\lambda+(n-1)\gamma\lambda-nl}{cxn}\right\}^{-\frac{1}{x+1}} \equiv \bar{e}$ from (6) when $\theta = 1$, whereas $\theta^{(0)}(\bar{e}) = 0$ due to (A2). Therefore, $\theta = 0$ or 1 can never be an equilibrium. When $\theta \in (0,1)$, $e_i = \frac{g}{nr} > 0$. By substituting (6) into $r\sum_{i\in N} e_i^{(0)} = g$ and solving for θ, we obtain the equilibrium supplier's probability of violation $\theta^{(0)}$ in the lemma. □

Proof of Proposition 1: First, we verify $v_{B^N}(N) \geq \sum_{k=1}^{m} v_B(B_k)$ for any $B \in \Pi$. From (9), $v_{B^N}(N) = \sum_{i\in N}\alpha_i - \theta n\beta\lambda\left(1 - e_N^{(1)}\right) - \theta nle_N^{(1)} - c(1-e_N^{(1)})^{-x} \geq \sum_{i\in N}\alpha_i - \theta n\beta\lambda\left(1-e_{B_{\max}}^{(1)}\right) - \theta nle_{B_{\max}}^{(1)} - c(1-e_{B_{\max}}^{(1)})^{-x} \geq \sum_{i\in N}\alpha_i - \sum_{k=1}^{m}\left\{\theta n_k\beta\lambda\left(1-\frac{\sum_{h=1}^m n_h e_{B_h}^{(1)}}{n}\right) + \theta n_k l e_{B_k}^{(1)} + c(1-e_{B_k}^{(1)})^{-x}\right\} = \sum_{k=1}^{m} v_B(B_k)$, where $e_N^{(1)}$ is the optimal audit effort of the grand coalition N under B^N and $e_{B_{\max}}^{(1)} = \max_h e_{B_h}^{(1)}$. The first inequality is due to the optimality of $e_N^{(1)}$ given B^N and the second inequality follows from the definition of $e_{B_{\max}}^{(1)}$ and $\beta\lambda > l$.

Next, we prove that if (i) $c \geq t_{cost}$ or (ii) $\beta/\gamma \geq 1$, then manufacturers in $S \subset N$ have no incentives to secede from the grand coalition N, by comparing the allocation to S under B with that under B^N. Suppose coalition structure B satisfies $B = \arg\min_{B'\ni S} v_{B'}(S)$. We consider allocation φ with $\sum_{i\in S}\varphi_i = v_B(S)$ such that it is the largest allocation that satisfies $\sum_{i\in S}\varphi_i \leq v_{B'}(S)$ for all $B' \ni S$. From (9), the allocation to S under B^N satisfies $\sum_{i\in S}\varphi_i^{Eg} = \sum_{i\in S}\alpha_i - \theta n_s\beta\lambda\left(1-e_N^{(1)}\right) - \theta n_s le_N^{(1)} - \frac{n_s}{n}c(1-e_N^{(1)})^{-x} \geq \sum_{i\in S}\alpha_i - \theta n_s\beta\lambda\left(1-e_S^{(1)}\right) - \theta n_s le_S^{(1)} - \frac{n_s}{n}c(1-e_S^{(1)})^{-x}$, where n_s is the number of manufacturers in S, $e_S^{(1)}$ is the optimal audit effort of coalition S under B, and the inequality is due to the optimality of $e_N^{(1)}$ given B^N. Similarly, the allocation to S under B satisfies $\sum_{i\in S}\varphi_i = \sum_{i\in S}\alpha_i - \theta n_s\left\{\beta\lambda\left(1-\frac{\sum_{k=1}^m n_k e_{B_k}^{(1)}}{n}\right) + \gamma\lambda\left(\frac{\sum_{k=1}^m n_k e_{B_k}^{(1)}}{n} - e_S^{(1)}\right) + le_S^{(1)}\right\} - c(1-e_S^{(1)})^{-x}$. Then, by solving $\sum_{i\in S}\alpha_i - \theta n_s\beta\lambda\left(1-e_S^{(1)}\right) - \theta n_s le_S^{(1)} - \frac{n_s}{n}c(1-e_S^{(1)})^{-x} \geq \sum_{i\in S}\varphi_i$, we obtain $c \geq \theta n_s\lambda(1-\frac{n_s}{n})^{-1}(1-e_S^{(1)})^x(\beta-\gamma)\left(\frac{\sum_{k=1}^m n_k e_{B_k}^{(1)}}{n} - e_S^{(1)}\right) \equiv t_{cost}$. Therefore, if $c \geq t_{cost}$, then $\sum_{i\in S}\varphi_i^{Eg} \geq \sum_{i\in S}\varphi_i$ so that S has no incentives to secede from the grand coalition.

Similarly, for condition (ii), we consider coalition structure B which minimizes $v_B(S)$ and allocation φ with $\sum_{i\in S}\varphi_i = v_B(S)$. Note from (8) that when $\frac{\beta}{\gamma} \geq 1$, $v_B(S)$ is increasing in $e_{B_k}^{(1)}$ and $e_{B_k}^{(1)}$ is increasing in n_k. Thus, except S, every coalition B_k in B includes only one manufacturer (i.e., $n_k = 1$). Then, we obtain $\sum_{i\in S}\varphi_i^{Eg} - \sum_{i\in S}\varphi_i \geq \theta n_s\lambda(\beta-\gamma)\left(e_S^{(1)} - \frac{\sum_{k=1}^m n_k e_{B_k}^{(1)}}{n}\right) \geq 0$. □

Proof of Proposition 2: By substituting $e_{B_k}^{(1)}(\theta)$ in (8) into $r\sum_{k=1}^m n_k e_{B_k}^{(1)}(\theta) = g$, we obtain the following equation that $\theta^{(1)}$ satisfies:

$$r\sum_{k=1}^{m} n_k \left[1-\left[\frac{\theta^{(1)} n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right]=g. \tag{14}$$

Since the left-hand side of (14) is increasing in $\theta^{(1)}$, when $r\sum_{k=1}^{m} n_k \left[1-\left[\frac{\theta^{(0)} n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right]$ $\geq g$, $\theta^{(1)} \leq \theta^{(0)}$. By substituting $\theta^{(1)}$ in the left-hand side of (14) with $\theta^{(0)}=\frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)}$, we obtain $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]$, where $X_k=\frac{n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{\beta\lambda+(n-1)\gamma\lambda-nl}$. When $X_k\geq 1$ for $k=1,2,...,m$, $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})\right]=r\sum_{k=1}^{m}\frac{n_k g}{nr}=g$. By solving $X_k\geq 1$ for $\frac{\beta}{\gamma}$, we obtain that $\frac{\beta}{\gamma}\geq 1+\frac{n}{n_k+1}(\frac{l}{\gamma\lambda}-1)$.

When $m=1$, define $\xi^{(1)}=1+\frac{n}{n+1}(\frac{l}{\gamma\lambda}-1)$. One can see that $\xi^{(1)}\in(0,1]$ and $\xi^{(1)}$ is decreasing in n because $\gamma\lambda>l$. If $\frac{\beta}{\gamma}\geq\xi^{(1)}$, then $X_k\geq 1$ and $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq r\sum_{k=1}^{m}\frac{n_k g}{nr}=g$. Similarly, if $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq g$, then $X_k\geq 1$ so $\frac{\beta}{\gamma}\geq\xi^{(1)}$. Therefore, $\theta^{(1)}\leq\theta^{(0)}$ if and only if $\frac{\beta}{\gamma}\geq\xi^{(1)}$.

When $m>1$, define $\overline{\xi}^{(1)}=\max_k\{1+\frac{n}{n_k+1}(\frac{l}{\gamma\lambda}-1)\}$ and $\underline{\xi}^{(1)}=\min_k\{1+\frac{n}{n_k+1}(\frac{l}{\gamma\lambda}-1)\}$. One can see that $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq g$ if $\frac{\beta}{\gamma}\geq\overline{\xi}^{(1)}$, and $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\leq g$ if $\frac{\beta}{\gamma}\leq\underline{\xi}^{(1)}$. Since we assume $1-\left\{\frac{\beta\lambda+(n-1)\gamma\lambda-nl}{cxn}\right\}^{-\frac{1}{x+1}}>\frac{g}{nr}$, $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]$ is increasing in X_k. Further, X_k is increasing in $\frac{\beta}{\gamma}$. Thus, there exists $\xi'^{(1)}\in[\underline{\xi}^{(1)},\overline{\xi}^{(1)}]$ such that $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq g$ and $\theta^{(1)}\leq\theta^{(0)}$ if and only if $\frac{\beta}{\gamma}\geq\xi'^{(1)}$. Since $\overline{\xi}^{(1)}\leq\xi^{(1)}$ and $\frac{\beta}{\gamma}>0$, we have $\xi'^{(1)}\in[0,\xi^{(1)}]$. □

Proof of Corollary 1: The proof is similar to that of Proposition 1 and is omitted.

Proof of Proposition 3: By substituting $e^{(2)}_{i,B_k}(\theta)$ in (11) into $r\sum_{k=1}^{m} n_k e^{(2)}_{i,B_k}(\theta)=g$, we obtain the following equation that $\theta^{(2)}$ satisfies:

$$r\sum_{k=1}^{m} n_k\left[1-\left[\frac{\theta^{(2)}\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{n_k}{x+n_k}}\right]=g. \tag{15}$$

Since the left-hand side of (15) is increasing in $\theta^{(2)}$, when $r\sum_{k=1}^{m} n_k\left[1-\left[\frac{\theta^{(0)}\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{n_k}{x+n_k}}\right]$ $\geq g$, $\theta^{(2)}\leq\theta^{(0)}$. By substituting $\theta^{(2)}$ in the left-hand side of (15) with $\theta^{(0)}=\frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)}$, we obtain $r\sum_{k=1}^{m} n_k Y_k$, where $Y_k=1-\left[\frac{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl}{\beta\lambda+(n-1)\gamma\lambda-nl}\right]^{-\frac{n_k}{x+n_k}}(1-\frac{g}{nr})^{\frac{n_k(x+1)}{x+n_k}}$. When $Y_k\geq\frac{g}{nr}$ for $k=1,2,...,m$, $r\sum_{k=1}^{m} n_k Y_k\geq r\sum_{k=1}^{m}\frac{n_k g}{nr}=g$. By solving $Y_k\geq\frac{g}{nr}$ for $\frac{\beta}{\gamma}$, we obtain that $\frac{\beta}{\gamma}\geq 1+n\left[1-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]\left[n_k-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]^{-1}(\frac{l}{\gamma\lambda}-1)\equiv A_k$. Define $\overline{\xi}^{(2)}=\max_k A_k$ and $\underline{\xi}^{(2)}=\min_k A_k$. One can see that $r\sum_{k=1}^{m} n_k Y_k\geq g$ if $\frac{\beta}{\gamma}\geq\overline{\xi}^{(2)}$, and $r\sum_{k=1}^{m} n_k Y_k\leq g$ if

$\frac{\beta}{\gamma} \le \underline{\xi}^{(2)}$. Since $r\sum_{k=1}^{m} n_k Y_k$ is increasing in Y_k and Y_k is increasing in $\frac{\beta}{\gamma}$ due to our assumption $1-\left\{\frac{\beta\lambda+(n-1)\gamma\lambda-nl}{cxn}\right\}^{-\frac{1}{x+1}} > \frac{g}{nr}$, there exists $\xi^{(2)} \in [\underline{\xi}^{(2)}, \overline{\xi}^{(2)}]$ such that $r\sum_{k=1}^{m} n_k Y_k \ge g$ and $\theta^{(2)} \le \theta^{(0)}$ if and only if $\frac{\beta}{\gamma} \ge \xi^{(2)}$. Since $\gamma\lambda > l$, we have $\xi^{(2)} \le \overline{\xi}^{(2)} \le 1$, and further $\frac{\beta}{\gamma} > 0$ so $\xi^{(2)} \in [0,1]$.

When $m=1$, $\xi^{(2)} = 1 + n\left[1-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\right]\left[n-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\right]^{-1}(\frac{l}{\gamma\lambda}-1)$. We obtain $\frac{\partial\xi^{(2)}}{\partial n} = (\frac{l}{\gamma\lambda}-1)(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}[n(g-nr)\{1-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\}-g(n-1)\{n-1-(1-\frac{nr}{g})\ln(1-\frac{g}{nr})\}]/[n\{n-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\}^2(nr-g)]$. Since $g<nr$ and $\gamma\lambda > l$, $\frac{\partial\xi^{(2)}}{\partial n} > 0$ if $n-1-(1-\frac{nr}{g})\ln(1-\frac{g}{nr}) > 0$. It is easy to see that $(1-\frac{nr}{g})\ln(1-\frac{g}{nr})$ is decreasing in $\frac{g}{nr}$ and $\lim_{\frac{g}{nr}\to 0}(1-\frac{nr}{g})\ln(1-\frac{g}{nr}) = 1$. Thus, $n-1-(1-\frac{nr}{g})\ln(1-\frac{g}{nr}) > n-2 \ge 0$, so $\xi^{(2)}$ is increasing in n.

When g/r is sufficiently small, $\left[1-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]\left[n_k-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]^{-1}$ is sufficiently small so $A_k > 0$ for $k=1,2,...,m$. Thus, $\underline{\xi}^{(2)} > 0$ and $\xi^{(2)} \ge \underline{\xi}^{(2)} > 0$. □

Proof of Corollary 2: Under the grand coalition, we obtain $\theta^{(1)} = \frac{cx}{\beta\lambda n - nl}\left(1-\frac{g}{nr}\right)^{-(x+1)}$ by solving $e_N^{(1)}(\theta) = \frac{g}{nr}$ similar to the base model. We obtain $\theta^{(2)} = \frac{cx}{\beta\lambda - l}\left(1-\frac{g}{nr}\right)^{-(x/n+1)}$ by solving $1-(1-e_{i,N}^{(2)}(\theta))^n = \frac{g}{nr}$. We simplify the inequality $\frac{cx}{\beta\lambda n-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)} \ge \frac{cx}{\beta\lambda-l}\left(1-\frac{g}{nr}\right)^{-(x/n+1)}$ and obtain $g/r \ge n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$.

By substituting $e_N^{(1)} = \frac{g}{nr}$ and $e_{i,N}^{(2)} = 1-(1-\frac{g}{nr})^{1/n}$ into $\Delta^{(1)}$ and $\Delta^{(2)}$, respectively, we obtain $\Delta^{(1)} - \Delta^{(2)} = -c\left(1-\frac{g}{nr}\right)^{-x} + nc\left(1-\frac{g}{nr}\right)^{-x/n} - \delta(\theta^{(1)}-\theta^{(2)})$. Since $-c\left(1-\frac{g}{nr}\right)^{-x} + nc\left(1-\frac{g}{nr}\right)^{-x/n} \le 0$ if and only if $g/r \ge n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$ and $\theta^{(1)}-\theta^{(2)} \ge 0$ under the same condition, $\Delta^{(1)} - \Delta^{(2)} \le 0$ if and only if $g/r \ge n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$. □

Proof of Proposition 4: From (12), we can see that $v'_{B^N}(N) \ge \sum\limits_{k=1}^{m} v'_B(B_k)$ for any $B \in \Pi$. In what follows, we prove in (i) and (ii) that manufacturers in $S \subset N$ have no incentives to secede from the grand coalition N if $\frac{\beta}{\gamma} \ge 1$ and φ^{Eg} is used, or if $\frac{n-2}{2n-2} \le \frac{\beta}{\gamma} < 1$ and φ^{Un} is used. Lastly, we prove in (iii) that manufacturers always have incentives to secede if $\frac{n-2}{2n-2} \le \frac{\beta}{\gamma} < 1$ and more than one manufacturers found the supplier's violation, or if $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$.

(i) From (12), $\sum_{i\in S}\varphi_i^{Eg} = \sum_{i\in S}\alpha_i$. We first prove that any coalition S, whose members fail to detect social responsibility risk (i.e., $s_i = 0$ for all $i \in S$), has no incentive to secede from the grand coalition N. For such a coalition S, since $\frac{\beta}{\gamma} \ge 1$, $\sum_{i\in S}\varphi_i = \sum_{i\in S}\alpha_i - n_s\left\{\beta\lambda\left(1-\frac{\sum_{k=1}^{m} n_k I(B_k)}{n}\right)\right.$ $\left.+\gamma\lambda\left(\frac{\sum_{k=1}^{m} n_k I(B_k)}{n}\right)\right\} \le \sum_{i\in S}\alpha_i = \sum_{i\in S}\varphi_i^{Eg}$. Next, we show that coalition S with at least one manufacturer who has detected social responsibility risk has no incentive to secede from the grand coalition N as well. For such a coalition S, $\sum_{i\in S}\varphi_i = \sum_{i\in S}\alpha_i - n_s\lambda(\beta-\gamma)\left(1-\frac{\sum_{k=1}^{m} n_k I(B_k)}{n}\right) \le \sum_{i\in S}\alpha_i = \sum_{i\in S}\varphi_i^{Eg}$, where the inequality holds because $\frac{\beta}{\gamma} \ge 1$. On the contrary, when $\frac{\beta}{\gamma} < 1$, $\sum_{i\in S}\varphi_i > \sum_{i\in S}\alpha_i = \sum_{i\in S}\varphi_i^{Eg}$ for coalition S with at least one manufacturer who has detected social responsibility risk, so φ^{Eg} is not in the core.

(ii) We first consider coalition S such that $i \notin S$. Under φ^{Un}, $\sum_{j\in S}\varphi_j^{Un} = \sum_{j\in S}\alpha_j - \frac{n_s}{n}\{\beta\lambda(n-1)+\gamma\lambda\}$. Similar to the proof of Proposition 1, we consider coalition structure B which minimizes $v'_B(S)$ and allocation φ with $\sum_{j\in S}\varphi_j = v'_B(S)$. Since $\frac{\beta}{\gamma} < 1$, the coalition structure B

that minimizes $v'_B(S)$ should have the highest social responsibility level. This can be achieved by letting all manufacturers that are not in S form one coalition (i.e., $m=2$). Thus, $\sum_{j\in S}\varphi_j = \sum_{j\in S}\alpha_j - \frac{n_s}{n}\{\beta\lambda n_s + \gamma\lambda(n-n_s)\} < \sum_{j\in S}\varphi_j^{Un}$ because $\frac{\beta}{\gamma}<1$. Next, for coalition S such that $i\in S$, $\sum_{j\in S}\varphi_j^{Un} = \sum_{j\in S}\alpha_j + \lambda\left(1-\frac{n_s}{n}\right)\{n\beta+\gamma-\beta\}$. Yet, $\sum_{j\in S}\varphi_j = \sum_{j\in S}\alpha_j + \lambda(\gamma-\beta)\left(1-\frac{n_s}{n}\right) = \sum_{j\in S}\varphi_j^{Un} - \lambda\left(1-\frac{n_s}{n}\right)n\beta < \sum_{j\in S}\varphi_j^{Un}$.
(iii) We first prove that when $\frac{\beta}{\gamma}<1$, the core is empty when there exists another manufacturer j $(\neq i)$ such that $s_j=1$. On the one hand, the allocation φ that is not dominated via $\{i,j\}$ satisfies $\varphi_i+\varphi_j \geq \alpha_i+\alpha_j+2\lambda\left(1-\frac{2}{n}\right)(\gamma-\beta) > \alpha_i+\alpha_j$, where the first inequality is because $\varphi_i+\varphi_j \geq v_B(\{i,j\})$ for all B for which $\{i,j\}\in B$ from the definition of domination and the second inequality holds because $\frac{\beta}{\gamma}<1$. On the other hand, the allocation φ that is not dominated via $N\backslash i$ satisfies $\sum_{z\in N\backslash i}\varphi_z \geq \sum_{z\in N\backslash i}\alpha_z$ because $\sum_{z\in N\backslash i}\varphi_z \geq v_B(N\backslash i)$ for all B for which $N\backslash i\in B$ from the definition of domination. Then, we obtain $\sum_{z\in N}\varphi_z = \sum_{z\in N\backslash i}\varphi_z + \varphi_i > \sum_{z\in N}\alpha_z$, which means that φ is not feasible. Therefore, the core is empty.

Next, we prove that when $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$, the core is empty even when there exists only one manufacturer i such that $s_i=1$. Similar to the case above, according to the definition of domination, the allocation φ that is not dominated via $N\backslash i$ satisfies $\sum_{z\in N\backslash i}\varphi_z \geq \sum_{z\in N\backslash i}\alpha_z - (n-1)\left\{\beta\lambda\left(1-\frac{1}{n}\right)+\frac{\gamma\lambda}{n}\right\}$. Suppose $j\in N\backslash i$ so that $s_j=0$. The allocation φ that is not dominated via $N\backslash j$ satisfies $\sum_{z\in N\backslash j}\varphi_z \geq \sum_{z\in N\backslash j}\alpha_z + \frac{n-1}{n}\lambda(\gamma-\beta)$ according to the definition of domination. Then, we obtain $\sum_{z\in N}\varphi_z = \alpha_j + \frac{1}{n-1}\left(\sum_{z\in N\backslash i}\varphi_z - \sum_{z\in N\backslash i}\alpha_z\right) + \sum_{z\in N\backslash j}\varphi_z \geq \alpha_j - \beta\lambda\left(1-\frac{1}{n}\right) - \frac{\gamma\lambda}{n} + \sum_{z\in N\backslash j}\alpha_z + \frac{n-1}{n}\lambda(\gamma-\beta) = \sum_{z\in N}\alpha_z - 2\beta\lambda\left(1-\frac{1}{n}\right) + \gamma\lambda\frac{n-2}{n} > \sum_{z\in N}\alpha_z$, where the last inequality holds because $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$. Thus, φ is not feasible, so the core is empty. □

Proof of Corollary 3: We prove that when $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$, the following allocation is in the core:

$$\begin{aligned}\varphi_i^{Sub} &= \alpha_i + \frac{n-1}{n}\{\beta\lambda(n-1)+\gamma\lambda\} + \frac{n-1}{n}\{(n-2)\gamma\lambda - (2n-2)\beta\lambda\} \text{ and}\\ \varphi_{i'}^{Sub} &= \alpha_i - \frac{n-1}{n}\{\beta\lambda + \frac{\gamma\lambda}{n-1}\} \text{ for } i'\in N\backslash i.\end{aligned}$$

First, consider coalition S such that $i\notin S$. For any allocation φ to dominate φ^{Sub} via S, $\sum_{z\in S}\varphi_z \leq \sum_{z\in S}\alpha_z - n_s\left\{\beta\lambda\left(1-\frac{n-n_s}{n}\right)+\gamma\lambda\left(\frac{n-n_s}{n}\right)\right\} \leq \sum_{z\in S}\alpha_z - n_s\left\{\beta\lambda\left(1-\frac{1}{n}\right)+\gamma\lambda\left(\frac{1}{n}\right)\right\} = \sum_{z\in S}\varphi_z^{Sub}$, where the first inequality is due to the definition of domination and the second inequality is due to $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$. For coalition S with $i\in S$, if any allocation φ dominates φ^{Sub} via S, then $\sum_{z\in S}\varphi_z - \sum_{z\in S}\varphi_z^{Sub} \leq \sum_{z\in S}\alpha_z - n_s\left\{\beta\lambda\left(1-\frac{n_s}{n}\right)+\gamma\lambda\left(\frac{n_s}{n}-1\right)\right\} - \sum_{z\in S}\varphi_z^{Sub} = (n-n_s)\lambda\left\{\frac{n_s-1}{n}\gamma - \frac{n_s-1+n}{n}\beta\right\} - \frac{n-1}{n}\lambda\{(n-2)\gamma-(2n-2)\beta\} \leq 0$, where the first inequality is due to the definition of domination and the second inequality is due to $1\leq n_s\leq n-1$ and $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$. Therefore, S has no incentives to secede from the grand coalition and φ^{Sub} is in the core. □

Proof of Corollary 4: (i) Similar to the proof of Proposition 1, we consider coalition structure B which minimizes $v_B^{(3)}(S)$ and allocation φ with $\sum_{i\in S}\varphi_i = v_B^{(3)}(S)$. When $\frac{\beta}{\gamma}\geq 1$, $v_B^{(3)}(S)$ is increasing in $e_{B_k}^{(3)}$ and $e_i^{(3)}$. Further, $e_{B_k}^{(3)}$ and $e_i^{(3)}$ are increasing in n_k. Thus, except S, every coalition B_k in B includes only one manufacturer (i.e., $n_k=1$). Then, we obtain $\sum_{i\in S}\varphi_i^{Eg} - \sum_{i\in S}\varphi_i \geq \theta\widehat{n}_s\lambda(\beta-\gamma)\left(e_S^{(3)} - \frac{\sum_{k=1}^{\widehat{m}} n_k e_{B_k}^{(3)}}{n}\right) + \theta(n_s-\widehat{n}_s)\lambda(\beta-\gamma)\left\{\left(1-\prod_{i\in S\backslash\widehat{S}}(1-e_i^{(3)})\right) - \sum_{k=\widehat{m}+1}^{m}\frac{\left(1-\prod_{j\in B_k}(1-e_j^{(3)})\right)n_k}{n}\right\} \geq 0$, where $\widehat{S} = S\cap\{1,2,...,\widehat{n}\}$ and $\widehat{n}_s$ is the number of manufacturers in $\widehat{S}$.

(ii) It can be easily shown that either $\theta^{(1)} \leq \theta^{(3)} \leq \theta^{(2)}$ or $\theta^{(2)} \leq \theta^{(3)} \leq \theta^{(1)}$. Define $\underline{\xi}^{(3)} = \min\{\xi^{(1)}, \xi^{(2)}\}$ and $\overline{\xi}^{(3)} = \max\{\xi^{(1)}, \xi^{(2)}\}$. According to Propositions 2 and 3, $\theta^{(3)} \leq \theta^{(0)}$ if $\beta/r \leq \underline{\xi}^{(3)}$ and $\theta^{(3)} \geq \theta^{(0)}$ if $\beta/r \geq \overline{\xi}^{(3)}$. Due to the monotonicity and continuity of $\theta^{(3)} - \theta^{(0)}$ with respect to β/r, we obtain that there exists $\xi^{(3)}$ such that $\theta^{(3)} \leq \theta^{(0)}$ if and only if $\beta/\gamma \geq \xi^{(3)}$. □

Proof of Corollary 5: (i) Under the grand coalition, by substituting $e_N^{(1)} = \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}$ and $e_{i,N}^{(2)} = 1-(1-\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L})^{1/n}$ into $v_{B^N}^{(1)}(N)$ and $v_{B^N}^{(2)}(N)$, respectively, we obtain $v_{B^N}^{(2)}(N) - v_{B^N}^{(1)}(N) = -nc\left(1-\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-x/n} + c\left(1-\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-x}$. Then we can obtain $v_{B^N}^{(2)}(N) - v_{B^N}^{(1)}(N) \geq 0$ if and only if $\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L} \geq n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$, which is more likely to hold as $\widetilde{n}$ decreases because $r_H > r_L$.

(ii) Under the grand coalition, we obtain $\theta^{(0)} = \frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-(x+1)}$, $\theta^{(1)} = \frac{cx}{\beta\lambda n-nl}\left(1-\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-(x+1)}$ and $\theta^{(2)} = \frac{cx}{\beta\lambda-l}\left(1-\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-(x/n+1)}$. Similar to the proof of part (i) above, we obtain $\theta^{(2)} \leq \theta^{(0)}$ and $\theta^{(2)} \leq \theta^{(1)}$ are more likely to hold as $\widetilde{n}$ decreases. □

Proof of Corollary 6: Under individual auditing, the expected profit of manufacturer i is given by

$$E\pi_i = (1-\theta)\alpha_i + \theta\left\{\alpha_i - \beta\lambda\left(1-\frac{\sum_{j\in N} e_j}{n}\right) + \gamma\lambda\left(e_i - \frac{\sum_{j\in N} e_j}{n}\right) - e_i l\right\} - (r_i+v) - C(e_i), \quad (16)$$

where (r_i+v) denotes manufacturer i's payment to the supplier given the supplier's production cost v. By substituting $e_i = \frac{g}{nr_i}$ and θ in (7) into (16), we obtain

$$E\pi_i = \alpha_i - \frac{[cxn(\beta\lambda-l)+c]\left(1-\frac{g}{nr_i}\right)^{-x} + cxnl\left(1-\frac{g}{nr_i}\right)^{-x-1}}{\beta\lambda+(n-1)\gamma\lambda-nl} - (r_i+v).$$

The first-order condition implies that $r_i^{(0)}$ satisfies $[cx^2n(\beta\lambda-l)+cx]\left(1-\frac{g}{nr_i^{(0)}}\right)^{-x-1}\frac{g}{n\{r_i^{(0)}\}^2} + cxnl(x+1)\left(1-\frac{g}{nr_i^{(0)}}\right)^{-x-2}\frac{g}{n\{r_i^{(0)}\}^2} - [\beta\lambda+(n-1)\gamma\lambda-nl] = 0$. Similarly, under joint auditing, $r_i^{(1)}$ satisfies $[cx^2n(\beta\lambda-l)+cx]\left(1-\frac{g}{nr_i^{(1)}}\right)^{-x-1}\frac{g}{n\{r_i^{(1)}\}^2} + cxnl(x+1)\left(1-\frac{g}{nr_i^{(1)}}\right)^{-x-2}\frac{g}{n\{r_i^{(1)}\}^2} - n_k[\beta\lambda n_k + (n-n_k)\gamma\lambda - nl] = 0$. Under ex-ante audit sharing, $r_i^{(2)}$ satisfies $[cx^2n(\beta\lambda-l)+cx]\left(1-\frac{g}{nr_i^{(2)}}\right)^{-x/n_k-1}\frac{g}{n\{r_i^{(2)}\}^2} + cxnl(x+n_k)\left(1-\frac{g}{nr_i^{(2)}}\right)^{-x/n_k-2}\frac{g}{n\{r_i^{(2)}\}^2} - n_k[\beta\lambda n_k + (n-n_k)\gamma\lambda - nl] = 0$.

When $\beta/\gamma \geq 1$, $\beta\lambda+(n-1)\gamma\lambda-nl \leq \beta\lambda n_k+(n-n_k)\gamma\lambda-nl$. Since the left-hand sides of the three equations above are all decreasing in r_i, we obtain that $r_i^{(0)} \geq r_i^{(1)}$ and $r_i^{(0)} \geq r_i^{(2)}$. Since $\left(1-\frac{g}{nr_i}\right)^{-x/n_k} \leq \left(1-\frac{g}{nr_i}\right)^{-x}$, we obtain that $r_i^{(2)} \geq r_i^{(1)}$. □

Proof of Corollary 7: When the number of manufacturers in every coalition is the same, the expected profit of manufacturer i with violation correction under ex-ante audit sharing is the same as that under individual auditing in (13). The expected profit of coalition B_k under joint auditing is given by

$$E\pi_{B_k}(e_{B_j}) = (1-\theta)\alpha_i + \theta\left\{\alpha_i - \beta\lambda\prod_{B_j\in B}(1-e_{B_j})\right\} - C(e_{B_j}).$$

By comparing this expression with (13), one can see that by taking one coalition under joint auditing as one manufacturer under individual auditing, we can analyze joint auditing similar to individual auditing (except that we have m instead of n manufacturers). Therefore, in the following, we show the proof for individual auditing only.

By solving the first-order condition of (13), we obtain manufacturer i's optimal audit effort $e^*(\theta) = 1-\left(\frac{\theta\beta\lambda}{cx}\right)^{-\frac{1}{x+n}}$ with violation correction. Then we obtain $\theta^* = \frac{cx}{\beta\lambda}\left(1-\frac{g}{nr}\right)^{-(x/n+1)}$ by solving $1-(1-e^*(\theta))^n = \frac{g}{nr}$. When $\beta \geq \gamma$, we obtain $\theta^* = \frac{cx}{\beta\lambda}\left(1-\frac{g}{nr}\right)^{-(x/n+1)} \leq \frac{cx}{\beta\lambda}\left(1-\frac{g}{nr}\right)^{-(x+1)} \leq \frac{cxn}{\beta\lambda+(n-1)\gamma\lambda}\left(1-\frac{g}{nr}\right)^{-(x+1)} = \theta^{(0)}$; i.e., the violation probability of the supplier is lower when manufacturers help the supplier to correct the violation.

By substituting θ^* into $e^*(\theta)$, we obtain $e^* = 1-\left(1-\frac{g}{nr}\right)^{\frac{1}{n}}$. According to (13), we have the equilibrium profit $E\pi_i(e^*,\theta^*) = (1-\theta^*)\alpha_i + \theta^*\left\{\alpha_i - \beta\lambda(1-\frac{g}{nr})\right\} - C(e^*) \geq (1-\theta^{(0)})\alpha_i + \theta^{(0)}\left\{\alpha_i - \beta\lambda(1-\frac{g}{nr})\right\} - C(e^*) \geq (1-\theta^{(0)})\alpha_i + \theta^{(0)}\left\{\alpha_i - \beta\lambda(1-\frac{g}{nr})\right\} - C(e_i^{(0)})$, where the last expression is the profit of manufacturer i in the base model. Therefore, the expected profits of manufacturers are higher. □

Proof of Corollary 8: Under individual auditing and audit sharing, the expected profit of the supplier is $E\pi_0 = n(r-f) - \theta\left\{(r-f)\sum_{i\in N} e_i - g\right\}$. Then the equilibrium decisions of the manufacturers and supplier are the same as those in the original model except that r is replaced by $r-f$. Therefore, Propositions 3 and 4 continue to hold.

Under joint auditing, the expected profit of the supplier is $E\pi_0 = nr - mf - \theta\{\sum_{k=1}^m (n_k r - f)e_{B_k} - g\}$. Proposition 1 continues to hold as θ is fixed. In the following, we show that Proposition 2 also holds. By substituting $e_{B_k}^{(1)}(\theta)$ in (8) into $\sum_{k=1}^m (n_k r - f)e_{B_k}^{(1)} = g$, we obtain the following equation that $\theta^{(1)}$ satisfies:

$$\sum_{k=1}^m (n_k r - f)\left[1 - \left[\frac{\theta^{(1)} n_k\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right] = g. \tag{17}$$

Since the left-hand side of (17) is increasing in $\theta^{(1)}$, when $\sum_{k=1}^m (n_k r - f)\left[1 - \left[\frac{\theta^{(0)} n_k\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right] \geq g$, $\theta^{(1)} \leq \theta^{(0)}$. By substituting $\theta^{(1)}$ in the left-hand side of (17) with $\theta^{(0)} = \frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{nr-nf}\right)^{-(x+1)}$, we obtain $\sum_{k=1}^m (n_k r - f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right]$, where $X_k = \frac{n_k\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{\beta\lambda+(n-1)\gamma\lambda-nl}$ is increasing in β/γ. When $m=1$, $X_k = \frac{n\{\beta\lambda n - nl\}}{\beta\lambda+(n-1)\gamma\lambda-nl}$ and $\theta^{(1)} \leq \theta^{(0)}$ if and only if $X_k \geq [\{1-g/(nr-f)\}/\{1-g/(nr-nf)\}]^{-(x+1)}$. One can see that in this case, X_k is increasing in n and the right-hand side of the inequality is decreasing in n, so the threshold for β/γ, $\xi^{(1)}$, is decreasing in n. Furthermore, if $\beta/\gamma = 1$, $X_k \geq 1$ so $\sum_{k=1}^m (n_k r - f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right] \geq \sum_{k=1}^m (n_k r - f)\frac{g}{nr-nf} \geq g$, and thus $\xi^{(1)} \leq 1$. The proof for the case when $m > 1$ is similar to that of Proposition 2. □

Proof of Corollary 9: In the proof of Corollary 8, one can see $\sum_{k=1}^{m}(n_k r - f)\frac{g}{nr-nf}$ is increasing in f. Since $\sum_{k=1}^{m}(n_k r - f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right]$ is increasing in X_k, there exists $x(f)$, which is decreasing in f, such that when $X_k \geq x(f)$ for $k = 1, 2, ..., m$, $\sum_{k=1}^{m}(n_k r - f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right] \geq g$ and $\theta^{(1)} \leq \theta^{(0)}$. By solving $X_k \geq x(f)$ for $\frac{\beta}{\gamma}$, we obtain that $\frac{\beta}{\gamma} \geq 1 + \frac{n}{n_k + x(f)}(\frac{l}{\gamma\lambda} - 1)$, which is decreasing in f because $\gamma\lambda > l$ and $x(f)$ is decreasing in f. □

A2 Additional Results

A2.1 Consumer Choice Model

Suppose there are two groups of socially conscious consumers for manufacturer i. The first group of γ' consumers is informed of manufacturer i's social responsibility level z_i and their utilities from purchasing manufacturer i's product are given by $v - z_i$, where $v \sim U[0,1]$ represents the brand loyalty to manufacturer i. A consumer in this group switches to other manufacturers if $v - z_i < 0$. The second group of β' consumers is uninformed of manufacturer i's social responsibility level z_i and their utilities from purchasing manufacturer i's product are given by $u - \widetilde{z}_i$, where $u \sim U[0,1]$ and $\widetilde{z}_i$ is the belief about manufacturer i's social responsibility level. Although these consumers are uninformed of one particular manufacturer's social responsibility level, they can learn the overall social responsibility level of the market from the media. We assume that they take the average level of social responsibility among all manufacturers as their belief for z_i; i.e., $\widetilde{z}_i = \sum_{j\in N} z_j/n$. A consumer in this group does not purchase any product in the market if $u - \widetilde{z}_i < 0$. The consumer does not switch to other manufacturers because given the consumer is uninformed, other manufacturers appear to have the same social responsibility level as manufacturer i to the consumer. With this setup, the number of consumers that switch to manufacturer i from other manufacturers is given by $\gamma' \sum_{j\in N\setminus i} z_j/(n-1)$. The number of consumers that switch from manufacturer i to others or choose not to purchase is given by $\gamma' z_i + \beta' \sum_{j\in N} z_j/n$. Therefore, manufacturer i's demand from socially conscious consumers is given by $\gamma' + \beta' + \gamma' \sum_{j\in N\setminus i} z_j/(n-1) - \gamma' z_i - \beta' \sum_{j\in N} z_j/n$. Let $\alpha_i = \gamma' + \beta'$, $\beta = \beta'$ and $\gamma = \gamma' n/(n-1)$. Then the demand can be rewritten as $\alpha_i - \beta \sum_{j\in N} z_j/n + \gamma\left(\sum_{j\in N} z_j/n - z_i\right)$. This supports the functional form of π_i in (2).

A2.2 Extension to Convex Cost and Supplier's Profit

In the base model, we assume that the supplier may adopt a pure strategy of either producing parts responsibly (i.e., $\theta = 0$) or irresponsibly (i.e., $\theta = 1$), or adopt a mixed strategy of choosing $\theta \in (0, 1)$. In this case, the expected cost saving of the supplier from producing parts irresponsibly is $g\theta$, which is linear in θ. It is plausible in some other cases that the compliance cost of the supplier is convex in his compliance effort. In the following, we first show that our insights from the base model continue to hold under the convex compliance cost and then analyze supplier's profit in this case.

Let $\varepsilon \in [0,1]$ denote the compliance effort of the supplier and we assume that the compliance cost takes a quadratic form as $g\varepsilon^2$. Then the expected profit of the supplier, $E\pi_0$, under individual auditing (cf. (1) in the base model) can be expressed as $E\pi_0(\varepsilon) = r\left\{n - (1-\varepsilon)\sum_{i\in N} e_i\right\} - g\varepsilon^2$, where the first term represents the expected revenue of the supplier from selling parts with compliance effort ε. By solving the first-order condition, we can obtain the optimal compliance effort of the supplier under individual auditing $\varepsilon^{(0)}(e_i) = r/(2g)\sum_{i\in N} e_i$.

For trackability, we assume that the audit costs of manufacturers are also quadratic in their audit efforts; i.e., $C(e_i) = ce_i^2$. Then we can obtain the following optimal audit effort of manufacturer i under individual auditing (cf. (6) in the base model) by solving the first-order condition of (3):

$$e_i^{(0)}(\varepsilon) = \frac{(1-\varepsilon)\{\beta\lambda + (n-1)\gamma\lambda - nl\}}{2cn}.$$

The optimal audit efforts under joint auditing and ex-ante audit sharing can be obtained similarly. Following the analysis similar to that of the base model, we obtain the following corollary:

Corollary 10 *With the quadratic compliance cost and audit cost (i.e., $g\varepsilon^2$ and ce_i^2), Propositions 1, 2, 3, and 4 continue to hold except that joint auditing (resp., ex-ante audit sharing) being more effective than individual auditing means $\varepsilon^{(1)} \geq \varepsilon^{(0)}$ (resp., $\varepsilon^{(2)} \geq \varepsilon^{(0)}$).*

Proof: The quadratic costs do not affect our analysis of the partition functions, and thus Propositions 1 and 4 continue to hold. For Proposition 2, by substituting $e_i^{(0)}(\varepsilon)$ into $\varepsilon = r/(2g)\sum_{i\in N} e_i$ and solving the equation for ε, we obtain the equilibrium compliance effort of the supplier under individual auditing $\varepsilon^{(0)} = r\{\beta\lambda + (n-1)\gamma\lambda - nl\}/[4gc + r\{\beta\lambda + (n-1)\gamma\lambda - nl\}]$. On the other hand, the equilibrium compliance effort under joint auditing $\varepsilon^{(1)}$ satisfies $\varepsilon^{(1)} = r/(2g)\sum_{k=1}^m n_k e_{B_k}^{(1)}$, where $e_{B_k}^{(1)} = n_k(1-\varepsilon^{(1)})\{n_k\beta\lambda + (n-n_k)\gamma\lambda - nl\}/(2cn)$. By replacing $\varepsilon^{(1)}$ in $\varepsilon^{(1)}/(1-\varepsilon^{(1)})$ with $\varepsilon^{(0)}$ and comparing it with $r\sum_{k=1}^m n_k^2\{n_k\beta\lambda + (n-n_k)\gamma\lambda - nl\}/(4cng)$, following a similar proof to that of Proposition 2, one can show that the proposition continues to hold. Proposition 3 can be proved similarly. □

Lastly, the following corollary compares the supplier's profit with and without the cooperation in auditing.

Corollary 11 *The expected profit of the supplier under joint auditing or ex-ante audit sharing is lower than that under individual auditing if β/γ is sufficiently large.*

Proof: By comparing the optimal audit effort under individual auditing $e_i^{(0)}(\varepsilon) = (1-\varepsilon)\{\beta\lambda + (n-1)\gamma\lambda - nl\}/(2cn)$ and that under joint auditing $e_{B_k}^{(1)}(\varepsilon) = n_k(1-\varepsilon^{(1)})\{n_k\beta\lambda + (n-n_k)\gamma\lambda - nl\}/(2cn)$, it is easy to see that $e_i^{(0)}(\varepsilon) < e_{B_k}^{(1)}(\varepsilon)$ if β/γ is sufficiently large. Thus, the expected profit of the supplier under individual auditing satisfies $E\pi_0^{(0)}(\varepsilon^{(0)}) = r\left\{n - (1-\varepsilon^{(0)})\sum_{i\in N} e_i^{(0)}(\varepsilon^{(0)})\right\} - g\varepsilon^{(0)2} \geq r\left\{n - (1-\varepsilon^{(1)})\sum_{i\in N} e_i^{(0)}(\varepsilon^{(1)})\right\} - g\varepsilon^{(1)2} > r\left\{n - (1-\varepsilon^{(1)})\sum_{k=1}^m n_k e_{B_k}^{(1)}(\varepsilon^{(1)})\right\} - g\varepsilon^{(1)2} = E\pi_0^{(1)}(\varepsilon^{(1)})$ if β/γ is sufficiently large, where the first inequality is due to the optimality of $\varepsilon^{(0)}$ under individual auditing. The result under ex-ante audit sharing can be proved similarly. □

When the negative externality is high or the positive externality is low, the manufacturers conduct more comprehensive audits when they cooperate than when they do not. As a result, the supplier has to choose a higher compliance effort under joint auditing or ex-ante audit sharing, which leads to a higher compliance cost and a lower profit.