

Technical Appendix

Proof of Proposition 1

Given that two firms are symmetric, we conduct the comparative statics from firm A's perspective. When firms adopt quality discrimination, second-period equilibrium quality levels are:

$$q_{ao}^* = \frac{12\gamma - 1}{12\gamma(9\gamma - 1)} \quad (\text{A1})$$

$$q_{an}^* = \frac{6\gamma - 1}{12\gamma(9\gamma - 1)} \quad (\text{A2})$$

It follows that $q_{ao}^* - q_{an}^* = \frac{1}{2(9\gamma - 1)} > 0$ for $\gamma > \frac{1}{9}$, the region that we consider. Therefore, firms reward repeat customers on the quality dimension. Second-period equilibrium prices are:

$$p_{ao}^* = \frac{12\gamma - 1}{2(9\gamma - 1)} \quad (\text{A3})$$

$$p_{an}^* = \frac{6\gamma - 1}{2(9\gamma - 1)} \quad (\text{A4})$$

We can obtain that $p_{ao}^* - p_{an}^* = \frac{3\gamma}{9\gamma - 1} > 0$ for $\gamma > \frac{1}{9}$. Therefore, firms reward new customers on the price dimension. The second-period equilibrium profits are

$$\Pi_{a2}^* = \Pi_{b2}^* = \frac{(18\gamma - 1)(90\gamma^2 - 18\gamma + 1)}{72\gamma(9\gamma - 1)^2} \quad (\text{A5})$$

From the benchmark model, we know that firm's second-period profits without quality discrimination is $\frac{1}{4}$.

$$\Pi_{a2}^* - \frac{1}{4} = \frac{162\gamma^3 - 90\gamma^2 + 18\gamma - 1}{72\gamma(9\gamma - 1)^2} \quad (\text{A6})$$

which is greater than zero. Therefore, quality discrimination increases firm's second-period profit.

Proof of Proposition 2

The benchmark model shows that without quality discrimination, firm's first-period prices are $\frac{4}{3}$. When firms adopt quality discrimination, first-period prices are

$$p_{a1}^* = \frac{(12\gamma - 1)(6\gamma - 1)}{6\gamma(9\gamma - 1)} \quad (\text{A7})$$

$$p_{a1}^* - \frac{4}{3} = -\frac{10\gamma - 1}{6\gamma(9\gamma - 1)} \quad (\text{A8})$$

which is less than zero for $\gamma > \frac{1}{9}$. Given that firms' first-period demand is $\frac{1}{2}$ with or without quality discrimination, lower first-period prices with quality discrimination lead to lower first-period profits.

Proof of Proposition 3

Let superscript PP and CC indicate the models with BBP only and with quality discrimination. We compare firms' total profits over two periods, consumer surplus, and social welfare between these two cases.

$$\begin{aligned} \Pi_{at}^{*CC} - \Pi_{at}^{*PP} &= \frac{5508\gamma^3 - 1818\gamma^2 + 198\gamma - 7}{72\gamma(9\gamma - 1)^2} - \frac{11}{12} \\ &= \frac{162\gamma^3 - 630\gamma^2 + 132\gamma - 7}{72\gamma(9\gamma - 1)^2} < 0 \end{aligned} \quad (\text{A9})$$

for $\gamma > \frac{1}{9}$. Therefore, quality discrimination decreases firm's total profits over two periods.

$$\begin{aligned} CS_{at}^{*CC} &= 2 \int_0^{\theta_a^*} (v - \theta - p_{a1}^* + v + q_{a2}^* - \theta - p_{ao}^*) d\theta + 2 \int_{\theta_a^*}^{\theta_1^*} (v - \theta - p_{a1}^* + v + q_{b2}^* - 1 + \theta - p_{bn}^*) d\theta \\ &= 2v - \frac{41}{18} \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} CS_{at}^{*PP} &= 2 \int_0^{\theta_a^*} (v - \theta - p_{a1}^* + v + q_{ao}^* - \theta - p_{ao}^*) d\theta + 2 \int_{\theta_a^*}^{\theta_1^*} (v - \theta - p_{a1}^* + v + q_{bn}^* - 1 + \theta - p_{bn}^*) d\theta \\ &= \frac{1296\gamma^3 v - 1584\gamma^3 - 288\gamma^2 v + 528\gamma^2 + 16\gamma v - 57\gamma + 2}{8\gamma(9\gamma - 1)^2} \end{aligned} \quad (\text{A11})$$

$$CS_{at}^{*CC} - CS_{at}^{*PP} = -\frac{972\gamma^3 - 1800\gamma^2 + 349\gamma - 18}{72\gamma(9\gamma - 1)^2} > 0 \quad (\text{A12})$$

for $\gamma > \frac{1}{9}$. Therefore, quality discrimination increases consumer surplus.

$$SW_{at}^{*CC} = CS_{at}^{*CC} + \Pi_{at}^{*CC} = 2v - \frac{4}{9} \quad (\text{A13})$$

$$\begin{aligned} SW_{at}^{*PP} &= CS_{at}^{*PP} + \Pi_{at}^{*PP} \\ &= \frac{11664\gamma^3 v - 3240\gamma^3 - 2592\gamma^2 v + 1116\gamma^2 + 144\gamma v - 117\gamma + 4}{72\gamma(9\gamma - 1)^2} \end{aligned} \quad (\text{A14})$$

$$SW_{at}^{*CC} - SW_{at}^{*PP} = -\frac{648\gamma^3 - 540\gamma^2 + 85\gamma - 4}{72\gamma(9\gamma - 1)^2} > 0 \quad (\text{A15})$$

if and only if $\gamma < 0.65$. Therefore, quality discrimination increases social welfare if and only if $\gamma < 0.65$.

Proof of Proposition 4

Let superscript PP and CC indicate the models with BBP only and with quality discrimination and superscript CP indicate the asymmetric case in which only firm A uses quality discrimination. We compare firms' second-period profits in these cases because firms decide whether to invest in quality discrimination at the beginning of the second period.

$$\begin{aligned}\Pi_{a2}^{*CP} - \Pi_{a2}^{*PP} &= \frac{14580\gamma^4 - 7776\gamma^3 + 1377\gamma^2 - 90\gamma + 2}{81\gamma(18\gamma - 1)(6\gamma - 1)^2} - \frac{1}{4} \\ &= \frac{5832\gamma^4 - 10692\gamma^3 + 3078\gamma^2 - 279\gamma + 8}{324\gamma(18\gamma - 1)(6\gamma - 1)^2}\end{aligned}\quad (\text{A16})$$

which is positive if $\gamma < 0.19$.

$$\begin{aligned}\Pi_{b2}^{*CC} - \Pi_{b2}^{*CP} &= \frac{(18\gamma - 1)(90\gamma^2 - 18\gamma + 1)}{72\gamma(9\gamma - 1)^2} - \frac{262440\gamma^5 - 113724\gamma^4 + 18954\gamma^3 - 1539\gamma^2 + 63\gamma - 1}{81\gamma(18\gamma - 1)(6\gamma - 1)^2} \\ &= \frac{7558272\gamma^6 - 3884112\gamma^5 + 798984\gamma^4 - 82620\gamma^3 + 4374\gamma^2 - 108\gamma + 1}{648\gamma(18\gamma - 1)^2(6\gamma - 1)^2(9\gamma - 1)^2}\end{aligned}\quad (\text{A17})$$

which is positive if $\gamma < 0.15$. Therefore for $\gamma < 0.15$, $\Pi_{a2}^{*CP} > \Pi_{a2}^{*PP}$ and $\Pi_{b2}^{*CC} > \Pi_{b2}^{*CP}$, which leads both firms to use quality discrimination in equilibrium. If $\gamma \in (0.15, 0.19)$, $\Pi_{a2}^{*CP} > \Pi_{a2}^{*PP}$ but $\Pi_{b2}^{*CC} < \Pi_{b2}^{*CP}$. In this case, the asymmetric equilibrium in which one firm uses quality discrimination and the other firm doesn't arises. If $\gamma > 0.19$, $\Pi_{a2}^{*CP} < \Pi_{a2}^{*PP}$ and $\Pi_{b2}^{*CC} < \Pi_{b2}^{*CP}$, both firms do not use quality discrimination.

Proof of Proposition 5

Let γ_o and γ_n indicate the cost of quality provision for repeat and new customers, respectively and $\delta = \frac{\gamma_o}{\gamma_n} > 0$ represent the cost efficiency of serving repeat customers relative to new customers. Then $\gamma_o = \delta\gamma_n$. We solve the main model as follows.

The second-period marginal consumers are located at θ_a and θ_b which take the same forms as in the main model.

$$v - \theta_a + q_{ao} - p_{ao} = v - (1 - \theta_a) + q_{bn} - p_{bn}, \quad (\text{A18})$$

$$\theta_a = \frac{1}{2} + \frac{q_{ao} - q_{bn}}{2} - \frac{p_{ao} - p_{bn}}{2}. \quad (\text{A19})$$

and

$$v - \theta_b + q_{an} - p_{an} = v - (1 - \theta_b) + q_{bo} - p_{bo}, \quad (\text{A20})$$

$$\theta_b = \frac{1}{2} + \frac{q_{an} - q_{bo}}{2} - \frac{p_{an} - p_{bo}}{2}. \quad (\text{A21})$$

Firms' profit functions in the second period are

$$\Pi_{a2} = p_{ao}\theta_a + p_{an}(\theta_b - \theta_1) - \gamma_o q_{ao}^2 - \gamma_n q_{an}^2, \quad (\text{A22})$$

$$\Pi_{b2} = p_{bo}(1 - \theta_b) + p_{bn}(\theta_1 - \theta_a) - \gamma_o q_{bo}^2 - \gamma_n q_{bn}^2. \quad (\text{A23})$$

We solve for second-period prices and obtain $p_{ao}^* = \frac{q_{ao} - q_{bn}}{3} + \frac{2\theta_1}{3} + \frac{1}{3}$, $p_{an}^* = \frac{q_{an} - q_{bo}}{3} - \frac{4\theta_1}{3} + 1$, $p_{bo}^* = \frac{q_{bo} - q_{an}}{3} - \frac{2\theta_1}{3} + 1$, and $p_{bn}^* = \frac{q_{bn} - q_{ao}}{3} + \frac{4\theta_1}{3} - \frac{1}{3}$. Then, we solve for the second-period quality and obtain $q_{an}^* = \frac{\theta_1 - 1 + 9\delta\gamma_n - 12\delta\gamma_n\theta_1}{3\gamma_n(18\delta\gamma_n - \delta - 1)}$, $q_{ao}^* = \frac{6\gamma_n\theta_1 + 3\gamma_n - \theta_1}{3\gamma_n(18\delta\gamma_n - \delta - 1)}$, $q_{bn}^* = \frac{12\delta\gamma_n\theta_1 - 3\delta\gamma_n - \theta_1}{3\gamma_n(18\delta\gamma_n - \delta - 1)}$, and $q_{bo}^* = \frac{\theta_1 - 1 + 9\gamma_n - 6\gamma_n\theta_1}{3\gamma_n(18\delta\gamma_n - \delta - 1)}$. The first-period consumer is located at θ_1 . We must have that

$$v - \theta_1 - p_{a1} + [v - (1 - \theta_1) + q_{bn}^{e*} - p_{bn}^{e*}] = v - (1 - \theta_1) - p_{b1} + [v - \theta_1 + q_{an}^{e*} - p_{an}^{e*}]. \quad (\text{A24})$$

We obtain

$$\theta_1 = \frac{1 - 6\gamma_n + 3\gamma_n(p_{a1} - p_{b1}) - 12\delta\gamma_n + 72\delta\gamma_n^2 + 3\delta\gamma_n(p_{a1} - p_{b1}) - 54\delta\gamma_n^2(p_{a1} - p_{b1})}{2(1 - 6\gamma_n + 72\delta\gamma_n^2 - 12\delta\gamma_n)} \quad (\text{A25})$$

Firms' profit functions to maximize in the first period are:

$$\Pi_{at} = p_{a1}\theta_1 + \Pi_{a2}^*, \quad (\text{A26})$$

$$\Pi_{bt} = p_{b1}(1 - \theta_1) + \Pi_{b2}^*. \quad (\text{A27})$$

We obtain first-period prices:

$$p_{a1}^* = p_{b1}^* = \frac{2(1944\delta^2\gamma_n^3 - 306\delta^2\gamma_n^2 + 9\delta^2\gamma_n - 396\delta\gamma_n^2 + 54\delta\gamma_n - \delta + 18\gamma_n - 2)}{9\gamma_n(18\delta\gamma_n - \delta - 1)^2} \quad (\text{A28})$$

In equilibrium, firms' second-period profits are

$$\Pi_{a2}^* = \Pi_{b2}^* = \frac{3240\delta^2\gamma_n^3 - 468\delta^2\gamma_n^2 + 18\delta^2\gamma_n - 360\delta\gamma_n^2 + 36\delta\gamma_n - \delta + 18\gamma_n - 1}{36\gamma_n(18\delta\gamma_n - \delta - 1)^2} \quad (\text{A29})$$

firms' total profits over two periods are

$$\Pi_{at}^* = \Pi_{bt}^* = \frac{11016\delta^2\gamma_n^3 - 1692\delta^2\gamma_n^2 + 54\delta^2\gamma_n - 1944\delta\gamma_n^2 + 252\delta\gamma_n - 5\delta + 90\gamma_n - 9}{36\gamma_n(18\delta\gamma_n - \delta - 1)^2} \quad (\text{A30})$$

The impact of δ on profits are as follows:

$$\frac{\partial \Pi_{a2}^*}{\partial \delta} = -\frac{(12\gamma_n - 1)(6\delta\gamma_n - \delta + 24\gamma_n - 1)}{36\gamma_n(18\delta\gamma_n - \delta - 1)^3} < 0 \quad (\text{A31})$$

$$\frac{\partial \Pi_{at}^*}{\partial \delta} = \frac{12960\delta\gamma_n^3 - 3096\delta\gamma_n^2 + 234\delta\gamma_n - 1296\gamma_n^2 - 5\delta + 252\gamma_n - 13}{36\gamma_n(18\delta\gamma_n - \delta - 1)^3} > 0 \quad (\text{A32})$$

in the region of parameters that we consider. Since total profits increase with δ but second-period profits decrease with δ , it follows that first-period profits increase with δ .

A1 Firm and Consumer Patience

Let δ_f and δ_c represent the discounting factor of the firms and consumers. We analyze the benchmark model with BBP and main model with quality discrimination. When firms and consumers discount the second-period outcome, the second-period equilibrium stay the same as without discounting. Thus, we focus on analyzing the first-period decisions.

A1.1 Benchmark Model

The first-period marginal consumer is located at θ_1 . With consumer discounting, we have

$$\begin{aligned} v - \theta_1 - p_{a1} + \delta_c [v - (1 - \theta_1) + q_{b2}^{e*} - p_{bn}^{e*}] &= \\ v - (1 - \theta_1) - p_{b1} + \delta_c [v - \theta_1 + q_{a2}^{e*} - p_{an}^{e*}] &= \\ \theta_1 &= \frac{1}{2} - \frac{7(p_{a1} - p_{b1})}{2(7 + 2\delta_c)} \end{aligned} \quad (\text{A33})$$

The total profits that firms maximize in the first period are

$$\Pi_{at} = p_{a1}\theta_1 + \Pi_{a2}^*, \quad (\text{A34})$$

$$\Pi_{bt} = p_{b1}(1 - \theta_1) + \Pi_{b2}^*. \quad (\text{A35})$$

We solve for the first-period equilibrium prices and quality and obtain $p_{a1}^* = p_{b1}^* = 1 + \frac{2\delta_c}{7} + \frac{\delta_f}{21}$.

A1.2 Main Model

Again, the second-period analysis is the same as in the main model because firm and consumer discounting does not affect the second-period decisions. The first-period marginal consumer at θ_1 is solved as follows:

$$\begin{aligned} v - \theta_1 + q_{a1} - p_{a1} + \delta_c [v - (1 - \theta_1) + q_{bn}^{e*} - p_{bn}^{e*}] &= v - (1 - \theta_1) + q_{b1} - p_{b1} + \delta_c [v - \theta_1 + q_{an}^{e*} - p_{an}^{e*}], \\ \theta_1 &= \frac{1}{2} - \frac{3[3\delta_c(q_{a1} - q_{b1}) + 7(p_{a1} - p_{b1} + q_{b1} - q_{a1})]}{21 - \delta_c} \end{aligned} \quad (\text{A36})$$

Firms set first-period quality and prices to maximize the following profit functions:

$$\Pi_{at} = p_{a1}\theta_1 + \Pi_{a2}^* - q_{a1}^2, \quad (\text{A37})$$

$$\Pi_{bt} = p_{b1}(1 - \theta_1) + \Pi_{b2}^* - q_{b1}^2. \quad (\text{A38})$$

First-period equilibrium quality levels are $q_{a1}^* = q_{b1}^* = \frac{9261 + 189\delta_c^2 + 1656\delta_c\delta_f - 7424\delta_f^2 - 4410\delta_c + 4200\delta_f}{42(1323 - 63\delta_c - 928\delta_f)}$.

A1.3 Verification of Main Results

With firm and consumer patience, proposition 1 remains the same as the second-period outcomes do not change. To see this, $q_{ao}^{*CC} - q_{an}^{*CC} = \frac{1}{7}$, $p_{ao}^{*CC} - p_{an}^{*CC} = \frac{3}{7}$, and

$\Pi_{a2}^{*CC} - \Pi_{a2}^{*PP} = \frac{23}{1764}$. Proposition 2 holds as $p_{a1}^{*CC} - p_{a1}^{*PP} = -\frac{\delta_c}{3} - \frac{\delta_f}{21} < 0$ and $\Pi_{at}^{*CC} - \Pi_{at}^{*PP} = \frac{U_1}{7056(63\delta_c+928\delta_f-1323)^2(14\delta_c-38\delta_f+49)^2} < 0$, where $U_1 = 244268136\delta_c^5\delta_f + 2382098985\delta_c^4\delta_f^2 - 11529899136\delta_c^3\delta_f^3 + 3807147008\delta_c^2\delta_f^4 + 39685259264\delta_c\delta_f^5 - 914838624\delta_c^5 - 27535138596\delta_c^4\delta_f - 46187599332\delta_c^3\delta_f^2 + 905390869712\delta_c^2\delta_f^3 - 1541574429696\delta_c\delta_f^4 - 106895622144\delta_f^5 + 31947782832\delta_c^4 + 191482856208\delta_c^3\delta_f - 3752254963786\delta_c^2\delta_f^2 + 7858122606432\delta_c\delta_f^3 + 440868841472\delta_f^4 - 143183135592\delta_c^3 + 5156406567900\delta_c^2\delta_f - 15993413416036\delta_c\delta_f^2 - 452079659696\delta_f^3 - 2364819724764\delta_c^2 + 14728301634816\delta_c\delta_f - 350037350551\delta_f^2 - 5126298229584\delta_c + 853188665616\delta_f - 386633673468 < 0$. $CS^{*CC} - CS^{*PP} = \frac{U_2}{1764(63\delta_c+928\delta_f-1323)(14\delta_c-38\delta_f+49)} > 0$, where $U_2 = 484659\delta_c^3\delta_f + 738528\delta_c^2\delta_f^2 + 532728\delta_c^3 + 5677805\delta_c^2\delta_f - 19901192\delta_c\delta_f^2 - 2962176\delta_f^3 - 9322740\delta_c^2 + 55604857\delta_c\delta_f + 7812168\delta_f^2 - 39155508\delta_c - 5092521\delta_f$. $SW^{*CC} - SW^{*PP} = \frac{U_3}{3528(63\delta_c+928\delta_f-1323)^2(14\delta_c-38\delta_f+49)^2} > 0$, where $U_3 = 1099206612\delta_c^5\delta_f + 13957694541\delta_c^4\delta_f^2 - 30058089600\delta_c^3\delta_f^3 - 48279755776\delta_c^2\delta_f^4 + 39685259264\delta_c\delta_f^5 + 24893568\delta_c^5 - 21189211218\delta_c^4\delta_f + 71491174244\delta_c^3\delta_f^2 + 219317898352\delta_c^2\delta_f^3 - 200769442816\delta_c\delta_f^4 + 102020726784\delta_f^5 - 871274880\delta_c^4 - 42413713020\delta_c^3\delta_f - 291898325418\delta_c^2\delta_f^2 + 313767351072\delta_c\delta_f^3 - 562934177792\delta_f^4 + 3964300704\delta_c^3 + 65020703076\delta_c^2\delta_f - 65943090444\delta_c\delta_f^2 + 1165856105456\delta_f^3 + 64038703680\delta_c^2 - 220188103632\delta_c\delta_f - 1074067015931\delta_f^2 + 134481277728\delta_c + 371380010022\delta_f > 0$ for $\delta_c, \delta_f \in [0, 1]$.

A2 Switching Costs

Let s denote the switching cost. We analyze the benchmark model and main model with switching costs.

A2.1 Benchmark Model

When firms use BBP and consumers incur switching costs, the marginal consumers in the second period become:

$$v - \theta_a + q_{a2} - p_{ao} = v - (1 - \theta_a) + q_{b2} - p_{bn} - s, \quad (\text{A39})$$

$$\theta_a = \frac{1+s}{2} + \frac{q_{a2} - q_{b2}}{2} - \frac{p_{ao} - p_{bn}}{2}. \quad (\text{A40})$$

$$v - \theta_b + q_{a2} - p_{an} - s = v - (1 - \theta_b) + q_{b2} - p_{bo}, \quad (\text{A41})$$

$$\theta_b = \frac{1-s}{2} + \frac{q_{a2} - q_{b2}}{2} - \frac{p_{an} - p_{bo}}{2}. \quad (\text{A42})$$

Firms maximize second-period profit functions:

$$\Pi_{a2} = p_{ao}\theta_a + p_{an}(\theta_b - \theta_1) - q_{a2}^2, \quad (\text{A43})$$

$$\Pi_{b2} = p_{bo}(1 - \theta_b) + p_{bn}(\theta_1 - \theta_a) - q_{b2}^2. \quad (\text{A44})$$

Second-period prices are $p_{an}^* = \frac{3-4\theta_1-s}{3} + \frac{q_{a2}-q_{b2}}{3}$, $p_{ao}^* = \frac{1+s+2\theta_1}{3} + \frac{q_{a2}-q_{b2}}{3}$, $p_{bn}^* = \frac{4\theta_1-1-s}{3} + \frac{q_{b2}-q_{a2}}{3}$, and $p_{bo}^* = \frac{3-2\theta_1+s}{3} + \frac{q_{b2}-q_{a2}}{3}$. Second-period quality levels are $q_{a2}^* = \frac{5}{21} - \frac{\theta_1}{7}$ and $q_{b2}^* = \frac{2}{21} + \frac{\theta_1}{7}$.

The first-period marginal consumer is located at θ_1 . We have

$$\begin{aligned} v - \theta_1 - p_{a1} + [v - (1 - \theta_1) + q_{b2}^{e*} - p_{bn}^{e*} - s] &= v - (1 - \theta_1) - p_{b1} + [v - \theta_1 + q_{a2}^{e*} - p_{an}^{e*} - s] \\ \theta_1 &= \frac{1}{2} + \frac{7(p_{b1} - p_{a1})}{18}. \end{aligned} \quad (\text{A45})$$

The total profits that firms maximize in the first period are

$$\Pi_{at} = p_{a1}\theta_1 + \Pi_{a2}^*, \quad (\text{A46})$$

$$\Pi_{bt} = p_{b1}(1 - \theta_1) + \Pi_{b2}^*. \quad (\text{A47})$$

We solve for the first-period equilibrium prices and obtain $p_{a1}^* = \frac{4}{3} - \frac{2s}{3}$ and $p_{b1}^* = \frac{4}{3} - \frac{2s}{3}$. We summarize the rest of the equilibrium outcomes in Table A2.

A2.2 Main Model

When both firms customize quality and consumers incur switching costs, the second-period marginal consumers are

$$v - \theta_a + q_{ao} - p_{ao} = v - (1 - \theta_a) + q_{bn} - p_{bn} - s, \quad (\text{A48})$$

$$\theta_a = \frac{1+s}{2} + \frac{q_{ao} - q_{bn}}{2} - \frac{p_{ao} - p_{bn}}{2}. \quad (\text{A49})$$

$$v - \theta_b + q_{an} - p_{an} - s = v - (1 - \theta_b) + q_{bo} - p_{bo}, \quad (\text{A50})$$

$$\theta_b = \frac{1-s}{2} + \frac{q_{an} - q_{bo}}{2} - \frac{p_{an} - p_{bo}}{2}. \quad (\text{A51})$$

Firms' profit functions in the second period are

$$\Pi_{a2} = p_{ao}\theta_a + p_{an}(\theta_b - \theta_1) - \gamma(q_{ao}^2 + q_{an}^2), \quad (\text{A52})$$

$$\Pi_{b2} = p_{bo}(1 - \theta_b) + p_{bn}(\theta_1 - \theta_a) - \gamma(q_{bo}^2 + q_{bn}^2). \quad (\text{A53})$$

Second-period prices are $p_{ao}^* = \frac{q_{ao} - q_{bn}}{3} + \frac{2\theta_1}{3} + \frac{1+s}{3}$, $p_{an}^* = \frac{q_{an} - q_{bo}}{3} - \frac{4\theta_1 + s}{3} + 1$, $p_{bo}^* = \frac{q_{bo} - q_{an}}{3} - \frac{2\theta_1}{3} + 1 + \frac{s}{3}$, and $p_{bn}^* = \frac{q_{bn} - q_{ao}}{3} + \frac{4\theta_1}{3} - \frac{1+s}{3}$. Then, we solve for the second-period quality and obtain $q_{ao}^* = \frac{1+s}{7} + \frac{4\theta_1}{21}$, $q_{an}^* = \frac{1}{3} - \frac{10\theta_1}{21} - \frac{s}{7}$, $q_{bo}^* = \frac{s}{7} + \frac{1}{3} - \frac{4\theta_1}{21}$, and $q_{bn}^* = \frac{10\theta_1}{21} - \frac{1+s}{7}$.

The first-period marginal consumer at θ_1 is solved as follows:

$$\begin{aligned} v - \theta_1 - p_{a1} + [v - (1 - \theta_1) + q_{bn}^{e*} - p_{bn}^{e*}] &= v - (1 - \theta_1) - p_{b1} + [v - \theta_1 + q_{an}^{e*} - p_{an}^{e*}], \\ \theta_1 &= \frac{1}{2} - \frac{21(p_{a1} - p_{b1})}{40}. \end{aligned} \quad (\text{A54})$$

Firms set first-period quality and prices to maximize the following profit functions:

$$\Pi_{at} = p_{a1}\theta_1 + \Pi_{a2}^*, \quad (\text{A55})$$

$$\Pi_{bt} = p_{b1}(1 - \theta_1) + \Pi_{b2}^*. \quad (\text{A56})$$

Table A1: Equilibrium Outcomes with Switching Costs ($\gamma = 0.12$)

	Benchmark Model	Main Model
Second-Period Quality	$q_{a2}^*: 0.167$	$q_{ao}^*: 0.238 + 0.143s$ $q_{an}^*: 0.095 - 0.143s$
Second-Period Prices	$p_{ao}^*: 0.667 + 0.333s$ $p_{an}^*: 0.333 - 0.333s$	$p_{ao}^*: 0.714 + 0.429s$ $p_{an}^*: 0.286 - 0.429s$
First-Period Prices	$p_{a1}^*: 1.333 + 0.667s$	$p_{a1}^*: 0.952 - 0.762s$
Second-Period Profits (Π_{a2}^*)	$0.250 + 0.111s + 0.111s^2$	$0.263 + 0.163s + 0.163s^2$
First-Period Profits (Π_{a1}^*)	$0.667 - 0.333s$	$0.476 - 0.381s$
Total Profits (Π_{at}^*)	$0.917 - 0.222s + 0.111s^2$	$0.739 - 0.218s + 0.163s^2$
Consumer Surplus (CS^*)	$17.722 + 0.222s + 0.056s^2$	$18.112 + 0.354s + 0.092s^2$
Social Welfare (SW^*)	$19.556 - 0.222s + 0.278s^2$	$19.591 - 0.082s + 0.418s^2$

First-period prices are $p_{a1}^* = \frac{20-16s}{21}$ and $p_{b1}^* = \frac{20-16s}{21}$. The rest of the equilibrium outcomes are summarized in Table A2. In equilibrium, $\theta_a^* = \frac{5+3s}{15}$, $\theta_1^* = \frac{1}{2}$, and $\theta_b^* = \frac{9-3s}{15}$. The equilibrium consumption pattern holds if $\theta_a^* < \theta_1^* < \theta_b^*$ that requires $s < \frac{2}{3}$.

We can verify that $q_{ao}^* - q_{an}^* = \frac{1+2s}{7}$ and $p_{ao}^* - p_{an}^* = \frac{3+6s}{7}$. Thus, firms reward repeat customers on the quality dimension and reward new customers on the price dimension. Comparison of profits in two periods show that second-period profits are higher, first-period profits are lower, and total profits are lower with quality customization. Therefore, all results in the main model continue to hold.