Online Supplement to "Promoting Solar Panel Investments: Feed-in-Tariff versus Tax-Rebate Policies"

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A. Simple Economic Model

Consider a representative household that can generate Q units of electricity in perpetuity (the same insights hold with a finite-life asset as well) after incurring an investment cost X. Let the retail price of electricity be P (e.g., this could be a point forecast of future prices). The NPV of the investment is then given by $\int_0^\infty PQe^{-rt}\,dt - X = \frac{P}{r}Q - X$, where r is the discount rate. Under the feed-in-tariff policy, the government purchases the generated electricity at rate F from a household, so that the NPV of the household's investment becomes $\int_0^\infty FQe^{-rt}\,dt - X = \frac{F}{r}Q - X$. Essentially, the government increases the household's NPV by $\frac{F-P}{r}Q$, compared to the no-subsidy case. Under the tax-rebate policy, the government reduces the household's investment cost by a fraction R, making the NPV of the investment $\int_0^\infty PQe^{-rt}\,dt - (X - RX) = \frac{P}{r}Q - X(1-R)$. Essentially, the government increases the household's NPV by RX, compared to the no-subsidy.

Formally, the investment timing problem of a household is the same as the household's problem in Section 4.1.1. In both settings, the price and the cost are constant. Therefore, a household with efficiency Q invests in a panel at time $t=0$ if $Q \geq Q^i$ under subsidy policy $i \in \{F, R\}$, where thresholds Q^F and Q^R are given in equations (11) and (12), respectively. If $Q < Q^i$, then the household never invests.

Accounting for a household's investment decisions, the government maximizes the value of each subsidy program. Recall that, for a given feed-in tariff level F, the government's value is given in equation (13). The government maximizes this value by choosing $F > P$. For a given tax-rebate level R, the government's value is given in equation (15). The government maximizes this value by choosing $R \in [0,1]$.

Proposition 5. *Assume $P(t) = P, X(t) = X$. Consider a homogeneous population of households with generating efficiency Q. For any Q, the government is indifferent between the feed-in-tariff and the tax-rebate policies, and prefers both policies to the no-subsidy policy.*

Intuitively, as discussed in Section 1, the government can match the benefit and the cost of the two subsidy policies exactly by choosing the feed-in tariff and the tax rebate accordingly.

B. Parameter Estimation

We annualize all problem parameters whenever relevant. First, we estimate the annual drift and volatility of the GBMs for the electricity price and the investment cost processes given in equations (1) and (2), respectively. We adopt the actuarial interpretation of these processes as being presented under the physical measure. For both processes, we use the monthly historical data from the period of February 2007 – December 2015 for the states of California, Arizona, and New Jersey[4], the three largest solar energy markets in the U.S. (Harrington and Gould 2016). We estimate the drift and the volatility of each process separately in each state and use the average values across the states. For the investment cost process $X(t)$, we find that the

[4] The cost data is based on The Open PV Project (NREL 2019). The price data is based on The Electricity Data Browser (EIA 2019).

drift is -6%, the volatility is 15%, and the initial value is \$4 per Watt (W). For the electricity price process $P(t)$, we find that drift is 5%, the volatility is 20%, and the initial value is \$0.11 per kilowatt hour (kWh). Based on our data, neither cost nor price changes are correlated with the market return (proxied by the return on the S&P500 index) in a statistically and economically significant way. This supports our actuarial interpretation (Dixit and Pindyck 1994, Chapter 4).

The correlation between the price and cost processes is not statistically significant and is estimated to be -0.03. This is an estimate for a particular data set and, in general, there could be correlation between price and cost processes. We assume that the interest rate is 10% per year and the generating efficiency follows a Gamma distribution with mean μ_Q and standard deviation σ_Q. In the base case, $\mu_Q = 0.3$ and $\sigma_Q = 0.2$, where μ_Q is consistent with the average capacity factor of solar panels in the U.S. (EIA 2018, Table 6.7.B).

Estimating the societal benefit parameter b is more challenging. This is because, as explained in Section 3, the benefit includes several factors that are difficult to quantify. We mainly focus on the value of avoided carbon emissions and set the social benefit parameter in accordance with the social cost of carbon. We take the social cost of carbon as \$70 per ton of carbon dioxide emissions (EPA 2016, p.4).

The units of the price and cost data are \$/kWh and \$/W, respectively, and the unit of the social cost of carbon is \$/ton of carbon dioxide. We next adjust these units. First, the investment cost $X(t)$ is a one-time cost and its unit is \$/installation. Because the average installation is approximately 5 kW, we scale the cost data up by multiplying it with $5,000$ W/installation. Second, the electricity price $P(t)$ is received annually and its unit is \$/year/installation. Because the most efficient household (with efficiency $Q = 1$) can produce solar energy for a maximum of approximately $3,500$ hours/year in the U.S.[5], we multiply the price data with $17,500 (= 3,500 \times 5)$ kWh/year/installation. Finally, the unit for the societal benefit rate b is also \$/year/installation. We multiply the social cost of carbon with $181 = (17,500 \times 0.01035)$ ton of carbon dioxide/year/installation, where we assume that the generation from an installation displaces the use of a coal-fired power plant with emission intensity 0.01035 tons/kWh (EIA 2016).

C. Proofs

Proof of Lemma 1. (i) Under condition (11), the NPV of the investment under the feed-in-tariff policy is positive so that the household undertakes the investment.

(ii) Under condition (12), the NPV of the investment under the tax-rebate policy is positive. ■

Proof of Proposition 1. Let F^* be the optimal feed-in tariff level with the corresponding threshold efficiency for investment Q^{F^*}. By the definition of the threshold efficiencies in equations (11) and (12), there exists R, computed from $Q^R = Q^{F^*}$:

$$\frac{rX}{F^*} = Q^{F^*} = Q^R = \frac{rX\,(1-R)}{P}, \tag{C.1}$$

as $R = 1 - P/F^*$. Then, for any $Q \geq Q^{F^*}$:

$$\frac{F^* - P}{r} Q \geq \frac{F^* - P}{r} Q^{F^*} = \frac{F^* - P}{r}\frac{rX}{F^*} = \frac{F^* - P}{F^*} X = RX. \tag{C.2}$$

Therefore,

$$\pi^{F^*} = \int_{Q^{F^*}}^{1} \left[\frac{b}{r} - \frac{F^* - P}{r}\right] Q\psi\,(Q)\,dQ \leq \int_{Q^R}^{1} \left[\frac{b}{r} Q - RX\right] \psi\,(Q)\,dQ = \pi^R. \tag{C.3}$$

Because R is a feasible tax rebate, $\pi^{R^*} \geq \pi^R \geq \pi^{F^*}$. Finally, $\pi^{F^*} \geq \pi^{F=0} = \pi^N$. ■

[5] See `https://perma.cc/U76S-KCEW` for the weather data of select U.S. cities.

Proof of Lemma 2. Part (i) follows directly from Lemma 1 part (i).

Proof of Part (ii) comes from the standard approach (Dixit and Pindyck 1994), with slight modifications to help us to reuse results in the subsequent proofs. First, the household's investment policy has a threshold structure such that a household with efficiency Q invests if $P(t) \geq P_Q^R$, where P_Q^R is the threshold value (to be computed). Second, instead of solving a free-boundary problem directly to compute the value of the household and the value of the threshold, we transform the problem to a more convenient form as follows.

Assuming that the current time is t and the investment has already been made, a household receives $P(t+s)Q$ with $s \in [0, \infty)$ in perpetuity, which corresponds to value

$$E_t\left[\int_0^\infty e^{-rs} P(t+s)Q\,ds\right] = \frac{P(t)Q}{r-\mu_p}, \tag{C.4}$$

where the expectation is taken under the pricing measure, given the information up to time t. Define the stopping time $\tau_Q^R = \inf\{t : P(t) \geq P_Q^R\}$. Under the pricing measure, the value of the household is

$$W_Q^R = E\left[e^{-r\tau_Q^R}\left\{E_{\tau_Q^R}\left[\int_0^\infty e^{-rs} P(\tau_Q^R+s)Q\,ds\right] - X(1-R)\right\}\right] = E\left[e^{-r\tau_Q^R}\left\{\frac{P\left(\tau_Q^R\right)}{r-\mu_p}Q - X(1-R)\right\}\right], \tag{C.5}$$

where equality follows from equation (C.4). We use the fact that $P\left(\tau_Q^R\right) = P_Q^R$ to write

$$W_Q^R = E\left[e^{-r\tau_Q^R}\left\{\frac{P_Q^R}{r-\mu_p}Q - X(1-R)\right\}\right] = E\left[e^{-r\tau_Q^R}\right]\left\{\frac{P_Q^R}{r-\mu_p}Q - X(1-R)\right\}. \tag{C.6}$$

Next, to compute $E\left[e^{-r\tau_Q^R}\right]$ we interpret it as the value $V(P)$ of an Arrow-Debreu security that pays \$1 at time τ_Q^R, given that the current price of electricity is P. Now, following the standard approach (Dixit and Pindyck 1994 or McDonald and Siegel 1986), this value satisfies the ordinary differential equation (ODE):

$$\frac{1}{2}\sigma_p^2 P^2 \frac{d^2V}{dP^2} + \mu_p P \frac{dV}{dP} - rV = 0 \tag{C.7}$$

for $P \leq P_Q^R$, with the boundary conditions: $V(0) = 0$ and $V(P_Q^R) = 1$. The solution approach for such ODEs is standard (Dixit and Pindyck 1994). It suffices to verify that

$$V(P) = \left(\frac{P}{P_Q^R}\right)^\theta \tag{C.8}$$

for $P \leq P_Q^R$ and $V(P) = 1$ for $P > P_Q^R$, with θ given in equation (19), satisfies both equation (C.7) and the boundary conditions.

Finally, we need to compute the value of the threshold P_Q^R. A household chooses the investment threshold P_Q^R to maximize the household's value W_Q^R given in equation (C.6). This is equivalent to applying a smooth-pasting condition. Solving optimization $\max_{P_Q^R} W_Q^R$, we derive value for P_Q^R given in equation (19). ■

Proof of Lemma 3. Quantity θ in equation (19) is a solution of the characteristic equation

$$\frac{1}{2}\sigma_p^2\theta^2 + \left(\mu_p - \frac{1}{2}\sigma_p^2\right)\theta - r = 0 \tag{C.9}$$

for ODE (C.7). Using implicit differentiation in equation (C.9), we show that $\frac{d\theta}{d\sigma_p} = -\frac{\theta(\theta-1)\sigma_p}{\left(\theta-\frac{1}{2}\right)\sigma_p^2+\mu_p} < 0$ so that θ decreases in σ_p. Consequently, $\frac{\theta}{\theta-1}$ increases in σ_p. Furthermore, $\frac{d\theta}{d\mu_p} = -\frac{\theta}{\left(\theta-\frac{1}{2}\right)\sigma_p^2+\mu_p} < 0$; thus, $\frac{\theta}{\theta-1}$ increases in μ_p. ■

Proof of Lemma 4. (i) By definition, $\pi_Q^N = \pi_Q^{R=0} = \frac{b}{r}Q\left\{\left[\frac{P(0)}{P_Q^{R=0}}\right]^\theta \wedge 1\right\}$, where P_Q^R is defined in equation (19). Using assumption (22) and substituting $R=0$, $\pi_Q^N = \frac{bQ}{r}\left[\frac{1}{X}\frac{P(0)Q}{r-\mu_p}\frac{\theta-1}{\theta}\right]^\theta$.

(ii) Under the feed-in-tariff policy, the government maximizes the function (20). This function is maximized when $F^* = \frac{rX}{Q}$ so that $\pi_Q^{F^*} = \left(\frac{b}{r} + \frac{P(0)}{r-\mu_p}\right)Q - X$, provided it is positive. Requirement that the optimal profit is non-negative produces the conditions on Q. Note that $F^* \geq P(0)$ by assumption (22).

(iii) Under the tax-rebate policy, the government maximizes the function (21). Value P_Q^R is given in equation (19) and it decreases in R with $P_Q^1 = 0$. There are two cases: $\frac{P(0)}{P_Q^0} > 1$ and $\frac{P(0)}{P_Q^0} \leq 1$.

Case 1: $\frac{P(0)}{P_Q^0} > 1$. It follows that $\frac{P(0)}{P_Q^R} > 1$ for all R. Thus, R^* should be 0. The condition $\frac{P(0)}{P_Q^0} > 1$ for $R=0$ is equivalent to $Q > \frac{X}{\frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}$. However, by assumption (22), this set is empty because $\frac{X}{\frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}} > 1$.

Case 2: $\frac{P(0)}{P_Q^0} \leq 1$ or equivalently $Q \leq \frac{X}{\frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}$. There exists $R^1 = 1 - \frac{P(0)Q}{X(r-\mu_p)}\frac{\theta-1}{\theta}$, such that $\frac{P(0)}{P_Q^{R^1}} = 1$ and $R^1 \geq 0$ if and only if $Q \leq \frac{X}{\frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}$. The condition $R^1 \leq 1$ is satisfied because $\frac{P(0)Q}{X(r-\mu_p)}\frac{\theta-1}{\theta} \geq 0$. Therefore, the objective function (21) is defined in two pieces: $R \in [0, R^1]$ and $R \in [R^1, 1]$.

For all $R \in [R^1, 1]$, the objective function (21) is $\frac{bQ}{r} - RX$ and it decreases in R. Therefore, on this interval, $R^* = R^1$. The corresponding objective value is $\pi_Q^{R^1} = \frac{bQ}{r} + \frac{P(0)Q}{r-\mu_p}\frac{\theta-1}{\theta} - X$.

For all $R \in [0, R^1]$, the objective function (21) is $\left(\frac{bQ}{r} - RX\right)\left[\frac{P(0)}{P_Q^R}\right]^\theta$ and it is unimodal. The derivative of this function at $R = R^1$ is positive if $Q \geq \frac{X}{\frac{b}{r} + \left(\frac{\theta-1}{\theta}\right)^2\frac{P(0)}{r-\mu_p}}$. Then, in this case, $R^* = R^1$. The derivative of this function is negative at $R=0$ if $Q \leq \frac{X}{\theta\frac{b}{r}}$. Then, in this case, $R^* = 0$ with the corresponding objective value of $\frac{bQ}{r}\left[\frac{1}{X}\frac{P(0)Q}{r-\mu_p}\frac{\theta-1}{\theta}\right]^\theta$. For $Q \in \left(\frac{X}{\theta\frac{b}{r}}, \frac{X}{\frac{b}{r} + \left(\frac{\theta-1}{\theta}\right)^2\frac{P(0)}{r-\mu_p}}\right]$, the first order conditions produce $R^2 = \frac{1}{X(\theta-1)}\left(\frac{bQ\theta}{r} - X\right)$. Hence, $R^* = R^2$ and the corresponding objective value is $\frac{1}{(\theta-1)\left(X - \frac{bQ}{r}\right)^{\theta-1}}\left[\frac{P(0)Q}{r-\mu_p}\left(\frac{\theta-1}{\theta}\right)^2\right]^\theta$. ■

Proof of Proposition 2. First, note that, $\pi_Q^{R^*} \geq \pi_Q^N$ for all Q because the no-subsidy case is a feasible tax-rebate policy where $R=0$.

(i) For all $Q \in \left[0, \frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}}\right]$, $\pi_Q^{R^*} \geq \pi_Q^N \geq \pi_Q^{F^*}$ as $\pi_Q^{F^*} = 0$ and $\pi_Q^{R^*}, \pi_Q^N \geq 0$ by Lemma 4.

(ii) For $Q \in \left[\frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}, 1\right]$, because $\frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}} < \frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}$, Lemma 4 yields

$$\pi_Q^{F^*} = \frac{bQ}{r} + \frac{P(0)Q}{r-\mu_p} - X. \tag{C.10}$$

The value of $\pi_Q^{R^*}$ is defined on two intervals: $Q \in \left(\frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}, \frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\left(\frac{\theta-1}{\theta}\right)^2}\right]$ and $Q \in \left(\frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\left(\frac{\theta-1}{\theta}\right)^2}, 1\right]$.

First, consider $Q \in \left(\frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}}, \frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\left(\frac{\theta-1}{\theta}\right)^2}\right]$. The assumption that $\theta\frac{b}{r} > \frac{P(0)}{r-\mu_p}$ is equivalent to $\theta\frac{b}{r} > \frac{b}{r} + \frac{P(0)}{r-\mu_p}\frac{\theta-1}{\theta}$. Therefore, $Q \geq \frac{X}{\theta\frac{b}{r}}$ and, by Lemma 4,

$$\pi_Q^{R^*} = \frac{1}{(\theta-1)\left(X - \frac{bQ}{r}\right)^{\theta-1}}\left[\frac{P(0)Q}{r-\mu_p}\left(\frac{\theta-1}{\theta}\right)^2\right]^\theta \leq \frac{1}{\theta-1}\left(X - \frac{bQ}{r}\right). \tag{C.11}$$

The inequality is constructed by replacing term $\left[\frac{P(0)}{P_Q^R}\right]^\theta$ evaluated at $R = \frac{1}{(\theta-1)X}\left(\theta\frac{bQ}{r} - X\right)$ with 1. Note that $\pi_Q^{F^*} = \frac{bQ}{r} + \frac{P(0)Q}{r-\mu_p} - X \geq \frac{1}{\theta-1}\left(X - \frac{bQ}{r}\right)$ if and only if $\frac{bQ}{r} + \frac{P(0)Q}{r-\mu_p}\frac{\theta-1}{\theta} - X \geq 0$, which is true on this interval.

Second, consider $Q \in \left(\frac{X}{\frac{b}{r} + \frac{P(0)}{r-\mu_p}\left(\frac{\theta-1}{\theta}\right)^2}, 1\right]$. From Lemma 4

$$\pi_Q^{R^*} = \frac{bQ}{r} + \frac{P(0)Q}{r-\mu_p}\frac{\theta-1}{\theta} - X < \frac{bQ}{r} + \frac{P(0)Q}{r-\mu_p} - X = \pi_Q^{F^*}. \tag{C.12}$$

Therefore, $\pi_Q^{F^*} \geq \pi_Q^{R^*} \geq \pi_Q^N$. ∎

Proof of Lemma 5. The proof is similar to that of Lemma 2.

(i) Under the feed-in-tariff policy, the household's investment policy has a threshold structure such that a household invests if $X(t) \leq X_Q^F$, where X_Q^F is the threshold value (to be computed). Assuming that the current time is t and the investment has already been made, a household receives FQ in perpetuity, which corresponds to value $\int_0^\infty e^{-rs} FQ\, ds = \frac{FQ}{r}$. Define the stopping time $\tau_Q^F = \inf\{t : X(t) \leq X_Q^F\}$. Under the pricing measure, the value of the household is

$$W_Q^F = E\left[e^{-r\tau_Q^F}\left\{\frac{FQ}{r} - X\left(\tau_Q^F\right)\right\}\right] = E\left[e^{-r\tau_Q^F}\right]\left\{\frac{FQ}{r} - X_Q^F\right\}, \tag{C.13}$$

where equality follows from $X\left(\tau_Q^F\right) = X_Q^F$. To compute $E\left[e^{-r\tau_Q^F}\right]$, we interpret it as the value $V(X)$ of an Arrow-Debreu security that pays \$1 at time τ_Q^F, given the current cost is X. This value satisfies the following ODE:

$$\frac{1}{2}\sigma_x^2 X^2 \frac{d^2V}{dX^2} + \mu_x X \frac{dV}{dX} - rV = 0, \tag{C.14}$$

for $X \geq X_Q^F$, with the boundary conditions that $V \to 0$ as $X \to \infty$ and $V(X_Q^F) = 1$. The solution of this ODE with these boundary conditions is

$$V(X) = \left(\frac{X_Q^F}{X}\right)^\eta, \tag{C.15}$$

if $\frac{X_Q^F}{X} \leq 1$ and $V(X) = 1$ if $\frac{X_Q^F}{X} > 1$, where η is given in equation (23). The last step of the proof is to compute the investment threshold X_Q^F that maximizes the household's value W_Q^F. This optimal X_Q^F is presented in equation (23).

(ii) Under the tax-rebate policy, the cost of the household is $C(t) = X(t)(1-R)$, which follows the same process (2) as $X(t)$. The benefit to a household comes from the constant price P from the time of investment into perpetuity. Thus, the household's problem has the same structure as in part (i) with C in place of X and P in place of F. That, is the value of the household is

$$W_Q^R = E\left[e^{-r\tau_Q^R}\left\{\frac{PQ}{r} - X\left(\tau_Q^R\right)(1-R)\right\}\right] = E\left[e^{-r\tau_Q^R}\right]\left\{\frac{PQ}{r} - X_Q^R(1-R)\right\}, \tag{C.16}$$

Adopting the proof from part (i) results follow. ∎

Proof of Lemma 6. (i) By definition, $\pi_Q^N = \pi_Q^{R=0} = \frac{b}{r}Q\left\{\left[\frac{X_Q^{R=0}}{X(0)}\right]^\eta \wedge 1\right\}$, where X_Q^R is defined in equation (24). Substituting $R=0$, if $Q \leq \frac{rX(0)}{P\frac{\eta}{\eta+1}}$, $\pi_Q^N = \frac{b}{r}Q\left[\frac{PQ\frac{\eta}{\eta+1}}{rX(0)}\right]^\eta$; otherwise, $\pi_Q^N = \frac{bQ}{r}$.

(ii) Under the feed-in-tariff policy, the government's objective function is in equation (25). Recall that threshold X_Q^F is given in equation (23) and it increases in F. There are two cases: $\left[\frac{X_Q^F}{X(0)}\right]^\eta > 1$ and $\left[\frac{X_Q^F}{X(0)}\right]^\eta \leq 1$.

If $\left[\frac{X_Q^F}{X(0)}\right]^\eta > 1$, then $F^* = P$. The condition $\left[\frac{X_Q^F}{X(0)}\right]^\eta > 1$ for $F = P$ is equivalent to $Q > \frac{rX(0)}{P\frac{\eta}{\eta+1}}$.

If $\left[\frac{X_Q^F}{X(0)}\right]^\eta \leq 1$, there are two candidate solutions: either F^1 or F^2, where $F^1 = \{F : X_Q^F = X(0)\}$ and F^2 is the maximizer of $f_2(F) = \left(\frac{b}{r} - \frac{F-P}{r}\right)Q\left[\frac{X_Q^F}{X(0)}\right]^\eta$. Note that $F^2 \leq F^1$ so that $F^* = F^2$ if and only if $Q \leq \frac{rX(0)}{(b+P)\left(\frac{\eta}{\eta+1}\right)^2}$. Finally, $f_2'(P) < 0$ if $b \leq \frac{P}{\eta}$. Therefore, in this case, $F^* = P$.

(iii) Under the tax-rebate policy, using the results of Lemma 5, the government's objective function is in equation (26) and the analysis is similar to the feed-in-tariff policy above. Specifically, there are two cases $\left[\frac{X_Q^R}{X(0)}\right]^\eta > 1$ and $\left[\frac{X_Q^R}{X(0)}\right]^\eta \leq 1$. Case $\left[\frac{X_Q^R}{X(0)}\right]^\eta > 1$ is equivalent to $Q > \frac{rX(0)}{P\frac{\eta}{\eta+1}}$. In this case, $R^* = 0$. If $\left[\frac{X_Q^R}{X(0)}\right]^\eta \leq 1$, there are two candidate solutions: $R^1 = \{R : X_Q^R = X(0)\}$ and R^2, which is the maximizer of $f_2(R) = \left(\frac{bQ}{r} - RX_Q^R\right)\left[\frac{X_Q^R}{X(0)}\right]^\eta$. The condition that ensures $R^2 \leq R^1$ so that $R^* = R^2$ is equivalent to $Q \leq \frac{rX(0)}{\left(b+P\frac{\eta}{\eta+1}\right)\frac{\eta}{\eta+1}}$. Finally, $f_2'(0) < 0$ if $b \leq \frac{P}{\eta+1}$. Therefore, in this case, $R^* = 0$. ■

Proof of Proposition 3. Based on the optimal policies given in Lemma 6, there are three intervals on b: $b \leq \frac{P}{\eta+1}$, $\frac{P}{\eta+1} < b \leq \frac{P}{\eta}$, and $b > \frac{P}{\eta}$. In the first interval, $\pi_Q^{F^*} = \pi_Q^{R^*} = \frac{bQ}{r}\left[\frac{PQ}{rX(0)}\frac{\eta}{\eta+1}\right]^\eta$. In the second interval, F^* remains the same, but R^* depends on Q. Nevertheless, $R = 0$ is still feasible. Thus, $\pi_Q^{F^*} = \pi_Q^{R=0} \leq \pi_Q^{R^*}$. In the third interval, we consider four cases.

Case 1: $0 \leq Q \leq \frac{rX(0)}{\left(b+P\frac{\eta}{\eta+1}\right)\frac{\eta}{\eta+1}}$. This condition and Lemma 6, yield

$$\frac{\pi_Q^{R^*}}{\pi_Q^{F^*}} = \frac{\left(\frac{b+P\frac{\eta}{\eta+1}}{(b+P)\frac{\eta}{\eta+1}}\right)^{\eta+1}}{\frac{\eta+1}{\eta}} \geq \frac{\left(1+\frac{1}{\eta^2+\eta}\right)^{\eta+1}}{\frac{\eta+1}{\eta}} \geq 1, \tag{C.17}$$

where the first inequality is due to $b > \frac{P}{\eta}$ and the second follows from the Bernoulli's inequality.

Case 2: $\frac{rX(0)}{\left(b+P\frac{\eta}{\eta+1}\right)\frac{\eta}{\eta+1}} \leq Q \leq \frac{rX(0)}{(b+P)\left(\frac{\eta}{\eta+1}\right)^2}$. $\pi_Q^{F^*}$ is constant, but $\pi_Q^{R^*}$ increases. Thus, $\pi_Q^{R^*} \geq \pi_Q^{F^*}$.

Case 3: $\frac{rX(0)}{(b+P)\left(\frac{\eta}{\eta+1}\right)^2} \leq Q \leq \frac{rX(0)}{P\frac{\eta}{\eta+1}}$. By Lemma 6, $\pi_Q^{R^*} - \pi_Q^{F^*} = \frac{X(0)}{\eta} - \frac{PQ}{r(\eta+1)} \geq 0$, where the inequality follows from $Q \leq \frac{rX(0)}{P\frac{\eta}{\eta+1}}$.

Case 4: $Q \geq \frac{rX(0)}{P\frac{\eta}{\eta+1}}$. From Lemma 6, it follows that $\pi_Q^{F^*} = \pi_Q^{R^*}$. ■

Proof of Lemma 7. (i) Under the feed-in-tariff policy, the price dynamics does not affect the household's investment decisions. Therefore, this part is identical to part (i) in Lemma 5.

(ii) First, we introduce cost process $C(t) = X(t)(1-R)$ and observe that it follows equation (2). Assuming that the current time is t and the household made an investment, the value of this investment in perpetuity is $V(t) = E_t\left[\int_0^\infty e^{-rs}P(t+s)Q\,ds\right] = \frac{P(t)Q}{r-\mu_p}$. This value follows process (1). Define new variable $Y_1(t) = \frac{V(t)}{C(t)}$. Now the direct application of results from McDonald and Siegel (1986) proves this lemma. ■

Proof of Lemma 8. This proof is omitted for brevity because it is similar to the proof of Lemma 3. ■

Proof of Lemma 9. (i) Under the feed-in-tariff policy, the government's value π_Q^F is given as

$$\pi_Q^F = E\left[\int_{\tau_Q^F}^\infty [b-(F-P(t))]\,Qe^{-rt}\,dt\right] = \frac{b-F}{r}QE\left[e^{-r\tau_Q^F}\right] + \frac{Q}{r-\mu_p}E\left[e^{-r\tau_Q^F}P\left(\tau_Q^F\right)\right], \tag{C.18}$$

where $\tau_Q^F = \inf\{t : X(t) \leq X_Q^F\}$ and X_Q^F is defined in equation (23). By the proof of Lemma 5, $E\left[e^{-r\tau_Q^F}\right] = \left[\frac{X_Q^F}{X(0)}\right]^\eta \wedge 1$, where η is defined in equation (23). Let $V(X,P) = E\left[e^{-r\tau_Q^F}P\left(\tau_Q^F\right)\right]$, using the standard approach (Dixit and Pindyck 1994), $V(X,P)$ satisfies the following partial differential equation (PDE):

$$\frac{1}{2}\sigma_x^2X^2\frac{\partial^2 V}{\partial X^2} + \frac{1}{2}\sigma_p P^2\frac{\partial^2 V}{\partial P^2} + \rho\sigma_x\sigma_p XP\frac{\partial^2 V}{\partial X\partial P} + \mu_x X\frac{\partial V}{\partial X} + \mu_p P\frac{\partial V}{\partial P} - rV = 0, \tag{C.19}$$

for $X \geq X_Q^F$, with the boundary conditions $V(X_Q^F, P) = P$, $\lim_{X\to\infty} V(X,P) = 0$. We verify by substitution that $V(P,X) = P\left\{\left[\frac{X_Q^F}{X}\right]^\nu \wedge 1\right\}$ solves this PDE with the boundary conditions. Substituting the values of $E\left[e^{-r\tau_Q^F}\right]$ and $E\left[e^{-r\tau_Q^F}P\left(\tau_Q^F\right)\right]$ in equation (C.18) yields π_Q^F given in equation (30).

(ii) Under the tax-rebate policy, the government's value π_Q^R is

$$\pi_Q^R = E\left[\int_{\tau_Q^R}^{\infty} bQe^{-rt}\,dt - e^{-r\tau_Q^R} RX\left(\tau_Q^R\right)\right] = \frac{b}{r}QE\left[e^{-r\tau_Q^R}\right] - RE\left[e^{-r\tau_Q^R} X\left(\tau_Q^R\right)\right], \tag{C.20}$$

where $\tau_Q^R = \inf\{t : Y(t) \geq Y_Q^R\}$, and $Y(t)$ and Y_Q^R are given in equations (27) and (29), respectively. Define function $V(Y) = E\left[e^{-r\tau_Q^R}\right]$. Quantity $Y(t)$ follows the process

$$dY(t) = \left(\mu_p - \mu_x + \sigma_x^2 - \rho\sigma_x\sigma_p\right) Y(t)dt + \sigma_p Y(t)dZ_p(t) - \sigma_x Y(t)dZ_x(t). \tag{C.21}$$

Function $V(Y)$ satisfies ODE:

$$\frac{1}{2}\sigma^2 Y^2 \frac{d^2V}{dY^2} + \left(\mu_p - \mu_x + \sigma_x^2 - \rho\sigma_x\sigma_p\right) Y \frac{dV}{dY} - rV = 0, \tag{C.22}$$

for $Y \leq Y_Q^R$, with the boundary conditions $V(0) = 0$ and $V(Y_Q^R) = 1$. Direct substitution verifies that $V(Y) = \left[\frac{Y}{Y_Q^R}\right]^{\zeta} \wedge 1$, with ζ defined in equation (33), satisfies this ODE and the boundary conditions.

Redefine function $V(P, X) = E\left[e^{-r\tau_Q^R} X\left(\tau_Q^R\right)\right]$ which satisfies PDE:

$$\frac{1}{2}\sigma_p^2 P^2 \frac{\partial^2 V}{\partial P^2} + \frac{1}{2}\sigma_x^2 X^2 \frac{\partial^2 V}{\partial X^2} + \rho\sigma_p\sigma_x PX \frac{\partial^2 V}{\partial P \partial X} + \mu_p P \frac{\partial V}{\partial P} + \mu_x X \frac{\partial V}{\partial X} - rV = 0, \tag{C.23}$$

for $\frac{P}{X} \leq Y_Q^R$, with the boundary conditions $V(0, X) = 0$ and $V(P, X)|_{\frac{P}{X} = Y_Q^R} = X$. Direct substitution proves that $V(P, X) = X\left\{\left[\frac{Y}{Y_Q^R}\right]^{\gamma} \wedge 1\right\}$, with γ defined in equation (29), satisfies both the PDE and the boundary conditions. Substituting the values of $E\left[e^{-r\tau_Q^R}\right]$ and $E\left[e^{-r\tau_Q^R} X\left(\tau_Q^R\right)\right]$ in equation (C.20) yields π_Q^R given in equation (32). ■

Proof of Lemma 10. (i) In the government's value π_Q^F, given in equation (30), the only term that depends on ρ is ν, which is obtained by the characteristic equation

$$\frac{1}{2}\sigma_x^2\nu^2 + \left(\mu_x - \rho\sigma_x\sigma_p - \frac{1}{2}\sigma_x^2\right)\nu - (r - \mu_p) = 0 \tag{C.24}$$

for PDE (C.19). Using implicit differentiation in equation (C.24), $\frac{d\nu}{d\rho} = \frac{\sigma_p}{\sigma_x}\frac{\nu}{\bar{\nu}} > 0$, where $\bar{\nu} := \sqrt{\left(\frac{\mu_x + \rho\sigma_x\sigma_p}{\sigma_x^2} - \frac{1}{2}\right)^2 + \frac{2(r-\mu_p)}{\sigma_x^2}}$. Because ν increases in ρ, π_Q^F decreases in ρ.

(ii) In π_Q^F, only ν depends on σ_p. Using implicit differentiation in equation (C.24), $\frac{d\nu}{d\sigma_P} = \rho\frac{\sigma_x v}{\bar{\nu}}$. The result then follows from that π_Q^F decreases in ν.

Proof of Proposition 4. (i) Under the tax-rebate policy, the government's optimization problem is given in equation (8), where π_Q^R is defined in equation (32). Let $b = 0$. Then $R^* = 0$ with the corresponding government's value $\pi^{R^*} = 0$. Under the feed-in-tariff policy, the government's optimization problem is given in equation (6), where π_Q^F is defined in equation (30). Using these definitions, consider $F = P(0)$ as a feasible solution with the objective value

$$\pi^{F=P(0)} = \int_0^1 \pi_Q^{F=P(0)}\psi(Q)dQ = \int_0^1 Q\left(\frac{P(0)}{r - \mu_p}\left[\frac{X_Q^{F=P(0)}}{X(0)}\right]^{\nu} - \frac{P(0)}{r}\left[\frac{X_Q^{F=P(0)}}{X(0)}\right]^{\eta}\right)\psi(Q)dQ, \tag{C.25}$$

where X_Q^F and η, and ν are given in equations (23) and (31), respectively. Here, the condition (34) ensures that $X_Q^{F=P(0)} \leq X(0)$. If $\rho < 0$, then $\eta > \nu$. Therefore, for $b = 0$, $0 = \pi^{R^*} < \pi^{F=P(0)} \leq \pi^{F^*}$. By continuity of the objective functions, the inequality holds as long as $b < \check{b}$ for some $\check{b} > 0$.

(ii) There exists some $\hat{b}$ such that, if $b \geq \hat{b}$, it is optimal for the government to provide sufficient subsidy under any policy that all households invest immediately. That is, under the feed-in-tariff policy, $X_{\bar{Q}}^{F^*} = X(0)$, where $\bar{Q}$ is the lowest efficiency level and X_Q^F is defined in equation (23). This implies that $F^* = \frac{rX(0)}{\bar{Q}} \frac{\eta+1}{\eta}$. Under the tax-rebate policy, $Y_{\bar{Q}}^{R^*} = Y(0)$, where Y_Q^R is defined in equation (29). This implies that $R^* = 1 - \frac{Y(0)\bar{Q}}{r-\mu_p} \frac{\gamma-1}{\gamma}$. Given the optimal subsidies, the difference between the government's values from a household with efficiency $Q \geq \bar{Q}$ is

$$\pi_Q^{R^*} - \pi_Q^{F^*} = \left(Q - \bar{Q}\frac{\gamma-1}{\gamma}\right)\left[X(0)\left(\frac{\frac{Q}{\bar{Q}}\left(1+\frac{1}{\eta}\right)-1}{Q-\bar{Q}\frac{\gamma-1}{\gamma}}\right) - \frac{P(0)}{r-\mu_p}\right]. \tag{C.26}$$

To show that $\pi_Q^{R^*} \geq \pi_Q^{F^*}$, because $X(0) \geq \frac{P(0)}{r-\mu_p}$ by condition (34), it suffices to show $\left(\frac{\frac{Q}{\bar{Q}}\left(1+\frac{1}{\eta}\right)-1}{Q-\bar{Q}\frac{\gamma-1}{\gamma}}\right) \geq 1$, or, equivalently, $\frac{Q}{\bar{Q}}\left(1+\frac{1}{\eta}\right) - Q \geq 1 - \bar{Q}\frac{\gamma-1}{\gamma}$. Note that, for any $Q \geq \bar{Q}$,

$$\frac{Q}{\bar{Q}}\left(1+\frac{1}{\eta}\right) - Q \geq 1 + \frac{1}{\eta} - \bar{Q} \geq 1 + \frac{1}{\gamma} - \bar{Q} \geq 1 - \bar{Q}\frac{\gamma-1}{\gamma}, \tag{C.27}$$

where the second inequality follows from the condition $\sigma_x^2 \geq r + \mu_x$, which implies that $\eta < 1 < \gamma$. Therefore, $\pi_Q^{R^*} \geq \pi_Q^{F^*}$ for any Q and

$$\pi^{R^*} = \int_0^1 \pi_Q^{R^*}\psi(Q)dQ \geq \pi^{F^*} = \int_0^1 \pi_Q^{F^*}\psi(Q)dQ. \tag{C.28}$$

■

Proof of Proposition 5. We derive the optimal feed-in-tariff and tax-rebate policies as follows.

Condition	F^*	R^*	$\pi_Q^{F^*} = \pi_Q^{R^*}$
$Q \in \left[0, \frac{rX}{b+P}\right]$	P	0	0
$Q \in \left(\frac{rX}{b+P}, \frac{rX}{P}\right]$	$\frac{rX}{Q}$	$1 - \frac{1}{X}\frac{P}{r}Q$	$\frac{b+P}{r}Q - X$
$Q \in \left(\frac{rX}{P}, 1\right]$	P	0	$\frac{b}{r}Q$

Furthermore, the value of the no-subsidy policy is $\pi_Q^N = \frac{b}{r}Q \times 1_{\{Q > \frac{rX}{P}\}}$. Therefore $\pi_Q^{F^*} = \pi_Q^{R^*} \geq \pi_Q^N$. ■

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