

Online Appendix for “Responsible Sourcing under Supplier-Auditor Collusion”

Li Chen

Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853
li.chen@cornell.edu

Shiqing Yao

Monash Business School, Monash University, Melbourne, VIC 3145, Australia
shiqing.yao@monash.edu

Kaijie Zhu

CUHK Business School, The Chinese University of Hong Kong, Shatin, Hong Kong
kzhu@baf.msmail.cuhk.edu.hk

Appendix A: Proofs for Main Results

Proof (Lemma 1) We first show that in equilibrium, $p_1 \geq c\lambda$ and $p_2 \leq c\lambda$. Suppose $p_1 < c\lambda$. The buyer’s profit keeps unchanged by increasing p_1 to $c\lambda$ since the supplier never chooses to take the audit. Similarly, if $p_2 > c\lambda$, by reducing p_2 to $c\lambda$, the buyer can lower the contract payment without changing the supplier’s behavior. Thus, it makes the buyer better off. Accordingly, the buyer’s optimal profit can be obtained by setting $p_1 \geq c\lambda$ and $p_2 \leq c\lambda$.

Next, according to the assumption of $l > 2v$, $\int_0^{p_2/c} (v - l + lx - p_2) dx < 0$ for any $p_2 > 0$. Therefore, based on $p_1 \geq c\lambda$ and $p_2 \leq c\lambda$, the buyer is better off by setting $p_2 = 0$. With this observation, the buyer’s problem can be simplified as $\Pi^0 = \max_{c\lambda \leq p_1 \leq c} \Pi^0(\lambda, p_1, 0)$, where $\Pi^0(\lambda, p_1, 0) = \int_{\lambda}^{\frac{p_1}{c}} (v - l + lx - p_1) dx$. We can compute $\Pi^0(\lambda, p, 0) - \Pi^0(\lambda + 1 - \frac{p}{c}, c, 0) = (\frac{l}{c} - 1)(\frac{p}{c} - \lambda)(p - c) < 0$ for any $c\lambda \leq p < c$, which indicates the buyer’s equilibrium contract payment must be $p_1^0 = c$. Furthermore, by noting $\frac{\partial^2 \Pi^0(\lambda, c, 0)}{\partial \lambda^2} < 0$, we obtain the equilibrium process quality requirement as $\lambda^0 = 1 - \frac{v-c}{l}$. Hence, $\Pi^0 = \frac{(v-c)^2}{2l}$. \square

Proof (Proposition 1) We divide the proof into three steps. First, we show that when collusion is possible, the buyer’s optimal profit can be obtained by setting $p_2 \leq c\lambda$. Suppose $p_2 > c\lambda$. Note that the supplier with process quality level $x < p_2/c$ will contract with the buyer because $p_2 - cx \geq 0$ whilst the supplier with process quality level $x > p_2/c$ will contract with the buyer with payment p_1 since $p_1 \geq p_2$. Hence, increasing λ will not impact the supplier’s participation decision but may

make the supplier choose to contract with the buyer with payment p_2 , reducing the buyer's expected contract payment and thus making the buyer better off. Therefore, the buyer's optimal profit can be obtained by setting $p_2 \leq c\lambda$.

Second, we show that the buyer's optimal profit can be obtained by setting $p_1 \geq c\lambda$. Suppose that $p_1 < c\lambda$. In that case, if $K > c/\varphi$ and $x < \lambda$, the gain from collusion is $p_1 - cx - \varphi K(\lambda - x) = (\varphi K - c)x + p_1 - \varphi K\lambda < (\varphi K - c)(x - \lambda) < 0$. That means that no supplier is willing to contract with the buyer with payment p_1 . Therefore, the buyer's optimal profit can only be obtained by setting $p_1 \geq c\lambda$ when $K > c/\varphi$. Suppose $K \leq c/\varphi$. In this case, the substandard supplier will contract with the buyer with payment p_1 if and only if $p_1 - cx - \varphi K(\lambda - x) \geq (p_2 - cx)^+$. From that, it can be shown that there must exist a $\xi > 0$ such that the buyer's profit is less than or equal to $(1 - \gamma) \int_0^{p_2/c} (v - l(1 - x) - p_2) dx + \gamma \int_0^\xi (v - l(1 - x)) dx$. Because $l > 2v$, the buyer's profit must be less than zero. Accordingly, the buyer's optimal profit can be obtained by setting $p_1 \geq c\lambda$.

Third, based on the above results (i.e., $p_1 \geq c\lambda \geq p_2$), the remaining discussion can be divided into three cases. We first consider the case $\lambda < \frac{p_1 - p_2}{\varphi K}$. In that case, we have $p_1 - cx - \varphi K(\lambda - x) \geq p_2 - cx$, which implies that $p_1 - cx - \varphi K(\lambda - x) \geq 0$ for all $0 \leq x \leq \lambda$ and the supplier with process quality level lower than p_2/c will contract with the buyer through collusion. Accordingly, the buyer's profit can be expressed as $\int_\lambda^{\min(1, p_1/c)} (v - l(1 - x) - p_1) dx + (1 - \gamma) \int_0^{p_2/c} (v - l(1 - x) - p_2) dx + \gamma \int_0^\lambda (v - l(1 - x) - p_1) dx$. Apparently, by reducing p_2 to 0, the inequality $\lambda < \frac{p_1 - p_2}{\varphi K}$ still holds while the buyer's profit increases, which contradicts the optimality. Secondly, we consider the case $\lambda - \frac{p_1 - p_2}{\varphi K} \geq \frac{p_2}{c}$. In that case, it can be shown that $K > c/\varphi$ and the buyer's profit function can be written as $\int_\lambda^{\min(1, p_1/c)} (v - l(1 - x) - p_1) dx + \int_0^{p_2/c} (v - l(1 - x) - p_2) dx + \gamma \int_{\frac{\varphi K \lambda - p_1}{\varphi K - c}}^\lambda (v - l(1 - x) - p_1) dx$. Apparently, by reducing p_2 to 0, the inequality $\lambda - \frac{p_1 - p_2}{\varphi K} \geq \frac{p_2}{c}$ still holds while the buyer's profit increases. Thirdly, we consider the most complicated case $0 \leq \lambda - \frac{p_1 - p_2}{\varphi K} < \frac{p_2}{c}$. In this case, the buyer's profit function can be expressed as $\Pi(\lambda, p_1, p_2) = \int_\lambda^{\min(1, p_1/c)} (v - l(1 - x) - p_1) dx + (1 - \gamma) \int_0^{p_2/c} (v - l(1 - x) - p_2) dx + \gamma \int_{\lambda - \frac{p_1 - p_2}{\varphi K}}^\lambda (v - l(1 - x) - p_1) dx + \gamma \int_0^{\lambda - \frac{p_1 - p_2}{\varphi K}} (v - l(1 - x) - p_2) dx$. It can be shown that $p_1 \leq c$ (otherwise, the buyer can be better off reducing p_1 slightly). Therefore, $\min(1, p_1/c) = p_1/c$. Consider two subcases, $K \leq c/\varphi$ and $K > c/\varphi$. Subcase 1: $K \leq c/\varphi$. It can be shown that $\Pi(\lambda, p_1, p_2) \leq \int_0^{p_1/c} (v - l + lx) dx - (1 - \gamma) \int_{p_2/c}^\lambda (v - l + lx) dx = \frac{p_1}{c} [v - l + \frac{l}{2} \frac{p_1}{c}] - (1 - \gamma) (\lambda - \frac{p_2}{c}) [v - l + \frac{l}{2} (\lambda + \frac{p_2}{c})]$. Note that $v - l + \frac{l}{2} \frac{p_1}{c} < 0$ and $\lambda - \frac{p_2}{c} > 0$. Hence, if $v - l + \frac{l}{2} (\lambda + \frac{p_2}{c}) \geq 0$, then $\Pi(\lambda, p_1, p_2) \leq 0$ and then a contradiction. If $v - l + \frac{l}{2} \frac{p_1}{c} < 0$, $\Pi(\lambda, p_1, p_2) < \frac{p_1}{c} [v - l + \frac{l}{2} \frac{p_1}{c}] - \frac{p_1}{c} [v - l + \frac{l}{2} (\lambda + \frac{p_2}{c})] = \frac{p_1}{c} \frac{l}{2} [\frac{p_1 - p_2}{c} - \lambda] \leq \frac{p_1}{c} \frac{l}{2} [\frac{p_1 - p_2}{\varphi K} - \lambda]$. Since $\lambda - \frac{p_1 - p_2}{\varphi K} \geq 0$, $\Pi(\lambda, p_1, p_2) < 0$. Subcase 2: $K > c/\varphi$. Let us consider shutout contract $(\lambda', p_1, 0)$, where $\lambda' = \lambda + \frac{c}{\varphi K} \left(\frac{p_2}{c} - \left(\lambda - \frac{p_1 - p_2}{\varphi K} \right) \right) > \lambda$. We will show this constructed contract yields

higher profit. Note that under the collusive state, the profit increases, because the supplier with process quality lower than $\lambda - \frac{p_1 - p_2}{\varphi K}$ is shut out and the supplier with process quality higher than $\lambda - \frac{p_1 - p_2}{\varphi K}$ contracts with the buyer. Under the non-collusive state, the increased profit by using the shutout contract is $\Delta = -\int_{\lambda}^{\lambda'} (v - l + lx - p_1) dx - \int_0^{p_2/c} (v - l + lx - p_2) dx$. Hence, $\Delta > -\int_{\lambda}^{\lambda'} (v - l + lx) dx - \int_0^{p_2/c} (v - l + lx) dx = -\int_0^{\lambda'} (v - l + lx) dx + \int_{p_2/c}^{\lambda} (v - l + lx) dx = -\lambda' [v - l + \frac{1}{2}\lambda'] + (\lambda - \frac{p_2}{c}) [v - l + \frac{1}{2}(\lambda + \frac{p_2}{c})]$. Note that $v - l + \frac{1}{2}\lambda < 0$ and $\lambda > \frac{p_2}{c}$. If $v - l + \frac{1}{2}(\lambda + \frac{p_2}{c}) > 0$, then $\Delta > 0$ and hence a contradiction. Suppose $v - l + \frac{1}{2}(\lambda + \frac{p_2}{c}) \leq 0$, $\Delta \geq -\lambda' [v - l + \frac{1}{2}\lambda'] + \lambda' [v - l + \frac{1}{2}(\lambda + \frac{p_2}{c})] = -\lambda' \frac{l}{2} [\lambda' - \lambda - \frac{p_2}{c}] = -\lambda' \frac{l}{2} \left[\frac{c - \varphi K}{\varphi K} \frac{p_2}{c} - \frac{c}{\varphi K} \left(\lambda - \frac{p_1 - p_2}{\varphi K} \right) \right]$. Since $\frac{c - \varphi K}{\varphi K} < 0$ and $\lambda - \frac{p_1 - p_2}{\varphi K} \geq 0$, we also have $\Delta > 0$. Therefore, the shutout contract $(\lambda', p_1, 0)$ dominates (λ, p_1, p_2) . Thus, the result follows. \square

Proof (Lemma 2) The result is straightforward to prove when the state is not collusive. Now assume that the state is collusive. Then, a supplier with process quality level $x \geq \lambda$ will participate when the production cost is no more than the contract payment, i.e., $cx \leq p$. Thus, a supplier with process quality $\lambda \leq x \leq \frac{p}{c}$ will participate. On the other hand, a supplier with process quality $x < \lambda$ will collude and participate if the condition $p - cx - \varphi K(\lambda - x) \geq 0$ holds, for which we discuss two cases. Case 1: $K > c/\varphi$. In this case, we obtain that $p - cx - \varphi K(\lambda - x) \geq 0$ is equivalent to $x \geq \lambda - \frac{p - c\lambda}{\varphi K - c}$. Case 2: $K \leq c/\varphi$. In this case, we can show that $p - cx - \varphi K(\lambda - x) \geq c\lambda - cx - \varphi K(\lambda - x) = (c - \varphi K)(\lambda - x) \geq 0$ holds. Thus, the result follows. \square

Proof (Proposition 2) We first consider the case: $K > c/\varphi$. We will show that the equilibrium payment is $p^* = c$ by the following two claims.

Claim 1: In equilibrium, $p^ \leq c$. Proof.* We prove the claim by contradiction. Consider the equilibrium (λ^*, p^*) and assume $p^* > c$. We show below that the solution is inferior to another, leading to a contradiction. We discuss two cases. Case 1: $p^* > \varphi K\lambda^*$. In this case, the buyer's profit function is

$$\Pi(\lambda, p) = (1 - \gamma) \int_{\lambda}^1 (v - l(1 - x) - p) dx + \gamma \int_0^1 (v - l(1 - x) - p) dx.$$

We can compute $\frac{\partial \Pi(\lambda^*, p^*)}{\partial p} = -1 < 0$, which implies that the buyer is better off reducing the payment p^* slightly. Hence, a contradiction. Case 2: $p^* \leq \varphi K\lambda^*$. In this case, the buyer's profit function is

$$\Pi(\lambda, p) = (1 - \gamma) \int_{\lambda}^1 (v - l(1 - x) - p) dx + \gamma \int_{\frac{\varphi K\lambda - p}{\varphi K - c}}^1 (v - l(1 - x) - p) dx,$$

from which we can compute

$$\frac{\partial \Pi(\lambda, p)}{\partial p} = -(1 - \gamma)(1 - \lambda) - \gamma \left(1 - \frac{\varphi K\lambda - p}{\varphi K - c} \right)$$

$$\begin{aligned} & +\gamma \frac{1}{\varphi K - c} \left[v - l \left(1 - \frac{\varphi K \lambda - p}{\varphi K - c} \right) - p \right] \\ \frac{\partial \Pi(\lambda, p)}{\partial \lambda} & = -(1 - \gamma)(v - l(1 - \lambda) - p) \\ & - \gamma \frac{\varphi K}{\varphi K - c} \left[v - l \left(1 - \frac{\varphi K \lambda - p}{\varphi K - c} \right) - p \right]. \end{aligned}$$

Then we divide the discussion into two subcases. Subcase 1: $p^* = \varphi K \lambda^*$. In this subcase, we have $\frac{\partial \Pi(\lambda^*, p^*)}{\partial p} = -(1 - \gamma)(1 - \lambda) - \gamma + \frac{\gamma}{\varphi K - c}(v - l - p^*) < 0$, due to $l > v$. Thus, the buyer will be better off reducing the payment p^* slightly, which leads to a contradiction. Subcase 2: $p^* < \varphi K \lambda^*$. In this subcase, due to the optimality of (λ^*, p^*) , we have $\frac{\partial \Pi(\lambda^*, p^*)}{\partial \lambda} \geq 0$ (more specifically, $\frac{\partial \Pi(\lambda^*, p^*)}{\partial \lambda} = 0$ when $\frac{p^*}{\varphi K} < \lambda^* < 1$, and $\frac{\partial \Pi(\lambda^*, p^*)}{\partial \lambda} \geq 0$ when $\lambda^* = 1$). Then, a detailed investigation of $\frac{\partial \Pi(\lambda^*, p^*)}{\partial \lambda}$ indicates $\frac{\varphi K}{\varphi K - c} \left[v - l \left(1 - \frac{\varphi K \lambda - p}{\varphi K - c} \right) - p \right] < 0$ because $v - l(1 - \lambda) - p > v - l \left(1 - \frac{\varphi K \lambda - p}{\varphi K - c} \right) - p > v - l \left(1 - \frac{\varphi K \lambda - p}{\varphi K - c} \right) - p$, where the inequalities come from $p > c \geq c\lambda$. As a result, $\frac{\partial \Pi(\lambda^*, p^*)}{\partial p} < 0$. That is, the buyer is better off reducing the payment p^* slightly. Hence, a contradiction.

Claim 2: In equilibrium, $p^ \geq c$. Proof.* We prove the claim by contradiction. Consider the equilibrium (λ^*, p^*) and assume $p^* < c$. We will show below that the solution is inferior to another, leading to a contradiction. To that end, we discuss two cases. Case 1: $p^* < \varphi K \lambda^*$. In this case, the buyer's profit function is

$$\Pi(\lambda, p) = (1 - \gamma) \int_{\lambda}^{\frac{p}{c}} (v - l(1 - x) - p) dx + \gamma \int_{\frac{\varphi K \lambda - p}{\varphi K - c}}^{\frac{p}{c}} (v - l(1 - x) - p) dx.$$

We will show $\Pi(\lambda^*, p^*) < \Pi\left(\lambda^* + 1 - \frac{p^*}{c}, c\right)$. To that end, we define

$$g(m) \triangleq \Pi\left(\lambda^* + m \left(1 - \frac{p^*}{c}\right), p^* + m(c - p^*)\right).$$

Thus, we aim to show equivalently $g(0) < g(1)$. By the Mean Value Theorem and the chain rule, we obtain that there exists $\tau \in (0, 1)$ such that

$$\begin{aligned} g(1) - g(0) & = \frac{\partial \Pi\left(\lambda^* + \tau \left(1 - \frac{p^*}{c}\right), p^* + \tau(c - p^*)\right)}{\partial \lambda} \left(1 - \frac{p^*}{c}\right) \\ & \quad + \frac{\partial \Pi\left(\lambda^* + \tau \left(1 - \frac{p^*}{c}\right), p^* + \tau(c - p^*)\right)}{\partial p} (c - p^*) \\ & = \left[(1 - \gamma) + \gamma \frac{\varphi K}{\varphi K - c} \right] \left(\frac{l}{c} - 1\right) \left(\frac{p^* + \tau(c - p^*)}{c} - \lambda^*\right) (c - p^*) > 0 \end{aligned}$$

where the inequality comes from $l > c$, $c > p^*$, and $p^* \geq c\lambda^*$. Hence, compared to (λ^*, p^*) , the buyer is better off with $\left(\lambda^* + 1 - \frac{p^*}{c}, c\right)$, which leads to a contradiction. Case 2: $p^* > \varphi K \lambda^*$. In this case, the buyer's profit function is

$$\Pi(\lambda, p) = (1 - \gamma) \int_{\lambda}^{\frac{p}{c}} [v - l(1 - x) - p] dx + \gamma \int_0^{\frac{p}{c}} [v - l(1 - x) - p] dx.$$

Then we divide the discussion into two subcases. Subcase 1: $\lambda^* > 0$. In this subcase, the optimal solution (λ^*, p^*) is an interior point of the region $\{(\lambda, p) : p < c, \lambda > 0, p > \varphi K \lambda\}$. Thus, the Hessian matrix $\nabla^2 \Pi(\lambda^*, p^*)$ should be negative semi-definite, for which a necessary condition is $|\nabla^2 \Pi(\lambda^*, p^*)| \geq 0$, or equivalently,

$$\frac{\partial^2 \Pi(\lambda^*, p^*)}{\partial \lambda^2} \frac{\partial^2 \Pi(\lambda^*, p^*)}{\partial p^2} - \left[\frac{\partial^2 \Pi(\lambda^*, p^*)}{\partial \lambda \partial p} \right]^2 = -(1-\gamma) l \frac{l-2c}{c^2} - (1-\gamma)^2 \geq 0.$$

However, the above inequality never holds since $l > 2c$. Hence, a contradiction. Subcase 2: $\lambda^* = 0$. Note we have $\frac{\partial \Pi(0, p^*)}{\partial \lambda} = (1-\gamma)[(l-v) + p^*] > 0$, which implies that the buyer can be better off increasing the value of λ slightly. Hence, a contradiction as well. In short, Claim 2 holds, that is, the buyer's payment p^* is no less than c .

Next, we proceed to solve for the equilibrium supplier process quality requirement λ^* . We consider two cases, $\varphi K \lambda \geq c$ and $\varphi K \lambda \leq c$, for which the respective optimizations that maximize the buyer's profit are

$$\max_{\lambda \geq \frac{c}{\varphi K}} \left\{ (1-\gamma) \int_{\lambda}^1 (v-c-l(1-x)) dx + \gamma \int_{\frac{\varphi K \lambda - c}{\varphi K - c}}^1 (v-c-l(1-x)) dx \right\},$$

and

$$\max_{\lambda \leq \frac{c}{\varphi K}} \left\{ (1-\gamma) \int_{\lambda}^1 (v-c-l(1-x)) dx + \gamma \int_0^1 (v-c-l(1-x)) dx \right\}.$$

The unconstrained solution to the first optimization is $\lambda_1 := 1 - \frac{(\varphi K - c + \gamma c)(\varphi K - c)}{(\varphi K - c + \gamma c)(\varphi K - c) + \gamma \varphi K c} \cdot \frac{v-c}{l}$, which yields a payoff of $\Pi_1 := \frac{(\varphi K - c + \gamma c)^2 l}{(\varphi K - c + \gamma c)^2 l + \gamma (l - \gamma) c^2} \cdot \frac{(v-c)^2}{2l}$. The unconstrained solution to the second optimization is $\lambda_2 := 1 - \frac{v-c}{l}$, which yields a payoff of $\Pi_2 := (1-\gamma) \frac{(v-c)^2}{2l} - \gamma \frac{l-2(v-c)}{2}$. Note that it is straightforward to prove $\lambda_2 < \lambda_1$. Based on it, our discussion can be divided into three cases. Case 1: $\lambda_2 < \frac{c}{\varphi K} < \lambda_1$. In this case, the buyer's profit function is bimodal in λ . Thus, we need to compare Π_1 and Π_2 . One can show that the optimal process quality requirement is $\lambda^* = \lambda_1$ if $\frac{\varphi K(2c-2\gamma c-\varphi K)}{(1-\gamma)lc^2} \leq \frac{l-2v+2c}{(l-v+c)^2}$. Case 2: $\frac{c}{\varphi K} \leq \lambda_2 < \lambda_1$. The buyer's profit function is unimodal in λ . In this case, the optimal process quality requirement is $\lambda^* = \lambda_1$. In this case, $\frac{\varphi K(2c-2\gamma c-\varphi K)}{(1-\gamma)lc^2} \leq \frac{l-2v+2c}{(l-v+c)^2}$. Case 3: $\lambda_2 < \lambda_1 \leq \frac{c}{\varphi K}$. The buyer's profit function has two maximal points in λ . Thus, the optimal quality requirement is $\lambda^* = \lambda_2$. In this case, $\frac{\varphi K(2c-2\gamma c-\varphi K)}{(1-\gamma)lc^2} > \frac{l-2v+2c}{(l-v+c)^2}$. Note that $\frac{\varphi K(2c-2\gamma c-\varphi K)}{(1-\gamma)lc^2} \leq \frac{l-2v+2c}{(l-v+c)^2}$ is equal to $\gamma > \underline{\gamma}(K)$ when $K > c/\varphi$. Moreover, it's easy to see that when $\lambda^* = \lambda_1$, λ^* is decreasing in K and increasing in γ . Combining the above results, we can obtain the desired result for $K > c/\varphi$.

Finally, we examine the case in which $K \leq c/\varphi$. We can show that the equilibrium payment $p^* = c$ continues to hold for this case by following the proof stated above. In addition, due to $K \leq c/\varphi$, the buyer's profit function is Π_2 if it yields a positive value. Therefore, the buyer's

equilibrium profit is Π_2 and the equilibrium process quality requirement is $\lambda^* = \lambda_2$ if $\Pi_2 > 0$, or equivalently, if $\gamma > \bar{\gamma}(K)$; otherwise, the buyer does not source from the supplier. Combining the above cases, our results follow. \square

Proof (Proposition 3) According to Proposition 2, it is straightforward to show $\mu^* = \gamma$ and $\eta^* = 0$ in region Ω_1 . In region Ω_2 , by definition, $\eta^* = \frac{(1-\gamma)\gamma\varphi Kc}{(\varphi K)^2 - 2(1-\gamma)\varphi Kc + (1-\gamma)c^2} < 1 - \gamma$, which is decreasing in K . Similarly, by definition, $\mu^* = \frac{v-c}{l-v+c} \frac{\gamma(1-\gamma)(\varphi K-c)c}{(1-\gamma)(\varphi K-c)^2 + \gamma(\varphi K)^2} < \gamma$. Note that $\text{sgn}\left[\frac{d\mu^*}{dK}\right] = \text{sgn}\left[-(\varphi K)^2 + 2c\varphi K - (1-\gamma)c^2\right]$. Then, we have our desired results for η^* in respect of K . Directly from formulations of η^* and μ^* derived, we have their respective asymptotic properties as $K \rightarrow \infty$. \square

Proof (Proposition 4) We can divide the discussion into the following three cases based on regions. In region Ω_1 , the efficiency loss can be written by $L = \Pi^0 - \Pi^* + \int_{\lambda^0}^1 (c - cx)dx - \gamma \int_{\lambda^0}^1 (c - cx)dx - (1 - \gamma) \int_{\lambda^*}^1 (c - cx)dx$. Note that $\Pi^0 - \Pi^* > 0$. In what follows, we show that $\int_{\lambda^0}^1 (c - cx)dx \geq \gamma \int_{\lambda^0}^1 (c - cx)dx + (1 - \gamma) \int_{\lambda^*}^1 (c - cx)dx$. According to the definition, $\int_{\lambda^0}^1 (c - cx)dx = \frac{(v-c)^2 c}{2l^2}$. When (K, γ) falls in Ω_1 , $\gamma \int_{\lambda^0}^1 (c - cx)dx + (1 - \gamma) \int_{\lambda^*}^1 (c - cx)dx = \frac{1}{2}\gamma c + (1 - \gamma) \frac{(v-c)^2 c}{2l^2}$. Because $l > 2(v - c)$, we always have $L > 0$. In this case, L are constant in K . When (K, γ) falls in Ω_2 , it can be shown that $\gamma \int_{\lambda^0}^1 (c - cx)dx + (1 - \gamma) \int_{\lambda^*}^1 (c - cx)dx = \frac{(\varphi K - c + \gamma c)^2 [\varphi^2 K^2 - 2(1-\gamma)\varphi Kc + (1-\gamma)c^2] (v-c)^2 c}{[(\varphi K - c + \gamma c)(\varphi K - c) + \gamma Kc]^2} \frac{(v-c)^2 c}{2l^2}$. It can be further shown that $\frac{(\varphi K - c + \gamma c)^2 [\varphi^2 K^2 - 2(1-\gamma)\varphi Kc + (1-\gamma)c^2]}{[(\varphi K - c + \gamma c)(\varphi K - c) + \gamma Kc]^2} \leq 1$ and the equality sign holds if and only if $\gamma = 1$. Hence, $L \geq 0$ and the equality sign holds if and only if $\gamma = 1$ or 0. Moreover, we can further show that L is decreasing in K . Then, the desired results follow. \square

Proof (Proposition 5) We start with a general screening contract. Following the analysis in §3.1, $p_1 \geq c\lambda + (1 - \tau)h$ and $p_2 \leq c\lambda$ in equilibrium. Then, the buyer's optimization problem can be written as follows

$$\Pi^h = \max_{\substack{0 \leq \lambda \leq 1 \\ p_1 \geq c\lambda + (1-\tau)h \\ 0 \leq p_2 \leq c\lambda}} \left\{ \int_{\lambda}^{\min\{(p_1 - (1-\tau)h)/c, 1\}} (v - l + lx - p_1 - \tau h) dx + \int_0^{\min\{\lambda, p_2/c\}} (v - l + lx - p_2) dx \right\}$$

Let $(\lambda^h, p_1^h, p_2^h)$ be the optimal solution to the above problem. The following claim summarizes the buyer's equilibrium contracting strategy with in-house audit.

Claim: When $v \leq c + h$, the buyer does not offer a contract. Otherwise, $\lambda^h = 1 - (v - c - h)/l$, $p_1^h = c + (1 - \tau)h$ and $p_2^h = 0$. In this case, the buyer's equilibrium profit is $\Pi^h = (v - c - h)^2 / (2l) > 0$.

Proof. According to the assumption of $l > 2v$ condition, $\int_0^{p_2/c} (v - l + lx - p_2) dx < 0$ for any $p_2 > 0$.

Therefore, the buyer is better off setting $p_2 = 0$. With this observation, the buyer's problem can be simplified as $\Pi^h = \max_{c\lambda + (1-\tau)h \leq p_1 \leq c + (1-\tau)h} \int_{\lambda}^{\frac{p_1 - (1-\tau)h}{c}} (v - l + lx - p_1 - \tau h) dx$. We denote the buyer's profit by $\Pi(\lambda, p; h)$, that is, $\Pi(\lambda, p; h) = \int_{\lambda}^{\frac{p - (1-\tau)h}{c}} (v - l + lx - p - \tau h) dx$. We can compute $\Pi(\lambda, p; h) - \Pi\left(\lambda + 1 - \frac{p - (1-\tau)h}{c}, c + (1-\tau)h; h\right) = \left(\frac{l}{c} - 1\right) \left(\frac{p - (1-\tau)h}{c} - \lambda\right) (p - (1-\tau)h - c) < 0$ for any $p < c + (1-\tau)h$, which indicates the buyer's equilibrium contract payment must be $p^h = c + (1-\tau)h$ if $\Pi^h > 0$. By noting $\frac{\partial^2 \Pi(\lambda, c + (1-\tau)h; h)}{\partial^2 \lambda} < 0$, we obtain the equilibrium process quality requirement as $\lambda^h = \min\left(1 - \frac{v-c-h}{l}, 1\right)$ if $\Pi^h > 0$. Hence, when $v > c + h$, $\lambda^h = 1 - \frac{v-c-h}{l}$ and $\Pi^h = \frac{(v-c-h)^2}{2l}$; otherwise, the buyer does not offer a contract.

Based on the above claim, we can see that when $h > v - c$, the buyer would never prefer in-house audit. When $h < v - c$, in region Ω_0 , the buyer prefers in-house audit since in-house audit can yield a positive profit. In Ω_1 , by comparing their respective profits, it can be shown that the buyer prefers in-house audit if and only if $\gamma > \frac{h(2v-2c-h)}{(v-c-l)^2}$. Similarly, in Ω_2 , it can be shown that the buyer prefers in-house audit if and only if $K < \frac{c\left[(1-\gamma)\sqrt{h(2v-2c-h)} + \sqrt{\gamma(1-\gamma)(v-c-h)}\right]}{\varphi\sqrt{h(2v-2c-h)}}$. Moreover, it can be further verified that the inequality becomes $\gamma \geq \left(\frac{v-c-h}{v-c}\right)^2$ when $K = c/\varphi$. As a result, the buyer prefers third-party audit when $\gamma \geq \left(\frac{v-c-h}{v-c}\right)^2$. Then, we are done. \square

Proof (Proposition 6) The result in region Ω_1 is straightforward since $\theta^* = \gamma\left(1 - \frac{v-c}{l}\right)$. In region Ω_2 , the equilibrium collusion probability can be written as $\theta^* = \frac{\gamma(\varphi K - c + \gamma c)c}{(\varphi K - c + \gamma c)^2 + \gamma(1-\gamma)c^2} \cdot \frac{v-c}{l}$. Note that θ^* is concave in K and decreasing in K if $K > \left(1 - \gamma + \sqrt{\gamma(1-\gamma)}\right)c/\varphi$. Also note that $\left(1 - \gamma + \sqrt{\gamma(1-\gamma)}\right)c \leq c$ if and only if $\gamma > \frac{1}{2}$. By noticing $\lim_{K \rightarrow c/\varphi^+} \gamma(K) < \frac{1}{2}$, we then can divide the discussion into three cases. Case 1: $\gamma \geq \frac{1}{2}$. θ^* is decreasing in K since $\left(1 - \gamma + \sqrt{\gamma(1-\gamma)}\right)c \leq c$. Case 2: $\lim_{K \rightarrow c/\varphi^+} \gamma(K) \leq \gamma < \frac{1}{2}$. From Proposition 2, $(K, \gamma) \in \Omega_2$ for any $K > c/\varphi$. Note that $\left(1 - \gamma + \sqrt{\gamma(1-\gamma)}\right)c > c$. We have θ^* is first increasing and then decreasing in K . Case 3: $\gamma < \lim_{K \rightarrow c/\varphi^+} \gamma(K)$. In this case, $\theta^* = \gamma\left(1 - \frac{v-c}{l}\right)$ if $K < K'$ and $\theta^* = \frac{\gamma(\varphi K - c + \gamma c)c}{(\varphi K - c + \gamma c)^2 + \gamma(1-\gamma)c^2} \cdot \frac{v-c}{l}$ if $K \geq K'$, where $\underline{\gamma}(K') = \gamma$. It can be shown that $\underline{\gamma}\left(\left(1 - \gamma + \sqrt{\gamma(1-\gamma)}\right)c/\varphi\right) < \gamma$ if and only if $\gamma < \frac{1}{2} \left(\frac{v-c}{l-v+c}\right)^2$. Therefore, θ^* is first increasing and then decreasing in K for $\frac{1}{2} \left(\frac{v-c}{l-v+c}\right)^2 < \gamma < \lim_{K \rightarrow c/\varphi^+} \gamma(K)$ and decreasing in K for $\gamma \leq \frac{1}{2} \left(\frac{v-c}{l-v+c}\right)^2$. Moreover, from Proposition 2, it is straightforward to see that Π^* is increasing in K . Combining all the above results, we have proved the result of Ω_2 . \square

Appendix B: Results and Proofs for §6 (Model Extensions)

The details can be found in the E-Companion of the paper, available at the following website: <http://ssrn.com/abstract=2812209>.