Appendix A: Omitted Details from Section 2

A.1. Additional Details on Empirical Analysis of FRUS Data (Section 2.1)

A.1.1. Donor Dropout We use a logistic regression to estimate the impact that matching outcomes have on donor dropout. We want to focus our analysis on the (large) subset of donations which are *intended* to recur; however, since we do not directly observe donors intentions, we instead restrict our attention to donations which have occurred for at least two consecutive weeks. Our analysis consists of 22,981 donation-weeks, which comprises 97 weeks and 906 different donations. For each donation d and week t, we observe the binary matching outcome $missed_{d,t}$, which is 1 if the donation was "missed" (i.e. if a donation route and time was posted to the FRUS website and app, but no volunteer signed up) and 0 if the donation was "picked up." A donation d is said to "drop out" after week t if it does not occur again for at least 6 weeks. This binary outcome is our response variable, $dropout_{d,t}$. We estimate the following model, where $\mathbf{X}_{d,t}$ are control variables which allow for location and seasonal fixed effects:

$$\text{logit}[\Pr(dropout_{d,t})] = \lambda_0 + \lambda_1 \cdot missed_{d,t} + \lambda_2 \cdot \mathbf{X}_{d,t}. \tag{18}$$

In Table 1, we report the results of our logistic regression, which estimates the impact of a missed donation on the dropout probability. The estimated effect is statistically significant at the $p < 0.001$ level, and is robust to controls on location and season. It is also robust to changes in our definition of dropout (e.g. if we assume a donation drops out if it does not occur again for 4 weeks). We omit further results for the sake of brevity.

	(1) *missed* only	(2) *missed* + Location FEs	(3) *missed* + Seasonal FEs	(4) *missed* + Loc./Seas. FEs
missed	1.232^{***}	1.375^{***}	1.221^{***}	1.373^{***}
	(0.267)	(0.273)	(0.267)	(0.273)
Constant	-5.759^{***}	-5.065^{***}	-5.783^{***}	-5.219^{***}
	(0.122)	(0.219)	(0.216)	(0.278)
Df Model	1	8	4	11
n	22981	22981	22981	22981

Note: $^{***}\, p < 0.001$

Table 1: Logistic regression estimating the impact of a missed donation on the likelihood of dropout.

A.1.2. Volunteer Dropout Due to the challenges of our censored dataset described in Section 2.1, we use the availability of donations as a link between match outcomes and volunteer dropout. If there are fewer availabilities, we expect volunteers who are not part of an adopted match to be less likely to find a compatible match, and thus be more likely to drop out. In contrast, we do not expect volunteers who are part of an adopted match to be impacted by the number of availabilities, since they are guaranteed a compatible match.

We focus our analysis on four large locations which together comprise a majority of the data. This allows us to track the size of the spot market, which is defined as the number of donations scheduled for within the next week that are currently available for sign-up. We use a logistic regression to estimate the impact of availabilities (i.e., the size of next week's spot market) on volunteer dropout, both for volunteers which are not part of an adopted match and for volunteers which are. For each volunteer v in location j that transports a donation in week t, we observe (i) whether they are part of an adopted match in week t, (ii) the size of the spot market in their location in week $t+1$ ($spot_{j,t+1}$), and (iii) whether they drop out following week t ($dropout_{v,j,t}$), which is defined as not transporting another donation for the next six weeks. Due to varying marketplace characteristics, we incorporate location-based fixed effects along with seasonal fixed effects into the vector $\mathbf{X}_{v,j,t}$. For volunteers who are not part of an adopted match in week t, we estimate the following:

$$\text{logit}[\Pr(dropout_{v,j,t})] = \lambda_0 + \lambda_1 \cdot spot_{j,t+1} + \lambda_2 \cdot \mathbf{X}_{v,j,t}. \tag{19}$$

	Vols. in one-time match		Vols. in adopted match	
	(1)	(2)	(3)	(4)
	spot + Location FEs	*spot* + Loc./Seas. FEs	*spot* + Location FEs	*spot* + Loc./Seas. FEs
spot	-0.035^{***}	-0.031^{***}	0.001	0.009
	(0.008)	(0.009)	(0.016)	(0.019)
Constant	-1.019^{***}	-1.123^{***}	-3.258^{***}	-3.394^{***}
	(0.134)	(0.176)	(0.256)	(0.353)
Df Model	4	7	4	7
n	3204	3204	5156	5156

Note: $^{***}p < 0.001$

Table 2: Logistic regression estimating the impact of the number of available donations in the spot market on the likelihood of volunteer dropout.

As reported in columns (1) and (2) of Table 2, we find that an additional donation in the spot market significantly decreases the dropout probability for volunteers who are not in an adopted match. We then repeat this analysis for volunteers in adopted matches, and we find that the impact of an additional available donation is negligible and not statistically significant (as seen in columns (3) and (4)). This supports our hypothesis that available donations only impact the dropout probability of volunteers who may not be able to find a match. As shown, these results are robust to the inclusion of seasonal fixed effects, and furthermore, the results are robust to changes in our definition of dropout (e.g. if we assume a volunteer drops out if they does not volunteer again for 4 weeks) and our temporal definition of availabilities (e.g. if we consider the size of the spot market over a slightly different seven-day window). We omit such results for the sake of brevity.

A.1.3. Asymmetric Volunteer Engagement We would like to directly assess the engagement of volunteers as a function of their match type. Unfortunately, we do not observe volunteers' visits to the platform unless they sign up for a donation (see Section 2.1), which means that we can only measure a volunteer's engagement by the number of donations they complete. Despite this limitation, in the following, we provide aggregate as well as location-specific evidence that suggests volunteers in non-adopted matches can be more engaged. We also discuss how the censored nature of our data may impact the analysis.

If volunteers are highly engaged but cannot find a compatible match, we will not observe their high level of engagement. To sidestep this limitation, we focus on the subset of volunteers who transported one donation d in week t and have the option to sign up for that same donation d in week $t+1$ (for volunteers in an adopted match, this is almost always the case). We then assess the number of donations completed by these volunteers in week $t+1$ as a function of their match type in period t. We focus on four large locations which together comprise a majority of the data, and for each location, we use a linear regression where the dependent variable $donations_{v,t+1}$ is the number of donations completed by volunteer v during week $t+1$.

Before presenting the regression analysis, in Figure 7 we show the distribution of $donations_{v,t+1}$ separated by match type. We observe that (i) a vast majority of volunteers in an adopted match only complete one match — the one they are committed to; (ii) those volunteers also have a higher retention rate; (iii) however, interestingly, volunteers in non-adopted matches are more likely to complete 2+ donations, compelling evidence that they are more engaged. In fact, due to this extra engagement, on average, those volunteers complete more donations in the subsequent week compared to their counterparts in adopted matches (1.09 vs 0.94).

For this subset, we now estimate a simple model which depends on an indicator variable $non\text{-}adopt_{v,t}$ that is 1 if v is not part of an adopted match in week t,

$$donations_{v,t+1} = \lambda_0 + \lambda_1 \cdot non\text{-}adopt_{v,t}. \tag{20}$$

Due to potential differences in volunteer characteristics across markets, we conduct this estimation separately for each of the four locations. The coefficient on $non\text{-}adopt_{v,t}$ represents the increased engagement provided by volunteers who are not part of an adopted match relative to volunteers who are part

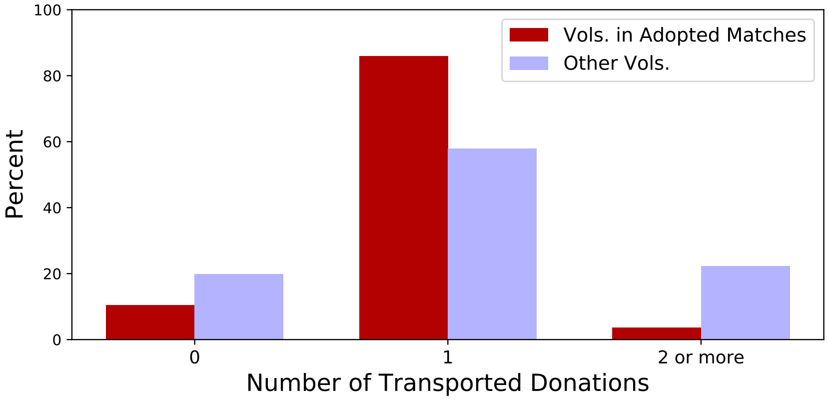

Figure 7: Of the volunteers in Locations (a), (b), (c), and (d) who transported one donation in week t and have the same donation available to them in week $t+1$, we show the percentage who sign up for 0, 1, or 2+ donations in week $t+1$, separated by their match type in week t.

	Location (a)	Location (b)	Location (c)	Location (d)
non-adopt	-0.292^{**}	0.249^{***}	-0.034	0.489^{***}
	(0.110)	(0.061)	(0.054)	(0.095)
Constant	1.019^{***}	0.899^{***}	0.948^{***}	0.900^{***}
	(0.013)	(0.014)	(0.011)	(0.030)
Df Model	1	1	1	1
n	753	1254	1468	357

Note: $^{**}p < 0.01$; $^{***}p < 0.001$

Table 3: Linear regression estimating the impact of match type on the number of completed donations.

of an adopted match. These results are robust to the inclusion of a seasonal fixed effect term, though we omit such results for the sake of brevity. Table 3 shows that in two of the four locations, the coefficient on $non\text{-}adopt_{v,t}$ is positive and statistically significant, which implies that in those locations, volunteers who were in one-time matches in period t are more engaged in period $t+1$ than volunteers who were in adopted matches.

We claim that the estimates in Table 3 represent a lower bound on the difference in engagement between volunteers in one-time matches and volunteers in adopted matches (for each location). To see why, note that the number of completed donations, $donations_{v,t+1}$, can only serve as a lower bound on volunteer v's true engagement level in period $t+1$, $engagement_{v,t+1}$. There is the potential for a gap between these two quantities when the engagement level of the volunteer is greater than 1: if the volunteer is willing to complete multiple donations, they may be unable to find enough compatible donations in the spot market. Volunteers in one-time matches are, on average, more likely to complete multiple donations (this is true in each location and shown in aggregate in Figure 7).

As a result, if we were able to directly observe $engagement_{v,t}$ and use it as the dependent variable in Equation (20) instead of $donations_{v,t}$, then we would expect our estimate for the coefficient on $non\text{-}adopt_{v,t}$ to be larger. In other words, because we cannot always observe the true engagement level of volunteers when their engagement is large, we are disproportionately underestimating the engagement level of volunteers in one-time matches.

Consistent with this claim, we observe that our estimates in Table 3 have a strong negative correlation with the percentage of donations that are adopted in that location. In Location (a), where 78% of donations are adopted, it is likely quite difficult for a volunteer to find a compatible match in the spot market, in which case we cannot observe an increase in their engagement. However, in locations where it is easier to find a compatible match in the spot market (e.g., in Location (d), where only 37% of donations are adopted), we are more likely to observe an increase in a volunteer's engagement. This suggests that the differences in estimates across locations may simply reflect different degrees of underestimation.

In a setting where volunteers are homogeneous and the selection into an adopted match can be viewed as a random assignment (i.e., the population of volunteers in adopted matches is identical to the population of volunteers in one-time matches), then this approach provides us with a reasonable lower bound on the non-adoption growth benefit in our base model, i.e., the causal impact of match type on volunteer engagement. However, we emphasize that if volunteers are heterogeneous in their engagement behavior and endogenously determine their match type (as we model in Section 4.1), then our approach cannot identify the causal impact of match type. In such a setting, disallowing adoption would only impact the volunteers *who chose to form an adopted match*, and we cannot estimate their unobserved counterfactual engagement behavior. In the absence of experimental evidence, different scenarios are plausible (they could become more engaged, less engaged, or not change their engagement behavior). This can have implications for the optimal platform design, as we elaborate on in Section 4.1.

To shed more light on the effects of asymmetric volunteer engagement, we would like to be able to directly attribute higher rates of missed donations to higher adoption rates. Unfortunately, there is a notable lack of temporal variation in the adoption level within locations. Since there is insufficient variation in the adoption level *within* locations, we instead look for differences *across* locations in terms of adoption rates and missed donations. We focus our analysis on one full year for the seven locations with at least 1000 total donations in our dataset.

For these locations, we find a correlation of $\rho = 0.604$ between the average adoption level (i.e., the fraction of matches which were adopted) in a location and the fraction of donations which were missed in that location in that year. We expect that the relationship between the adoption fraction and missed donations is caused by a lack of engagement in the spot market. To lend credibility to this hypothesis, we find a correlation of $\rho = 0.824$ between the adoption level in a location and the fraction of donations *in the spot market* which were missed in that location. We note that there are limitations to this analysis since we only have sufficient data on a handful of locations, and there can be other differences across locations which could confound the relationship.

A.2. Proof of Proposition 1

To aid in this proof, we will use constants Δ_t and Φ_t in bounding the deviation of the state variables from their expectation. We will also use of the following lemma, which we prove in Appendix B.1.

LEMMA 2. *Let* $S_t^n = M_t^n - K_t^n$. *For each* $t \in [T]$, *if there exists some* C *such that* $D_t^n \le Cn$ *and* $V_t^n \le Cn$, *then there exists a constant* Δ_t *such that with probability* $1 - O(n^{1/4} e^{-c^6 n^{1/4}})$,

$$\left| \frac{S_t^n}{n} - \mu\left(\frac{D_t^n - K_t^n}{n}, \frac{V_t^n - K_t^n}{n} \right) \right| \le \Delta_t n^{-1/4}.$$

Given Δ_t, we define $\Phi_{-1} = 0$ and $\Phi_t := \sum_{\tau=0}^{t} 6^{t-\tau} \Delta_\tau$ for all $t \in [T]$. We now prove by induction that for all $t \in [T]$, with probability $1 - O(n^{1/4} e^{-c^6 n^{1/4}})$,

$$\left| \frac{K_t^n}{n} - k_t \right| \le \Phi_{t-1} n^{-1/4}, \quad \left| \frac{D_t^n}{n} - d_t \right| \le 2\Phi_{t-1} n^{-1/4}, \quad \left| \frac{V_t^n}{n} - v_t \right| \le 3\Phi_{t-1} n^{-1/4}, \quad \left| \frac{M_t^n}{n} - m_t \right| \le \Phi_t n^{-1/4}. \tag{21}$$

As a base case, when $t = 0$ the first three inequalities hold by definition and the final inequality follows from an application of Lemma 2, using the facts that $\frac{M_0^n}{n} - m_0 = \frac{S_0^n + K_0^n}{n} - (s_0 + k_0) = \frac{S_0^n}{n} - s_0$ and $\Phi_0 = \Delta_0$. We now assume that all four inequalities hold for $t = \tau$. We will prove that they also hold for $t = \tau + 1$. Now

$$\begin{aligned} \left| \frac{K_{\tau+1}^n}{n} - k_{\tau+1} \right| &\le \left| \frac{K_{\tau+1}^n}{n} - \mathbb{E}\left[\frac{K_{\tau+1}^n}{n} \mid M_\tau^n \right] \right| + \left| \mathbb{E}\left[\frac{K_{\tau+1}^n}{n} \mid M_\tau^n \right] - k_{\tau+1} \right| \\ &= \frac{1}{n} \left| K_{\tau+1}^n - (1-\gamma) z_\tau M_\tau^n \right| + (1-\gamma) z_\tau \left| \frac{M_\tau^n}{n} - m_\tau \right|. \end{aligned}$$

Recall that $K_{\tau+1}^n$ is a binomial random variable. Thus, using a Chernoff bound, with probability $1 - 2e^{n^{-1/2}}$ the first term is at most $n^{-5/4} \sqrt{3(1-\gamma) z_\tau M_\tau^n}$. By the inductive hypothesis, the second term is at most $(1-\gamma) z_\tau \Phi_\tau n^{-1/4}$ with probability $1 - O(n^{1/4} e^{-c^6 n^{1/4}})$, and we note that this implies $M_\tau = O(n)$. We now take a union bound over those two upper bounds. With probability $1 - O(e^{n^{-1/2}}) - O(n^{1/4} e^{-c^6 n^{1/4}})$, the distance between $\frac{K_{\tau+1}^n}{n}$ and $k_{\tau+1}$ is at most the sum of our two upper bounds. For large enough n, the second upper bound dwarfs the first, implying that with probability $1 - O\left(n^{1/4} e^{-c^6 n^{1/4}} \right)$, $\left| \frac{K_{\tau+1}^n}{n} - k_{\tau+1} \right| \le \Phi_\tau n^{-1/4}$. This proves by induction that the first inequality in Line (21) holds for all $t \in [T]$.

For the sake of brevity, we omit the nearly identical proofs for the second and third inequalities, which use Chernoff bounds two and three times, respectively, due to the additional binomial processes.

For the final inequality, we leverage the sublinearity of the matching function to bound the distance between the matching resulting from the actual state and the matching resulting from the deterministic approximation of the state.

$$\begin{aligned}\left|\frac{M^n_{\tau+1}}{n}-m_{\tau+1}\right| &\leq \left|\frac{M^n_{\tau+1}}{n}-\frac{K^n_{\tau+1}}{n}-\mu\left(\frac{D^n_{\tau+1}-K^n_{\tau+1}}{n},\frac{V^n_{\tau+1}-K^n_{\tau+1}}{n}\right)\right| \\ &\quad+\left|\frac{K^n_{\tau+1}}{n}+\mu\left(\frac{D^n_{\tau+1}-K^n_{\tau+1}}{n},\frac{V^n_{\tau+1}-K^n_{\tau+1}}{n}\right)-m_{\tau+1}\right| \\ &\leq \left|\frac{S^n_{\tau+1}}{n}-\mu\left(\frac{D^n_{\tau+1}-K^n_{\tau+1}}{n},\frac{V^n_{\tau+1}-K^n_{\tau+1}}{n}\right)\right|+\left|\frac{D^n_{\tau+1}}{n}-d_{\tau+1}\right|+\left|\frac{K^n_{\tau+1}}{n}-k_{\tau+1}\right|+\left|\frac{V^n_{\tau+1}}{n}-v_{\tau+1}\right| \qquad (22)\end{aligned}$$

The first term of (22) comes from cancelling out the certain adopted matches. The next three terms come from the fact that $\frac{\partial m_t}{\partial d_t}$, $\frac{\partial m_t}{\partial k_t}$ and $\frac{\partial m_t}{\partial v_t}$ are all in [0,1] (see Appendix B.4). We have concentration results for all three of these terms, and we thus take a union bound over those concentration results. With probability $1-O(n^{1/4}e^{-c^6 n^{1/4}})$, the sum of those three terms is upper-bounded by $6\Phi_\tau n^{-1/4}$, which also implies that $D^n_{\tau+1}\leq Cn$ and $V^n_{\tau+1}\leq Cn$ for some large C. This allows us to apply Lemma 2 to the first term of (22): with probability $1-O\left(n^{1/4}e^{-c^6 n^{1/4}}\right)$, it is at most $\Delta_{\tau+1}n^{-1/4}$. Combining these two bounds via a union bound, we have with probability $1-O(n^{1/4}e^{-c^6 n^{1/4}})$,

$$\left|\frac{M^n_{\tau+1}}{n}-m_{\tau+1}\right|\leq 6\Phi_\tau n^{-1/4}+\Delta_{\tau+1}n^{-1/4}=\Phi_{\tau+1}n^{-1/4}.$$

This completes the proof by induction (we note that the number of union bounds taken is $O(T)=O(1)$). To show that our convergence results hold for all $t\in[T]$, we take a union bound over the $4T$ state variables. Again noting that $T=O(1)$, we have with probability $1-O(n^{1/4}e^{-c^6 n^{1/4}})$, for all $t\in[T]$,

$$\left|\frac{K^n_t}{n}-k_t\right|\leq O\left(n^{-1/4}\right), \qquad \left|\frac{D^n_t}{n}-d_t\right|\leq O\left(n^{-1/4}\right), \qquad \left|\frac{V^n_t}{n}-v_t\right|\leq O\left(n^{-1/4}\right), \qquad \left|\frac{M^n_t}{n}-m_t\right|\leq O\left(n^{-1/4}\right).$$

This completes the proof of almost sure convergence.

A.3. Discussion of Figure 3

Figure 3 provides an example where different static policies for the adoption level lead to extremely divergent trajectories of market growth. To develop intuition into the key drivers of this divergence, first note that in our model of a random matching market with homogeneous compatibility, we would expect the efficiency of the spot market to improve with market thickness. (We formally establish the positive impact of market thickness on matching outcomes in Proposition 2.) Under a static policy of no adoption ($z=0$) — where everyone participates in the spot market in each period — this leads to a positive feedback loop: as the spot market grows, it becomes more efficient and the fraction of missed donations decreases, which leads to increased engagement and further increases the size of the market in the next period. Under a static policy of full adoption ($z=1$), the fraction of missed donations is initially quite low due to the benefits of match certainty, which leads to sharp initial growth. However, its spot market remains small and relatively inefficient, which inhibits long-term growth. As we formalize in Section 3.4, the difference in the eventual growth rates of these two policies is driven by the positive non-adoption growth benefit in this example: $\gamma - \alpha + \alpha' = 0.09$. In other words, the size of the volunteer pool increases (in expectation) if an adopted match is replaced with a one-time match.

In contrast to the policies at the two extremes of the adoption level decision, a static policy which sets a fractional adoption level of $z = 0.5$ leads to the decay of the marketplace. This market failure stems from the somewhat surprising observation that the fraction of missed donations is initially higher under this policy than the two alternatives. (In Theorem 1, we formally show that the number of matches in the first period is quasi-convex in the adoption level decision.) This sub-optimal initial policy leads to a vicious cycle: missed donations harm engagement, which reduces market thickness and consequently market efficiency, and eventually leads to a further increase in the fraction of missed donations.

Appendix B: Omitted Details from Section 3

B.1. Proof of Proposition 2

Based on the greedy random matching process, if there are na available donations when a volunteer arrives, the volunteer will form a match with probability $1 - (1 - \frac{c}{n})^{na}$. As a consequence, the expected number of available donations when the next volunteer arrives is given by $na - 1 + (1 - \frac{c}{n})^{na}$. This enables us to write a stochastic differential equation for the number of matched donations. We then show that the size of the matching converges to the solution of a deterministic differential equation as the market size grows, holding the ex ante number of compatible matches for donations and volunteers (i.e., the market thickness) fixed.

Suppose that there are na donations and nb volunteers in the spot market, where the match probability is given by $\frac{c}{n}$. Let $Y(Z) \in [na]$ be the number of matched donations right after the Z^{th} volunteer arrives, where $Z \in [nb]$. At the beginning of the process, there are no matches ($Y(0) = 0$), and according to the greedy matching process described above, the expected number of matches evolves as follows:

$$\mathbb{E}[Y(Z+1)|Y(Z)] = Y(Z) + 1 - \left(1 - \frac{c}{n}\right)^{n(a - \frac{Y(Z)}{n})}$$

If we define $y(z) = \frac{Y(nz)}{n}$, then for large n, $\mathbb{E}[y(z + \frac{1}{n}) - y(z)|y(z)]/(1/n) = 1 - e^{-c(a-y(z))} + o(1)$. As $n \to \infty$, this corresponds to the differential equation $\frac{dy}{dz} = 1 - e^{-c(a-y(z))}$. Given the initial condition $y(0) = 0$, this differential equation has the unique solution $y(z) = a + z - \frac{1}{c}\log(e^{ca} + e^{cz} - 1)$.

To show convergence (which will complete the proof of Proposition 2), we apply Theorem 5.1 from Wormald et al. (1999), and use the fact that the expected change in the number of matches can be characterized by

$$\mathbb{E}\left[Y(Z+1) - Y(Z) \mid Y(Z)\right] = 1 - \left(1 - \frac{c}{n}\right)^{n\left(a - \frac{Y(Z)}{n}\right)}.$$

Theorem 5.1 (Wormald et al. 1999) *Let $Q^{(n)+}$ represent the set of all possible sequences of matching outcomes for the nb volunteers, and let $g : Q^{(n)+} \to \mathbb{R}$ and $f : \mathbb{R}^2 \to \mathbb{R}$ such that $|g(h_z)| \leq nb$ for all $h_z \in Q^{(n)+}$ for all n. Let $Y(Z)$ denote the random counterpart of $g(h_z)$, and assume the following three conditions hold:*

(i) (Boundedness Hypothesis) For some $\beta \geq 1$, $|Y(Z+1) - Y(Z)| \leq \beta$.

(ii) (Trend Hypothesis) For some $\lambda_1 = o(1)$, $|E[Y(Z+1) - Y(Z)|Y(Z)] - f(\frac{Z}{n}, \frac{Y(Z)}{n})| \leq \lambda_1$ for all $Z \leq bn$.

(iii) (Lipschitz Hypothesis) The function f is continuous and satisfies a Lipschitz condition for a domain $D := \{(Z, Y(Z)) : Z \in (-\delta, bn + \delta), Y(Z) \in (-\delta, an + \delta)\}$.

Then the following are true:

(a) The differential equation $\frac{dy}{dz} = f(z, y(z))$ has a unique solution passing through $y(0) = 0$ which extends arbitrarily close to the boundary of D.

(b) Let $\lambda > \lambda_1$ with $\lambda = o(1)$. For a sufficiently large constant C, with probability $1 - O(\frac{\beta}{\lambda} exp(-\frac{n\lambda^3}{\beta^3}))$, $\frac{Y(Z)}{n} = y(z) + O(\lambda)$ for all $Z \in [0, \sigma n]$ where $y(z)$ is the solution described in (a) and σ is the supremum of those z to which the solution can be extended before reaching within l^∞-distance $C\lambda$ of the boundary of D.

We first note that for all Z, $\mathbb{E}[Y(Z+1) - Y(Z)] \leq 1$, since no more than one match can occur for each volunteer who visits the platform. This satisfies (i) the boundedness hypothesis. For (ii) the trend hypothesis, we define $f(\frac{Z}{n}, \frac{Y(Z)}{n}) = e^{(-c)\left(a - \frac{Y(Z)}{n}\right)}$, and we claim that $1 - \left(1 - \frac{c}{n}\right)^{n\left(a - \frac{Y(Z)}{n}\right)}$ is always within $\lambda_1 := \frac{c^2}{n}$ of $f(\frac{Z}{n}, \frac{Y(Z)}{n})$. To see this, note that for any $x \geq 0$, $e^{-x-x^2} \leq 1 - x \leq e^{-x}$. Setting $x = \frac{c}{n}$ and raising each term to the $n\left(a - \frac{Y(Z)}{n}\right)$ power yields

$$e^{\left(-c - \frac{c^2}{n}\right)\left(a - \frac{Y(Z)}{n}\right)} \leq \left(1 - \frac{c}{n}\right)^{n\left(a - \frac{Y(Z)}{n}\right)} \leq e^{-c\left(a - \frac{Y(Z)}{n}\right)}$$

$$e^{(-c)(a - \frac{Y(Z)}{n})}\left(1 - \frac{c^2(a - \frac{Y(Z)}{n})}{n}\right) \leq \left(1 - \frac{c}{n}\right)^{n\left(a - \frac{Y(Z)}{n}\right)} \leq e^{-c\left(a - \frac{Y(Z)}{n}\right)}$$

$$e^{(-c)\left(a - \frac{Y(Z)}{n}\right)}(1 - \frac{c^2}{n}) \leq \left(1 - \frac{c}{n}\right)^{n\left(a - \frac{Y(Z)}{n}\right)} \leq e^{-c\left(a - \frac{Y(Z)}{n}\right)}$$

Noting that $\frac{c^2}{n} e^{(-c)\left(a - \frac{Y(Z)}{n}\right)} \leq \frac{c^2}{n}$ for $Y(Z) \in [0, an]$ validates our claim that $1 - \left(1 - \frac{c}{n}\right)^{n\left(a - \frac{Y(Z)}{n}\right)}$ is always within λ_1 of $f(\frac{Z}{n}, \frac{Y(Z)}{n})$.

Finally, we note that the constant $L = c^2 e^{c\delta}$ satisfies (iii) the Lipschitz hypothesis for the function $1 - e^{-c(a - y(z))}$ over domain D. Given that the three conditions of Theorem 5.1 in Wormald et al. (1999) are met, we define $\lambda = c^2 n^{-1/4} > \lambda_1$ and apply the theorem to yield that for all $z \in [0, b]$, with probability $1 - O(n^{1/4} e^{-c^6 n^{1/4}})$, $\frac{Y(nz)}{n} = y(z) + O(n^{-1/4})$. Plugging in $z = b$ completes that proof that the expected matching converges almost surely to $a + b - \frac{1}{c}\log(e^{ca} + e^{cb} - 1)$.

Proof of Lemma 2 As a consequence of the proof of Proposition 2, we know that for any constants a and b, there exists a constant $\Delta(a,b)$ such that with probability $1-O(n^{1/4}e^{-c^6 n^{1/4}})$, $\left|\frac{S^n}{n}-\mu(a,b)\right| \le \Delta(a,b)n^{-1/4}$. If $D_t^n \le Cn$ and $V_t^n \le Cn$, then $\frac{D_t^n - K_t^n}{n} \le C$ and $\frac{V_t^n - K_t^n}{n} \le C$. In that case, if we define $\Delta_t = \max_{a,b\in[0,C]} \Delta(a,b)$, then with probability $1-O(n^{1/4}e^{-c^6 n^{1/4}})$, $\left|\frac{S_t^n}{n} - \mu\left(\frac{D_t^n - K_t^n}{n}, \frac{V_t^n - K_t^n}{n}\right)\right| \le \Delta_t n^{-1/4}$. This completes the proof.

B.2. Proof of Theorem 1

We begin by showing that $m_1(z_0)$ does not attain a local maximum. From (2)-(4), we have $\frac{\partial d_1}{\partial z_0} = 0$, $\frac{\partial k_1}{\partial z_0} = (1-\gamma)m_0 = D_k$, and $\frac{\partial v_1}{\partial z_0} = (\alpha - \alpha' - \gamma)m_0 = D_v$. Combining these derivatives with (1) and (5), we have

$$\begin{aligned} m_1 &= d_1 - k_1 + v_1 - \frac{1}{c}\log(e^{c(d_1-k_1)} + e^{c(v_1-k_1)} - 1) \\ \frac{\partial m_1}{\partial z_0} &= \frac{D_v e^{c(d_1-k_1)} + D_k - D_v}{e^{c(d_1-k_1)} + e^{c(v_1-k_1)} - 1} && (23) \\ \frac{\partial^2 m_1}{\partial z_0^2} &= \left(\frac{D_k^2 e^{c(d_1-k_1)} + (D_k - D_v)^2 e^{c(v_1-k_1)} - D_v^2 e^{c(d_1+v_1-2k_1)}}{(e^{c(d_1-k_1)} + e^{c(v_1-k_1)} - 1)^2}\right) c && (24) \end{aligned}$$

The first-order condition prescribed by (23) is equivalent to $e^{c(d_1-k_1)} = \frac{D_v - D_k}{D_v} = -\frac{1+\alpha'-\alpha}{\alpha-\alpha'-\gamma}$. When the FOC holds, the numerator of (24) reduces to $D_k^2 e^{c(d_1-k_1)} + D_k(D_k - D_v)e^{c(v_1-k_1)}$. This must be positive, since $D_k > D_v$ and $D_k > 0$. Thus, m_1 is convex at any critical point and consequently can have no local maxima as a function of z_0. This implies that the optimal solution must be at a boundary, i.e., $z_0 = 0$ or $z_0 = 1$.

To compare the values $m_1(0)$ and $m_1(1)$, we use a slight abuse of notation to augment (d_1, k_1, v_1) by z_t to compare them for $z_t = 0$ and $z_t = 1$. Since $d_1(0) = d_1(1) := d_1$ and $k_1(0) = 0$, the following are necessary and sufficient conditions for $m_1(1) \ge m_1(0)$:

$$\begin{aligned} & 0 \ge e^{c(v_1(0)-v_1(1)+k_1(1))} - \frac{e^{cv_1(0)} + e^{cd_1} - 1}{e^{c(v_1(1)-k_1(1))} + e^{c(d_1-k_1(1))} - 1} && (25) \\ \Leftrightarrow\quad & 0 \ge e^{c(D_k-D_v)}(e^{cd_1}e^{-cD_k} - 1) - e^{cd_1} + 1 && (26) \\ \Leftrightarrow\quad & 0 \ge e^{cd_1}(e^{-cD_v} - 1) - (e^{c(D_k-D_v)} - 1) && (27) \end{aligned}$$

Line (25) comes from applying the definitions in (1)-(5), Line (26) is algebraic, using the equalities $v_1(0) - v_1(1) + k_1(1) = D_k - D_v$ and $k_1(1) = D_k$, and Line (27) follows from rearranging. It is satisfied when either of the two conditions in (6) is met, which completes the proof.

B.3. Accuracy of Deterministic Approximations

Our insights in Section 3 are based on deterministic approximations to the underlying multi-dimensional Markov process. Propositions 1 and 2 imply that our deterministic approximations are accurate when the system is large. In this section, we numerically demonstrate the accuracy of our approximations for moderate system sizes, and we discuss how some of our main structural results continue to apply.

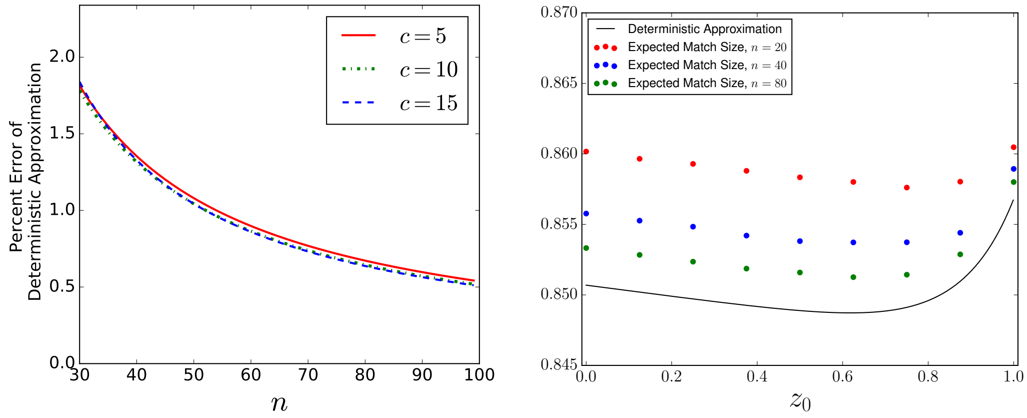

Figure 8: *Left*: Convergence to the deterministic approximation of the matching function when both sides of the market are of size n and compatibility probability is c/n. *Right*: Comparing the expected value of $m_1(z_0)$ to its deterministic approximation in an identical setting to the left panel of Figure 4, except with initial condition $(d_0, k_0, v_0) = (0.8, 0.8, 0.8)$ to ensure the integrality of nm_0.

Our numerical analysis shows that the matching function $\mu(a,b)$ defined in (5) is always an underestimate of the actual expected matching, but becomes increasingly accurate as the scaling factor n gets large. This is evident in the left panel of Figure 8, in which we show the normalized gap between the expected size of the matching in a market with two sides of equal size n and its corresponding deterministic approximation ($\mu(1,1)$, as defined in (5)). We observe that the normalized gap is always positive, and is below 2% when the size of both sides is at least 30.

Our numerical analysis also suggests that the structure of the optimal myopic policy remains unchanged: either fully allowing adoption or disallowing adoption will be optimal. In the right panel of Figure 8, we use simulation to find the expected number of matches for various levels of adoption and different values of the scaling factor n, and we compare those results to our deterministic approximation. We make the following observations from the plot: (i) the shape of $m_1(z_0)$, i.e., the deterministic approximation, is broadly consistent with the simulated results. However, (ii) the deterministic matching function is always an underestimate, and (iii) the gap is smallest at $z_0 = 1$, when most of the matching is pre-determined due to adoption. Observations (ii) and (iii) imply that the deterministic approximation relatively overvalues adoption. Hence we conjecture if the optimal myopic policy according to the deterministic approximation is $z_t = 0$ (i.e., if neither condition in (6) holds), then the optimal myopic policy is $z_t = 0$ when considering the actual expected matching.

B.4. Proof of Lemma 1

Consider two policies $\hat{\mathbf{z}} = \{\hat{z}_t : t \in [T]\}$ and $\tilde{\mathbf{z}} = \{\tilde{z}_t : t \in [T]\}$ that differ only at time τ, where $\hat{z}_\tau = z'_\tau$ and $\tilde{z}_\tau \neq z'_\tau$. Using identical notation to denote the state variables when following each policy, we have $\hat{m}_{\tau+1} \geq \tilde{m}_{\tau+1}$. When the condition in Lemma 1 holds, we have $\hat{v}_{\tau+1} \geq \tilde{v}_{\tau+1}$, and by the dynamics described in (2), $\hat{d}_{\tau+1} \geq \tilde{d}_{\tau+1}$. To complete the proof, we use Claim 1 to show these conditions imply $\hat{m}_t \geq \tilde{m}_t \, \forall t \in [T] \setminus [\tau]$.

Claim 1 **(Pathwise Dominance of Superior State)**. *Suppose the platform's decisions* z_t *are fixed for all* $t \in [T']$ *where* $T' \leq T$. *In that case, for any two initial states* (d_0, k_0, v_0) *and* (d_0', k_0', v_0'), *if* $d_0 \geq d_0'$, $v_0 \geq v_0'$, *and* $m_0 \geq m_0'$, *then for all* $t \in [T']$, $d_t \geq d_t'$, $v_t \geq v_t'$, *and* $m_t \geq m_t'$.

Proof of Claim 1 We proceed via induction. We have as a base case $d_0 \geq d_0'$, $v_0 \geq v_0'$, and $m_0 \geq m_0'$. Suppose this holds for all periods $t \leq \tau$. We will show it holds for $t = \tau + 1$. Based on the dynamics described in (2)-(4), since the platform's policy decision z_τ is assumed to be fixed, we have

$$\begin{aligned}
d_{\tau+1} &= (1-\beta)d_\tau + \beta' m_\tau && \geq \quad (1-\beta)d_\tau' + \beta' m_\tau' && \geq d_{\tau+1}' \\
k_{\tau+1} &= (1-\gamma)z_\tau m_\tau && \geq \quad (1-\gamma)z_\tau m_\tau' && \geq k_{\tau+1}' \\
v_{\tau+1} &= (1-\alpha)v_\tau + (\alpha'+\gamma')m_\tau + (\alpha-\alpha'-\gamma)z_\tau m_\tau && && \\
&\geq (1-\alpha)v_\tau' + (\alpha'+\gamma')m_\tau' + (\alpha-\alpha'-\gamma)z_\tau m_\tau' && && \geq v_{\tau+1}'
\end{aligned}$$

To complete the proof of the claim, we need to show that $m_{\tau+1}$ is non-decreasing in $d_{\tau+1}$, $v_{\tau+1}$, and $k_{\tau+1}$. We establish this directly, using the matching function defined in (5).

$$\begin{aligned}
\frac{\partial m_{\tau+1}}{\partial d_{\tau+1}} &= \quad 1 - \frac{e^{c(d_{\tau+1}-k_{\tau+1})}}{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})} - 1} && = \frac{e^{c(v_{\tau+1}-k_{\tau+1})} - 1}{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})} - 1} \in [0,1] \\
\frac{\partial m_{\tau+1}}{\partial v_{\tau+1}} &= \quad 1 - \frac{e^{c(v_{\tau+1}-k_{\tau+1})}}{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})} - 1} && = \frac{e^{c(d_{\tau+1}-k_{\tau+1})} - 1}{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})} - 1} \in [0,1] \\
\frac{\partial m_{\tau+1}}{\partial k_{\tau+1}} &= -1 + \frac{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})}}{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})} - 1} && = \frac{1}{e^{c(d_{\tau+1}-k_{\tau+1})} + e^{c(v_{\tau+1}-k_{\tau+1})} - 1} \in [0,1]
\end{aligned}$$

This shows that $m_{\tau+1} \geq m_{\tau+1}'$, which completes the proof of Claim 1. □

Since $\hat{z}_t = \tilde{z}_t$ for all $t \in [T] \setminus [\tau]$, we can apply Claim 1 to the initial states $(\hat{d}_{\tau+1} \, \hat{k}_{\tau+1}, \hat{v}_{\tau+1})$ and $(\tilde{d}_{\tau+1}, \tilde{k}_{\tau+1}, \tilde{v}_{\tau+1})$ for $T' = T - \tau - 1$ to show that $\hat{m}_t \geq \tilde{m}_t$ for all $t \in [T] \setminus [\tau]$. Therefore, by replacing the decision in period τ with z_τ', the platform can only increase the total number of completed donations. Using a contradiction argument, no policy with $z_\tau \neq z_\tau'$ can be strictly optimal. This completes the proof that z_τ' (the optimal myopic policy) is the optimal policy in period τ (in the presence of multiple optimal policies, we follow the convention of choosing the myopically optimal one).

B.5. Proof of Theorem 3

We first show that (MMT) has a unique solution in the AGN regime. Then we show that if $d_\tau \geq \bar{d}$, $z_\tau = 0$ is optimal. Finally, we show that the thresholds define an absorbing state, i.e., if $d_\tau \geq \bar{d}$ and $v_\tau \geq \bar{v}$, then $d_{\tau+1} \geq \bar{d}$ and $v_{\tau+1} \geq \bar{v}$. This implies that once $d_t \geq \bar{d}$ and $v_t \geq \bar{v}$, $z_\tau = 0$ must be optimal for all $\tau \geq t$.

To show that (MMT) has a unique solution, it is sufficient to prove that the right hand sides of (7) and (8) have slope less than c and that the right hand sides of (9) and (10) have slopes in $[0,c]$ with respect to the decision variables. In that case, (10) must be tight at any optimal solution. Applying the chain rule, the right hand sides of (7)-(9) must have slopes less than 1 as a function of d. Thus, a linear search for the first value of d to satisfy all three constraints solves the problem.

Clearly the right hand side of (7) is increasing with slope less than c (it is a constant). For the other three constraints, we can exploit their common structure. We will prove that the general function $f(x) = \frac{1}{c}\log(\frac{e^{c\zeta_1 x}-1}{e^{c\zeta_2 x}-1})$ satisfies $f'(x) < 1$ when $0 < \zeta_2 < \zeta_1 < 1 + \zeta_2$.

$$\begin{aligned}\frac{df}{dx} &= \frac{\zeta_1 e^{c\zeta_1 x}}{e^{c\zeta_1 x}-1} - \frac{\zeta_2 e^{c\zeta_2 x}}{e^{c\zeta_2 x}-1} \\ &= \zeta_1 + \frac{\zeta_1}{e^{c\zeta_1 x}-1} - \zeta_2 - \frac{\zeta_2}{e^{c\zeta_2 x}-1} \qquad \leq \qquad \zeta_1 - \zeta_2 \qquad < \qquad 1 \end{aligned} \tag{28}$$

Line (28) comes from noting that $\frac{\zeta}{e^{cx\zeta}-1}$ is decreasing in ζ, so $\frac{\zeta_1}{e^{c\zeta_1 x}-1} - \frac{\zeta_2}{e^{c\zeta_2 x}-1} \leq 0$. This immediately shows that the right hand sides of (9) and (10) both have slopes less than c. Similarly, the right hand side of (8) must have a slope less than $\frac{1-\beta'}{1-\beta}c < c$. Note also that $\zeta + \frac{\zeta}{e^{cx\zeta}-1}$ is increasing in ζ, which means that according to (28), $\frac{df}{dx} \geq 0$. This implies that (9) and (10) are weakly increasing in both decision variables. This completes the proof there is a unique solution to (MMT) which can be found via a linear search on d.

We now show that if $d_\tau \geq \bar{d}$, $z_\tau = 0$ is optimal. By appealing to Lemma 1 in the AGN regime, it is sufficient to show that $cd_{\tau+1} \geq \log\left(\frac{e^{c(1+\alpha'-\alpha)m_\tau}-1}{e^{c(\gamma+\alpha'-\alpha)m_\tau}-1}\right)$. Using (2) to rewrite $d_{\tau+1}$ in terms of d_τ and m_τ, we find that the above condition is equivalent to $cd_\tau \geq \frac{1}{1-\beta}\log\left(\frac{e^{c(1+\alpha'-\alpha)m_\tau}-1}{e^{c(\gamma+\alpha'-\alpha)m_\tau}-1}\right) - \frac{\beta'}{1-\beta}cm_\tau$. It can be shown that this threshold is convex in m_τ, so its maximum value over its domain must occur when $m_\tau = d_\tau$ or when $m_\tau = 0$. Constraint (8) ensures that cd_τ exceeds the threshold when $m_\tau = d_\tau$, while constraint (7) ensures that cd_τ exceeds the threshold when $m_\tau = 0$ (recall that we define the threshold to equal its limiting value when $m_\tau = 0$). This means that if these two constraints are satisfied, the optimal myopic policy is $z_\tau = 0$. Since we are in the AGN regime, applying Lemma 1 proves that the optimal policy is $z_\tau = 0$.

We now show that $\bar{d}$ and $\bar{v}$ define an absorbing state when the platform follows a policy of no adoption. Suppose $d_t \geq \bar{d}$ and $v_t \geq \bar{v}$. Since the matching function in (5) is increasing in each side of the market, m_t is lower-bounded by $\bar{m} := \mu(\bar{d},\bar{v}) = \bar{d} + \bar{v} - \frac{1}{c}\log(e^{c\bar{d}} + e^{c\bar{v}} - 1)$. When following a policy of $z_t = 0$, $v_{t+1} = (1-\alpha)v_t + (\alpha'+\gamma')m_t \geq (1-\alpha)\bar{v} + (\alpha'+\gamma')\bar{m}$. Though we omit the details for the sake of brevity, constraint (9) is algebraically equivalent to a constraint $(1-\alpha)\bar{v} + (\alpha'+\gamma')\bar{m} \geq \bar{v}$. Thus, $v_{t+1} \geq \bar{v}$.

Similarly, $d_{t+1} = (1-\beta)d_t + \beta' m_t \geq (1-\beta)\bar{d} + \beta'\bar{m}$. Constraint (10) is algebraically equivalent to a constraint $(1-\beta)\bar{d} + \beta'\bar{m} \geq \bar{d}$ (we again omit the details for brevity). Thus, when following a policy of no adoption, $\bar{v}$ and $\bar{d}$ define an absorbing state where no adoption remains the optimal policy.

B.6. Proof of Theorem 4

For ease of notation throughout the proof, we define constants $A_1 := \max\{1-\beta+\beta', 1-\alpha+\alpha'+\gamma'\}$, $A_2 := \min\{1-\beta+\beta', 1-\alpha+\alpha'+\gamma'\}$ and $A_3 := \max\{\beta', \alpha'+\gamma'\}$. Note that $A_1 \geq A_2 \geq 1$.

We first establish an upper bound on the value of the platform design problem. Consider a modified setting where the matching in each period is equal to the small side in the matching market, e.g., $\hat{m}_t(d_t, k_t, v_t) = \min\{d_t, v_t\}$. We will call this the *match-min* setting, and we will use consistent notation to indicate the state variables in the match-min setting, e.g. $\hat{d}_{t+1} = (1-\beta)\hat{d}_t + \beta'\hat{m}_t$. In this setting, the platform's decision at time t only impacts $\hat{m}_{t+1}$ through its impact on $\hat{v}_{t+1}$. Thus, intuitively in the AGN regime, the platform's optimal policy is to set $\hat{z}_t = 0$ for all $t \in [T]$, which maximizes the growth of the volunteer side of the market. Based on a similar argument as in the proof of Lemma 1, we claim the value of completed donations in the match-min setting when $\hat{z}_t = 0$ for all $t \in [T]$ is an upper bound on the value of the platform design problem. The value of this upper bound is given by $\sum_{t=1}^{T} \delta^{t-1}\hat{m}_t$.

Now suppose m_t represents the number of matches in period t when the platform follows a static policy of no adoption and the matching output is determined by (5). We aim to lower-bound the performance ratio of this policy by lower-bounding $1 - \frac{\sum_{t=1}^{T}\delta^{t-1}(\hat{m}_t - m_t)}{\sum_{t=1}^{T}\delta^{t-1}\hat{m}_t}$. First, we place a lower bound on the denominator:

$$\sum_{t=1}^{T}\delta^{t-1}\hat{m}_t \geq \sum_{t=1}^{T}\delta^{t-1}(A_2)^t \min\{d_0, v_0\} = A_2 \min\{d_0, v_0\}\frac{1-(A_2\delta)^T}{1-A_2\delta} \geq \min\{d_0, v_0\}\frac{1-(A_1\delta)^T}{1-\delta} \tag{29}$$

Next, we place an upper bound on $\sum_{t=1}^{T}\delta^{t-1}(\hat{m}_t - m_t)$. Using the matching function defined in (5),

$$\begin{aligned}\hat{m}_t - m_t &= \min\{\hat{d}_t, \hat{v}_t\} - d_t - v_t + k_t + \frac{1}{c}\log(e^{c(d_t-k_t)} + e^{c(v_t-k_t)} - 1)\\ &\leq \min\{\hat{d}_t, \hat{v}_t\} - \min\{d_t, v_t\} + \frac{\log(2)}{c} \quad \leq \quad \max\{\hat{d}_t - d_t, \hat{v}_t - v_t\} + \frac{\log(2)}{c}\end{aligned} \tag{30}$$

We will now show via induction that $\max\{\hat{d}_t - d_t, \hat{v}_t - v_t\} \leq \frac{A_3\log(2)(A_1^t - 1)}{c(A_1-1)}$. This holds by equality at $t=0$. Now we assume it holds for $t \leq k$ and we aim to show that it continues to hold at $t = k+1$. We begin with the volunteer side of the market:

$$\begin{aligned}\hat{v}_{k+1} - v_{k+1} &= (1-\alpha)(\hat{v}_k - v_k) + (\alpha'+\gamma')(\hat{m}_k - m_k)\\ &\leq (1-\alpha)\max\{\hat{v}_k - v_k, \hat{d}_k - d_k\} + (\alpha'+\gamma')(\max\{\hat{d}_k - d_k, \hat{v}_k - v_k\} + \frac{\log(2)}{c})\\ &\leq A_1\max\{\hat{d}_k - d_k, \hat{v}_k - v_k\} + \frac{A_3\log(2)}{c}\end{aligned}$$

Repeating a nearly identical process for donors, we can show $\hat{d}_{k+1} - d_{k+1} \leq A_1\max\{\hat{d}_k - d_k, \hat{v}_k - v_k\} + \frac{A_3\log(2)}{c}$. Thus, applying our inductive hypothesis:

$$\max\{\hat{d}_{k+1} - d_{k+1}, \hat{v}_{k+1} - v_{k+1}\} \quad \leq \quad A_1\left(\frac{A_3\log(2)(A_1^k - 1)}{c(A_1-1)}\right) + \frac{A_3\log(2)}{c} \quad = \quad \frac{A_3\log(2)(A_1^{k+1}-1)}{c(A_1-1)}$$

This completes the proof by induction. We now plug our upper bound on $\max\{\hat{d}_t - d_t, \hat{v}_t - v_t\}$ into (30) and take a discounted sum over all $t \in [T]$:

$$\sum_{t=1}^{T} \delta^{t-1}(\hat{m}_t - m_t) \quad \leq \quad \sum_{t=1}^{T} \delta^{t-1} \left(\frac{\log(2)}{c} + \frac{A_3 \log(2)(A_1^t - 1)}{c(A_1 - 1)} \right) \quad \leq \quad \frac{\log(2)}{c(1-\delta)} + \frac{A_3 \log(2)(1 - (\delta A_1)^T)}{c(1-\delta)(1-\delta A_1)} \tag{31}$$

The second inequality in (31) is purely algebraic. We omit the details for brevity. Using this result and the lower bound from (29), we can lower bound the approximation factor of a static no-adoption policy (assuming $\delta A_1 < 1$).

$$1 - \frac{\sum_{t=1}^{T} \delta^{t-1}(\hat{m}_t - m_t)}{\sum_{t=1}^{T} \delta^{t-1}(\hat{m}_t)} \quad \geq \quad 1 - \frac{\frac{\log(2)}{c(1-\delta)} + \frac{A_3 \log(2)(1-(\delta A_1)^T)}{c(1-\delta)(1-\delta A_1)}}{\min\{d_0, v_0\} \frac{1-(A_1\delta)^T}{1-\delta}} \quad \geq \quad 1 - \frac{\log(2)(\frac{1}{1-(\delta A_1)^T} + \frac{A_3}{1-\delta A_1})}{c \min\{d_0, v_0\}}$$

This is equivalent to the approximation factor reported in Theorem 4.

Appendix C: Omitted Details from Section 4

C.1. Discussion of Model Dynamics (Heterogeneous Volunteers)

We now provide additional details about the model dynamics of Section 4.1 given in Equations (11) through (14), which respectively govern the evolution of (i) matches, (ii) donations, (iii) adopted matches, and (iv) volunteers.

Matches. We first consider the number of matches involving volunteers of each type, which includes both their adopted matches and their fraction of the spot market matches. Based on our assumption that compatibility is re-drawn identically and independently for all visits to the spot market, volunteers of type j form a number of matches in the spot market which is proportional to their prevalence in the volunteer side of the spot market. Consequently, the dynamics for the type-specific number of matches converges to Equation (11).

We note that the assumption that compatibility is re-drawn each time the volunteer visits the spot market is imperative for tractability. Without it, we will have a high-dimensional state space, where we need to track the state of compatibility for each donation and each volunteer. In particular, for the x^{th} visit of a volunteer A, we need to know if a donation B has been considered by A in any of their previous $x-1$ visits. Characterizing the matching outcome in such a high-dimensional system is prohibitively challenging; as such, we make the assumption that compatibility is re-drawn. However, we highlight that even under this simplifying assumption, the matching function qualitatively preserves important properties, such as monotonicity in the size of both sides of the market as well as monotonicity in the compatibility probability.

Donations. We next consider the evolution of the number of donations. As their growth only depends on being matched (as opposed to the type of match or the type of volunteer in the match), the deterministic dynamics remain unchanged from our base model, i.e., Equation (12) is identical to Equation (2).

Adopted Matches. Because the engagement level of a volunteer can change from period to period, it is particularly challenging to track the dynamics for a fixed volunteer type. We illustrate this using a simple example. Suppose that there is only one group (i.e., $|\mathcal{G}| = 1$), and hence a volunteer's type j can simply be denoted by their engagement level r, which we assume is at most 2, i.e., $\mathcal{J} = \{1, 2\}$. To specify the evolution of adopted matches in period $t+1$ involving volunteers of type 1, i.e., k^1_{t+1}, we need to sum two terms.

First, we need to account for adopted matches involving volunteers that *were* of type 1 in period t and *remained* type 1 in period $t+1$. Suppose volunteer A was of type 1 in period t and formed an adopted match. Recall that A will decrease their engagement with probability $\tilde{\gamma}^1$. Furthermore, A's *expected* decrease in engagement is given by γ^1. This implies that A will *increase* their engagement by 1 with probability $\tilde{\gamma}^1 - \gamma^1 := \hat{\gamma}^1$, and they will *maintain* their engagement with probability $1 - \tilde{\gamma}^1 - \hat{\gamma}^1$. Using this notation, the number of adopted matches involving volunteers that *were* of type 1 in period t and *remained* type 1 in period $t+1$ is given by $(1 - \tilde{\gamma}^1 - \hat{\gamma}^1)\,(k^1_t + z_t\rho^1(m^1_t - k^1_t))$. We highlight that this expression reflects two additional differences from to our base model: (i) newly-formed matches are adopted matches if and only if adoption is allowed for the donation (which happens with probability z_t) *and* the volunteer is willing to participate in an adopted match (which happens with probability ρ^j). (ii) Once an adopted match is formed, the platform cannot dissolve it, which more accurately captures the set of feasible policies in practice.

Then, we need to account for adopted matches involving volunteers that *were* of type 2 in period t and *changed* to type 1 in period $t+1$. For this to occur, they need to have and maintain one adopted match, while also decreasing their engagement level due to their other matching outcome (which can happen in one of three ways, based on the three possible matching outcomes). Enumerating over these possible combinations of outcomes is already challenging (and would require additional notation); this challenge is exacerbated in settings where volunteers may have an engagement level that exceeds 2.

To avoid this issue, we keep track of a closely related quantity, $\tilde{k}^j_{t+1}$, which denotes the number of fixed matches at the start of period $t+1$ involving volunteers that *were* of type j in period t. The evolution of $\tilde{k}^j_{t+1}$ follows a similar logic as Equation (3) and is given as follows:

$$\tilde{k}^j_{t+1} = (1 - \tilde{\gamma}^j)\left(k^j_t + z_t\rho^j(m^j_t - k^j_t)\right) \tag{32}$$

In a myopic setting, it is sufficient to focus on the total number of adopted matches, which is the sum of the adopted matches over all volunteer types. To establish Equation (13), we leverage the fact that this sum is equivalent regardless of whether we focus on the volunteer's current type or their previous type, i.e., $k_t = \sum_{j\in\mathcal{J}} k^j_t = \sum_{j\in\mathcal{J}} \tilde{k}^j_t$.

Volunteers. Following the same motivation as above, we choose to express the dynamics in terms of $\tilde{v}^j_{t+1}$, which represents the *aggregate* engagement level of volunteers that *were* of type j in period t. Note that, as explained in Section 4.1, this can differ from the *number* of volunteers that were of type j in period t.

The deterministic dynamics of $\tilde{v}^j_{t+1}$ are given by:

$$\begin{aligned}
\tilde{v}^j_{t+1} \;=\; & (1-\alpha^j)(v^j_t - m^j_t) && \text{(failed matches)}\\
& + (1+\gamma'^j-\gamma^j)\left(k^j_t + z_t\rho^j(m^j_t - k^j_t)\right) && \text{(adopted matches)}\\
& + (1+\gamma'^j+\alpha'^j-\alpha^j)(1-z_t\rho^j)(m^j_t-k^j_t) && \text{(other matches)}\\
\;=\; & (1-\alpha^j)v^j_t + (\alpha'^j+\gamma'^j)m^j_t - (\gamma^j-\alpha^j+\alpha'^j)\left(k^j_t + z_t\rho^j(m^j_t-k^j_t)\right),
\end{aligned}$$

Summing over all volunteer types, this corresponds to the dynamics in Equation (14).

C.2. Proof of Proposition 3

The proof of Proposition 3 follows quite naturally from the proof of Theorem 1. In this extended setting, we can once again show that $m_1(z_0)$ does not attain a local maximum.

As before, $\frac{\partial d_1}{\partial z_0} = 0$. Based on equation (13) and the definition of $\hat{D}_k$, $\frac{\partial k_1}{\partial z_0} = \sum_{j\in\mathcal{J}}(1-\tilde{\gamma}^j)\rho^j(m^j_0 - k^j_0) = \hat{D}_k$. Based on equation (14) and the definition of $\hat{D}_v$, $\frac{\partial v_1}{\partial z_0} = \sum_{j\in\mathcal{J}}(\alpha^j - \alpha'^j - \gamma^j)\rho^j(m^j_0 - k^j_0) = \hat{D}_v$. We emphasize that $\hat{D}_k$ and $\hat{D}_v$ depend only on instance primitives and are constant with respect to z_0.

Combining these derivatives with (1) and (5), we have

$$\begin{aligned}
m_1 &= d_1 - k_1 + v_1 - \frac{1}{c}\log(e^{c(d_1-k_1)} + e^{c(v_1-k_1)} - 1)\\
\frac{\partial m_1}{\partial z_0} &= \frac{\hat{D}_v e^{c(d_1-k_1)} + \hat{D}_k - \hat{D}_v}{e^{c(d_1-k_1)} + e^{c(v_1-k_1)} - 1} && (33)\\
\frac{\partial^2 m_1}{\partial z_0^2} &= \left(\frac{\hat{D}_k^2 e^{c(d_1-k_1)} + (\hat{D}_k - \hat{D}_v)^2 e^{c(v_1-k_1)} - \hat{D}_v^2 e^{c(d_1+v_1-2k_1)}}{(e^{c(d_1-k_1)} + e^{c(v_1-k_1)} - 1)^2}\right) c && (34)
\end{aligned}$$

Note that $\hat{D}_k \geq 0$. Hence, if $\hat{D}_v \geq 0$, then $\partial m_1/\partial z_0$ can never be negative, and $z_0 = 1$ is myopically optimal.

Otherwise, if $\hat{D}_v < 0$, the first-order condition prescribed by (33) is equivalent to $e^{c(d_1-k_1)} = \frac{\hat{D}_v - \hat{D}_k}{\hat{D}_v}$. When that FOC holds, the numerator of (34) reduces to $\hat{D}_k^2 e^{c(d_1-k_1)} + \hat{D}_k(\hat{D}_k - \hat{D}_v)e^{c(v_1-k_1)}$. This must be positive whenever $\hat{D}_v < 0$, since $\hat{D}_k > 0$. Thus, m_1 is convex at any critical point and consequently can have no local maxima as a function of z_0. This implies that the optimal myopic solution must be at a boundary, i.e., $z_0 = 0$ or $z_0 = 1$.

We now provide conditions under which $z_0 = 1$ is myopically optimal. Recall that if $\hat{D}_v \geq 0$, then $\partial m_1/\partial z_0 \geq 0$, which implies $z_0 = 1$ is myopically optimal. We use the definition of m^j_t in Line (11) to show that $\hat{D}_v \geq 0$ is equivalent to the first condition in Equation (15):

$$0 \leq \sum_{j\in\mathcal{J}}(\alpha^j - \alpha'^j - \gamma^j)\rho^j(m^j_0 - k^j_0)$$

$$\Leftrightarrow \quad 0 \le \sum_{j\in\mathcal{J}} (\alpha^j - \alpha'^j - \gamma^j)\rho^j \left(\frac{v_0^j - k_0^j}{v_0 - k_0}\right) \mu(d_0 - k_0, v_0 - k_0)$$

$$\Leftrightarrow \quad 0 \le \sum_{j\in\mathcal{J}} (\alpha^j - \alpha'^j - \gamma^j)\rho^j \left(\frac{v_0^j - k_0^j}{v_0 - k_0}\right)$$

We now consider cases where $\hat{D}_v < 0$. To compare the values $m_1(0)$ and $m_1(1)$, we use a slight abuse of notation to augment (d_1, k_1, v_1) by z_t to compare them for $z_t = 0$ and $z_t = 1$. The adoption level decision does not influence the subsequent number of donations; hence, we define $d_1(0) = d_1(1) := d_1$. As such, the following are necessary and sufficient conditions for $m_1(1) \ge m_1(0)$:

$$0 \ge e^{c(v_1(0) - v_1(1) + k_1(1) - k_1(0))} - \frac{e^{c(v_1(0) - k_1(0))} + e^{c(d_1 - k_1(0))} - 1}{e^{c(v_1(1) - k_1(1))} + e^{c(d_1 - k_1(1))} - 1} \tag{35}$$

$$\Leftrightarrow \quad 0 \ge e^{c(d_1 - k_1(0) - \hat{D}_v)} - e^{c(\hat{D}_k - \hat{D}_v)} - e^{c(d_1 - k_1(0))} + 1 \tag{36}$$

$$\Leftrightarrow \quad 0 \ge e^{c(d_1 - k_1(0))}(e^{-c\hat{D}_v} - 1) - (e^{c(\hat{D}_k - \hat{D}_v)} - 1) \tag{37}$$

Line (35) comes from applying definitions, Line (36) is algebraic, using the equalities $v_1(1) - v_1(0) = \hat{D}_v$ and $k_1(1) - k_1(0) = \hat{D}_k$. Inequality (37) follows from rearranging, and — for cases where $\hat{D}_v < 0$ — it is satisfied if and only if the second condition in Equation (15) is met. (To solidify the connection to the second condition in (15), note that $k_1(0) = \sum_{j\in\mathcal{J}} (1 - \tilde{\gamma}^j) k_0^j$.)

C.3. Proof of Proposition 4

The proof follows the same argument used in the proof of Proposition 2 (presented in Appendix B.1). To adjust the proof for this setting, it is sufficient to show that after the arrival of each volunteer to the spot market, the expected change in the number of matches involving each type of donation converges to the differential equations presented in Equations (16) and (17).

To see this, let $Y^j(Z) \in [nd^j]$ be the number of matched donations of type j right after the Z^{th} volunteer arrives. At the beginning of the process, there are no matches ($Y^j(0) = 0$), and according to the greedy matching process, the expected number of *total* matches evolves as follows:

$$\mathbb{E}[Y^h(Z+1) + Y^e(Z+1) | Y^h(Z), Y^e(Z)] = Y^h(Z) + Y^e(Z) + 1 - \left(1 - \frac{c^h}{n}\right)^{n(d^h - \frac{Y^h(Z)}{n})} \left(1 - \frac{c^e}{n}\right)^{n(d^e - \frac{Y^e(Z)}{n})}$$

This expression comes from the fact that an arriving volunteer fails to match if and only if they are incompatible with *every* remaining donation. If we define $y^j(z) = \frac{Y^j(nz)}{n}$, then for large n,

$$\frac{\mathbb{E}[y^h(z + 1/n) + y^e(z + 1/n) - y^h(z) - y^e(z) | y^h(z), y^e(z)]}{1/n} \;=\; 1 - e^{-c^h(d^h - y^h(z)) - c^e(d^e - y^e(z))} + o(1).$$

This gives the probability that an arriving volunteer finds a match, but it does not say anything about the type of the donation that the volunteer matches with. Recall that arriving volunteers consider donations in a random order. Thus, as n approaches infinity, the number of donations a volunteer needs to consider before finding a compatible hard-to-match (resp. easy-to-match) donation is an exponential random variable with rate $\frac{c^h(d^h-y^h(z))}{d^h-y^h(z)+d^e-y^e(z)}$ (resp. $\frac{c^e(d^e-y^e(z))}{d^h-y^h(z)+d^e-y^e(z)}$). Note that these two random variables are independent. Furthermore, since the volunteer sees donations in a random order and greedily matches with the first compatible donation that they see, the minimum of these two random variables determines whether they find a match, and if so, the type of the matched donation.

Based on properties of exponential random variables, conditional on the volunteer forming a match, the probability that it forms a match with a hard-to-match (resp. easy-to-match) donation is proportional to its rate of finding the first compatible match of that type. In other words, conditional on finding a match, the probability of matching with a hard-to-match donation is $\frac{c^h(d^h-y^h(z))}{c^h(d^h-y^h(z))+c^e(d^e-y^e(z))}$. Therefore,

$$\begin{aligned}&\frac{\mathbb{E}[y^h(z+1/n)-y^h(z)|y^h(z),y^e(z)]}{1/n}\\&\qquad=\frac{c^h(d^h-y^h(z))}{c^h(d^h-y^h(z))+c^e(d^e-y^e(z))}\left(1-e^{-c^h(d^h-y^h(z))-c^e(d^e-y^e(z))}\right)+o(1)\end{aligned}$$

$$\begin{aligned}&\frac{\mathbb{E}[y^e(z+1/n)-y^e(z)|y^h(z),y^e(z)]}{1/n}\\&\qquad=\frac{c^e(d^e-y^e(z))}{c^h(d^h-y^h(z))+c^e(d^e-y^e(z))}\left(1-e^{-c^h(d^h-y^h(z))-c^e(d^e-y^e(z))}\right)+o(1)\end{aligned}$$

As $n\to\infty$, these equations correspond to the differential equations given in Equations (16) and (17). Applying Theorem 5.1 from Wormald et al. (1999) establishes convergence and uniqueness, which completes the proof.

C.4. Sensitivity to Volunteers' Adoption Probability

In practice, when platforms such as FRUS *allow* adoption, volunteers may still choose to form a one-time match. In such settings, the optimal policy (and its performance) depends on the probability that a volunteer will form an adopted match when such an option is allowed. In the following, we study the impact of adoption probability on the optimal policy and its performance in a special case of the model introduced in Section 4.1 where all volunteer types have the same parameters, e.g. $\rho^j=\rho$ for all j.

Intuitively, a lower adoption probability can only harm the performance of the optimal policy. If volunteers are less willing to adopt, this gives the platform less control over the type of matches which are formed, which is equivalent to reducing the action space of the platform. For instance, in a myopic setting, suppose allowing adoption for all donations is the optimal policy when the adoption probability is given by ρ. Then, if the adoption probability decreases to $\hat{\rho}<\rho$, the performance of the policy that allows adoption for all donations will decrease, to the extent that such a policy may even become sub-optimal. However, if disallowing adoption is the optimal policy when the adoption probability is

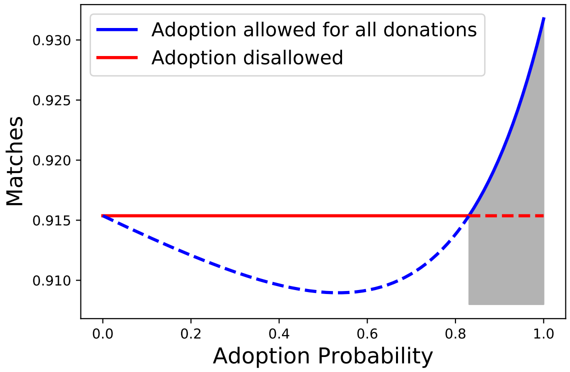

Figure 9: For model primitives $(\alpha, \alpha', \gamma, \gamma', \beta, \beta') = (0.1, 0.1, 0.05, 0.1, 0.05, 0.1)$, $v_0 = d_0 = 1$, $k_0 = 0$, and $c = 5$, the plot shows the myopic number of matches achieved by the only two potentially-optimal policies, as a function of the adoption probability ρ.

given by ρ, then a decrease in the adoption probability to $\hat{\rho} < \rho$ will not impact the performance or the optimality of the policy that disallows adoption.

We provide numerical support for this intuition in Figure 9, which plots the myopic number of matches achieved by policies of $z_0 = 1$ (fully allowing adoption) and $z_0 = 0$ (disallowing adoption) as a function of the adoption probability. Specifically, we use our model for heterogeneous volunteers (presented in Section 4.1) which includes a parameter for adoption probability. For simplicity, we assume that all volunteer types have identical parameters (thus, we drop the superscript from our notation). In the image shown, $(\alpha, \alpha', \gamma, \gamma', \beta, \beta') = (0.1, 0.1, 0.05, 0.1, 0.05, 0.1)$, $v_0 = d_0 = 1$, $k_0 = 0$, and $c = 5$. We note that these parameters are equal to the expected value of the simulation space described in Appendix C.5.

As shown by the solid blue curve, fully allowing adoption is the myopically optimal policy when $\rho = 1$ (i.e., when the platform has complete control over the types of matches formed). As long as the adoption probability lies within the shaded region on the right, this policy remains optimal, but we highlight that its performance degrades noticeably as the adoption probability decreases. If the adoption probability is below a threshold (which occurs at an adoption probability of 0.831 in this instance), then disallowing adoption is myopically optimal, as shown by the solid red line. Below this threshold, changes in the adoption probability do not impact the performance of the optimal policy.

We remark that the platform has significant incentive to try to increase the adoption probability whenever fully allowing adoption is optimal, and each percentage-point improvement is increasingly valuable. Improving the number of matches, even by a small amount, can make a significant difference in the growth trajectory of the platform.

C.5. Performance of a Repeated Optimal Myopic Policy (Heterogeneous Volunteers)

All Simulations		Heterogeneous Volunteers		Heterogeneous Donors	
Primitive	Distribution	Primitive	Distribution	Primitive	Distribution
k_0	Deterministically 0	d_0	Deterministically 1	v_0	Deterministically 1
β	Uniform$[0, 0.1]$	v_0^p	Uniform$[0, 1]$	d_0^h	Uniform$[0, 1]$
β'	Uniform$[\beta, \beta + 0.1]$	v_0^s	Uniform$[0, 1]$	d_0^e	Uniform$[0, 1]$
γ	Uniform$[0, 0.1]$	c	Uniform$[0, 10]$	c^h	Uniform$[0, 5]$
γ'	Uniform$[\gamma, \gamma + 0.1]$	α'^p	Uniform$[0, 0.2]$	c^e	Uniform$[c^h, 2c^h]$
α	Uniform$[0, 0.2]$	α'^s	Uniform$[0, 0.2]$	α'	Uniform$[0, 0.2]$
δ	Uniform$[0.70, 1 - B_1]$	ρ^p	Uniform$[0, 1]$		
		ρ^s	Uniform$[0, 1]$		

Table 4: Distribution for model primitives in the simulated instances of Section C.5 (Columns 1 and 2) and C.6 (Columns 1 and 3). All primitives are drawn independently for each instance, aside from γ', β', δ, and c^e which are drawn from ranges which depend on other primitives.

As we elaborate on in Appendix C.1, characterizing the optimal long-run policy in a general setting with heterogeneous volunteers is intractable; even specifying the type-specific dynamics is challenging without additional structural assumptions on the type space of volunteers. Consequently, to study a long-run setting with heterogeneous volunteers, here we focus on instances with two distinct groups of volunteers, students (s) and professionals (p), and we assume that volunteers have engagement level at most 1. Hence, we denote the set of possible volunteer types as $\mathcal{J} = \{s, p\}$. We focus on instances where the two types $j \in \mathcal{J}$ differ only in their adoption probability ρ^j and their engagement from being in a one-time match α'^j, as varying these quantities is sufficient to control the non-adoption growth benefit of each type as well as the relative importance of their non-adoption growth benefit in determining the optimal myopic policy (see Proposition 3 and the subsequent discussion). We assume the two types have identical engagement behavior otherwise (i.e., $\gamma^j = \gamma$ and $\alpha^j = \alpha$ for $j \in \mathcal{J}$).

In such a setting, we compare the performance of a repeated optimal myopic policy to the performance of the optimal policy across 10,000 simulations over a horizon $T = 10$. For each of these simulations, we generated instance primitives according to Table 4. We normalized the initial donation side of the market to size 1, and for simplicity, we assumed no initial adoption (i.e., $k_0^p = k_0^s = 0$). Most instance primitives were generated independently. The only exceptions are γ' and β', which were chosen to be large enough to ensure growth is possible, as well as the discount rate δ, which is upper bounded by $1 - B_1$ (where $B_1 := \max\{\gamma' - \gamma, \gamma' - \alpha + \alpha'^p, \gamma' - \alpha + \alpha'^s, \beta' - \beta\}$ is the maximum possible growth rate) to ensure that the discounted size of the market remains bounded.

For each of the 10,000 instances, we found the policy that, in each period, optimally chooses either full adoption or no adoption via exhaustive search. We used this solution as a proxy for the optimal policy. Separately, we randomly selected and tested 500 of the instances and verified that in each one, the optimal policy $\vec{z}^*$ was indeed always either full adoption or no adoption in each period, i.e., $\vec{z}^* \in \{0, 1\}^T$.

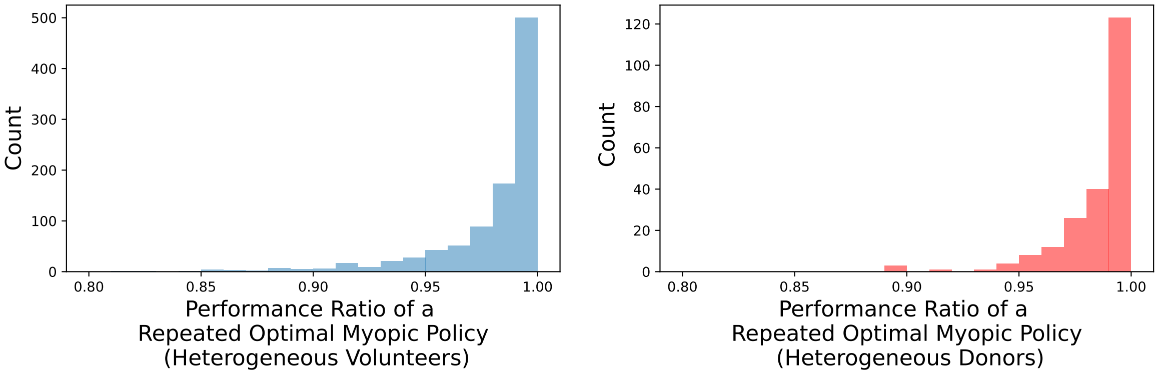

Figure 10: Histogram of the performance of a repeated myopic policy compared to our benchmark: (left) in a setting with heterogeneous volunteers, for the 962 instances out of 10,000 where the benchmark is strictly better, and (right) in a setting with heterogeneous donors, for the 218 instances out of 1000 where the benchmark is strictly better.

We highlight that a repeated optimal myopic policy performs quite well: in 90.4% of the 10,000 simulated instances, it achieves the same performance as our benchmark. In the left panel of Figure 10, we focus our attention on the subset of 962 instances where a repeated optimal myopic policy is strictly worse than our benchmark. We display a histogram showing the performance ratio of a repeated optimal myopic policy compared to the benchmark. Even in the worst case that we observe, the repeated optimal myopic policy achieves 81.2% of the benchmark value.

C.6. Performance of a Repeated Optimal Myopic Policy (Heterogeneous Donations)

Given the lack of a closed-form matching function for even a single period, it is intractable to characterize the optimal long-run policy in a setting with heterogeneous donations. Instead, we use simulation to compare the performance of a repeated optimal myopic policy to the performance of the same benchmark used in Section C.5, i.e., the optimal policy among $(z^h, z^e) \in \{0,1\}^{2\times T}$. Due to the computational challenges of (i) numerically estimating the matching outcomes and (ii) searching for a two-dimensional optimal policy, we limit ourselves to 1000 simulations with a horizon $T = 5$. For each of these simulations, we generate instance primitives according to Columns 1 and 3 of Table 4. We normalize the initial donation side of the market to size 1, and for simplicity, we assume no initial adoption (i.e., $k_0^h = k_0^e = 0$). Most instance primitives are generated independently. The only exceptions are γ' and β', which are chosen to be large enough to ensure growth is possible, the discount rate δ, which is upper bounded by $1 - B_1$ (where $B_1 := \max\{\gamma' - \gamma, \gamma' - \alpha + \alpha', \beta' - \beta\}$ is the maximum possible growth rate) to ensure that the discounted size of the market remains bounded, and the compatibility parameter c^e, which is chosen to be greater than c^h but within a factor of at most 2.

For each of the 1000 instances, we conduct a grid search in each period to identify the myopic policy, which involves computing the matching outcomes for each candidate solution by numerically solving the simultaneous differential equations using a step size of 0.0001. In all periods across all simulated instances, we find that the optimal myopic policy consists of either allowing for full adoption or disallowing adoption within each donation type, although the decision can be different across the two types of donations.

We compare the repeated myopic policy against our benchmark policy, which we remind optimally chooses either full adoption or no adoption for each donation type in each period via exhaustive search (i.e., the optimal policy among all $\mathbf{z} \in \{0,1\}^{2\times T}$). We use this benchmark as a proxy for the optimal policy, as conducting a full grid search over $[0,1]^{2\times T}$ is computationally intractable. To select the best of these 1024 candidate policies, we likewise computed the matching outcomes in each period for all candidate policies by numerically solving the simultaneous differential equations using a step size of 0.0001.

We highlight that a repeated optimal myopic policy performs quite well compared to this benchmark: in 78.2% of the 1000 simulated instances, they are identical. In the right panel of Figure 10, we focus our attention on the subset of 218 instances where a repeated optimal myopic policy is strictly worse than our benchmark. We display a histogram showing the performance ratio of a repeated optimal myopic policy compared to the benchmark. Even in the worst case that we observe, the repeated optimal myopic policy achieves 89.5% of the benchmark.