Online Appendix to "Intertemporal Segmentation via Flexible-Duration Group Buying"

A. Premium Group-Buying Product

LEMMA A.1 (CUSTOMERS' SIGN-UP BEHAVIOR). *For any $\theta \neq 1$ and N, if high-end (low-end) customers sign up at state $n\,(1 \leqslant n \leqslant N)$, then all high-end (low-end) customers sign up at any subsequent state $n'\,(1 \leqslant n' \leqslant n)$.*

Proof of Lemma A.1. See online supplement. □

PROPOSITION A.1 (NO-GROUP-BUYING BENCHMARKS). *In the absence of group buying as an option, there exists a threshold for the inventory holding cost, $\bar{h}_1$, such that*

(i) if $h \leqslant \bar{h}_1$, it is optimal for the firm to offer both products by using the product-line strategy, and high-end customers purchase the premium product while low-end customers purchase the regular product;

(ii) if $h > \bar{h}_1$, the firm offers only the regular product,

(ii-1) when $H/L \leqslant 1/\gamma$, it is optimal for the firm to adopt the volume strategy, and both high- and low-end customers purchase the regular product;

(ii-2) when $H/L > 1/\gamma$, it is optimal for the firm to adopt the margin strategy, and only high-end customers purchase the regular product.

Proof of Proposition A.1. See online supplement. □

Proof of Theorem 1. Consider a customer with valuation v who arrives at pledge-to-go state $n\,(1 \leqslant n \leqslant N)$. She would like to sign up if and only if $v - p - c \cdot w(n) \geqslant 0$. Denote the threshold for customers' valuation at state n as $\bar{v}_n$.

Note that v follows either a continuous or a discrete distribution. First, consider the case of a general continuous distribution. We assume $v \in [0, v_m]$ with a finite upper bound v_m or $v \in [0, \infty)$. Thus, $\bar{v}_n = p + c \cdot w(n)$, where $w(n) = \frac{1}{\lambda}\sum_{k=1}^{n-1} \frac{1}{\bar{F}(\bar{v}_k)}$ is the expected waiting time at state n and $\bar{F}(v) = 1 - F(v)$. Then, define the difference between two neighboring thresholds as

$$\Delta v_n \equiv \bar{v}_{n+1} - \bar{v}_n = c \cdot w(n+1) - c \cdot w(n) = \frac{c}{\lambda}\sum_{k=1}^{n} \frac{1}{\bar{F}(\bar{v}_k)} - \frac{c}{\lambda}\sum_{k=1}^{n-1} \frac{1}{\bar{F}(\bar{v}_k)} = \frac{c}{\lambda \bar{F}(\bar{v}_n)} > 0,$$

where $1 \leqslant n < N$. Thus, $\bar{v}_n$ is increasing in n, and Δv_n is also increasing in n. Moreover, we have

$$\bar{v}_n = \bar{v}_1 + \frac{c}{\lambda}\left[\frac{1}{\bar{F}(\bar{v}_1)} + \frac{1}{\bar{F}(\bar{v}_2)} + \cdots + \frac{1}{\bar{F}(\bar{v}_{n-1})}\right] = \bar{v}_1 + \frac{c}{\lambda}\sum_{k=1}^{n-1} \frac{1}{\bar{F}(\bar{v}_k)}. \tag{A.1}$$

The firm's long-run average profit can be written as

$$\begin{aligned}\pi(\bar{v}_1,\bar{v}_2,\cdots,\bar{v}_N) &= \sum_{n=1}^{N}\Big[p^*(\bar{v}_1,\bar{v}_2,\cdots,\bar{v}_N)\cdot\lambda\cdot\bar{F}(\bar{v}_n)\cdot P(n)\Big]\\ &= \frac{\bar{v}_1\lambda\sum_{n=1}^{N}\bar{F}(\bar{v}_n)\big[w(n+1)-w(n)\big]}{w(N+1)}\\ &= \frac{cN\lambda\bar{v}_1\bar{F}(\bar{v}_N)}{\lambda(\bar{v}_N-\bar{v}_1)\bar{F}(\bar{v}_N)+c},\end{aligned}$$

where the second equation follows from $p^*(\bar{v}_1,\bar{v}_2,\cdots,\bar{v}_N)=\bar{v}_n-\frac{c}{\lambda}\sum_{k=1}^{n-1}\frac{1}{\bar{F}(\bar{v}_k)}$ for any $n\,(1\leqslant n\leqslant N)$ and the last equation follows from the expression of $w(n)$ and (A.1). For ease of exposition, define $\pi(\bar{v}_1,\bar{v}_N)\equiv\frac{cN\lambda\bar{v}_1\bar{F}(\bar{v}_N)}{\lambda(\bar{v}_N-\bar{v}_1)\bar{F}(\bar{v}_N)+c}$. Therefore, the firm's optimization problem can be reduced to

$$\begin{aligned}\max_{\bar{v}_1,\bar{v}_N}\quad & \pi(\bar{v}_1,\bar{v}_N)=\frac{cN\lambda\bar{v}_1\bar{F}(\bar{v}_N)}{\lambda(\bar{v}_N-\bar{v}_1)\bar{F}(\bar{v}_N)+c}\\ \text{s.t.}\quad & \bar{v}_n=\bar{v}_{n-1}+\frac{c}{\lambda\bar{F}(\bar{v}_{n-1})},\ \text{for any } n\,(1<n\leqslant N),\end{aligned}$$

which demonstrates that the problem for an arbitrary $N\geqslant 2$ under a general continuous distribution can be technically challenging. Moreover, $\bar{v}_N<v_m$ always holds, because otherwise, if $\bar{v}_N=v_m$, $\bar{F}(\bar{v}_N)=0$, and then $\pi(\bar{v}_1,\bar{v}_N)=0$ always holds.

Second, consider the case of a general discrete distribution. We assume $v\in\{v_1,v_2,\ldots,v_M\}$, where $0\leqslant v_1<v_2<\cdots<v_M$ and M is arbitrary. Thus, $\bar{v}_n=v_j$, where $v_{j-1}<p+c\cdot w(n)\leqslant v_j$, $0<j\leqslant M$, and $w(n)=\frac{1}{\lambda}\sum_{k=1}^{n-1}\frac{1}{P(v\geqslant\bar{v}_k)}$. Then, for any $n\,(1\leqslant n<N)$, $\bar{v}_{n+1}=v_{j'}$, where $v_{j'-1}<p+c\cdot w(n+1)\leqslant v_{j'}$ and $j\leqslant j'\leqslant M$. If $j'=j$, then $\bar{v}_{n+1}=\bar{v}_n$. If $j'>j$, then there must exist some $i\,(j+1\leqslant i\leqslant M)$ such that $v_j<p+c\cdot w(n+1)\leqslant v_i$, and hence, $\bar{v}_{n+1}=v_i>v_j=\bar{v}_n$. Thus, $\bar{v}_n$ is increasing in n. Moreover, $\bar{v}_N\leqslant v_m$. $\square$

Proof of Proposition 1. By Lemma A.1, if the high-end customers sign up first, there are three possible scenarios: $\{H\}$, $\{H;H+L\}$, and $\{H+L\}$, which are defined as Case I for ease of analysis. Similarly, if the low-end customers sign up first, there are also three possible scenarios: $\{L\}$, $\{L;H+L\}$, and $\{L+H\}$, which are defined as Case II. Here we rule out the scenario $\{L+H\}$ in Case II to avoid repetition. We use the tipping state $\bar{x}^{\mathrm{G}}$ to stand for different scenarios.[1] Specifically, in Case I, $\bar{x}^{\mathrm{G}}=0$, $1\leqslant\bar{x}^{\mathrm{G}}<N$, and $\bar{x}^{\mathrm{G}}=N$ represent scenarios $\{H\}$, $\{H;H+L\}$, and $\{H+L\}$, respectively. In Case II, $\bar{x}^{\mathrm{G}}=0$ and $1\leqslant\bar{x}^{\mathrm{G}}<N$ represent scenarios $\{L\}$ and $\{L;H+L\}$, respectively.

For Case II, the IR and IC constraints are

[1] $\bar{n}^{\mathrm{G}}$ denotes the largest integer that is less than or equal to $\bar{x}^{\mathrm{G}}$, thus we use $\bar{x}^{\mathrm{G}}$ instead of $\bar{n}^{\mathrm{G}}$ in the Online Appendices and Supplements for preciseness.

$$\begin{cases} IR_L: \theta L - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant 0 & 1\leqslant n\leqslant N,\\ IC_L: \theta L - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant L - r^{\mathrm{G}} & 1\leqslant n\leqslant N,\\ IR_{H1}: H - r^{\mathrm{G}} \geqslant 0 & \bar{x}^{\mathrm{G}} < n\leqslant N,\\ IC_{H1}: H - r^{\mathrm{G}} \geqslant \theta H - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) & \bar{x}^{\mathrm{G}} < n\leqslant N,\\ IR_{H2}: \theta H - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant 0 & 1\leqslant n\leqslant \bar{x}^{\mathrm{G}},\\ IC_{H2}: \theta H - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant H - r^{\mathrm{G}} & 1\leqslant n\leqslant \bar{x}^{\mathrm{G}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{G}} < N$ and the expected waiting time $w^{\mathrm{G}}(n)$ is

$$w^{\mathrm{G}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1\leqslant n\leqslant \bar{x}^{\mathrm{G}},\\ \frac{\bar{x}^{\mathrm{G}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{G}}-1}{(1-\gamma)\lambda} & \bar{x}^{\mathrm{G}} < n\leqslant N. \end{cases}$$

In equilibrium, IR_L is binding at state $n=N$. Thus, $p^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = \theta L - c\cdot w^{\mathrm{G}}(N)$. Define two prices $\bar{r}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) \equiv p^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) - (\theta-1)H + c\cdot w^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$, $\bar{r}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) \equiv p^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) - (\theta-1)L + c\cdot w^{\mathrm{G}}(N)$. $\bar{r}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) < \bar{r}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ always holds because $H > L$ and $w^{\mathrm{G}}(n)$ is monotonously increasing in n. To satisfy IC_{H1} and IC_L, the price r^{G} should meet the constraints $r^{\mathrm{G}} < \bar{r}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ and $r^{\mathrm{G}} \geqslant \bar{r}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$. Since $\bar{r}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) < \bar{r}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ when $\theta > 1$, it is impossible to satisfy these two constraints at the same time. Therefore, Case II cannot become the equilibrium.

Using similar logic, we can show that Case I can become the equilibrium, which proves the proposition. Refer to the proof of Proposition 2 for details of Case I. ☐

Proof of Proposition 2. We continue the detailed analysis for Case I in this part (see the definition in the proof of Proposition 1). In the base model, the IR and IC constraints are

$$\begin{cases} IR_H: \theta H - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant 0 & 1\leqslant n\leqslant N,\\ IC_H: \theta H - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant H - r^{\mathrm{G}} & 1\leqslant n\leqslant N,\\ IR_{L1}: L - r^{\mathrm{G}} \geqslant 0 & \bar{x}^{\mathrm{G}} < n\leqslant N,\\ IC_{L1}: L - r^{\mathrm{G}} \geqslant \theta L - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) & \bar{x}^{\mathrm{G}} < n\leqslant N,\\ IR_{L2}: \theta L - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant 0 & 1\leqslant n\leqslant \bar{x}^{\mathrm{G}},\\ IC_{L2}: \theta L - p^{\mathrm{G}} - c\cdot w^{\mathrm{G}}(n) \geqslant L - r^{\mathrm{G}} & 1\leqslant n\leqslant \bar{x}^{\mathrm{G}}, \end{cases}$$

where $0 \leqslant \bar{x}^{\mathrm{G}} \leqslant N$ and the expected waiting time $w^{\mathrm{G}}(n)$ is

$$w^{\mathrm{G}}(n) = \begin{cases} \frac{n-1}{\lambda} & 1\leqslant n\leqslant \bar{x}^{\mathrm{G}},\\ \frac{\bar{x}^{\mathrm{G}}}{\lambda} + \frac{n-\bar{x}^{\mathrm{G}}-1}{\gamma\lambda} & \bar{x}^{\mathrm{G}} < n\leqslant N. \end{cases}$$

In equilibrium, IR_{L1} is binding. Thus, $r^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = L$. Whether IC_H or IC_{L2} is binding in equilibrium depends on the relative size of $\bar{x}^{\mathrm{G}}$. Since $w^{\mathrm{G}}(n)$ is monotonously increasing in n, no matter whether IC_H or IC_{L2} is binding, the binding must happen at the largest possible state n. Define two prices $\bar{p}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) \equiv \theta L - c\cdot w^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$, $\bar{p}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) \equiv (\theta-1)H + L - c\cdot w^{\mathrm{G}}(N)$, and we know that $\bar{p}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) < \bar{p}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ if and only if $\bar{x}^{\mathrm{G}} > \bar{x}_1^{\mathrm{G}}$, where $\bar{x}_1^{\mathrm{G}} \equiv N - 1 + \gamma - (\theta-1)(H-L)\gamma\lambda/c$. Then, we can write the price p^{G} as the function of tipping state $\bar{x}^{\mathrm{G}}$:

$$p^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = \begin{cases} \bar{p}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) & 1\leqslant \bar{x}^{\mathrm{G}}\leqslant \bar{x}_1^{\mathrm{G}},\\ \bar{p}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) & \bar{x}_1^{\mathrm{G}} < \bar{x}^{\mathrm{G}}\leqslant N. \end{cases}$$

We then derive the firm's long-run average profit π^{G}, also as the function of tipping state $\bar{x}^{\mathrm{G}}$:

$$\begin{aligned}\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) &= \left[p^{\mathrm{G}}(\bar{x}^{\mathrm{G}})\cdot\gamma\lambda + r^{\mathrm{G}}(\bar{x}^{\mathrm{G}})\cdot(1-\gamma)\lambda\right]\cdot P(n>\bar{x}^{\mathrm{G}}) + \left[p^{\mathrm{G}}(\bar{x}^{\mathrm{G}})\cdot\lambda\right]\cdot P(n\leqslant\bar{x}^{\mathrm{G}})\\ &= \frac{[p^{\mathrm{G}}(\bar{x}^{\mathrm{G}})\cdot\gamma\lambda + L(1-\gamma)\lambda]\cdot\frac{N-\bar{x}^{\mathrm{G}}}{\gamma\lambda} + [p^{\mathrm{G}}(\bar{x}^{\mathrm{G}})\cdot\lambda]\cdot\frac{\bar{x}^{\mathrm{G}}}{\lambda}}{\frac{\bar{x}^{\mathrm{G}}}{\lambda}+\frac{N-\bar{x}^{\mathrm{G}}}{\gamma\lambda}}\\ &= \frac{p^{\mathrm{G}}(\bar{x}^{\mathrm{G}})\cdot\gamma\lambda N + L(1-\gamma)\lambda(N-\bar{x}^{\mathrm{G}})}{N-(1-\gamma)\bar{x}^{\mathrm{G}}}.\end{aligned}$$

For $\bar{x}_1^{\mathrm{G}} < \bar{x}^{\mathrm{G}} \leqslant N$, plugging $p^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = \bar{p}_1^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ into $\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$, we have

$$\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = \frac{\theta L\gamma\lambda N + L(1-\gamma)\lambda(N-\bar{x}^{\mathrm{G}}) - c\gamma N(\bar{x}^{\mathrm{G}}-1)}{N-(1-\gamma)\bar{x}^{\mathrm{G}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ w.r.t. $\bar{x}^{\mathrm{G}}$, we have

$$\frac{\partial\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})}{\partial\bar{x}^{\mathrm{G}}} = \frac{\gamma N\left[L(1-\gamma)(\theta-1)\lambda - c(N-1+\gamma)\right]}{\left[N-(1-\gamma)\bar{x}^{\mathrm{G}}\right]^2}.$$

We can see that $\frac{\partial\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})}{\partial\bar{x}^{\mathrm{G}}} \geqslant 0$ if and only if $N \leqslant \bar{N}_1$. Therefore, for $\bar{x}_1^{\mathrm{G}} < \bar{x}^{\mathrm{G}} \leqslant N$, when $N \leqslant \bar{N}_1$, the firm sets $\bar{x}^{\mathrm{G}} = N$; when $N > \bar{N}_1$, the firm sets $\bar{x}^{\mathrm{G}} = \bar{x}_1^{\mathrm{G}}$. Note that $\bar{x}_1^{\mathrm{G}} > 0$ if and only if $N > \bar{N}_2$.

For $1 \leqslant \bar{x}^{\mathrm{G}} \leqslant \bar{x}_1^{\mathrm{G}}$, plugging $p^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = \bar{p}_2^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ into $\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$, we have

$$\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}}) = \frac{[(\theta-1)H+L]\,\gamma\lambda N + L(1-\gamma)\lambda(N-\bar{x}^{\mathrm{G}}) - cN(N-1) + c(1-\gamma)N\bar{x}^{\mathrm{G}}}{N-(1-\gamma)\bar{x}^{\mathrm{G}}}.$$

Taking the first-order derivative of $\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})$ w.r.t. $\bar{x}^{\mathrm{G}}$, we have

$$\frac{\partial\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})}{\partial\bar{x}^{\mathrm{G}}} = \frac{(1-\gamma)N\left[c+(\theta-1)H\gamma\lambda\right]}{\left[N-(1-\gamma)\bar{x}^{\mathrm{G}}\right]^2} > 0.$$

Since $\frac{\partial\pi^{\mathrm{G}}(\bar{x}^{\mathrm{G}})}{\partial\bar{x}^{\mathrm{G}}} > 0$ always holds, for $1 \leqslant \bar{x}^{\mathrm{G}} \leqslant \bar{x}_1^{\mathrm{G}}$, the firm always sets $\bar{x}^{\mathrm{G}} = \bar{x}_1^{\mathrm{G}}$.

Comparing the results above, we define two thresholds for the batch size N:

$$\begin{aligned}\bar{N}_1 &\equiv \frac{(\theta-1)(1-\gamma)L\lambda}{c} + 1 - \gamma,\\ \bar{N}_2 &\equiv \frac{(\theta-1)(H-L)\gamma\lambda}{c} + 1 - \gamma,\end{aligned}$$

which determines the optimal tipping state $\bar{x}^{\mathrm{G}}$. Besides, $\bar{N}_1$ increases in L. For a given L, $\bar{N}_2$ increases in H/L. $\bar{N}_2 > \bar{N}_1$ if and only if $H/L > 1/\gamma$. Thus, the REE when offering group buying is

(i) when $H/L \leqslant 1/\gamma$,

 (1) if $N \leqslant \bar{N}_1$, $\bar{x}^{\mathrm{G}} = N$, $r^{\mathrm{G}} = L$, $p^{\mathrm{G}} = \theta L - c(N-1)/\lambda$, and $\pi^{\mathrm{G}} = \theta L\lambda - c(N-1)$;

 (2) if $N > \bar{N}_1$, $\bar{x}^{\mathrm{G}} = \bar{x}_1^{\mathrm{G}}$, $r^{\mathrm{G}} = L$, $p^{\mathrm{G}} = \theta L + \gamma(\theta-1)(H-L) - c(N-2+\gamma)/\lambda$, and $\pi^{\mathrm{G}} = \pi_1^{\mathrm{G}}$;

(ii) when $H/L > 1/\gamma$,

 (1) if $N \leqslant \bar{N}_2$, $\bar{x}^{\mathrm{G}} = 0$, $r^{\mathrm{G}} = L$, $p^{\mathrm{G}} = (\theta-1)H + L - c(N-1)/(\gamma\lambda)$, and $\pi^{\mathrm{G}} = (\theta-1)H\gamma\lambda + L\lambda - c(N-1)$;

(2) if $N > \bar{N}_2$, $\bar{x}^{\mathrm{G}} = \bar{x}_1^{\mathrm{G}}$, $r^{\mathrm{G}} = L$, $p^{\mathrm{G}} = \theta L + \gamma(\theta-1)(H-L) - c(N-2+\gamma)/\lambda$, and $\pi^{\mathrm{G}} = \pi_1^{\mathrm{G}}$;

where

$$\pi_1^{\mathrm{G}} \equiv \frac{(\theta-1)(H-L)L(1-\gamma)\gamma\lambda^2 + (\theta-1)cHN\gamma^2\lambda + cL\lambda\left[1+\theta(1-\gamma)\gamma N + \gamma^2(N+1) - 2\gamma\right] - c^2\gamma N(N-2+\gamma)}{c\left[1+\gamma(N-2+\gamma)\right] + (\theta-1)(H-L)(1-\gamma)\gamma\lambda}.$$

Proof of Theorem 2. The range in which one strategy dominates the others follows directly by comparing the profits. Define the following thresholds for the inventory holding cost h:

$$\begin{aligned}
\bar{h}_2 &\equiv \frac{2c(N-1) + 2(\theta-1)(H\gamma - L)\lambda}{N+1}, \\
\bar{h}_3 &\equiv \frac{2c(N-1)}{N+1}, \\
\bar{h}_4 &\equiv \frac{2\lambda A_1[(\theta-1)H\gamma + L] - 2A_2}{(N+1)A_1}, \\
\bar{h} &\equiv \min\left\{\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4\right\},
\end{aligned}$$

where $A_1 \equiv c[1+\gamma(N-2+\gamma)] + (\theta-1)(H-L)(1-\gamma)\gamma\lambda$ and $A_2 \equiv (\theta-1)(H-L)L(1-\gamma)\gamma\lambda^2 + (\theta-1)cHN\gamma^2\lambda + cL\lambda\left[1+\theta(1-\gamma)\gamma N + \gamma^2(N+1) - 2\gamma\right] - c^2\gamma N(N-2+\gamma)$.

Define the following four thresholds for the batch size N:

$$\begin{aligned}
\bar{N}_3 &\equiv \frac{(\theta-1)L\lambda}{c} + 1, \\
\bar{N}_4 &\equiv \frac{(\theta-1)\left[(1-\gamma)L + \gamma H\right]\lambda}{c} + 2 - \gamma, \\
\bar{N}_5 &\equiv \frac{\left[L + (\theta-2)H\gamma\right]\lambda}{c} + 1, \\
\bar{N}_6 &\equiv \frac{\left[(\gamma + (1-\gamma)\theta)L + \gamma(\theta-2)H\right]\lambda}{2c} + 1 - \frac{\gamma}{2} + \frac{\sqrt{A_3}}{2c\gamma},
\end{aligned}$$

where $A_3 \equiv \left[4(1-\gamma)(L-H\gamma)\lambda\left(c(1-\gamma) + (H-L)(\theta-1)\gamma\lambda\right) + \gamma(c(2-\gamma) + (L(\gamma+(1-\gamma)\theta) + H\gamma(\theta-2)\lambda)^2)\right]\gamma$. $\bar{N}_3$ increases in L. For a given L, $\bar{N}_4$ increases in H/L, $\bar{N}_5$ increases in H/L when $\theta > 2$ while decreasing in H/L when $1 < \theta \leqslant 2$, and $\bar{N}_6$ decreases in H/L. Note that $\bar{N}_4 > \bar{N}_3 > \bar{N}_1$ always holds. For simplicity of notation, we further define the following threshold for the batch size N:

$$\bar{N} \equiv \begin{cases} \bar{N}_4 & H/L \leqslant 1/\gamma, \\ \max\{\bar{N}_5, \bar{N}_6\} & H/L > 1/\gamma. \end{cases}$$

When $\theta > 2$, $\bar{N}_5 > 0$ always holds; while when $1 < \theta \leqslant 2$, $\bar{N}_5 > 0$ if and only if $H/L < \bar{m}$, where $\bar{m} > 1/\gamma$ is a threshold for the valuation heterogeneity H/L, defined as the unique solution to the equation $\bar{N}_2 = \bar{N}_6$. □

Proof of Theorem 3. The customer surpluses under the volume, margin, and product-line strategies are: $S^{\mathrm{V}} = S_H^{\mathrm{V}} = (H-L)\gamma\lambda$, $S_L^{\mathrm{V}} = 0$; $S^{\mathrm{M}} = S_H^{\mathrm{M}} = S_L^{\mathrm{M}} = 0$; $S^{\mathrm{P}} = S_H^{\mathrm{P}} = (H-L)\gamma\lambda$, $S_L^{\mathrm{V}} = 0$. As for the group buying, the customer surpluses are

$$S^{\mathrm{G}} = \sum_{n=1}^{N} S_H^{\mathrm{G}}(n) \cdot \gamma\lambda \cdot P(n) + \sum_{n=1}^{\bar{x}_1^{\mathrm{G}}} S_L^{\mathrm{G}}(n) \cdot (1-\gamma)\lambda \cdot P(n),$$
$$S_H^{\mathrm{G}} = \sum_{n=1}^{N} S_H^{\mathrm{G}}(n) \cdot \gamma\lambda \cdot P(n),$$
$$S_L^{\mathrm{G}} = \sum_{n=1}^{\bar{x}_1^{\mathrm{G}}} S_L^{\mathrm{G}}(n) \cdot (1-\gamma)\lambda \cdot P(n),$$

where $S_H^{\mathrm{G}}(n) = \theta H - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n)$ and $S_L^{\mathrm{G}}(n) = \theta L - p^{\mathrm{G}} - c \cdot w^{\mathrm{G}}(n)$ are the individual surpluses for the high- and low-end customers at state n, respectively. By IC_H, IR_{L1} and IR_{L2}, we know $S_H^{\mathrm{G}}(n) \geqslant H - L$ and $S_L^{\mathrm{G}}(n) \geqslant 0$ hold for any $n\,(1 \leqslant n \leqslant N)$. The former equation does not always hold for all states, while when $H/L \geqslant 1/\gamma$ and $N < \bar{N}_2$, $S_L^{\mathrm{G}}(n) = 0$ holds for all states. In addition, since $\sum_{n=1}^{N} P(n) = 1$, and $P(n) > 0$ for any $n\,(1 \leqslant n \leqslant N)$, we have $S^{\mathrm{G}} > (H-L)\gamma\lambda$, $S_H^{\mathrm{G}} > (H-L)\gamma\lambda$, and $S_L^{\mathrm{G}} \geqslant 0$, which prove the theorem. □

B. Contingent Pricing

Proposition B.1 (REE under Contingent Pricing). *Under contingent pricing, for any given $\theta > 1$ and N, there exist two thresholds for the batch size, $\bar{N}_2^{\mathrm{C}} \equiv (\theta-1)(L - H\gamma)\lambda/c$ and $\bar{N}_3^{\mathrm{C}} \equiv (\theta-1)(H\gamma - L)\gamma\lambda/c$, such that*

(i) when $H/L \leqslant 1/\gamma$, the firm sets prices so that

(i-1) if $N \leqslant \bar{N}_2^{\mathrm{C}}$, $\{H+L\}$ is an REE;

(i-2) if $N > \bar{N}_2^{\mathrm{C}}$, $\{H; H+L\}$ is an REE;

(ii) when $H/L > 1/\gamma$, the firm sets prices so that

(ii-1) if $N \leqslant \bar{N}_3^{\mathrm{C}}$, $\{H\}$ is an REE;

(ii-2) if $N > \bar{N}_3^{\mathrm{C}}$, $\{H; H+L\}$ is an REE.

Proposition B.2 (Profitability of Contingent Pricing). *Compared with uniform pricing, contingent pricing always increases the firm's profit and enhances the firm's incentive to offer group buying.*

Theorem B.1 (Profit Comparison under Contingent Pricing). *Suppose $\theta > 1$.*

(i) If $H/L \leqslant 1/\gamma$, as N increases, the firm's optimal group-buying strategy changes from $\{G\,(H+L),\, R\,(\varnothing)\} \to \{G\,(H; H+L),\, R\,(L;\varnothing)\} \to \{NG,\, R\,(H+L)\}$.

(ii) If $H/L > 1/\gamma$, as N increases, the firm's optimal group-buying strategy changes from $\{G\,(H),\, R\,(L)\} \to \{G\,(H; H+L),\, R\,(L;\varnothing)\} \to \{NG,\, R\,(H)\}$.

C. Unobservable Group Buying

Proposition C.1 (REE in Unobservable Group Buying). *In unobservable group buying, for any given $\theta > 1$ and N, the firm sets prices so that*

(i) when $H/L \leqslant 1/\gamma$, $\{H+L\}$ is an REE;

(ii) when $H/L > 1/\gamma$, $\{H\}$ is an REE.

Theorem C.1 (Profit Comparison in Unobservable Group Buying). *Suppose $\theta > 1$.*

(i) If $H/L \leqslant 1/\gamma$, as N increases, the firm's optimal group-buying strategy changes from $\{G(H+L), R(\varnothing)\} \rightarrow \{NG, R(H+L)\}$.

(ii) If $H/L > 1/\gamma$, as N increases, the firm's optimal group-buying strategy changes from $\{G(H), R(L)\} \rightarrow \{NG, R(H)\}$.

Proposition C.2 (Profitability of Unobservable Group Buying). *When θ is sufficiently large, compared with observable group buying, unobservable group buying increases the firm's profit and its incentive to offer group buying.*

D. Heterogeneous Waiting Costs

Proposition D.1 (Customer Segmentation with Heterogeneous Waiting Costs). *Suppose $\theta > 1$. If customers have heterogeneous waiting costs, there exists a threshold for the waiting cost, $\bar{c}_H$, such that*

(i) when $c_H \leqslant \bar{c}_H$, with group buying, customer segmentation in equilibrium must be one of the following three scenarios: $\{H\}$, $\{H; H+L\}$, or $\{H+L\}$;

(ii) when $c_H > \bar{c}_H$, with group buying, customer segmentation in equilibrium must be one of the following three scenarios: $\{L\}$, $\{L; L+H\}$, or $\{L+H\}$.

Proposition D.2 (REE with Heterogeneous Waiting Costs). *If customers have heterogeneous waiting costs, for any given $\theta > 1$ and N, there exists a threshold for the waiting cost, $\bar{c}_H$, and three thresholds for the batch size, $\bar{N}_1^{\mathrm{D}} \equiv (\theta-1)(1-\gamma)L\lambda/c_L + 1 - \gamma$, $\bar{N}_2^{\mathrm{D}} \equiv (\theta-1)(H-L)\gamma\lambda/c_H + 1 - \gamma c_L/c_H$, and $\bar{N}_3^{\mathrm{D}} \equiv (\theta-1)H\gamma\lambda/c_H + \gamma$, such that*

(i) when $c_H \leqslant \bar{c}_H$, the firm sets prices so that

(i-i) when $H \leqslant [\gamma c_L + (1-\gamma)c_H]L/(\gamma c_L) - (c_H - c_L)/[(\theta-1)\lambda]$,

(i-i-1) if $N \leqslant \bar{N}_1^{\mathrm{D}}$, $\{H+L\}$ is an REE;

(i-i-2) if $N > \bar{N}_1^{\mathrm{D}}$, $\{H; H+L\}$ is an REE;

(i-ii) when $H/L > [\gamma c_L + (1-\gamma)c_H]L/(\gamma c_L) - (c_H - c_L)/[(\theta-1)\lambda]$,

(i-ii-1) if $N \leqslant \bar{N}_2^{\mathrm{D}}$, $\{H\}$ is an REE;

(i-ii-2) if $N > \bar{N}_2^{\mathrm{D}}$, $\{H; H+L\}$ is an REE;

(ii) when $c_H > \bar{c}_H$, the firm sets prices so that

(ii-i) if $N \leqslant \bar{N}_3^{\mathrm{D}}$, $\{L+H\}$ is an REE;

(ii-ii) if $N > \bar{N}_3^{\mathrm{D}}$, $\{L; L+H\}$ is an REE.

Corollary D.1 (Effect of Heterogeneous Waiting Costs on Customer Segmentation). *If customers have heterogeneous waiting costs, compared with the base model,*

(i) when $c_H \leqslant c_L$, the attractiveness of $\{H; H+L\}$ to the firm compared with $\{H\}$ decreases;

(ii) when $c_L < c_H \leqslant \bar{c}_H$, the attractiveness of $\{H; H+L\}$ to the firm compared with $\{H\}$ increases.

E. Inferior Group-Buying Product

Proposition E.1 (Customer Segmentation with Group Buying). *Suppose $\theta < 1$. With group buying, the customer segmentation must be one of the following three scenarios: $\{L\}$, $\{L; L+H\}$, or $\{L+H\}$.*

Proposition E.2 (REE with Inferior Group-Buying Product). *With inferior group buying, for any given $\theta < 1$ and N, there exists a threshold for the batch size $\bar{N}^{\mathrm{I}}$, and the firm sets prices so that*

(i) if $N \leqslant \bar{N}^{\mathrm{I}}$, $\{L\}$ is an REE;

(ii) if $N > \bar{N}^{\mathrm{I}}$, $\{L; L+H\}$ is an REE.

Theorem E.1 (Profit Comparison with Inferior Group-Buying Product). *Compared with the volume and margin strategies, it is profitable for the firm to offer group buying with an inferior product (i.e., $\theta < 1$) if and only if N is in an intermediate range, which is not empty if γ is sufficiently high.*

F. Horizontally Differentiated Products

Proposition F.1 (REE with Horizontally Differentiated Products). *In the context of horizontally differentiated products, there exists a threshold for the batch size $\bar{N}^{\mathrm{H}} \equiv (H - L)\gamma\lambda/c + 1 - \gamma$, and the firm sets prices so that*

(i) if $N \leqslant \bar{N}^{\mathrm{H}}$, $\{G\}$ is an REE;

(ii) if $N > \bar{N}^{\mathrm{H}}$, $\{G; G+R\}$ is an REE.

Theorem F.1 (Profit Comparison with Horizontally Differentiated Products). *In the context of horizontally differentiated products, there exists a threshold for the inventory holding cost $\bar{h}^{\mathrm{H}}$, above which it is optimal for the firm to offer the product line via flexible-duration group buying rather than doing so noncontingently, and below which the opposite is ture.*