Online Appendix to
Contract Unobservability and Downstream Competition

A. Proofs

Proof of Lemma 1: In a centralized channel, the manufacturer's profit is $\Pi = p_i D_i + p_j D_j$. The manufacturer chooses p_i and p_j that maximize Π. Solving the manufacturer's profit maximization problem we have $p_i = p_j = \frac{1}{2}$ and $\Pi = \frac{1}{2}$. Q.E.D.

Proof of Lemma 2: Following the discussion in the main text, in equilibrium, the manufacturer optimally extracts all the retailers' profits, i.e., $\pi_i^o = \pi_j^o$ in equilibrium. We can derive that

$$K_i^o = \frac{(1+\theta)(2-2w_i^o+\theta(3-(4+\theta)w_i^o+w_j^o+\theta w_j^o))^2}{(4+8\theta+3\theta^2)^2},$$

$$K_j^o = \frac{(1+\theta)(2-2w_j^o+\theta(3-(4+\theta)w_j^o+w_i^o+\theta w_i^o))^2}{(4+8\theta+3\theta^2)^2},$$

and that the manufacturer's profit is

$$\Pi^o = \frac{(1+\theta)(w_i^o(2+3\theta-(2+4\theta-\theta^2)w_i^o)+w_j^o(2+3\theta-(2+4\theta+\theta^2)w_j^o)-2\theta(1+\theta)w_i^o w_j^o)}{(2+\theta)(2+3\theta)} + K_i^o + K_j^o.$$

Maximizing the manufacturer's profit proves the lemma. Q.E.D.

Proof of Lemma 3: Following the discussion in the main text, we derive that the manufacturer's profit is

$$\Pi^o = \frac{(1+\theta)(w_i^o(2+3\theta-(2+4\theta-\theta^2)w_i^o)+w_j^o(2+3\theta-(2+4\theta+\theta^2)w_j^o)-2\theta(1+\theta)w_i^o w_j^o)}{(2+\theta)(2+3\theta)},$$

which is maximized at $w_i^o = w_j^o = \frac{1}{2}$. The rest of the proof follows immediately. Q.E.D.

Proof of Proposition 1: In the main text we have solved the retailer's problem. Now we solve the manufacturer's problem. The manufacturer chooses (K_i, w_i) and (K_j, w_j) to maximize the total profit from the retailers,

$$\Pi = w_i \cdot (1 - p_i + \theta(\tilde{p}_j - p_i)) + K_i + \Pi_j,$$

subject to the retailers' participation constraints $\tilde{\pi}_i \geq 0, \tilde{\pi}_j \geq 0$, where Π_j is the manufacturer's profit from retailer j. Note that Π_j is independent of K_i and w_i as retailer j does not observe the contract terms between the manufacturer and retailer i. Using back substitution and solving for the manufacturer's problem, we obtain the following solution:

$$K_i = \frac{(1+\theta\tilde{p}_j)^2}{4(1+\theta)}, \quad w_i = 0.$$

Similarly, we have

$$K_j = \frac{(1+\theta\tilde{p}_i)^2}{4(1+\theta)}, \quad w_j = 0.$$

Plugging $w_i = w_j = 0$ into p_i, p_j, we obtain

$$p_i = \frac{1+\theta\tilde{p}_j}{2(1+\theta)}, \;\; p_j = \frac{1+\theta\tilde{p}_i}{2(1+\theta)}.$$

The retailers' beliefs must be correct along the equilibrium path, i.e., $p_i = \tilde{p}_i, p_j = \tilde{p}_j$. It follows that $p_i = p_j = \frac{1}{2+\theta}$. The proposition follows immediately. Q.E.D.

Proof of Corollary 1: The proof follows from $\frac{\partial \Pi}{\partial \theta} = -\frac{2\theta}{(2+\theta)^3} < 0$. Q.E.D.

Proof of Proposition 2: Let $\tilde{p}_j$ denote retailer i's conjecture of p_j, which, according to passive beliefs, is not a function of w_i. As such, the problem facing retailer i is to choose p_i that maximizes its conjectured profit π_i, i.e.,

$$\tilde{\pi}_i = (p_i - w_i)(1 - p_i + \theta(\tilde{p}_j - p_i)),$$

where $1 - p_i + \theta(\tilde{p}_j - p_i)$ is retailer i's belief of the demand. Solving the retailer's problem we come up with

$$p_i = \frac{1+\theta\tilde{p}_j}{2+2\theta} + \frac{w_i}{2}.$$

Now we solve the manufacturer's decision. The manufacturer chooses w_i and w_j that maximize the total profit,

$$\Pi = w_i \cdot (1 - p_i + \theta(\tilde{p}_j - p_i)) + \Pi_j,$$

where Π_j is the manufacturer's profit from retailer j, which is independent of w_i. Solving the manufacturer's problem we have

$$w_i = \frac{1+\theta\tilde{p}_j}{2+2\theta}.$$

It follows that $p_i = \frac{3+3\theta\tilde{p}_j}{4(1+\theta)}$, and similarly, we can show that $p_j = \frac{3+3\theta\tilde{p}_i}{4(1+\theta)}$. The retailers' beliefs must be correct, i.e., $\tilde{p}_i = p_i, \tilde{p}_j = p_j$. Solving for p_i, p_j we get $p_i = p_j = \frac{3}{4+\theta}$. The proposition follows immediately. Q.E.D.

Proof of Corollary 2: For the manufacturer, we have

$$\Pi - \Pi^o = -\frac{\theta^2(1+\theta)}{2(2+\theta)(4+\theta)^2)} < 0.$$

For the retailer, we have

$$\pi_i - \pi_i^o = \frac{\theta(1+\theta)(8+3\theta)}{4(2+\theta)^2(4+\theta)^2} > 0.$$

This completes the proof. Q.E.D.

Proof of Proposition 3: The proposition follows immediately from comparing the firms' profits that are summarized in Table 2 and Table 3. Q.E.D.

Proof of Lemma 4: The analysis is analogous to that of the basic model. Let $\tilde{p}_j$ denote retailer i's conjecture of p_j, which, according to passive beliefs, is independent of w_i. Retailer i chooses p_i that maximizes its (conjectured) profit, i.e.,

$$\tilde{\pi}_i = (p_i - w_i)(1 - p_i + \theta(\tilde{p}_j - p_i)),$$

where $1 - p_i + \theta(\tilde{p}_j - p_i)$ is retailer i's belief of its demand. Solving the retailer i's problem we come up with

$$p_i = \frac{1+\theta\tilde{p}_j}{2+2\theta} + \frac{w_i}{2},$$

and symmetrically, we have

$$p_j = \frac{1+\theta\tilde{p}_i}{2+2\theta} + \frac{w_j}{2}.$$

Now we solve the manufacturer's decision. The manufacturer chooses w_i and w_j that maximize the total profit,

$$\Pi = w_i \cdot (1 - p_i + \theta(p_j - p_i)) + w_j \cdot (1 - p_j + \theta(p_i - p_j)).$$

Note that in the manufacturer's profit function above, the sales (the realized demands) to each retailer are determined by the true retail prices, and thus they are different from the conjectured demands of each retailer. Solving the manufacturer's problem we have

$$w_i = \frac{1+2\theta-\theta\tilde{p}_j}{2+2\theta}, \;\; w_j = \frac{1+2\theta-\theta\tilde{p}_i}{2+2\theta}.$$

The beliefs must coincide with the equilibrium actions, i.e., $p_i = \tilde{p}_i, p_j = \tilde{p}_j$, and this leads to the equilibrium solution summarized in the lemma. Q.E.D.

Proof of Proposition 4: The proof follows from direct profit comparison. Q.E.D.

Proof of Lemma 5: Under two-part tariff contracts, let $\tilde{p}_j$ be retailer i's belief of p_j. Given (K_i, w_i), if retailer i accepts the offer, p_i is chosen to maximize

$$\tilde{\pi}_i = (1 - p_i + \theta(\tilde{p}_j - p_i)) \cdot (p_i - w_i) - K_i.$$

Solving retailer i's problem we have

$$p_i = \frac{1 + \theta\tilde{p}_j}{2(1+\theta)} + \frac{w_i}{2}.$$

Similarly, for retailer j, we have

$$p_j = \frac{1 + \Delta + \theta\tilde{p}_i}{2(1+\theta)} + \frac{w_j}{2}.$$

The manufacturer maximizes the total profit,

$$\Pi = (1 - p_i + \theta(\tilde{p}_j - p_i)) \cdot w_i + K_i + (1 + \Delta - p_j + \theta(\tilde{p}_i - p_j)) \cdot w_j + K_j,$$

subject to the retailers' participation constraints $\tilde{\pi}_i \geq 0$ and $\tilde{\pi}_j \geq 0$. Solving the manufacturer's profit maximization problem, we obtain

$$w_i = 0, \;\; K_i = \frac{(1 + \theta\tilde{p}_j)^2}{4(1+\theta)}.$$

Similarly, for retailer j, we obtain the following results,

$$w_j = 0, \;\; K_i = \frac{(1 + \Delta + \theta\tilde{p}_i)^2}{4(1+\theta)}.$$

The retailers' beliefs must be correct, i.e., $p_i = \tilde{p}_i, p_j = \tilde{p}_j$. Solving the model yields the following equilibrium outcome:

$$(K_i, w_i) = \left(\frac{(1+\theta)(2+3\theta+\Delta\theta)^2}{(4+8\theta+3\theta^2)^2}, 0 \right), \;\; (K_j, w_j) = \left(\frac{(1+\theta)(2+3\theta+2\Delta+2\theta\Delta)^2}{(4+8\theta+3\theta^2)^2}, 0 \right),$$

and the retail prices are

$$p_i = \frac{2+3\theta+\Delta\theta}{(2+\theta)(2+3\theta)}, \;\; p_j = \frac{2+3\theta+2\Delta(1+\theta)}{(2+\theta)(2+3\theta)}.$$

The manufacturer's equilibrium profit is

$$\Pi = \frac{(1+\theta)(2(2+3\theta)^2 + 2(2+3\theta)^2\Delta + (4+8\theta+5\theta^2)\Delta^2)}{(4+8\theta+3\theta^2)^2},$$

and the retailers' equilibrium profits are $\pi_i = \pi_j = 0$.

We now consider wholesale price contracts, and let $\tilde{p}_j$ be retailer i's belief of p_j. Given w_i, retailer i chooses p_i that maximizes its conjectured profit

$$\tilde{\pi}_i = (1 - p_i + \theta(\tilde{p}_j - p_i)) \cdot (p_i - w_i).$$

Solving retailer i's problem we come up with

$$p_i = \frac{1 + \theta \tilde{p}_j}{2(1+\theta)} + \frac{w_i}{2}.$$

The manufacturer maximizes its total profit

$$\Pi = (1 - p_i + \theta(\tilde{p}_j - p_i)) \cdot w_i + \Pi_j,$$

where Π_j is the manufacturer's profit from retailer j, which is, according to passive beliefs, independent of w_i. Solving the manufacturer's problem, we obtain

$$w_i = \frac{1 + \theta \tilde{p}_j}{2(1+\theta)}.$$

Similarly, for retailer j, we have the following result,

$$w_j = \frac{1 + \theta \tilde{p}_i}{2(1+\theta)} + \frac{\Delta}{2}.$$

The retailers' beliefs must be correct, i.e., $p_i = \tilde{p}_i, p_j = \tilde{p}_j$. Solving the model yields the following equilibrium outcome: The contracts are

$$w_i = \frac{2(4 + 7\theta + 3\theta\Delta)}{(4+\theta)(4+7\theta)}, \quad w_j = \frac{2(4 + 7\theta + 4\Delta + 4\theta\Delta)}{(4+\theta)(4+7\theta)},$$

and the retail prices are

$$p_i = \frac{3(4 + 7\theta + 3\theta\Delta)}{(4+\theta)(4+7\theta)}, \quad p_j = \frac{3(4 + 7\theta + 4\Delta + 4\theta\Delta)}{(4+\theta)(4+7\theta)}.$$

The manufacturer's equilibrium profit is

$$\Pi = \frac{2(1+\theta)(2(4+7\theta)^2 + 2(4+7\theta)^2\Delta + (16 + 32\theta + 25\theta^2)\Delta^2)}{(4+\theta)^2(4+7\theta)^2},$$

and the retailers' equilibrium profits are

$$\pi_i = \frac{(1+\theta)(4 + 7\theta + 3\theta\Delta)^2}{(4+\theta)^2(4+7\theta)^2}, \quad \pi_j = \frac{(1+\theta)(4 + 7\theta + 4\Delta + 4\theta\Delta)^2}{(4+\theta)^2(4+7\theta)^2}.$$

This completes the proof. Q.E.D.

Proof of Proposition 5: Consider the manufacturer's profit under alternative supply chain contracts. Let $r = \frac{\Pi_{\text{wholesale}}}{\Pi_{\text{2PT}}} - 1$ be the manufacturer's profit improvement under wholesale price over two-part tariffs. Simple calculation yields that

$$\lim_{\theta \to \infty} r = \frac{1}{49}\left(41 + \frac{144(1+\Delta)}{18 + 18\Delta + 5\Delta^2}\right)$$

It follows that $\lim_{\theta \to \infty} r > 0$ holds for any Δ. Therefore, the proposition follows immediately. Q.E.D.

Proof of Lemma 6: Given wholesale price w, the retailer's profit is

$$\pi = (1 - p_i + \theta(p_j - p_i)) \cdot (p_i - w) + (1 - p_j + \theta(p_i - p_j)) \cdot (p_j - w) - K.$$

Given w, the retailer's optimal decisions are $p_i = p_j = (1 + w)/2$. In anticipation of the retailer's actions, the manufacturer chooses K and w that maximize

$$\Pi = (1 - p_i + \theta(p_j - p_i)) \cdot w + (1 - p_j + \theta(p_i - p_j)) \cdot w + K,$$

subject to the retailer's participation constraint $\pi \geq 0$. Solving the manufacturer's proves the lemma. Q.E.D.

Proof of Lemma 7: Given the wholesale price w, the merged retailer again chooses p_i, p_j to maximize its total profit, which leads to $p_i = p_j = (1 + w)/2$. Given the retailer's best response functions, the manufacturer chooses w to maximize its total profit,

$$\Pi = (1 - p_i + \theta(p_j - p_i)) \cdot w + (1 - p_j + \theta(p_i - p_j)) \cdot w.$$

Solving the manufacturer's problem, we come up with the lemma. Q.E.D.

Proof of Proposition 6: The proof follows from direct profit comparison. Q.E.D.

B. Wary Beliefs

So far, we assumed that retailers hold passive beliefs regarding offers made to their rivals. There are, however, alternative types of beliefs so as *wary beliefs*, which were initially introduced by McAfee and Schwartz (1994). Under wary beliefs, a retailer facing an out-of-equilibrium contract offer believes that, upon acceptance, the manufacturer will adjust the offers to other retailers so as to maximize the manufacturer's own profit and, importantly, each retailer is convinced that the other retailers share the same reasoning. We show that our results are robust under wary beliefs, and the firms' actions are the same under wary beliefs and passive beliefs.

Wholesale Prices

Suppose that the manufacturer offers a wholesale price w_i to retailer i. Let $R(w_i)$ be the optimal wholesale price that the manufacturer offers to retailer j given that the wholesale price to retailer i is w_i, and let $\tilde{R}(w_i)$ be retailer i's belief of w_j when it is offered a wholesale price w_i, which coincides with $R(w_i)$ in equilibrium.

Let $\Pi(w_i, w_j)$ be the manufacturer's profit given two offers w_i and w_j, wary beliefs require that

$$R(w_i) = \arg\max_{v} \ \Pi(w_i, v).$$

In other words, $(w_i, w_j) = (w_i, R(w_i))$ maximizes the manufacturer's profit given that the manufacturer offers w_i to retailer i. This is translated into

$$R(w_i) = \arg\max_{v} \quad w_i \cdot q(w_i, \tilde{R}(w_i)) + v \cdot q(v, \tilde{R}(v)),$$

where $q(w_i, w_j)$ is retailer i's procurement when the wholesale price is w_i and retailer i's rival is offered a wholesale price w_j. Note that v is absent in the first term on the right-hand side, and hence

$$R(w_i) = \arg\max_{v} \quad v \cdot q(v, \tilde{R}(v)).$$

Therefore $R(w_i)$ is independent of w_i. Following this, $\tilde{p}_j$, retailer i's conjecture of p_j, is also independent of w_i. As such, retailer i's profit maximization problem is

$$\pi_i = \max_{p_i} \ (p_i - w_i) \cdot (1 - p_i + \theta(\tilde{p}_j - p_i)).$$

Solving the retailer's problem yields that $p_i = \frac{1+\theta\tilde{p}_j}{2+2\theta} + \frac{w_i}{2}$. The manufacturer chooses w_i, w_j to maximize the total profit

$$\Pi = w_i \cdot (1 - p_i + \theta(\tilde{p}_j - p_i)) + \Pi_j,$$

where Π_j, the manufacturer's profit from retailer j, is independent of w_i. It follows that

$$w_i = \frac{1 + \theta\tilde{p}_j}{2(1+\theta)}.$$

The retailers' beliefs must be correct, i.e., $p_i = p_j = \tilde{p}_i = \tilde{p}_j$. Solving this yields the equilibrium strategy, which is equal to the equilibrium outcome under passive beliefs.

Two-part Tariffs

Suppose that the manufacturer offers a two-part tariff contract (K_i, w_i) to retailer i. Similarly, given contract (K_i, w_i), let $(T(w_i), R(w_i))$ be the optimal contract that the manufacturer offers to retailer j. Note that K_i is absent in the contract because it has nothing to do with the rival's strategy as long as the contract is accepted.

Let $\Pi(K_i, w_i, K_j, w_j)$ be the manufacturer's profit given two offers (K_i, w_i) and (K_j, w_j), wary beliefs require that

$$(T(w_i), R(w_i)) = \arg\max_{(g,\, v)} \ \Pi(K_i, w_i, g, v),$$

which can be further expressed in the following:

$$(T(w_i), R(w_i)) = \arg\max_{(g,\, v)} \quad w_i \cdot q(w_i, \tilde{R}(w_i)) + K_i + \Pi_j(g, v),$$

where $\Pi_j(g, v)$ is the manufacturer's profit earned from retailer j for a given two-part tariff contract (g, v). Note that the manufacturer extracts the retailer's entire profit from sales through the fixed fee. [1] Therefore,

$$\Pi_j(g, v) = v \cdot q(v, \tilde{R}(v)) + \pi(v, \tilde{R}(v)),$$

where $\pi(v, \tilde{R}(v))$ is retailer j's profit from sales when it is offered a wholesale price v and the rival retailer is offered a wholesale price $\tilde{R}(v)$. We can then determine

$$R(w_i) = \arg\max_{v} \quad w_i \cdot q(w_i, \tilde{R}(w_i)) + K_i + v \cdot q(v, \tilde{R}(v)) + \pi(v, \tilde{R}(v)).$$

Again, v is absent in the first two terms on the right-hand side, and thus $R(w_i)$ is independent of w_i, and so is $T(w_i)$. We can then show that the equilibrium strategies simply replicate the results of the basic model, and all our insights hold.

The above analysis suggests that our main findings are robust under wary beliefs: the manufacturer could prefer a wholesale price contract to a two-part tariff contract when contract terms are unobservable.

References

McAfee, R. P., and Schwartz, M. 1994. Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *American Economic Review*, 210–230.

[1] The retailer's profit from sales refers to the retailer's revenue net of its procurement cost.