Online Appendix – Manufacturer's Entry in the Product-Sharing Market

This online appendix contains the proofs of all the lemmas and propositions in the paper. Note that to show the proofs of Propositions 1~5, we first need solve the manufacturer's profit-maximization problem as defined in (P1). Due to space constraint, we omit the analysis of (P1) here. For details, one can refer to Appendix A in the Supplemental Materials file (*available upon request*).

PROOF OF LEMMA 1. This lemma examines the equilibrium of the product-sharing market. We solve the manufacturer's demand and the equilibrium market-clearing prices in the sharing market as a function of the manufacturer's retail price p and rental quantity q. We list below consumer i's possible options for her buying and rental decisions in the first period, with the corresponding total expected utility (across n periods).

(a) Buy the product, use it in the current usage period 1: $U_{ia} = v_{i1} - p + \sum_{j=2}^{n} E[\max(r_j - \beta, v_{ij})]$.

(b) Buy the product and rent it out in the sharing market in period 1:

$$U_{ib} = r_1 - \beta - p + \sum_{j=2}^{n} E[\max(r_j - \beta, v_{ij})].$$

(c) Do not buy the product but rent a product in the sharing market in period 1:

$$U_{ic} = v_{i1} - r_1 + \sum_{j=2}^{n} E[\max(v_{ij} - r_j, 0)].$$

(d) Do not buy the product and do not rent a product in the sharing market in period 1:

$$U_{id} = \sum_{j=2}^{n} E[\max(v_{ij} - r_j, 0)].$$

Note that in each future usage period $j \geq 2$, the supply in the product-sharing market comes from those products sold to the consumers before and the rental units offered by the manufacturer (i.e., $D(p, q) + q$). The expected equilibrium sharing prices will be the same for all periods $j \geq 2$, i.e., $E[r_2] = E[r_3] = \cdots = E[r_n]$. So, we can simplify the utility expressions (noting that both v_{ij} and r_j are stochastic variables; the expectation below is first taken with respect to the self-use value v_{ij}):

$\sum_{j=2}^{n} E[\max(r_j-\beta, v_{ij})] = (n-1)E[\max(r_j-\beta, v_{ij})] = (n-1)E\{\int_0^{r_j-\beta}(r_j-\beta)dx + \int_{r_j-\beta}^{1} xdx\} = \frac{n-1}{2}E[1+(r_j-\beta)^2]$, and

$\sum_{j=2}^{n} E[\max(v_{ij}-r_j, 0)] = (n-1)E[\max(v_{ij}-r_j, 0)] = (n-1)E\{\int_{r_j}^{1}(x-r_j)dx\} = \frac{n-1}{2}E[(1-r_j)^2]$.

Plugging the above expressions for $\sum_{j=2}^{n} E[\max(r_j-\beta, v_{ij})]$ and $\sum_{j=2}^{n} E[\max(v_{ij}-r_j, 0)]$ into the utility expressions, we have: (a) $U_{ia} = v_{i1} - p + \frac{n-1}{2}E[1+(r_j-\beta)^2]$; (b) $U_{ib} = r_1 - \beta - p + \frac{n-1}{2}E[1+(r_j-\beta)^2]$; (c) $U_{ic} = v_{i1} - r_1 + \frac{n-1}{2}E[(1-r_j)^2]$; (d) $U_{id} = \frac{n-1}{2}E[(1-r_j)^2]$. Next, we examine the consumers' optimal options.

If $r_1 - \beta - p + \frac{n-1}{2}E[1+(r_j-\beta)^2] - \frac{n-1}{2}E[(1-r_j)^2] \geq 0$ (e.g., $r_1 - p + \frac{n-1}{2}E[1+\left(r_j-\beta\right)^2] - \frac{n-1}{2}E[(1-r_j)^2] > 0$), then $U_{ia} > U_{ic}$, i.e., option (a) dominates option (c) and no consumer will rent the product from the sharing market in the first period. If $r_1 - \beta - p + \frac{n-1}{2}E[1+(r_j-\beta)^2] - \frac{n-1}{2}E[(1-r_j)^2] < 0$, then $U_{ib} < U_{id}$, i.e., option (b) is dominated by option (d). No consumer will rent the product out in the sharing market in the first period. In summary, there will be no C2C product-sharing transactions in the first period in equilibrium; option (b) is ruled out. So, in equilibrium, consumers in the first period choose option (a), option (c), or option (d). Next we break the analysis into two parts based on the comparisons between option (a) and option (c).

(i) First, if $r_1 - p + \frac{n-1}{2}E[1+(r_j-\beta)^2] - \frac{n-1}{2}E[(1-r_j)^2] < 0$, then $U_{ia} < U_{ic}$, so option (a) is dominated by option (c) and no consumer will buy the product. Note that when no consumer buy the product, the sharing price only depends on the manufacturer's rental units q. Specifically, there will be $r_1 = r_2 = \cdots = r_n = 1-q$. Plugging in, the constraint $r_1 - p + \frac{n-1}{2}E[1+(r_j-\beta)^2] - \frac{n-1}{2}E[(1-r_j)^2] < 0$ is equivalent to $\frac{(n+1)+(n-1)(1-\beta)^2}{2} - p - [1+(n-1)(1-\beta)]q < 0$. Thus when $p + [1+(n-1)(1-\beta)]q > \frac{(n+1)+(n-1)(1-\beta)^2}{2}$, we have $D(p,q) = 0$.

(ii) Second, if $r_1 - p + \frac{n-1}{2}E[1 + (r_j - \beta)^2] - \frac{n-1}{2}E[(1 - r_j)^2] \geq 0$, then $U_{ia} \geq U_{ic}$, i.e., option (c) is dominated by option (a). So, consumers choose either option (a) or option (d). In this case, there will be no transactions in the sharing market (neither B2C nor C2C transactions) in the first period in equilibrium. If $v_{i1} - p + \frac{n-1}{2}E[1 + (r_j - \beta)^2] \geq \frac{n-1}{2}E[(1 - r_j)^2]$ (i.e., $v_{i1} \geq p - \frac{n-1}{2}E[1 + (r_j - \beta)^2] + \frac{n-1}{2}E[(1 - r_j)^2]$), the consumer will choose option (a) and buy the product. If $v_{i1} < p - \frac{n-1}{2}E[1 + (r_j - \beta)^2] + \frac{n-1}{2}E[(1 - r_j)^2]$, the consumer will choose option (d) and not buy the product. We obtain the market demand function

$$D(p, q) = 1 - p + \frac{n-1}{2}E[1 + (r_j - \beta)^2] - \frac{n-1}{2}E[(1 - r_j)^2] = 1 - p + \frac{n-1}{2}[2(1 - \beta)E(r_j) + \beta^2].$$

Note that in any future period $j \geq 2$, the equilibrium market-clearing sharing price r_j is a random variable determined by the realizations of all consumers' self-use values v_{ij}. The total demand and the total supply for sharing in period j are both functions of the sharing price r_j. In equilibrium, the supply and the demand for sharing will be equal, which is the condition we use to determine the expected sharing price. We separate consumers into three groups based on their realized self-use value (v_{ij}). The product owners with a realized self-use value $v_{ij} < r_j - \beta$ will rent out their products in the sharing market, i.e., they contribute to the supply for sharing. The non-owners with $v_{ij} > r_j$ will rent a product from the sharing market, i.e., they represent the demand in the sharing market. For consumers with $v_{ij} \in (r_j - \beta, r_j)$, the product owners—those who bought the product before—will use the product themselves and the non-owners will neither use nor rent the product. Since the consumer's v_{ij} is independently drawn across periods, the expected number of non-owners with $v_{ij} > r_j$ is simply $(1 - r_j)[1 - D(p, q)]$. That is, the total demand in the sharing market is $(1 - r_j)[1 - D(p, q)]$. Meanwhile, the expected number of owners with $v_{ij} < r_j - \beta$ is simply $(r_j - \beta)D(p, q)$, so the sharing supply from these consumers is $(r_j - \beta)D(p, q)$. Since the manufacturer's supply in the sharing market is q, so at r_j, the total supply in the sharing market is $q + (r_j - \beta)D(p,q)$. Thus, in equilibrium of the sharing market, we must have in expectation $(1 - r_j)[1 - D(p,q)] = q + (r_j - \beta)D(p,q)$, which yields $E[r_j] = 1 - q - (1 - \beta)D(p,q)$. Substituting $E[r_j]$ into equation $D(p,q)$, one can obtain $D(p,q) = 1 + \frac{(n-1)(2\beta-\beta^2)}{2[1+(n-1)(1-\beta)^2]} - \frac{p+(n-1)(1-\beta)q}{1+(n-1)(1-\beta)^2}$. When $p + [1 + (n-1)(1-\beta)]q < \frac{(n+1)+(n-1)(1-\beta)^2}{2}$, one can easily show $D(p,q) > 0$. Meanwhile, if $p + (n-1)(1-\beta)q \leq \frac{(n-1)-(n-1)(1-\beta)^2}{2}$, $D(p,q) \geq 1$.

Summarizing (i) and (ii), we obtain the manufacturer's demand function:

$$D(p,q) = \begin{cases} 1 & \text{if} \quad p + (n-1)(1-\beta)q \leq \frac{(n-1)-(n-1)(1-\beta)^2}{2}, \\ 0 & \text{if} \quad p + [1 + (n-1)(1-\beta)]q \geq \frac{(n+1)+(n-1)(1-\beta)^2}{2}, \\ 1 + \frac{(n-1)(2\beta-\beta^2)}{2[1+(n-1)(1-\beta)^2]} - \frac{p+(n-1)(1-\beta)q}{1+(n-1)(1-\beta)^2} & \text{otherwise.} \end{cases}$$

In equilibrium, if some consumers choose to buy the product (i.e., $D(p,q) > 0$), there will be no sharing transactions in the first period. Furthermore, if there are sharing transactions in future period $j(\geq 2)$, the expected equilibrium market-clearing prices will be $E[r_j] = 1 - q - (1 - \beta)D(p,q)$. □

PROOF OF PROPOSITION 1. Note that $q^* = 0$ indicates that the manufacturer does not enter the sharing market to provide any rental service. We need to find the sufficient conditions for $q^* = 0$. As analyzed in Appendix A (provided in the Supplemental Materials file), when $I(\beta) \equiv \frac{(n-1)^2\beta^3-(n-1)[(n-1)(1+2\gamma)+2c]\beta^2+(n-1)[4(n-1)\gamma+2+2c]\beta-2n(n-1)\gamma-2c}{4(n-1)} \leq 0$ and $\pi^R = \frac{[(1-\gamma)n-c]^2}{4n} \leq \pi^S = \frac{[(n+1-2c)+(n-1)(1-\beta)^2]^2}{16[1+(n-1)(1-\beta)^2]}$, we get $q^* = 0$. When $\beta \to 0$, it is easily observed that $I(\beta) < 0$. Define $\beta_1 \in (0,1)$ as the smallest solution of the equation $\frac{(n-1)\beta^3-[(n-1)(1+2\gamma)+2c]\beta^2+[4(n-1)\gamma+2+2c]\beta}{4} - \frac{n(n-1)\gamma+c}{2(n-1)} = 0$ (i.e., $I(\beta) = 0$). Note that $I(\beta) \leq 0$ when $\beta \leq \beta_1$. Define $c_1 \equiv \frac{\sqrt{n}[n+1+(n-1)(1-\beta)^2]-2n(1-\gamma)\sqrt{1+(n-1)(1-\beta)^2}}{2\left[\sqrt{n}-\sqrt{1+(n-1)(1-\beta)^2}\right]} > 0$. One can show that when

$c \leq c_1$, $\pi^R \leq \pi^S$; when $c > c_1$, $\pi^R > \pi^S$. In summary, $q^* = 0$ when $\beta \leq \beta_1$ and $c \leq c_1$. This concludes the proof. □

PROOF OF PROPOSITION 2. Note that $q^* > 0$ indicates that the manufacturer will provide the rental service. Meanwhile, $D^* > 0$ and $E^*[r_j] > \beta$ indicate the existence of C2C sharing transactions. We need to find the sufficient conditions for $q^* > 0$, $D^* > 0$, and $E^*[r_j] > \beta$. Define $\beta_3 \equiv \frac{(n-1)(1+\gamma)+c}{2(n-1)}$. As analyzed in Appendix A (provided in the Supplemental Materials file), when $I(\beta) > 0$ and $\pi^{SR} > \pi^R$, we obtain $q^* > 0$ and $D^* > 0$. Note also that when $I(\beta) > 0$ and $\pi^{SR} > \pi^R$, we have $E^*[r_j] > \beta$ when $\beta < \beta_3$, and $E^*[r_j] \leq \beta$ when $\beta \geq \beta_3$. Recall that when $I(\beta) > 0$, $\pi^{SR} > \pi^S$ (see the analysis in Appendix A); when $c \leq c_1$, $\pi^S \geq \pi^R$ (see the proof of Proposition 1). Thus, given $I(\beta) > 0$, we know $\pi^{SR} > \pi^R$ when $c < c_1$. In summary, when $I(\beta) > 0$, $\beta < \beta_3$, and $c < c_1$, we have $q^* > 0$, $D^* > 0$, and $E^*[r_j] > \beta$. Given $\beta < \beta_3$ and $c < c_1$, one can show that there exist $\beta_2 \in [\beta_1, \beta_3)$ and $c_2 < c_1$ such that $I(\beta) > 0$ when $\beta_2 < \beta < \beta_3$ and $\max[0, c_2] < c < c_1$. Thus, we have $q^* > 0$, $D^* > 0$, and $E^*[r_j] > \beta$ when $\beta_2 < \beta < \beta_3$ and $\max[0, c_2] < c < c_1$. This completes the proof. □

PROOF OF PROPOSITION 3. (i) When $D^* = 0$, the manufacturer does not sell the product in the retail market, i.e., the manufacturer only provides the rental services. We need to find the sufficient condition for $D^* = 0$. As shown in Appendix A (provided in the Supplemental Materials file), when $q^* = 0$, we have $D^* = \frac{(n+1-2c)+(n-1)(1-\beta)^2}{4[1+(n-1)(1-\beta)^2]} > 0$. So, we only need to consider the case of $q^* > 0$. Define $c_3 \equiv \frac{(n-1)\beta^2+2(n-1)\gamma(1-\beta)+2}{2\beta} > c_1$. As shown in Appendix A, conditional on $q^* > 0$, when $c \geq c_3$, $D^* = 0$. Note that when $c \geq c_3 > c_1$, $\pi^R > \pi^S$ (see the proof of Proposition 1), which implies $q^* > 0$. Thus, we obtain $D^* = 0$ when $c \geq c_3$.

(ii) When $q^* > 0$ and $D^* > 0$, the manufacturer not only sells some products at retail but also provides rental services. However, if $E^*[r_j] < \beta$, the expected sharing price in the market cannot offset the C2C transaction cost and therefore C2C sharing transaction will not happen in expectation. We need to find the sufficient conditions for $q^* > 0$, $D^* > 0$, and $E^*[r_j] < \beta$. Recall that when $c < c_1$, $\pi^R < \pi^S$ (see the proof of Proposition 1), thus we have $D^* > 0$. As analyzed in Appendix A and in the proof of Proposition 2, when $I(\beta) > 0$ and $\beta \geq \beta_3$, we have $q^* > 0$ and $E^*[r_j] < \beta$. Conditional on $c < c_1$, one can show that there exist $\beta_4 \geq \beta_3$ such that $I(\beta) > 0$ when $\beta > \beta_4$. In summary, we have $q^* > 0$, $D^* > 0$, and $E^*[r_j] < \beta$ when $\beta > \beta_4$ and $c < c_1$. This completes the proof. □

PROOF OF PROPOSITION 4. (i) Note that $D(p,q) = 1 + \frac{(n-1)(2\beta-\beta^2)}{2[1+(n-1)(1-\beta)^2]} - \frac{p+(n-1)(1-\beta)q}{1+(n-1)(1-\beta)^2}$ (see Lemma 1). Plugging in, we get $D(p,q) + q = 1 + \frac{(n-1)(2\beta-\beta^2)}{2[1+(n-1)(1-\beta)^2]} - \frac{1}{1+(n-1)(1-\beta)^2}p + \frac{1-(n-1)(1-\beta)\beta}{1+(n-1)(1-\beta)^2}q$. As shown in Appendix A (provided in the Supplemental Materials file), $p^{SR} = p^S = \frac{n+1+2c+(n-1)(1-\beta)^2}{4}$ and $q^{SR} > q^S = 0$. Obviously, when $1-(n-1)(1-\beta)\beta > 0$, $D^{SR} + q^{SR} > D^S + q^S = D^S$; when $1-(n-1)(1-\beta)\beta < 0$, $D^{SR} + q^{SR} < D^S + q^S = D^S$.

(ii) First, $p^{SR} = p^S = \frac{n+1+2c+(n-1)(1-\beta)^2}{4}$. Second, from Lemma 1, $E[r_j] = 1 - q - (1-\beta)D(p,q)$. One can easily show $q + (1-\beta)D(p,q) = \frac{1}{1+(n-1)(1-\beta)^2}q + (1-\beta)\{1 + \frac{(n-1)(2\beta-\beta^2)}{2[1+(n-1)(1-\beta)^2]} - \frac{1}{1+(n-1)(1-\beta)^2}p\}$. Since $p^{SR} = p^S$ and $q^{SR} > q^S = 0$, we have $q^{SR} + (1-\beta)D^{SR} > q^R + (1-\beta)D^R$. Then, $E^{SR}[r_j] = 1 - q^{SR} - (1-\beta)D^{SR} > E^R[r_j] = 1 - q^R - (1-\beta)D^R$.

(iii) Note that $cs^S = \frac{1}{2}\pi^S$, $sw^S = \frac{3}{2}\pi^S$; $cs^{SR} = \frac{1}{2}\pi^{SR}$, $sw^{SR} = \frac{3}{2}\pi^{SR}$. When $\pi^{SR} > \pi^S$, we obtain $cs^{SR} > cs^S$ and $sw^{SR} > sw^S$. This ends the proof. □

PROOF OF PROPOSITION 5. We need to prove that $\frac{q^{SR}}{q^{SR}+(E^{SR}[r_j]-\beta)D^{SR}}$ can be non-monotonic in c. Equivalently, we can prove $\frac{q^{SR}}{(E^{SR}[r_j]-\beta)D^{SR}}$ can be non-monotonic in c. Without loss of generality, we assume $\gamma = 0$ for this proof. Define $R(c) = \frac{q^{SR}}{(E^{SR}[r_j]-\beta)D^{SR}}$. Plugging q^{SR}, $E^{SR}[r_j]$, and D^{SR}, we have $R(c) = \frac{q^{SR}}{(E^{SR}[r_j]-\beta)D^{SR}} = \frac{[2(n-1)^2\beta^3-2(n-1)^2\beta^2+4(n-1)\beta]-[4(n-1)\beta^2-4(n-1)\beta+4]c}{[n-1+2c-2(n-1)\beta][(n-1)\beta^2-2c\beta+2]}$. Taking the derivative gives $\frac{\partial R(c)}{\partial c} = \frac{1}{[n-1+2c-2(n-1)\beta]^2[(n-1)\beta^2-2c\beta+2]^2}H(c)$, where

$$H(c) = [8(n-1)\beta^3 - 8(n-1)\beta^2 + 8\beta]c^2 - [12(n-1)^2\beta^4 - 4(n-1)(5n-9)\beta^3 + 4(n-1)(2n-3)\beta^2 - 16(n-2)\beta + 8]c - [2(n-1)^3\beta^5 - 2(n-1)^2(n+9)\beta^4 + 28(n-1)^2\beta^3 + 4(n-1)(2n-9)\beta^2 - 8(n-1)(n-2)\beta + 8(n-2)].$$

One can show that $H(c)$ can be negative or positive. For example, when $\beta \to 1$, $H(c) \to 8c^2 + 4(n-7)c - 4(2n^2-7n+3)$, where $H(c)|_{c=0} = -4(2n^2-7n+3) < 0$ and $H(c)|_{c=n} = 4n^2 - 12 > 0$. Thus, we conclude $\frac{q^{SR}}{(E^{SR}[r_j]-\beta)D^{SR}}$ can be non-monotonic in c (as illustrated in Figure 4 within the parameter region of our focus). This completes the proof. □