Online Appendix

EC.1. Asymmetric Pure-Strategy Nash Equilibrium

In this section, we discuss the robustness of our results when considering asymmetric pure-strategy Nash equilibria. In the following lemma, we show that when $N = 2$, any pure-strategy Nash equilibrium is symmetric, and hence all our results under symmetric Nash equilibria follow.

LEMMA EC.A1. *Let e_i^* be agent $i \in \{1,2\}$'s equilibrium effort. Then, $e_1^* = e_2^*$.*

Proof. We first suppose that $e_1^* > 0$ and $e_2^* > 0$. Then, given that agent 2 exerts her equilibrium effort e_2^*, agent 1's utility when exerting effort e_1 is

$$U(e_1,T) = A(1-\gamma_{(1)}) + A(2\gamma_{(1)}-1)\int_{s\in\Xi} H(e_1 - e_2^* + s)h(s)ds - c(e_1)^b\tau(T)^{1-b} - F.$$

Evaluating the first derivative of $U_1(e_1,T)$ with respect to e_1 at $e_1 = e_1^*$ yields

$$\left.\frac{\partial U(e_1,T)}{\partial e_1}\right|_{e_1=e_1^*} = A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^* - e_2^* + s)h(s)ds - cb(e_1^*)^{b-1}\tau(T)^{1-b} = 0. \tag{EC.1}$$

Similarly, noting that $P(e_2 + \widetilde{\xi}_2 > e_1 + \widetilde{\xi}_1) = 1 - P(e_1 + \widetilde{\xi}_1 > e_2 + \widetilde{\xi}_2)$, given that agent 1 exerts her equilibrium effort e_1^*, agent 2's utility when exerting e_2 can be written as

$$U(e_2,T) = A(1-\gamma_{(1)}) + A(2\gamma_{(1)}-1)\left[1 - \int_{s\in\Xi} H(e_1^* - e_2 + s)h(s)ds\right] - c(e_2)^b\tau(T)^{1-b} - F.$$

Evaluating the first derivative of $U_2(e_2,T)$ with respect to e_2 at $e_2 = e_2^*$ gives

$$\left.\frac{\partial U_2(e_2,T)}{\partial e_2}\right|_{e_2=e_2^*} = -A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^* - e_2^* + s)h(s)ds - cb(e_2^*)^{b-1}\tau(T)^{1-b} = 0. \tag{EC.2}$$

From (EC.1) and (EC.2), agent 1's and agent 2's equilibrium efforts are

$$e_1^* = e_2^* = \left(\frac{A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^* - e_2^* + s)h(s)ds}{cb}\right)^{\frac{1}{b-1}}\tau(T).$$

Thus, there does not exist an asymmetric pure-strategy Nash equilibrium where $e_1^* > 0$ and $e_2^* > 0$.

We next suppose that $e_1^* > 0$ and $e_2^* = 0$. Then, $\frac{\partial U_2(e_2,T)}{\partial e_2}\big|_{e_2=0} = -A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^* + s)h(s)ds \le 0$. Thus, $\gamma_{(1)} = 0.5$ because $\int_{s\in\Xi} h(e_1^* + s)h(s)ds > 0$. However, when $e_2^* = 0$ and $\gamma_{(1)} = 0.5$, $\frac{\partial U_1(e_1,T)}{\partial e_1}\Big|_{e_1=e_1^*} = -cb(e_1^*)^{b-1}\tau(T)^{1-b} \neq 0$ for $e_1^* > 0$. Thus, by symmetry, there does not exist an asymmetric pure-strategy Nash equilibrium such that $e_1^* > 0$ and $e_2^* = 0$ or $e_2^* > 0$ and $e_1^* = 0$. ■

We next discuss the case where $N > 2$. Specifically, we are interested in whether an asymmetric equilibrium emerges when there is no symmetric one (i.e., $T > \overline{T}$) and how this asymmetric pure-strategy Nash equilibrium changes with T. For ease of illustration, we focus on the WTA award scheme. Let e_i^* be agent i's equilibrium effort. Given that all other agents $j \in \{1,2,...,N\} \setminus i$ exert their equilibrium effort e_j^*, agent i determines her effort e_i to maximize her expected utility

$$U(e_i,T) = A \int_{s\in\Xi} \prod_{j\in\{1,2,...,N\}\setminus i} H(e_i - e_j^* + s)h(s)ds - ce_i^b \tau(T)^{1-b} - F.$$

Let $I(e_i^*|e_{j\neq i}^*) \equiv \int_{s\in\Xi} \sum_{j\in\{1,2,...,N\}\setminus i} h(e_i^* - e_j^* + s)h(s) \prod_{k\in\{1,2,...,N\}\setminus\{i,j\}} H(e_i^* - e_k^* + s)ds$. Evaluating the first-derivative of $U_i(e_i,T)$ with respect to e_i at $e_i = e_i^*$ yields

$$AI(e_i^*|e_{j\neq i}^*) - cb(e_i^*)^{b-1}\tau(T)^{1-b} = 0 \text{ for all } i \in \{1,2,...,N\}. \tag{EC.3}$$

In the following lemma, we show that for a sufficiently large T, the agent's participation condition is violated. Thus, consistent with our finding in §3, T^* is bounded even when $\delta = 0$.

Lemma EC.A2. *There exists $\overline{T}_a$ such that when the contest duration $T > \overline{T}_a$, an agent's participation condition is violated under a solution to (EC.3).*

Proof. Let $P(e_i^*|e_{j\neq i}^*) \equiv \int_{s\in\Xi} \prod_{j\in\{1,2,...,N\}\setminus i} H(e_i^* - e_j^* + s)h(s)ds$. In equilibrium, all agents choose to participate in the contest if and only if the following participation condition is satisfied:

$$AP(e_i^*|e_{j\neq i}^*) - c\left(\frac{AI(e_i^*|e_{j\neq i}^*)}{cb}\right)^{\frac{b}{b-1}} \tau(T) - F \geq 0 \text{ for all } i \in \{1,2,...,N\}. \tag{EC.4}$$

Since $AP(e_i^*|e_{j\neq i}^*) \leq A$, as T approaches ∞, agent i's participation condition is violated unless $I(e_i^*|e_{j\neq i}^*)$ approaches 0 because $\tau(T)$ approaches ∞. Suppose that $\lim_{T\to\infty} I(e_i^*|e_{j\neq i}^*) = 0$. Then, there should exist e_k such that $k \neq i$ and e_k approaches ∞ as T approaches ∞. As e_k approaches ∞ and agent k's expected award is bounded by A, her participation condition is violated. So, for any solution to (EC.3), the agent's participation condition is violated for a sufficiently large T. ■

An important implication of Lemma EC.A2 is that even if an asymmetric pure-strategy Nash equilibrium emerges when $T > \overline{T}$, the agent's participation still becomes an issue as T increases. Thus, we next study whether a patient organizer has an incentive to increase the contest duration T up to $\overline{T}_a$ where the agent's participation condition binds, consistent with the effort-participation tradeoff in §3. As it is analytically intractable to analyze the impact of T on the organizer's profit Π under an asymmetric pure-strategy Nash equilibrium, we conduct an extensive numerical analysis. We take $\theta(t) = \theta$, and randomly generate 10,000 instances. In each instance, we randomly select parameters according to our numerical analysis setting in footnote 13 (in addition, we select N from Uniform(2,10) and θ from Uniform(0,5)). To focus on the case where there is no symmetric pure-strategy Nash equilibrium, we randomly generate T from Uniform($\overline{T}, 1.05\overline{T}$); and to focus on the case of a patient organizer, we assume that the discount factor $\delta = 0$. In each random instance, we solve (EC.3) numerically. Because the symmetric equilibrium effort in (5) is a solution to (EC.3), to prevent the numerical solver from getting stuck in this symmetric solution, we randomize the initial solutions that we feed to the solver. In 672 instances, we obtain a "valid" asymmetric solution where the sum of squared deviations of agents' first-order conditions from zero $\Big($i.e., $\sum_{i=1}^{N}\left(AI(e_i^*|e_{j\neq i}^*)-cb(e_i^*)^{b-1}\tau(T)^{1-b}\right)^2\Big)$ is less than 10^{-15}. (In 250 of these instances, all agents' utilities are non-negative, so there exists an asymmetric solution that satisfies (EC.3) and the participation condition (EC.4).) To check if Π increases with T at each of these 672 instances, we incrementally increase T to $1.0001T$, $1.001T$, $1.01T$, and $1.1T$, and check if Π increases. We observe that in *all* of these 672 instances, Π increases with T. Thus, we conclude that under an asymmetric pure-strategy Nash equilibrium, the organizer's profit Π increases with the contest duration T when the organizer is patient, and hence by Lemma EC.A2, the agent's participation condition drives the optimal contest duration.

EC.2. Mixed-Strategy Nash Equilibrium

In this section, we consider the case where agents play mixed strategies. For ease of illustration and following the contest literature (e.g., Hu and Wang 2020, Mihm and Schlapp 2018, Seel 2018, Bimpikis et al. 2019), we assume that $N=2$. Each agent $i\in\{1,2\}$ participates in the contest with probability $p_i\in[0,1]$, and exerts effort e_i if both agents participate, and exerts zero effort otherwise. We derive the equilibrium using the best-response argument in a two-stage game. In the second stage, if both agents participate, each agent exerts e^* as in (5). In the first stage, given that the other agent participates in the contest with the equilibrium probability of participation p^* and both agents exert e^*, agent i decides on p_i ($\in[0,1]$) to maximize her expected utility

$$U_i(p_i)=p_ip^*\left[\frac{A}{2}-c(e^*)^b\tau(T)^{1-b}-F\right]+p_i(1-p^*)[A_{(1)}-F]. \tag{EC.5}$$

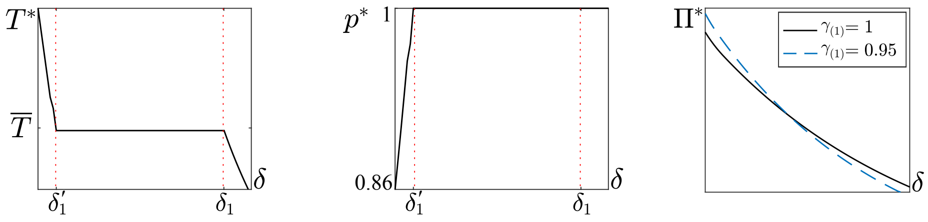
The second component in (EC.5) (i.e., $p_i(1-p^*)[A_{(1)}-F]$) is always non-negative, so whenever the first component (i.e., $p_ip^*\left[\frac{A}{2}-c(e^*)^b\tau(T)^{1-b}-F\right]$) is also non-negative, the best-response of the agent is to set $p^*=1$. Thus, the agent plays a non-pure strategy (i.e., $p^*<1$) only if she gets negative utility when both agents participate. The following lemma formally shows this result.

Lemma EC.A3. *For any $A_{(1)},A_{(2)}$, and T such that $\frac{A_{(1)}+A_{(2)}}{2}-c(e^*)^b\tau(T)^{1-b}-F\geq 0$, $p^*=1$.*

Proof. Suppose that $\frac{A_{(1)}+A_{(2)}}{2}-c(e^*)^b\tau(T)^{1-b}-F\geq 0$. When agent i chooses to participate in the contest, she gets a nonnegative utility. However, when the agent does not participate, she gets zero utility. Therefore, she cannot improve her utility by reducing p_i, and hence $p^*=1$. ■

Given e^* and p^*, the organizer's profit $\Pi=\exp(-\delta T)(p^*)^2\left(e^*+E\left[\widetilde{\xi}_{(1)}^2\right]\right)-(p^*)^2(A_{(1)}+A_{(2)})-2p^*(1-p^*)A_{(1)}$. The following corollary extends Lemma 2.

Corollary EC.1. *(a) When Π is non-monotonic, $T^*=\widehat{T}$ for any $\delta\geq\delta_1$, and each agent's equilibrium probability of participation $p^*=1$. (b) There exists $\delta_1'(\leq\delta_1)$ such that for any $\delta<\delta_1'$, $T^*>\overline{T}$ and $p^*<1$.*



(a) T^*. (b) p^*. (c) Π^*.

Figure EC.1 (a) T^* under $\gamma_{(1)} = 1$, (b) p^* under $\gamma_{(1)} = 1$, (c) Π^* under $\gamma_{(1)} = 1$ and $(\gamma_{(1)}, \gamma_{(2)}) = (0.95, 0.05)$ when agents play mixed strategies. The setting is the same as Figure 1.

Proof. **(a)** When Π is non-monotonic, by Lemma 2, $T^* = \widehat{T}$ for any $\delta \geq \delta_1$, and hence $T^* = \widehat{T}$ as in (16) and $\frac{A}{2} - c(e^*)^b \tau(T^*)^{1-b} - F \geq 0$. Thus, by Lemma EC.A3, $p^* = 1$.

(b) Suppose that $\delta < \delta_1$ or Π is monotonic. For agent $i \in \{1, 2\}$, given $p_j = p_j^*$ for $j \neq i$, taking the first derivative of $U_i(p_i)$ with respect to p_i and evaluating it at $p_i = p_i^*$ yields $\frac{\partial U_i(p_i)}{\partial p_i}\Big|_{p_i = p_i^*} = p_j^* \left[\frac{A}{2} - c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}} \tau(T) - F\right] + (1 - p_j^*)[A_{(1)} - F] = 0$. Thus, agent 1's and agent 2's equilibrium probabilities of participation are

$$p_1^* = p_2^* = p^* \equiv \frac{A_{(1)} - F}{A_{(1)} - A/2 + c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}} \tau(T)}. \tag{EC.6}$$

When $T^* = \overline{T}$ as in (15), $p^* = 1$ from (EC.6), and since p^* decreases with T, $p^* < 1$ if only if $T^* > \overline{T}$.

Since $\lim_{T\to\infty} p^* = 0$, $\lim_{T\to\infty} \Pi = 0$. Thus, $T^* (> \overline{T})$ is interior, and hence $\frac{\partial \Pi}{\partial T}\big|_{T=T^*} = 0$ is necessary for optimality. The first derivative of the organizer's profit Π with respect to T is

$$\frac{\partial \Pi}{\partial T} = \exp(-\delta T)\left[\left(2p^* \frac{\partial p^*}{\partial T} - \delta (p^*)^2\right)\left(e^* + E\left[\widetilde{\xi}_{(1)}^N\right]\right) + (p^*)^2 \frac{\partial e^*}{\partial T}\right] + \frac{\partial p^*}{\partial T}\left[A_{(1)}(2p^* - 2) - 2p^* A_{(2)}\right].$$

Under $\overline{T}$ in (15), $p^* = 1$. Thus, a sufficient condition for $p^* < 1$ is that $\frac{\partial \Pi}{\partial T}\big|_{T=\overline{T}} > 0$. We have

$$\frac{\partial \Pi}{\partial T}\bigg|_{T=\overline{T}} = \exp(-\delta T)\left[\left(2 \frac{\partial p^*}{\partial T}\bigg|_{T=\overline{T}} - \delta\right)\left(e^* + E\left[\widetilde{\xi}_{(1)}^N\right]\right) + \frac{\partial e^*}{\partial T}\bigg|_{T=\overline{T}}\right] - 2 \frac{\partial p^*}{\partial T}\bigg|_{T=\overline{T}} A_{(2)} > 0$$

if $\left(2 \frac{\partial p^*}{\partial T}\Big|_{T=\overline{T}} - \delta\right)\left(e^* + E\left[\widetilde{\xi}_{(1)}^N\right]\right) + \frac{\partial e^*}{\partial T}\Big|_{T=\overline{T}} \geq 0$, i.e., $\delta \leq \frac{2 \frac{\partial p^*}{\partial T}\big|_{T=\overline{T}}\left(e^* + E\left[\widetilde{\xi}_{(1)}^N\right]\right) + \frac{\partial e^*}{\partial T}\big|_{T=\overline{T}}}{e^* + E\left[\widetilde{\xi}_{(1)}^N\right]}$ since $\frac{\partial p^*}{\partial T}\Big|_{T=\overline{T}} < 0$. Thus, for any $\delta < \delta_1'$, $\frac{\partial \Pi}{\partial T}\big|_{T^*=\overline{T}} > 0$, and hence $T^* > \overline{T}$ and $p^* < 1$. ■

T being larger than $\overline{T}$ has the following opposing effects on Π. It improves Π by increasing e^*, but it reduces Π by decreasing p^* and discounting the organizer's payoff more. When $\delta = 0$, the organizer still limits T to balance the positive effect of a larger e^* and the negative effect of a smaller p^*. Thus, the effort-participation tradeoff we identify in §3 persists when agents play mixed strategies, and this tradeoff drives T^* for the patient organizer. Supplementary to Corollary EC.1, Figures EC.1(a) and EC.1(b) illustrate that for $\delta \in [\delta_1', \delta_1]$, $T^* = \overline{T}$ and $p^* = 1$ because negative effects of a smaller p^* and more discounting outweigh the positive effect of a larger e^*.

We next discuss the robustness of Theorem 1. When $T^* = \overline{T}$ and $p^* = 1$, Theorem 1 directly applies. The following corollary extends Theorem 1(a) to the case where $T^* > \overline{T}$ and $p^* < 1$. To analyze a patient organizer while retaining analytical tractability, we assume that $\delta = 0$ and $\theta(t) = \theta$ as in the innovation-contest literature (e.g., Hu and Wang 2020, Mihm and Schlapp 2018).

COROLLARY EC.2. *There exists $\underline{\alpha}$ such that T^* is increasing in $\alpha > \underline{\alpha}$.*

Proof. Suppose that $\delta = 0$ and $\theta(t) = \theta$. By Corollary EC.1, $p^* < 1$, and under the scale parameter α, the optimal contest duration $T^*[\alpha]$ that solves $\frac{\partial \Pi}{\partial T}\big|_{T=T^*[\alpha]} = 0$ is

$$T^*[\alpha] = \frac{\left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)\left(A_{(1)} - A/2\right) - 4\left(A_{(1)} - F\right)A_{(1)} - 2\left(A_{(1)} - F\right)\left(\alpha E\left[\widetilde{\xi}^N_{(1)}\right] - A\right)}{\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\alpha^{\frac{-1}{b-1}}\frac{(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\alpha^{\frac{-b}{b-1}}\right)c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}}.$$

The first derivative of $T^*[\alpha]$ with respect to α is

$$\begin{aligned}
\frac{\partial T^*[\alpha]}{\partial \alpha} = {} & \frac{\left(\frac{(A_{(1)}-F)\theta b}{xA}\right)\left(A_{(1)} - A/2\right) - 2\left(A_{(1)} - F\right)E\left[\widetilde{\xi}^N_{(1)}\right]}{\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)c\alpha^{\frac{-b}{b-1}}\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}} \\
& + \frac{1}{(b-1)}\frac{\left(\frac{(A_{(1)}-F)\theta b}{xA}\right)\left(A_{(1)} - A/2\right) - 2\left(A_{(1)} - F\right)E\left[\widetilde{\xi}^N_{(1)}\right]}{\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)c\alpha^{\frac{-b}{b-1}}\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}} \\
& + \frac{\left(\frac{(A_{(1)}-F)\theta b}{xA}\right)\left(A_{(1)} - A/2\right) - 2\left(A_{(1)} - F\right)E\left[\widetilde{\xi}^N_{(1)}\right]}{\left(\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)c\alpha^{\frac{-b}{b-1}}\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\right)^2}\left[-\left(2A_{(1)}\alpha^{\frac{-b}{b-1}}\right)c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\right].
\end{aligned}$$

$\lim_{\alpha\to\infty}\frac{\partial T^*[\alpha]}{\partial \alpha} > 0$, so by continuity, there exists $\underline{\alpha}$ such that T^* is increasing in any $\alpha > \underline{\alpha}$. ■

We next discuss how our results about the award scheme extend to the case where agents play mixed strategies. From Corollary EC.1(a) and Figure EC.1(b), we can also deduce that when $\delta > \delta'_1$, $p^* = 1$, and hence Theorem 2(a) directly apply. To analyze the case where $p^* < 1$, we conduct an extensive numerical analysis. We show that for a sufficiently small δ, the WTA award scheme is not optimal.[15] For instance, Figure EC.1(c) illustrates that up to some threshold on δ, Π^* is larger under the award scheme $(0.95A, 0.05A)$ than Π^* under the WTA award scheme. The intuition is similar to Theorem 2(a). Specifically, offering multiple awards increases p^*, and hence allows the organizer to set a longer T to elicit a larger expected effort from agents. Opposed to this positive effect, a longer T also leads to more discounting. When δ is small, the former positive effect dominates the latter negative effect, so offering multiple awards is optimal, as in Theorem 2(a). Also, from Figures EC.1(b) and EC.1(c), we can deduce that as δ decreases, it first becomes optimal to give multiple awards, and then as δ keeps decreasing, it becomes optimal to set $T^* > \overline{T}$ such that agents play non-pure strategies. Thus, $\underline{\delta}_0$, below which giving multiple awards is optimal, does not change when agents can play mixed strategies, and hence even giving multiple awards is more likely to be optimal as ρ increases, as in Theorem 2(b). Finally, in our numerical analysis, we also observe that when giving multiple awards is optimal, giving unequal awards is almost always better than giving equal awards, as in Proposition 2.

[15] We take $\theta(t) = \exp(\rho t)$, and randomly generate 10,000 instances where $p^* < 1$. In each instance, we select parameters according to our numerical analysis setting in footnote 13 (and we select δ from Uniform(0,0.0001)). We observe that in all instances, Π is larger under the award scheme $(0.95A, 0.05A)$ than Π under the WTA award scheme.

EC.3. Existence of Pure-Strategy Nash Equilibrium

In this section, we provide sufficient conditions for e^* in (5) to be a pure-strategy Nash equilibrium under $\overline{T}$. We first show sufficient conditions for an interim property in the following lemma, and then use this property in the main result of this section.

Lemma EC.A4. *Suppose that* $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\Big|_{e_i=\underline{e}} < 0$ *for some* $\underline{e}$ *and* $b > 2$. *For any* $e_i > \underline{e}$, *when* F *is sufficiently large or when* α *is sufficiently small, we have* $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$.

Proof. Under the scale parameter α, given that all other agents exert equilibrium efforts e^*, from (8) and $\overline{T}$ in (15), the second derivative of agent i's utility $U_i(e_i,\overline{T})$ with respect to e_i is

$$\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} = \sum_{j=1}^{N} \frac{\partial^2 P_{(j)}^N[e_i,e^*]}{\partial e_i^2}\left(\frac{A_{(j)}}{\alpha^2}\right) - cb(b-1)e_i^{b-2}\left(\frac{A-NF}{cN}\right)^{1-b}\left(\frac{xA}{\alpha cb}\right)^{b}.$$

Suppose that $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$ for some $e_i = \underline{e}$, but there exists $\widehat{e} > \underline{e}$ such that $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} \geq 0$ for $e_i = \widehat{e}$. Then, $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\big|_{e_i=\widehat{e}} > \frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\big|_{e_i=\underline{e}}$, i.e.,

$$\sum_{j=1}^{N}\left[\frac{\partial^2 P_{(j)}^N[e_i,e^*]}{\partial (e_i)^2}\bigg|_{e_i=\widehat{e}} - \frac{\partial^2 P_{(j)}^N[e_i,e^*]}{\partial (e_i)^2}\bigg|_{e_i=\underline{e}}\right]\frac{A_{(j)}}{\alpha^2} > b(b-1)\left[(\widehat{e})^{b-2} - (\underline{e})^{b-2}\right]\left(\frac{A-NF}{N}\right)^{1-b}\left(\frac{xA}{\alpha b}\right)^{b}. \quad \text{(EC.7)}$$

Suppose that $b > 2$. Since $\widehat{e} > \underline{e}$, the right-hand side of (EC.7) approaches ∞ as F approaches A/N. Thus, when F is sufficiently large, (EC.7) cannot be satisfied. Also, as α approaches 0, the right-hand side of (EC.7) approaches ∞ faster than the left-hand side of (EC.7) (when the left-hand side of (EC.7) is positive). Thus, regardless of the sign of the left-hand side of (EC.7), when α is sufficiently small, (EC.7) cannot be satisfied. Therefore, for any $e_i > \underline{e}$, when F is sufficiently large or when α is sufficiently small, we have $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$. ■

The following lemma shows that when the property in Lemma EC.A4 holds for any $\underline{e}$, e^* in (5) is a pure-strategy Nash equilibrium under $\overline{T}$.

Lemma EC.A5. *Suppose that for all* $\underline{e}$ *such that* $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\Big|_{e_i=\underline{e}} < 0$, *we have* $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$ *when* $e_i > \underline{e}$. *Then,* $U_i(e_i,\overline{T})$ *is pseudo concave. Thus,* e^* *in (5) is a pure-strategy Nash equilibrium under* $\overline{T}$.

Proof. Suppose that for all $\underline{e}$ such that $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\Big|_{e_i=\underline{e}} < 0$, we have $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$ when $e_i > \underline{e}$. First, we have $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i}\Big|_{e_i=0} = \sum_{j=1}^{N}\frac{\partial P_{(j)}^N[e_i,e^*]}{\partial e_i}A_{(j)}\Big|_{e_i=0} > 0$ and $\lim_{e_i\to\infty} U_i(e_i,\overline{T}) = -\infty$, so there should exist some e_i such that $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} < 0$ and $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$. So, there exists a threshold $e_0 (\geq 0)$ such that for any $e_i < e_0$, $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} \geq 0$; and for any $e_i > e_0$, $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$. So, we should have $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} > 0$ for any $e_i < e_0$, and there should exist another threshold $e_{00} (> e_0)$ such that for any $e_i < e_{00}$, $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} > 0$; and for any $e_i > e_{00}$, $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} < 0$. Thus, $U_i(e_i,\overline{T})$ is unimodal with mode e_{00}, and has a unique critical (maximum) point, so it is pseudo concave. Therefore, the first-order condition of the agent's utility-maximization problem in (7) is sufficient for optimality. Since e^* in (5) satisfies this first-order condition, e^* is the solution to the agent's utility-maximization problem in (7). As e^* under $\overline{T}$ also satisfies (8), e^* is a pure-strategy Nash equilibrium under $\overline{T}$. ■

EC.4. Additional Results

Lemma EC.A6. *$I^N_{(j)} \geq I^N_{(j+1)}$ for any $j \in \{1,2,...,N-1\}$. Furthermore, $\sum_{j=1}^N I^N_{(j)}\gamma_{(j)} \geq 0$ under any distribution of awards $(\gamma_{(1)},\gamma_{(2)},...,\gamma_{(N)})$ such that $\gamma_{(1)} \geq \gamma_{(2)} \geq ... \geq \gamma_{(N)}$.*

Proof. Let $W^N_{(j)}(s) = \frac{(N-1)!}{(N-j)!(j-1)!}H(s)^{N-j}(1-H(s))^{j-1}$. From (4), integration by parts yields

$$I^N_{(j)} = \theta\int_{s\in\Xi}\left(W^N_{(j)}\right)'(s)h(s)ds = \theta\lim_{s\to\overline{s}}W^N_{(j)}(s)h(s) - \int_{s\in\Xi}W^N_{(j)}(s)h'(s)ds.$$

$h^N_{(j)}(s) = \frac{N!}{(N-j)!(j-1)!}(1-H(s))^{j-1}H(s)^{N-j}h(s)$, so $W^N_{(j)}(s) = \frac{h^N_{(j)}(s)}{Nh(s)}$. Letting $w_j \equiv \lim_{s\to\overline{s}}\frac{h^N_{(j)}(s)}{N}$, we have $I^N_{(j)} - I^N_{(j+1)} = (w_j - w_{j+1}) + \frac{1}{N}\int_{s\in\Xi}\left[h^N_{(j+1)}(s) - h^N_{(j)}(s)\right]\frac{h'(s)}{h(s)}ds$, $\forall j \in \{1,2,...,N-1\}$.

Noting that $w_1 \geq 0$ and $w_j = 0$ for any $j \in \{2,3,...,N\}$, integration by parts yields

$$I^N_{(j)} - I^N_{(j+1)} \geq \frac{1}{N}\left(\lim_{s\to\overline{s}}\left[H^N_{(j+1)}(s) - H^N_{(j)}(s)\right]\frac{h'(s)}{h(s)} - \int_{s\in\Xi}\left[H^N_{(j+1)}(s) - H^N_{(j)}(s)\right]\left(\frac{h'(s)}{h(s)}\right)'ds\right),$$

for all $j \in \{1,2,...,N-1\}$. Because h is log-concave, $\lim_{s\to\overline{s}}\left[H^N_{(j+1)}(s) - H^N_{(j)}(s)\right]\frac{h'(s)}{h(s)} = 0$ and $\left(\frac{h'(s)}{h(s)}\right)' \leq 0$. Also, $H^N_{(j+1)}(s) - H^N_{(j)}(s) \geq 0$ since $\widetilde{\xi}^N_{(j)}$ first-order stochastically dominates $\widetilde{\xi}^N_{(j+1)}$ for any $j \in \{1,2,...,N-1\}$. Thus, $I^N_{(j)} - I^N_{(j+1)} \geq 0$ for any $j \in \{1,2,...,N-1\}$. Let $k = \max\{j|I^N_{(j)} \geq 0\}$. Because $I^N_{(j)} - I^N_{(j+1)} \geq 0$ for any $j \in \{1,2,...,N-1\}$, we have

$$\sum_{j=1}^N I^N_{(j)}\gamma_{(j)} \geq \sum_{j=1}^k I^N_{(j)}\gamma_{(k)} + \sum_{j=k+1}^N I^N_{(j)}\gamma_{(k)} = \gamma_{(k)}\sum_{j=1}^N I^N_{(j)} = \gamma_{(k)}\sum_{j=1}^N \frac{\partial P^N_{(j)}[e_i,e^*]}{\partial e_i}\bigg|_{e_i=e^*} = 0. \quad ■$$

Lemma EC.A7. *When Π is non-monotonic in T and unimodal, $\frac{\theta'(\widehat{T})}{\theta(\widehat{T})} < \frac{\delta(b-1)}{b}$.*

Proof. When Π is non-monotonic in T and unimodal, Π is unimodal with mode $T^* = \widehat{T}$ by Lemma 2, and hence $\frac{\partial\Pi}{\partial T} < 0$ when $T > \widehat{T}$. This is possible only when $\frac{\partial^2\Pi}{\partial T^2}\Big|_{T=\widehat{T}} < 0$ since $\frac{\partial\Pi}{\partial T}\big|_{T=\widehat{T}} = 0$. Thus, we should have $\frac{\partial^2\Pi}{\partial T^2}\Big|_{T=\widehat{T}} < 0$. The second derivative of Π with respect to T is

$$\frac{\partial^2\Pi}{\partial T^2} = \exp(-\delta T)\left(-\delta\left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}(-\delta\tau(T) + \tau'(T)) - \delta E\left[\widetilde{\xi}^N_{(1)}\right]\right] + \left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}(-\delta\tau'(T) + \tau''(T))\right]\right),$$

and hence $\frac{\partial^2\Pi}{\partial T^2}\Big|_{T=\widehat{T}} = \exp(-\delta T)\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}(-\delta\tau'(T) + \tau''(T))$. Thus, $\frac{\partial^2\Pi}{\partial T^2}\Big|_{T=\widehat{T}} < 0$ if and only if $-\delta\tau'(T) + \tau''(T) < 0$, i.e., $\frac{\theta'(\widehat{T})}{\theta(\widehat{T})} < \frac{\delta(b-1)}{b}$. When $\theta(t) = \exp(\rho t)$, this condition becomes $\rho < \frac{\delta(b-1)}{b}$. ■

Lemma EC.A8. *Let $\overrightarrow{\gamma_m} = (\gamma^m_{(1)},\gamma^m_{(2)},...,\gamma^m_{(N)})$, and $\overrightarrow{\gamma_1}$ and $\overrightarrow{\gamma_2}$ be such that $\sum_{j=1}^N I^N_{(j)}\gamma^1_{(j)} < \sum_{j=1}^N I^N_{(j)}\gamma^2_{(j)}$, $\Phi \leq 0$ under $\overrightarrow{\gamma_1}$ and $\overrightarrow{\gamma_2}$, and Π is non-monotonic in T under $\overrightarrow{\gamma_2}$. Then, Π is also non-monotonic in T under $\overrightarrow{\gamma_1}$, and Π under T^* is smaller under $\overrightarrow{\gamma_1}$ than that under $\overrightarrow{\gamma_2}$.*

Proof. Since Π is non-monotonic in T under $\overrightarrow{\gamma_2}$, there exists some $T=\dot{T}$ such that $\frac{\partial \Pi}{\partial T}\big|_{T=\dot{T}} = \exp(-\delta\dot{T})\left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\left(-\delta\tau(\dot{T})+\tau'(\dot{T})\right)-\delta E\left[\widetilde{\xi}^{N}_{(1)}\right]\right] < 0$. Because x under $\overrightarrow{\gamma_1}$ is smaller than x under $\overrightarrow{\gamma_2}$, $\frac{\partial \Pi}{\partial T}\big|_{T=\dot{T}} < 0$ under $\overrightarrow{\gamma_1}$, and hence, Π is also non-monotonic in T under $\overrightarrow{\gamma_1}$. Thus, given $\overrightarrow{\gamma_1}$ or $\overrightarrow{\gamma_2}$, $T^* = \widehat{T}$. Noting that $\frac{\partial \Pi}{\partial T}\big|_{T=\widehat{T}} = 0$, $\frac{\partial \Pi}{\partial x}\big|_{T=\widehat{T}} = \frac{\partial \Pi}{\partial x}\big|_{T=\widehat{T}} + \frac{\partial \Pi}{\partial T}\big|_{T=\widehat{T}}\frac{\partial T}{\partial x}\big|_{T=\widehat{T}} = \exp(-\delta\widehat{T})\left(\left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\tau(\widehat{T})\frac{x^{\frac{2-b}{b-1}}}{b-1}\right) > 0$. Then, since x under $\overrightarrow{\gamma_1}$ is smaller than x under $\overrightarrow{\gamma_2}$, Π under $T^*=\widehat{T}$ is smaller given $\overrightarrow{\gamma_1}$ than Π given $\overrightarrow{\gamma_2}$. ■