Online Supplement

Product Price, Quality and Service Decisions under Consumer Choice Models

In this online supplement, we provide other technical proofs for the paper titled “Product Price, Quality and Service Decisions under Consumer Choice Models.”

Proof of Proposition 5. For the price-only optimization problem, the total optimal profit, denoted by $r^\dagger$, is the unique solution to the following equation based on equation (17) in the proof of Theorem 1:

$$r = \sum_{i\in\mathcal{N}} \exp\left(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1\right).$$

Again, based on equation (17), the joint optimization problem on price and service duration can be rewritten as $r = \max_{\mathbf{t}} \sum_{i\in\mathcal{N}} \exp\left(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1\right)$. Note that the RHS of this equation can be separated for each product i for any given $r > 0$. Then, each function $\exp\left(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1\right)$ is strictly increasing or decreasing in the service duration of product $i \in \mathcal{N}$, because

$$\begin{aligned}&\frac{\partial}{\partial t_i}\exp(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1)\\ &= \left(s_i - (a_i - b_i q_i)\right)\exp(\alpha_i q_i - c_i(q_i) + t_i(s_i - (a_i - b_i q_i)) - r - 1).\end{aligned}$$

In particular, if $q_i \geq (a_i - s_i)/b_i$, the upper bound (resp., lower bound) duration of service is attached to product i for $b_i > 0$ (resp., $b_i < 0$); otherwise, the lower bound (resp., upper bound) will be offered for $b_i > 0$ (resp., $b_i < 0$). For $b_i = 0$, the firm attaches t_s or t_l to product i depending on the difference between a_i and s_i, i.e., t_l for $s_i \geq a_i$, whereas t_s for $s_i < a_i$. Denote the optimal service duration by $t_i^\ddagger$ for each $i \in \mathcal{N}$, and therefore $t_i^\ddagger$ equals t_s or t_l depending on the pre-determined quality level of product i and the sign of b_i.

Then, for the joint price and service duration optimization problem, the total optimal profit, denoted by $r^\ddagger$, is the unique solution to the following equation:

$$r = \sum_{i\in\mathcal{N}} \exp\left(\alpha_i q_i - c_i q_i^2 + t_i^\ddagger(s_i - (a_i - b_i q_i)) - r - 1\right).$$

For the joint price, service and quality optimization problem, the total optimal profit r^* is the unique solution to the following equation:

$$r^* = \sum_{i\in\mathcal{N}} \exp\left(\alpha_i q_i^* - c_i q_i^{*2} + t_i^*(s_i - (a_i - b_i q_i^*)) - r^* - 1\right),$$

where t_i^* is equal to either t_s or t_l, and $q_i^* = (\alpha_i + t_i^* b_i)/(2c_i)$ (see Theorem 1).

By Anderson et al. (1992) consumer surplus can be expressed as $E[\max_{i\in S^+} U_i] = \log\left(1 + \sum_{i\in S}\exp(\alpha_i q_i - p_i + t_i s_i)\right)$, so the comparison of consumer surplus in the above three optimization problems is given by

$$\begin{aligned}E[\max_{i\in\mathcal{N}^+} U_i^\dagger] &= \log\left(1 + \sum_{i\in\mathcal{N}}\exp(\alpha_i q_i - p_i^\dagger + t_i s_i)\right) = \log\left(1 + \sum_{i\in\mathcal{N}}\exp\left(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r^\dagger - 1\right)\right)\\ &= \log(1 + r^\dagger) \leq \log(1 + r^\ddagger) \leq \log(1 + r^*).\end{aligned}$$

The second equality holds because $p_i^\dagger = 1 + r^\dagger + c_i q_i^2 + t_i(a_i - b_i q_i)$ in the price optimization problem. The last equality holds because $r^\dagger = \sum_{i\in\mathcal{N}} \exp\left(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r^\dagger - 1\right)$; the inequalities hold because $r^\dagger \le r^\ddagger \le r^*$, i.e., in the joint price, service and quality optimization problem, the firm earns more profit than in the joint price and service optimization problem, which generates more profit than in the price-only optimization problem. Immediately, we have the comparison result: $E[\max_{i\in\mathcal{N}^+} U_i^\dagger] \le E[\max_{i\in\mathcal{N}^+} U_i^\ddagger] \le E[\max_{i\in\mathcal{N}^+} U_i^*]$. □

Proof of Theorem 2. Given the prices, quality levels and service durations for other firms, we consider the joint price, quality and service optimization problem for the firm i who aims to maximize its expected profit by choosing the optimal price, quality level and service duration. The problem can be formulated as $\max_{p_i,q_i,t_i} \Pi_i(\mathbf{p},\mathbf{q},\mathbf{t};\mathcal{N})$. Then, we have

$$\max_{p_i,q_i,t_i} \left\{ r_i : \ r_i = \Pi_i(\mathbf{p},\mathbf{q},\mathbf{t};\mathcal{N}) \right\} = \max_{p_i,q_i,t_i} \left\{ r_i : \text{ equation (EC.1) } \right\},$$

where the equation (EC.1) is defined as follows:

$$r_i \cdot \Big(1 + \sum_{j\neq i} \exp(\alpha_j q_j - p_j + t_j s_j)\Big) = \big[p_i - c_i q_i^2 - t_i(a_i - b_i q_i) - r_i\big] \cdot \exp(\alpha_i q_i - p_i + t_i s_i). \tag{EC.1}$$

Notice that for given $(p_j, q_j, t_j)_{j\neq i}$, the LHS of the above equation (EC.1) is increasing in r_i, while the RHS is decreasing in r_i. This leads to a unique solution to equation (EC.1) w.r.t. r_i. Therefore, the above joint optimization for firm i can be rewritten as follows:

$$r_i \cdot \Big(1 + \sum_{j\neq i} \exp(\alpha_j q_j - p_j + t_j s_j)\Big) = \max_{p_i,q_i,t_i} \Big\{ \big[p_i - c_i q_i^2 - t_i(a_i - b_i q_i) - r_i\big] \cdot \exp(\alpha_i q_i - p_i + t_i s_i) \Big\}.$$

Again, the function $\big[p_i - c_i q_i^2 - t_i(a_i - b_i q_i) - r_i\big] \cdot \exp(\alpha_i q_i - p_i + t_i s_i)$ is unimodal in p_i for any given q_i, t_i and r_i, and it achieves its maximum at $p_i = c_i q_i^2 + t_i(a_i - b_i q_i) + r_i + 1$. Thus, the above problem can be further simplified as follows:

$$r_i \cdot \Big(1 + \sum_{j\neq i} \exp(\alpha_j q_j - p_j + t_j s_j)\Big) = \max_{q_i,t_i} \Big\{ \exp\left(\alpha_i q_i - c_i q_i^2 - t_i(a_i - b_i q_i - s_i) - r_i - 1\right) \Big\}. \tag{EC.2}$$

The equilibrium quality level in the simultaneous competition in quality, service and price is $q_i = (\alpha_i + t_i b_i)/(2c_i)$, thus, the above problem can be further rewritten as

$$r_i \cdot \Big(1 + \sum_{j\neq i} \exp(\alpha_j q_j - p_j + t_j s_j)\Big) = \max_{t_i} \Big\{ \exp\left(b_i^2 t_i^2/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_i + \alpha_i^2/(4c_i) - r_i - 1\right) \Big\}. \tag{EC.3}$$

We also find that the function $\exp(b_i^2 t_i^2/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_i + \alpha_i^2/(4c_i) - r_i - 1)$ reaches its minimum at $t_i = \big(2(a_i - s_i)c_i - \alpha_i b_i\big)/b_i^2$, and reaches its maximum at one of the boundary points. i.e., t_s or t_l, which still depends on the comparison between $\big(2(a_i - s_i)c_i - \alpha_i b_i\big)/b_i^2$ and $(t_s + t_l)/2$. This indicates that both service duration and product quality in the competitive setting are the same as the monopolistic service duration and quality level, respectively. Thus, the joint competition reduces to the standard price competition under the MNL model.

Given the equilibrium quality levels and service durations $\mathbf{q}^*$ and $\mathbf{t}^*$, which are equal to the optimal results in the monopolistic case respectively, the joint service duration, quality and price competition becomes

$$\Pi_i(\mathbf{p},\mathbf{q}^*\mathbf{t}^*;\mathcal{N}) = \left[p_i - c_i q_i^{*2} - t_i^*(a_i - b_i q_i^*)\right]\cdot d_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N}). \tag{EC.4}$$

Consider the first-order derivative for $\Pi_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N})$ w.r.t. p_i:

$$\frac{\partial \Pi_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N})}{\partial p_i} = \Big(1 - \left[p_i - c_i q_i^{*2} - t_i^*(a_i - b_i q_i^*)\right]\cdot\big(1 - d_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N})\big)\Big)\cdot d_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N}).$$

We find $\Pi_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N})$ is quasi-concave in p_i, so there exists a Nash equilibrium; see, e.g., Anderson et al. (1992). In fact, we can show its uniqueness and obtain it in “closed-form”, which can be explicitly expressed by the unique root of a monotone function. By $\partial\Pi_i(\mathbf{p},\mathbf{q}^*,\mathbf{t}^*;\mathcal{N})/\partial p_i = 0$, we have $1 = \left[p_i - c_i q_i^{*2} - t_i^*(a_i - b_i q_i^*)\right]\cdot \big(1-\exp(\alpha_i q_i^* - p_i + t_i^* s_i)/A\big)$, where $A = 1 + \sum_{j\in\mathcal{N}}\exp(\alpha_j q_j^* - p_j + t_j^* s_j)$. Note that the RHS is increasing in p_i for any given A, so we denote the unique corresponding p_i for A by $\phi_i(A)$. Moreover, $\phi_i(A)$ is decreasing in A. Also, $\exp(\alpha_i q_i^* - \phi_i(A) + t_i^* s_i)/A$ is decreasing in A. Then, by $A = 1 + \sum_{j\in\mathcal{N}}\exp(\alpha_j q_j^* - p_j + t_j^* s_j)$ we have $1/A + \sum_{j\in\mathcal{N}}\exp(\alpha_j q_j^* - \phi_j(A) + t_j^* s_j)/A = 1$. Because each term of the LHS is decreasing in A from 1 to 0 as A increases from 1 to ∞, then there exists a unique solution to the above equation, denoted by A^o. Thus, the unique Nash equilibrium in the price competition is $p_i^o = \phi_i(A^o)$.

Again, by $A = 1 + \sum_{j\in\mathcal{N}}\exp(\alpha_j q_j^* - p_j + t_j^* s_j)$, we have $A - \exp(\alpha_i q_i^* - p_i + t_i^* s_i) = 1 + \sum_{j\neq i}\exp(\alpha_j q_j^* - p_j + t_j^* s_j) \geq 1$. By $1 = \left[p_i - c_i q_i^{*2} - t_i^*(a_i - b_i q_i^*)\right]\cdot\big(1 - \exp(\alpha_i q_i^* - p_i + t_i^* s_i)/A\big)$, we have

$$A = \left[p_i - c_i q_i^{*2} - t_i^*(a_i - b_i q_i^*)\right]\cdot\big(A - \exp(\alpha_i q_i^* - p_i + t_i^* s_i)\big) \geq \phi_i(A) - c_i q_i^{*2} - t_i^*(a_i - b_i q_i^*).$$

Then, $\phi_i(A) \leq A + c_i q_i^{*2} + t_i^*(a_i - b_i q_i^*)$ for any $i\in\mathcal{N}$.

Now, we compare the following equations

$$A = 1 + \sum_{i\in\mathcal{N}}\exp(\alpha_i q_i^* - \phi_i(A) + t_i^* s_i) \text{ and } m = 1 + \sum_{i\in\mathcal{N}}\exp\big(\alpha_i q_i^* - c_i q_i^{*2} - t_i^*(a_i - b_i q_i - s_i) - m\big).$$

The first one has a unique solution A^o as discussed above. The LHS of the second equation is a linear increasing line while its RHS is decreasing in m, so it also has a unique solution, denoted by m^*. By $\phi_i(A) \leq A + c_i q_i^{*2} + t_i^*(a_i - b_i q_i^*)$, we have $\exp(\alpha_i q_i^* - \phi_i(A) + t_i^* s_i) \geq \exp(\alpha_i q_i^* - c_i q_i^{*2} - t_i^*(a_i - b_i q_i - s_i) - A)$ for any $i\in\mathcal{N}$. Therefore, $A^o \geq m^*$. Immediately, the total market share (which can be expressed by $(A^o-1)/A^o$ and $(m^*-1)/m^*$ for the oligopolistic and monopolistic scenarios, respectively) and consumer surplus (which can be expressed by $\log(A^o)$ and $\log(m^*)$, respectively) under the oligopoly are higher than in the monopoly.

Recall that the optimal price can be expressed as $p_i^* = m^* + c_i q_i^{*2} + t_i^*(a_i - b_i q_i^*)$ in the monopolistic case. So we have $p_i^o = \phi_i(A^o) \leq \phi_i(m^*) \leq m^* + c_i q_i^{*2} + t_i^*(a_i - b_i q_i^*) = p_i^*$. The first inequality holds because $\phi_i(A)$ is decreasing in A for any i. Therefore, the prices in the oligopolistic scenario are lower than those in the monopolistic scenario. □

Proof of Proposition 6. Based on the result in Theorem 1, with service differentiation, the optimal total expected profit, denoted by r^o, is the unique solution to the following equation w.r.t. r:

$$r := \sum_{i\in S_l} \exp\left(b_i^2 t_l^2/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_l + \alpha_i^2/(4c_i) - r - 1\right) + \sum_{i\in S_s} \exp\left(b_i^2 t_s^2/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_s + \alpha_i^2/(4c_i) - r - 1\right),$$

where S_l and S_s are the subsets consisting of the products that match t_l and t_s respectively, according to Proposition 2.

Without service differentiation, the firm's optimal total expected profit is r^*, where r^* is the unique solution to the following equation:

$$r = \sum_{i\in\mathcal{N}} \exp\left(b_i^2 t^{*2}/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t^* + \alpha_i^2/(4c_i) - r - 1\right),$$

where $t^* = t_s$ or t_l as shown in Theorem 3.

Regarding product quality level, we have the optimal quality structure $q_i^* = (\alpha_i + t_i^* b_i)/(2c_i)$ with service differentiation. For the product set matching the longest duration of service t_l, the quality level for each product $i \in S_l$ is $q_i^* = (\alpha_i + t_l b_i)/(2c_i)$; for product $i \in S_s$, its optimal quality level is $q_i^* = (\alpha_i + t_s b_i)/(2c_i)$. Without service differentiation, the uniform optimal service duration t^* for all products would be a boundary point either t_s or t_l by Theorem 3, and therefore the quality level for each product $i \in \mathcal{N}$ is $q_i^* = (\alpha_i + t^* b_i)/(2c_i)$;

If $t^* = t_s$, the quality levels of the products in S_l will be influenced by service differentiation. In particular, the quality level of product $i \in S_l$ is higher for $b_i > 0$, whereas the level is lower for product $i \in S_l$ for $b_i < 0$. Similarly, if $t^* = t_l$, the quality levels of the products in S_s will be affected by service differentiation. Specifically, the quality level of product $i \in S_s$ is lower for $b_i > 0$, whereas is higher for product $i \in S_s$ for $b_i < 0$.

By Proposition 2, if $b_i > 0$, the product quality and service duration tend to go hand in hand for product i; otherwise, they show a tendency to oppose each other. This reveals that the high-quality product with $b_i > 0$ or/and the low-quality product with $b_i < 0$ highly tend to be included in S_l. In a similar fashion, the low-quality product with $b_i > 0$ or/and the high-quality product with $b_i < 0$ would be included in S_s. Therefore, for the products whose qualities are influenced by service differentiation, the quality levels of the products with relatively low input in quality tend to go down, whereas go up for the products with relatively high input in quality after incorporating service differentiation. Thus, we can say heterogenous services allow product quality levels to be more dispersed. Note that for product i with $b_i = 0$, the product quality level keeps unchanged with service differentiation.

Moreover, for any given $r > 0$, we have $\exp\left(b_i^2 t_i^{*2}/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_i^* + \alpha_i^2/(4c_i) - r - 1\right) \geq \exp\left(b_i^2 t^{*2}/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t^* + \alpha_i^2/(4c_i) - r - 1\right)$ for each product i with service differentiation. In particular, if $t^* = t_s$, the above inequality holds for any product $i \in S_l$, and the equality holds for each $i \in S_s$. Same thing happens with the case of $t^* = t_l$.

Thus, the total optimal profit with service differentiation is higher regardless of $t^* = t_s$ or $t^* = t_l$. That is $r^o \geq r^*$. Immediately, in the join optimization problem on price, quality and service, consumer surplus comparison is shown below:

$$\begin{aligned} E[\max_{i\in\mathcal{N}^+} U_i^*] &= \log\Big(1+\sum_{i\in\mathcal{N}}\exp(\alpha_i q_i^* - p_i^* + t^* s_i)\Big) \\ &= \log\Big(1+\sum_{i\in\mathcal{N}}\exp(b_i^2 t^{*2}/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t^* + \alpha_i^2/(4c_i) - r^* - 1)\Big) = \log(1+r^*) \\ &\leq \log(1+r^o) = \log\Big(1+\textstyle\sum_{i\in S_l}\exp(b_i^2 t_l^2/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_l + \alpha_i^2/(4c_i) - r^o - 1) \\ &\quad + \textstyle\sum_{i\in S_s}\exp(b_i^2 t_s^2/(4c_i) + (s_i - a_i + \alpha_i b_i/(2c_i))t_s + \alpha_i^2/(4c_i) - r^o - 1)\Big) = E[\max_{i\in\mathcal{N}^+} U_i^\dagger], \end{aligned}$$

where $E[\max_{i\in\mathcal{N}^+} U_i^\dagger]$ and $E[\max_{i\in\mathcal{N}^+} U_i^*]$ are consumer surplus with and without service differentiation, respectively. Therefore, both the firm and consumers are better off with service differentiation.

Recall the optimal price structure $p_i^* = 1 + r^* + c_i q_i^{*2} + t^*(a_i - b_i q_i^*)$ still holds with a uniform duration of service. Then, the same markup property holds regardless of whether service differs or not for different products. Because the total optimal profit with service differentiation is higher, then the product margin for each product $i \in \mathcal{N}$ is higher with service differentiation. □

Proof of Theorem 4. Taking the derivative of the total expected profit w.r.t. p_{xi} yields

$$\begin{aligned} \frac{\partial \Pi_M(\mathbf{p},\mathbf{q})}{\partial p_{xi}} &= d_{xi}\cdot\Big(1 - \big[p_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi})\big] + (1-1/\mu_1)\sum_{j=1}^{m_x}\big[p_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xi} - b_{xi}q_{xj})\big]d_{j|x} \\ &\quad + 1/\mu_1\sum_{x\in\{s,l\}}\sum_{i=1}^{m_x}\big[p_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi})\big]d_{xi}\Big). \end{aligned}$$

Consider the FOC for optimality, and we have

$$p_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi}) = 1 + r/\mu_1 + (1-1/\mu_1)r_x,$$

where $r = \sum_x \sum_{i=1}^{m_x}\big[p_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi})\big]d_{xi}$ denotes the total expected profit and $r_x = \sum_{i=1}^{m_x}\big[p_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi})\big]d_{i|x}$ denotes the average profit of nest x. We find that the markup for all products in the same nest is the same, which is independent of product index i in nest x.

Let ρ_x denote the constant markup for all the products in nest x, i.e.,

$$\rho_x = p_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi}). \tag{EC.5}$$

By the above result, the high-dimensional price optimization problem can be significantly simplified to a low-dimensional markup optimization problem, and the dimension is in accordance with the number of nests, i.e., two in this case.

For the sake of notational simplicity, let $d_x(\boldsymbol{\rho},\mathbf{q})$ be the probability that a consumer selects nest x at the first stage, where vector $\boldsymbol{\rho} = (\rho_s, \rho_l)$. Let $d_{i|x}(\rho_x, \mathbf{q}_x)$ denote the probability that product i of nest x is selected at the second stage, given that the consumer has chosen nest x at the first stage. Then we have

$$d_x(\boldsymbol{\rho},\mathbf{q}) = \frac{(e_x(\rho_x,\mathbf{q}_x))^{1/\mu_1}}{1 + (e_s(\rho_s,\mathbf{q}_s))^{1/\mu_1} + (e_l(\rho_l,\mathbf{q}_l))^{1/\mu_1}},$$

$$d_{i|x}(\rho_x, \mathbf{q}_x) = \frac{\exp(\alpha_{xi}q_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi}) + t_x s_{xi} - \rho_x)}{\sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xj} - b_{xj}q_{xj}) + t_x s_{xj} - \rho_x)},$$

where $e_x(\rho_x, \mathbf{q}_x) = \sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xj} - b_{xj}q_{xj}) + t_x s_{xj} - \rho_x)$. Therefore, the total expected profit corresponding to the markups $\boldsymbol{\rho}$ can be written as follows:

$$\Pi_M(\boldsymbol{\rho}, \mathbf{q}) =: \sum_{x \in \{s,l\}} \rho_x \cdot d_x(\boldsymbol{\rho}, \mathbf{q}). \tag{EC.6}$$

Then, considering the FOC of $\Pi_M(\boldsymbol{\rho}, \mathbf{q})$ w.r.t. the markup ρ_x yields $\partial \Pi_M(\boldsymbol{\rho}, \mathbf{q})/\partial \rho_x = d_x\big(1 - \rho_x/\mu_1 + 1/\mu_1 \sum_x \rho_x \cdot d_x\big) = 0$. We immediately obtain $\rho_x - \mu_1 = r$.

Plug $\rho_x = r + \mu_1$ into equation (EC.6). After some algebra and rearranging terms, $\max_{\mathbf{p},\mathbf{q}} \Pi_M(\mathbf{p}, \mathbf{q})$ is equivalent to the problem below:

$$r = \max_{\mathbf{q}} \sum_{x \in \{s,l\}} \mu_1 \bigg(\sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xj} - b_{xj}q_{xj}) + t_x s_{xj} - r - \mu_1) \bigg)^{1/\mu_1}. \tag{EC.7}$$

Considering the derivative w.r.t. each product quality level q_{xi} leads to the result that the optimal quality level for product i in any nest x satisfies $q_{xi}^\dagger = (\alpha_{xi} + t_x b_{xi})/(2c_{xi})$. In fact, we have

$$\begin{aligned} &\frac{\partial}{\partial q_{xi}} \sum_{x \in \{s,l\}} \mu_1 \bigg(\sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xj} - b_{xj}q_{xj}) + t_x s_{xj} - r - \mu_1) \bigg)^{1/\mu_1} \\ &= \frac{\big(\alpha_{xi} - 2c_{xi}q_{xi} + t_x b_{xi}\big) \exp\big(\alpha_{xi}q_{xi} - c_{xi}q_{xi}^2 - t_x(a_{xi} - b_{xi}q_{xi}) + t_x s_{xi} - r - \mu_1\big)}{\big(\sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xj} - b_{xj}q_{xj}) + t_x s_{xj} - r - \mu_1)\big)^{(1-1/\mu_1)}}, \end{aligned}$$

for any $r > 0$. We find that for the joint price and quality optimization problem under the NL model framework, it is still optimal for the firm to set the quality level for each product i independently of the quality levels and the prices of other products in the same nest, let alone the products across nests, despite that theses products are substitutable items for the consumers.

Therefore, we would update the problem $\max_{\mathbf{p},\mathbf{q}} \Pi_M(\mathbf{p}, \mathbf{q})$ as follows:

$$r = \sum_{x \in \{s,l\}} \mu_1 \bigg(\sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj}^\dagger - c_{xj}q_{xj}^{\dagger 2} - t_x(a_{xj} - b_{xj}q_{xj}^\dagger) + t_x s_{xj} - r - \mu_1) \bigg)^{1/\mu_1}. \tag{EC.8}$$

Notice that the LHS of the above equation is linearly increasing in r, whereas the RHS is strictly decreasing in r because y^g is concave increasing in y for $y > 0$ and $0 < g \leq 1$, where $y = \big(\sum_{j=1}^{m_x} \exp(\alpha_{xj}q_{xj}^\dagger - c_{xj}q_{xj}^{\dagger 2} - t_x(a_{xj} - b_{xj}q_{xj}^\dagger) + t_x s_{xj} - r - \mu_1)\big)$ is a strictly decreasing function in r and $g = 1/\mu_1$. Thus, we can obtain the total optimal profit, denoted by $r^\dagger$. Immediately, the optimal price for each product i in nest x can be expressed by $p_{xi}^\dagger = c_{xi}q_{xi}^{\dagger 2} + t_x(a_{xi} - b_{xi}q_{xi}^\dagger) + r^\dagger + \mu_1$. □

Proof of Proposition 7. Similar to the case under the MNL model, consumer surplus under the two-stage nested logit model can be expressed by

$$\mu_1 \log\Big(1 + \sum_{x \in \{s,l\}} \Big(\sum_{i \in \mathcal{N}_x} \exp(\alpha_{xi}q_{xi} - p_{xi} + t_x s_{xi}) \Big)^{1/\mu_1} \Big).$$

By the proof of Theorem 4, we can plug the optimal results for $q_{xi}^\dagger$ and $p_{xi}^\dagger$ into the above expression. Then, we have

$$W(r^\dagger) = \mu_1 \log\Big(1 + \sum_{x \in \{s,l\}} \Big(\sum_{i \in \mathcal{N}_x} \exp\big(\alpha_{xj}q_{xj}^\dagger - c_{xj}q_{xj}^{\dagger 2} - t_x(a_{xj} - b_{xj}q_{xj}^\dagger) + t_x s_{xj} - r^\dagger - \mu_1\big) \Big)^{1/\mu_1} \Big) = \mu_1 \log(1 + r^\dagger/\mu_1),$$

where $W(r^\dagger)$ denotes consumer surplus under the two-stage nested logit model, and $r^\dagger$ is the total optimal profit calculated by equation (EC.8).

In a similar fashion, we have

$$W(\tilde{r}) = \mu_1 \log\Big(1 + \sum_{x\in\{s,l\}} \Big(\sum_{i\in\mathcal{N}_x} \exp\big(\alpha_{xj}q_{xj} - c_{xj}q_{xj}^2 - t_x(a_{xj} - b_{xj}q_{xj}) + t_x s_{xj} - \tilde{r} - \mu_1\big)\Big)^{1/\mu_1}\Big) = \mu_1 \log(1 + \tilde{r}/\mu_1),$$

where $\tilde{r}$ is the total optimal profit when the firm can only determine prices, and product quality levels are pre-determined but may not be optimal.

By the similar arguments in Proposition 5, we have $\tilde{r} \le r^\dagger$, and therefore $W(\tilde{r}) \le W(r^\dagger)$. Thus, we still can conclude that consumer surplus improves and the firm earns more revenue when more decisions are controllable under the NL model. In other words, both the firm and consumers are better off when the firm jointly chooses the prices and qualities compared to the situation, under which the firm can only determine the prices under the NL model. □

Proof of Proposition 8. Based on equation (17), the joint optimization problem on price and service duration can be rewritten as follows:

$$r = \max_{\mathbf{t}} \sum_{i\in\mathcal{N}} \exp\big(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1\big). \tag{EC.9}$$

Note that the RHS of this equation can be separated for each product i for any given $r > 0$, and each function $\exp\big(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1\big)$ is strictly either increasing or decreasing in the service duration of product $i \in \mathcal{N}$, because

$$\begin{aligned} &\frac{\partial}{\partial t_i} \exp(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1) \\ &= \big(s_i - (a_i - b_i q_i)\big) \exp(\alpha_i q_i - c_i(q_i) + t_i(s_i - (a_i - b_i q_i)) - r - 1). \end{aligned}$$

In particular, (i) if $b_i > 0$, for $0 < q_i < (a_i - s_i)/b_i, i \in \mathcal{N}$, $\exp(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1)$ decreases in t_i, thus the optimal service duration for product i is the lower bound t_s; for $(a_i - s_i)/b_i \le q_i < a_i/b_i$, $\exp(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1)$ increases in t_i, and therefore the optimal service duration is the upper bound t_l.

(ii) If $b_i < 0$, for $0 < q_i < (a_i - s_i)/b_i, i \in \mathcal{N}$, $\exp(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1)$ increases in t_i, so the optimal service duration for product i is the upper bound t_l; for $q_i \ge (a_i - s_i)/b_i$, $\exp(\alpha_i q_i - c_i q_i^2 + t_i(s_i - (a_i - b_i q_i)) - r - 1)$ decreases in t_i, thus the optimal service duration is the lower bound t_s.

(iii) If $b_i = 0$, the selection of the optimal service duration for product i only depends on the comparison between a_i and s_i. In particular, if $a_i \le s_i$, the optimal service duration for product i is t_l; otherwise, the optimal service duration is t_s.

Then, the joint optimization problem (10) can be updated as follows:

$$\begin{aligned} r = \max_{S\subseteq\mathcal{N}} \Big(&\sum_{i\in\mathcal{S}_l} \exp(\alpha_i q_i - c_i q_i^2 + t_l(s_i - (a_i - b_i q_i)) - r - 1) \\ &+ \sum_{i\in S_s} \exp(\alpha_i q_i - c_i q_i^2 + t_s(s_i - (a_i - b_i q_i)) - r - 1)\Big), \end{aligned}$$

where $S_s \cup S_l = S \subseteq \mathcal{N}$, and subset S_s includes the product $i \in \mathcal{N}$ whose quality level satisfies $0 \le q_i \le (a_i - s_i)/b_i$ if $b_i > 0$ or $q_i \ge (a_i - s_i)/b_i$ if $b_i < 0$; subset S_l includes the product $i \in \mathcal{N}$ whose quality level satisfies $(a_i - s_i)/b_i \le q_i \le a_i/b_i$ if $b_i > 0$ or $0 \le q_i \le (a_i - s_i)/b_i$ if $b_i < 0$. All products will be offered in the joint optimization problem because each $\exp(\cdot)$ is always nonnegative, i.e., $S_s \cup S_l = S = \mathcal{N}$.

By the above equation, the LHS is a linear increasing line whereas the RHS is strictly decreasing in r for any $r > 0$, given quality vector $\mathbf{q}$. Thus, there exists a unique total optimal profit, denoted by r^*. Immediately, the corresponding optimal price is $p_i^* = 1 + r^* + c_i q_i^2 + t_s(a_i - b_i q_i)$ for product $i \in S_s$, and the corresponding optimal price is $p_i^* = 1 + r^* + c_i q_i^2 + t_l(a_i - b_i q_i)$ for product $i \in S_l$. □

Proof of Theorem 5. Let $\Pi^s(\mathbf{p}, \mathbf{q}; S_l, S_s) = r$ for notation convenience. Then, problem (13) can be reformulated as follows:

$$\max \Big\{ r = \sum_{i \in S_l} \big[p_i - c_i q_i^2 - t_l(a_i - b_i q_i)\big] \cdot d_i(\mathbf{p}, \mathbf{q}; S_l, S_s) + \sum_{i \in S_s} \big[p_i - c_i q_i^2 - t_s(a_i - b_i q_i)\big] \cdot d_i(\mathbf{p}, \mathbf{q}; S_l, S_s) \Big\}. \quad \text{(EC.10)}$$

Rearranging terms and using some algebra for the inside equation of problem (EC.10) yields the following equation:

$$r = \sum_{i \in S_l} h_i^l(r) + \sum_{i \in S_s} h_i^s(r), \quad \text{(EC.11)}$$

where $h_i^l(r) = \big[p_i - c_i q_i^2 - t_l(a_i - b_i q_i) - r\big] \cdot \exp(\alpha_i q_i - p_i + t_l s_i)$ and $h_i^s(r) = \big[p_i - c_i q_i^2 - t_s(a_i - b_i q_i) - r\big] \cdot \exp(\alpha_i q_i - p_i + t_s s_i)$. Again, the LHS of equation (EC.11) is a linear increasing line while the RHS is decreasing in r, so there exists a unique solution w.r.t. r for given sets S_l and S_s.

Therefore, problem (EC.10) can be further simplified to find the unique solution to equation $r = H(r)$, where for any r, function $H(r)$ is defined as follows:

$$H(r) = \max_{S_l, S_s \subseteq \mathcal{N}} \Big\{ \sum_{i \in S_l} h_i^l(r) + \sum_{i \in S_s} h_i^s(r) \Big\} = \sum_{i \in \mathcal{N}} h_i(r),$$

where $h_i(r) = \max\{h_i^l(r), h_i^s(r), 0\}$ for any $i \in \mathcal{N}$. Notice that $h_i^l(r)$ and $h_i^s(r)$ are both linearly decreasing functions w.r.t. r. Let x_i, y_i and z_i be the solutions to the equations $h_i^l(x) = h_i^s(x)$, $h_i^l(x) = 0$ and $h_i^s(x) = 0$ respectively, i.e.,

$$\begin{aligned} x_i &= (p_i - c_i q_i^2) - (a_i - b_i q_i) \cdot \frac{t_l \cdot \exp(t_l s_i) - t_s \cdot \exp(t_s s_i)}{\exp(t_l s_i) - \exp(t_s s_i)}; \\ y_i &= p_i - c_i q_i^2 - t_l(a_i - b_i q_i); \;\; z_i = p_i - c_i q_i^2 - t_s(a_i - b_i q_i). \end{aligned}$$

We observe that $x_i \le y_i \le z_i$. Therefore, $h_i(r)$ can be characterized as follows: $h_i(r) = h_i^l(r)$ for any $r \le x_i$; $h_i(r) = h_i^s(r)$ for any $x_i < r \le z_i$; and $h_i(r) = 0$ for any $r > z_i$.

Because $H(r)$ is decreasing in r, there exists a unique solution to the equation $r = H(r)$, denoted by r^*. Immediately, all product i's with $h_i(r^*) \ge 0$ or equivalently $z_i = p_i - c_i q_i^2 - t_s(a_i - b_i q_i) \ge r^*$ form an optimal offer set. In other words, a markup-ordered assortment based on the markup $p_i - c_i q_i^2 - t_s(a_i - b_i q_i)$, denoted by S_{i^*}, is the optimal offer set for problem (13).

Moreover, any product $i \in S_{i^*}$ with $x_i \ge r^*$ should be provided with the long duration of service; all other products in S_{i^*} should be provided with the short duration of service. By the definition of x_i's, an adjusted markup-ordered assortment should be provided with the long duration of service. □