Appendix to "Optimal Contract Design for a National Brand Manufacturer under Store Brand Private Information"

Appendix A

In this appendix, we list all mathematical notations used in the main paper in Table $A1$ below.

Notation	Definition
Π_n $[\hat{\Pi}_n]$	NB manufacturer's profit function under asymmetric [symmetric] information
Π_r $[\hat{\Pi}_r]$	Retailer's profit function under asymmetric [symmetric] information
Q_n $[Q_s]$	NB [SB] production quantity
p_n $[p_s]$	NB [SB] product retail price
q_n $[q_s]$	NB [SB] product quality
θ	Consumer's quality evaluation parameter
c_n $[c_s]$	NB [SB] unit production cost
c_s^H $[c_s^L]$	Estimated high-type [low-type] SB unit production cost
w	Unit wholesale price paid to the NB manufacturer
T	Fixed lump-sum payment paid to the NB manufacturer
w^H $[w^L]$	Unit wholesale price for high-type [low-type] retailer
T^H $[T^L]$	Fixed lump-sum payment for high-type [low-type] retailer
V_n $[V_r]$	Value of information to NB manufacturer [retailer]
CS	Consumer surplus

Table A1 List of mathematical notations

Appendix B

In this appendix, we provide proofs for all the results mentioned in the main paper.

Proof. **Lemma 1.** We start with deriving the local optima under cases I, II, and III. Under case I, direct differentiation on $\hat{\Pi}_r^I$ with respect to p_n yields $\frac{d\hat{\Pi}_r^I}{dp_n} = \frac{w+q_n-2p_n}{q_n}$ and $\frac{d^2\hat{\Pi}_r^I}{d{p_n}^2} = -\frac{2}{q_n} < 0$. Solving first-order condition equal to 0, we can get $p_n^{I*} = \frac{w+q_n}{2}$. It follows that $Q_n^{I*} = \frac{1}{2} - \frac{w}{2q_n}$ and $\hat{\Pi}_r^{I*} = \frac{(q_n-w)^2}{4q_n} - T$.

Under case III, the derivation of p_s^{III*} is obtained in a similar manner. We have $p_s^{III*} = \frac{c_s+q_s}{2}$, $Q_s^{III*} = \frac{1}{2} - \frac{c_s}{2q_s}$, and $\hat{\Pi}_r^{III*} = \frac{(q_s-c_s)^2}{4q_s}$.

Under case II, taking derivatives of $\hat{\Pi}_r^{II}$ with respect to p_n and p_s, we obtain $\frac{\partial\hat{\Pi}_r^{II}}{\partial p_n} = 1 - \frac{2(p_n-p_s)-w+c_s}{q_n-q_s}$, $\frac{\partial\Pi_r^{II}}{\partial p_s} = \frac{2(p_n-p_s)-w+c_s}{q_n-q_s} - \frac{2p_s-c_s}{q_s}$, $\frac{\partial^2\hat{\Pi}_r^{II}}{\partial p_n^2} = \frac{\partial^2\hat{\Pi}_r^{II}}{\partial p_s^2} = \frac{-2}{q_n-q_s} < 0$, $\frac{\partial^2\Pi_r^{II}}{\partial p_n^2}\frac{\partial^2\Pi_r^{II}}{\partial p_s^2} - (\frac{\partial^2\Pi_r^{II}}{\partial p_n \partial p_s})^2 = \frac{4}{q_s(q_n-q_s)} > 0$. Hence, the Hessian matrix of $\hat{\Pi}_r^{II}$ is negative definite. Then, by setting $\frac{\partial\hat{\Pi}_r^{II}}{\partial p_n} = 0$ and $\frac{\partial\hat{\Pi}_r^{II}}{\partial p_s} = 0$, we get the interior optimal solution for case II as $p_n^{II*} = \frac{w+q_n}{2}, p_s^{II*} = \frac{c_s+q_s}{2}$. It follows that $Q_n^{II*} = \frac{1}{2} - \frac{w-c_s}{2(q_n-q_s)}, Q_s^{II*} = \frac{w-c_s}{2(q_n-q_s)} - \frac{c_s}{2q_s}$, and $\hat{\Pi}_r^{II*} = \frac{q_n-w}{2}\left[\frac{1}{2} - \frac{w-c_s}{2(q_n-q_s)}\right] + \frac{q_s-c_s}{2}\left[\frac{w-c_s}{2(q_n-q_s)} - \frac{c_s}{2q_s}\right] - T$. Recall that the condition for the occurrence of case II is $\frac{p_s}{q_s} \leq \frac{p_n-p_s}{q_n-q_s} \leq 1$. We reorganize this condition as an interval of p_s, i.e., $p_s \in [p_n - q_n + q_s, \frac{p_n q_s}{q_n}]$. If $w \leq \frac{c_s q_n}{q_s}$, $\frac{p_n-p_s}{q_n-q_s} < 1$ is satisfied but $\frac{p_s}{q_s} < \frac{p_n-p_s}{q_n-q_s}$

does not hold. Since $\hat{\Pi}_r^{II}$ is quasi-concave and monotonically increasing in p_s within the closed interval $[p_n-q_n+q_s, \frac{p_n q_s}{q_n}]$, the local maximum resides at $p_s=p_n-q_n+q_s$ or $p_s=\frac{p_n q_s}{q_n}$. It follows from $w\le\frac{c_s q_n}{q_s}$ that $\hat{\Pi}_r^{II}|_{p_s=\frac{p_n q_s}{q_n}}=\frac{(q_n-w)^2}{4q_n}-T\ge\hat{\Pi}_r^{II}|_{p_s=p_n-q_n+q_s}=\frac{(q_s-c_s)^2}{4q_s}-T$. Hence, $\forall w\in(0,\frac{c_s q_n}{q_s}]$, $\hat{\Pi}_r^{II*}=\hat{\Pi}_r^{II}|_{p_s=\frac{p_n q_s}{q_n}}=\frac{(q_n-w)^2}{4q_n}-T=\hat{\Pi}_r^{I*}$. If w is in $(\frac{c_s q_n}{q_s}, c_s+q_n-q_s]$, the condition $\frac{p_s}{q_s}\le\frac{p_n-p_s}{q_n-q_s}\le 1$ is fully satisfied. Then $(p_n^{II*},p_s^{II*})=(\frac{w+q_n}{2},\frac{c_s+q_s}{2})$ is indeed the local optimum for case II. If $w>c_s+q_n-q_s$, $\hat{\Pi}_r^{II*}$ occurs at $p_s=p_n-q_n+q_s$ such that $\hat{\Pi}_r^{II*}=\Pi_r^{II*}|_{p_s=p_n-q_n+q_s}=\frac{(q_s-c_s)^2}{4q_s}-T$. However, $\hat{\Pi}_r^{II*}|_{p_s=p_n-q_n+q_s}<\hat{\Pi}_r^{III*}$ for any $T>0$. Then if $p_s=p_n-q_n+q_s$, it is no longer profitable for the retailer to carry both brands.

We now proceed to search for the global optimal profit, i.e., $\hat{\Pi}_r^*=\max\{\hat{\Pi}_r^{I*},\hat{\Pi}_r^{II*},\hat{\Pi}_r^{III*}\}$, through the pairwise comparisons among the three local optima. We have shown that when $w\le\frac{c_s q_n}{q_s}$, if the retailer decides to carry both brands, he has to set the retail prices to satisfy the condition $\frac{p_s}{q_s}<\frac{p_n-p_s}{q_n-q_s}<1$, resulting in a profit that is equivalent to carrying NB only. Therefore, given any $w\in(0,\frac{c_s q_n}{q_s}]$, the retailer is better off to carry NB only, i.e., $\hat{\Pi}_r^*=\Pi_r^{I*}$. Note that Π_r^{I*} is decreasing in w and continuous with $\hat{\Pi}_r^{II*}$ at $w=\frac{c_s q_n}{q_s}$.

When comparing $\hat{\Pi}_r^{I*}$and $\hat{\Pi}_r^{III*}$, we need to discuss how the retailer's decision differentiates based on T. Note the lump-sum transfer is not applicable in case III. For any $w\le\frac{c_s q_n}{q_s}$, there exists a threshold transfer $\bar{T}^I\ge 0$, such that $\hat{\Pi}_r^{I*}|_{T=\bar{T}^I}=\Pi_r^{III*}$. Then we have $\bar{T}^I=\frac{(q_n-w)^2}{4q_n}-\frac{(q_s-c_s)^2}{4q_s}$, which is a decreasing function in w. Hence, if the NB manufacturer (referred as NBM hereafter) charges a wholesale price $w\in[0,\frac{c_s q_n}{q_s}]$ and a reasonable fixed payment that is lower than the threshold value $\bar{T}^I(w)$, it is more profitable for the retailer to not introduce the SB and carry NB only; otherwise, if the fixed payment is higher than the threshold, it would be best for the retailer to only carry SB. Given any $w\in(\frac{c_s q_n}{q_s}, c_s+q_n-q_s)$, we first compare $\hat{\Pi}_r^{I*}$ with $\hat{\Pi}_r^{II*}$. We have proved that $\hat{\Pi}_r^{I*}$ and $\hat{\Pi}_r^{II*}$ are continuous at $w=\frac{c_s q_n}{q_s}$. It is straightforward to show that $\hat{\Pi}_r^{I*}$ with $\hat{\Pi}_r^{II*}$ are both monotonically decreasing in w, and $\frac{d\hat{\Pi}_r^{II*}(w)}{dw}<\frac{d\hat{\Pi}_r^{I*}(w)}{dw}$. Therefore, given any $w\in(\frac{c_s q_n}{q_s}, c_s+q_n-q_s)$, it is more profitable for the retailer to carry both products than carry NB only. To compare $\hat{\Pi}_r^{II*}$ with $\hat{\Pi}_r^{III*}$, we again recognize a threshold transfer $\bar{T}^{II}=\frac{(c_s-w+q_n-q_s)^2}{4(q_n-q_s)}$ such that $\hat{\Pi}_r^{II*}|_{T=\bar{T}^{II}}=\Pi_r^{III*}$. Note that $\bar{T}^{II}$ is decreasing in w and $\bar{T}^{II}|_{w=c_s+q_n-q_s}=0$. Furthermore, $\bar{T}^I$ and $\bar{T}^{II}$ are continuous at $w=\frac{c_s q_n}{q_s}$. Thus, for any $w\in(\frac{c_s q_n}{q_s}, c_s+q_n-q_s)$, if $0\le T\le\bar{T}^{II}(w)$, $\hat{\Pi}_r^*=\hat{\Pi}_r^{II*}$; otherwise, $\hat{\Pi}_r^*=\hat{\Pi}_r^{III*}$. When $w>c_s+q_n-q_s$, we can directly claim that for any $T\ge 0$, it is never profitable for the retailer to carry NB, i.e., $\hat{\Pi}_r^*=\hat{\Pi}_r^{III*}$. □

Proof. **Proposition 1. Case I**: Based on Lemma 1(ii), the occurrence of case I implies that NBM offers a (w,T) such that $w\le\frac{c_s q_n}{q_s}$ and $T\le\bar{T}^I(w)$, following which the $Q_n=Q_n^{I*}$. Since Π_n^I is increasing in T, we have $T^*=\bar{T}^I(w)$. The retailer consequently receives the reservation profit Π_r^{III*}. Substituting p_n^{I*}, Q_n^{I*} and T^* into Π_n^I, we get $\frac{d\Pi_n^I(w)}{dw}=\frac{c_n-\ w}{2q_n}$ and $\frac{d^2\Pi_n^I(w)}{dw^2}=-\frac{1}{2q_n}<0$. We then have the interior optimum $w^*=c_n$. If $c_n\le\frac{c_s q_n}{q_s}$, then $w^*\le\frac{c_s q_n}{q_s}$ is indeed satisfied. As a result, the NBM's optimal profit under case I becomes $\Pi_n^{I*}=\Pi_n^I|_{w=c_n}=\frac{(q_n-c_n)^2}{4q_n}-\frac{(q_s-c_s)^2}{4q_s}$. Otherwise, if $c_n>\frac{c_s q_n}{q_s}$, since the profit function is concave and decreasing on w for any $w \le \frac{c_s q_n}{q_s}$, the optimal solution exists at $w^* = \frac{c_s q_n}{q_s}$. Then We have $\Pi_n^{I*} = \Pi_n^I|_{w=\frac{c_s q_n}{q_s}} = \frac{2(c_s q_n - c_n q_s)(q_s - c_s) + (q_n - q_s)(q_s - c_s)^2}{4q_s^2}$.

Case II: Following Lemma 1(iii) , the occurrence of case II suggests that $\frac{c_s q_n}{q_s} < w < c_s + q_n - q_s$ and $T \le \bar{T}^{II}(w)$. Again, NBM will set the transfer T such that $T^* = \bar{T}^{II}(w)$ and the retailer only earns his reservation profit Π_r^{III*}.

We obtain $w^* = c_n$ by solving $\frac{d\Pi_n^{II}(w)}{dw} = 0$, noting $\frac{d^2\Pi_n^{II}(w)}{dw^2} < 0$. If $c_n \le \frac{c_s q_n}{q_s}$, Π_n^{II} is quasi-concave and decreasing in w for any $w \in (\frac{c_s q_n}{q_s}, c_s + q_n - q_s]$. w^* then has to be set as $\frac{c_s q_n}{q_s}$. Thus $\Pi_n^{II*} = \Pi_n^{II}|_{w=\frac{c_s q_n}{q_s}} = \frac{2(c_s q_n - c_n q_s)(q_s - c_s) + (q_n - q_s)(q_s - c_s)^2}{4q_s^2} = \Pi_n^I|_{w=\frac{c_s q_n}{q_s}}$. Otherwise, i.e., $\frac{c_s q_n}{q_s} < c_n \le c_s + q_n - q_s$, $w^* = c_n$ is indeed the optimal solution. Then we have $\Pi_n^{II*} = \Pi_n^{II}|_{w=c_n} = \frac{(q_n - c_n - q_s + c_s)^2}{4(q_n - q_s)}$. If $c_n > c_s + q_n - q_s$, w^* then equals to $c_s + q_n - q_s$. Thus, $\Pi_n^{II*} = \Pi_n^{II}|_{w=c_s+q_n-q_s} = 0$.

To search for the NBM's global optimal profit Π_n^*, we compare her profit functions under three cases across three intervals of c_n as follows.

(i) If $c_n \le \frac{c_s q_n}{q_s}$, then $\Pi_n^{I*} = \Pi_n^I|_{w^*=c_n}$ and $\Pi_n^{II*} = \Pi_n^{II}|_{w^*=\frac{c_s q_n}{q_s}}$. Thus, we have $\Pi_n^{I*} - \Pi_n^{II*} = \frac{(c_s q_n - c_n q_s)^2}{4q_n q_s^2} \ge 0$, and $\Pi_n^{I*} = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s)^2}{4q_s} \ge 0 = \Pi_n^{III}$. Therefore, if $c_n \le \frac{c_s q_n}{q_s}$, it is best for the NBM to offer a $(w, T) = (c_n, \bar{T}^I(c_n))$ such that the retailer will carry NB only.

(ii) If $\frac{c_s q_n}{q_s} < c_n < c_s + q_n - q_s$, $\Pi_n^{I*} = \Pi_n^I|_{w^{I*}=\frac{c_s q_n}{q_s}}$, $\Pi_n^{II*} = \Pi_n^{II}|_{w^{II*}=c_n}$. Again, it can be shown that $\Pi_n^{II*} - \Pi_n^{I*} = \frac{(c_s q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} > 0$ and $\Pi_n^{II*} = \frac{(q_n - c_n - q_s + c_s)^2}{4(q_n - q_s)} \ge 0 = \Pi_n^{III}$. Hence, if $\frac{c_s q_n}{q_s} < c_n \le c_s + q_n - q_s$, NBM's optimal strategy is to offer $(w, T) = (c_n, \bar{T}^{II}(c_n))$ such that the retailer will agree to carry both NB and SB.

(iii) If $c_n \ge c_s + q_n - q_s$, we have $\Pi_n^{I*} = \Pi_n^I|_{w^{I*}=\frac{c_s q_n}{q_s}} < \Pi_n^{II*} = \Pi_n^{II}|_{w^{II*}=c_s+q_n-q_s} = 0$. Hence, the NBM will not be profitable from selling her NB product to the retailer, and the retailer will carry SB only. □

Proof. **Corollary 1.** The results follow directly from differentiation on $\hat{\Pi}_n^*$ ($\hat{\Pi}_r^*$) with respect to q_n, c_s, c_n, and q_s, respectively. □

Proof. **Lemma 2.** Lemma 1 implies that if $w \le c_s^i + q_n - q_s$ and $T \le \bar{T}^i(w), \forall i = L, H$, then such (w, T) satisfies $\Pi_r(w, T, c_s^i) \ge \Pi_r^{III}(c_s^i)$. Next, we want to show that for any $w \in [0, c_s^L + q_n - q_s]$, $\Delta(w) \overset{def}{=} \bar{T}^H(w) - \bar{T}^L(w) \ge 0$. The function of $\Delta(w)$ varies depending on the range of w.

For any $0 \le w \le \frac{c_s^L q_n}{q_s}$, we also have $w < \frac{c_s^H q_n}{q_s}$ due to $c_s^L \le c_s^H$. Then $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IL}(w) = \frac{(q_s - c_s^L)^2 - (q_s - c_s^H)^2}{4q_s} \ge 0$.

For any $\frac{c_s^L q_n}{q_s} < w \le c_s^L + q_n - q_s$, we consider the following two cases:

(1) If $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$, then we have $\frac{c_s^L q_n}{q_s} < w \le c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s} < c_s^H + q_n - q_s$ due to $q_n > q_s > c_s^H$. Therefore, $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IIL}(w) = \frac{(q_n - w)^2}{4q_n} - \frac{(q_s - c_s^H)^2}{4q_s} - \frac{(c_s^L - w + q_n - q_s)^2}{4(q_n - q_s)}$. Since $\frac{\partial \Delta}{\partial w} = \frac{c_s^L q_n - q_s w}{2q_n^2 - 2q_n q_s} < 0, \forall w > \frac{c_s^L q_n}{q_s}$ and $\Delta|_{w=c_s^L+q_n-q_s} = \frac{(q_s - c_s^L)^2}{4q_n} - \frac{(q_s - c_s^H)^2}{4q_s} > 0$, we have $\Delta(w) > 0, \forall w \in (\frac{c_s^L q_n}{q_s}, c_s^L + q_n - q_s]$.

(2) If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, then we have $\frac{c_s^L q_n}{q_s} < \frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s < c_s^H + q_n - q_s$. Given any $\frac{c_s^L q_n}{q_s} < w \le \frac{c_s^H q_n}{q_s}$, we have $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IIL}(w)$ and $\frac{\partial \Delta}{\partial w} < 0, \forall w > \frac{c_s^L q_n}{q_s}$. In conjunction with $\Delta|_{w=\frac{c_s^H q_n}{q_s}} = \frac{(q_s - c_s^H)^2(q_n - q_s)^2 - (c_s^L q_s + q_n q_s - q_s^2 - c_s^H q_n)^2}{4(q_n - q_s)q_s^2} > 0$, we have $\Delta(w) > 0, \forall w \in (\frac{c_s^L q_n}{q_s}, \frac{c_s^H q_n}{q_s}]$. Moreover, for any $w \in$

$(\frac{c_s^H q_n}{q_s}, c_s^L + q_n - q_s]$, we again have $\Delta(w) = \bar{T}^{IIH}(w) - \bar{T}^{IIL}(w) = \frac{(c_s^H - c_s^L)(c_s^H + c_s^L + 2q_n - 2q_s - 2w)}{4(q_n - q_s)} \geq 0$ due to $c_s^L \leq c_s^H$.

In summary, we have shown that $\bar{T}^H(w) \geq \bar{T}^L(w), \forall w \in [0, c_s^L + q_n - q_s]$. Therefore, for any (w^L, T^L) that satisfies (IRL), it follows that $w^L \leq c_s^L + q_n - q_s < c_s^H + q_n - q_s$ and $T^L \leq \bar{T}^L(w^L) < \bar{T}^H(w^L)$, which implies that such (w^L, T^L) satisfies $\Pi_r(w^L, T^L, c_s^H) \geq \Pi_r^{III}(c_s^H)$ as well. Together with (ICH), we have $\Pi_r(w^H, T^H, c_s^H) \overset{(ICH)}{\geq} \Pi_r(w^L, T^L, c_s^H) \geq \Pi_r^{III}(c_s^H)$. □

Proof. **Lemma 3.** The proof is embedded in the paragraph right above Lemma 3 in the main body, thus it is omitted here. □

Proof. **Lemma 4.** To prove the result here is equivalent to show that under the (H^{II}, L^I) contract, (ICH), (ICL), (IRL), and $T^H \geq 0$ cannot be simultaneously satisfied. By offering a (H^{II}, L^I) contract, the NBM guides the high-type retailer to carry both brands, and the low-type retailer to carry NB only. In addition, it implies that (w^H, T^H) and $(w^L,\ T^L)$ must satisfy $\frac{c_s^H q_n}{q_s} < w^H \leq c_s^H + q_n - q_s, T^H \leq \bar{T}^{IIH}(w^H), w^L \leq \frac{c_s^L q_n}{q_s}$, and $T^L \leq \bar{T}^{IL}(w^L)$. Lemma 1 implies that if a high-type retailer lies about his type by choosing the L^I contract, he will carry NB only, i.e., $\Pi_r(w^L, T^L, c_s^H) \equiv \Pi_r^I(w^L, T^L, c_s^H)$. Hence, (ICH) can be equivalently written as $\Pi_r^{II}(w^H,\ T^H,\ c_s^H) \geq \Pi_r^I(w^L, T^L, c_s^H)$. Together with (IRL), we have $T^H \leq \frac{q_n - w^H}{2}[\frac{1}{2} - \frac{w^H - c_s^H}{2(q_n - q_s)}] + \frac{q_s - c_s^H}{2}[\frac{w^H - c_s^H}{2(q_n - q_s)} - \frac{c_s^H}{2q_s}] - \frac{(q_s - c_s^L)^2}{4q_s} \overset{def}{=} \hat{T}^H(w^H)$, which is a decreasing function in w^H.

If $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$, then $T^H \leq \hat{T}^H(\frac{c_s^H q_n}{q_s}) = \frac{q_n (q_s - c_s^H)^2 - (q_s - c_s^L)^2 q_s}{4q_s^2} < 0$, which contradicts with $T^H \geq 0$. If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$ and $w^H \leq c_s^L + q_n - q_s$, then Lemma 1 implies that if the low-type retailer falsely chooses H^I contract, he will carry both brands, i.e., $\Pi_r(w^H, T^H, c_s^L) \equiv \Pi_r^{II}(w^H, T^H, c_s^L)$. In order to satisfy (ICL), i.e., $\Pi_r^I(w^L, T^L, c_s^L) \geq \Pi_r^{II}(w^H, T^H, c_s^L)$, T^H has to satisfy $T^H \geq \frac{q_n - w^H}{2}[\frac{1}{2} - \frac{w^H - c_s^L}{2(q_n - q_s)}] + \frac{q_s - c_s^L}{2}[\frac{w^H - c_s^L}{2(q_n - q_s)} - \frac{c_s^L}{2q_s}] - \frac{(q_s - c_s^L)^2}{4q_s} > \hat{T}^H$, which contradicts with $T^H \leq \hat{T}^H$. In this scenario, a (H^{II}, L^I) contract cannot simultaneously satisfy (ICH), (IRL) and (ICL), thus it is infeasible. If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$ and $w^H > c_s^L + q_n - q_s$, then $T^H \leq \hat{T}^H(w^H) < \hat{T}^H(c_s^L + q_n - q_s) = \frac{(c_s^H - c_s^L)(c_s^H q_n + c_s^L q_n - 2(c_s^L + q_n - q_s) q_s)}{4(q_n - q_s) q_s} \leq 0$, which again contradicts with $T^H \geq 0$. □

Proof. **Lemma 5.** Let Π_n^{SD*} and Π_n^{NSD*} represent NBM's optimal profits from contracts with and without shutdown, respectively. Recall from Proposition 1 that Π_n^{SD*} is equal to $v\bar{T}^{IH}(c_n)$ if $c_n \leq \frac{c_s^H q_n}{q_s}$, or $\Pi_n^{SD*} = v\bar{T}^{IIH}(c_n)$ if $\frac{c_s^H q_n}{q_s} < c_n \leq c_s^H + q_n - q_s$.

(i) If $\tilde{c}_n < \frac{c_s^L q_n}{q_s}$, the optimal contract without shutdown is (H^I, L^{II}) with $(w^{H*}, w^{L*}) = (c_n, c_s^L + q_n - q_s)$. Under this contract, Π_n^{NSD*} is strictly less than Π_n^{SD*}. Now suppose $\tilde{c}_n > \frac{c_s^L q_n}{q_s}$. Let $\delta \overset{def}{=} \Pi_n^{SD*} - \Pi_n^{NSD*}$. We can show that δ is increasing in c_n, $\forall c_n \in (0, q_n)$. Therefore, there exists a unique $\bar{c}_n$ such that $\Pi_n^{SD*}(\bar{c}_n) = \Pi_n^{NSD*}(\bar{c}_n)$. For any $c_n < \bar{c}_n$, $\Pi_n^{SD*}(c_n) < \Pi_n^{NSD*}(c_n)$, and for any $c_n \geq \bar{c}_n$, $\Pi_n^{SD*}(c_n) \geq \Pi_n^{NSD*}(c_n)$.

(ii) Similar to above, we can prove that δ is decreasing in c_s^L. Suppose c_s^L increases from $\hat{c}_s^L$ to $\check{c}_s^L$. Following (i), there are unique $\hat{c}_n$ and $\check{c}_n$ such that $\delta(c_n = \hat{c}_n | c_s^L = \hat{c}_s^L) = 0$, and $\delta(c_n = \check{c}_n | c_s^L = \check{c}_s^L) = 0$. Due to $\hat{c}_s^L \leq \check{c}_s^L$, $\delta(c_n = \check{c}_n | c_s^L = \hat{c}_s^L) \geq \delta(c_n = \check{c}_n | c_s^L = \check{c}_s^L) = 0 = \delta(c_n = \hat{c}_n | c_s^L = \hat{c}_s^L)$. Since δ is increasing in c_n, we have $\check{c}_n \geq \hat{c}_n$. Likewise, we can also show that $\bar{c}_n$ is decreasing in c_s^H. □

Proof. **Proposition 2.** (i) Suppose $\frac{c_s^L q_n}{q_s} \le c_n < \bar{c}_n$. Then, NBM will offer a contract without shutdown to the retailer. By substituting T^H from Table 1 into Equation (5), we can obtain detailed forms for Π_n, defined as below:

$$\Pi_n(w^H, w^L) = \begin{cases} \Pi_n^{(1)}(w^H, w^L), \text{ if offering}(H^I, L^{II}) \ \& \ w^L \le \frac{c_s^H q_n}{q_s}, \\ \Pi_n^{(2)}(w^H, w^L), \text{ if offering}(H^I, L^{II}) \ \& \ \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s, \\ \Pi_n^{(3)}(w^H, w^L), \text{ if offering}(H^{II}, L^{II}) \ \& \ w^L \le \frac{c_s^H q_n}{q_s}, \\ \Pi_n^{(4)}(w^H, w^L), \text{ if offering}(H^{II}, L^{II}) \ \& \ \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s. \end{cases}$$

Differentiation of $\Pi_n(w^H, w^L)$ with respect to w^H and w^L yields:

$$\frac{\partial \Pi_n^{(1)}}{\partial w^H} = \frac{\partial \Pi_n^{(2)}}{\partial w^H} = \frac{v(c_n - w^H)}{2q_n}, \ \frac{\partial^2 \Pi_n^{(1)}}{\partial {w^H}^2} = \frac{\partial^2 \Pi_n^{(2)}}{\partial {w^H}^2} = -\frac{v}{2q_n} < 0,$$

$$\frac{\partial \Pi_n^{(3)}}{\partial w^H} = \frac{\partial \Pi_n^{(4)}}{\partial w^H} = \frac{v(c_n - w^H)}{2(q_n - q_s)}, \ \frac{\partial^2 \Pi_n^{(3)}}{\partial {w^H}^2} = \frac{\partial^2 \Pi_n^{(4)}}{\partial {w^H}^2} = -\frac{v}{2(q_n - q_s)} < 0,$$

$$\frac{\partial \Pi_n^{(1)}}{\partial w^L} = \frac{\partial \Pi_n^{(3)}}{\partial w^L} = \frac{q_n(c_n - vc_n - vc_s^L) - w^L(q_n - vq_n - vq_s)}{2q_n(q_n - q_s)}, \ \frac{\partial^2 \Pi_n^{(1)}}{\partial {w^L}^2} = \frac{\partial^2 \Pi_n^{(3)}}{\partial {w^L}^2} = -\frac{q_n - vq_n - vq_s}{2q_n(q_n - q_s)},$$

$$\frac{\partial \Pi_n^{(2)}}{\partial w^L} = \frac{\partial \Pi_n^{(4)}}{\partial w^L} = \frac{(1-v)c_n + v(c_s^H - c_s^L) - (1-v)w^L}{2(q_n - q_s)}, \ \frac{\partial^2 \Pi_n^{(2)}}{\partial {w^L}^2} = \frac{\partial^2 \Pi_n^{(4)}}{\partial {w^L}^2} = -\frac{1-v}{2(q_n - q_s)} < 0.$$

Hence, Π_n is concave in w^H regardless of its contract forms. $\Pi_n^{(2)}$ and $\Pi_n^{(4)}$ are concave in w^L and the interior optimal w^{L*} is $c_n + \frac{v(c_s^H - c_s^L)}{1-v}$. Whether $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are concave in w^L depends on the sign of $q_n - vq_n - vq_s$. If $q_n - vq_n - vq_s \ge 0$, then $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are concave in w^L, and the interior optimal w^{L*} is $\frac{q_n(c_n - vc_n - vc_s^L)}{q_n - vq_n - vq_s}$.

We compare the NBM's expected profits from (H^I, L^{II}) and (H^{II}, L^{II}) under the following three parameter settings: (1) $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$ and $q_n - vq_n - vq_s \ge 0$, (2) $\frac{c_s^H q_n}{q_s} \le c_s^L + q_n - q_s$ and $q_n - vq_n - vq_s \ge 0$, (3) $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$ and $q_n - vq_n - vq_s < 0$. Note that the condition of $\frac{c_s^L q_n}{q_s} \le c_n < \bar{c}_n$ cannot be met if $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$ and $q_n - vq_n - vq_s < 0$, thus this parameter setting is not discussed here.

Parameter setting 1: $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$ **and** $q_n - vq_n - vq_s \ge 0$

Under this parameter setting, a feasible w^L automatically satisfies $w^L \le \frac{c_s^H q_n}{q_s}$, regardless of the form of contract offered. Furthermore, $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are concave in w^L. We also have $\bar{c}_n < c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$. Given $\frac{c_s^L q_n}{q_s} \le c_n < \bar{c}_n$, the respective optimal (w^{H*}, w^{L*}) that maximizes $\Pi_n^{(1)}(w^H, w^L)$ and $\Pi_n^{(3)}(w^H, w^L)$ are $(c_n, \frac{q_n(c_n - vc_n - vc_s^L)}{q_n - vq_n - vq_s})$ and $(\frac{c_s^H q_n}{q_s}, \frac{q_n(c_n - vc_n - vc_s^L)}{q_n - vq_n - vq_s})$. $\Pi_n^{(1)*} - \Pi_n^{(3)*} = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_n q_s^2} \ge 0$. Thus, (H^I, L^{II}) contract is optimal in this case.

Parameter setting 2: $\frac{c_s^H q_n}{q_s} \le c_s^L + q_n - q_s$ **and** $q_n - vq_n - vq_s \ge 0$

Under this parameter setting, $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are concave in w^L. In order to distinguish the feasible boundary for w^H and w^L according to Lemma 1, we further recognize two cases within this parameter setting: (i) $\frac{c_s^H q_n}{q_s} \le \tilde{c}_n$, (ii) $\tilde{c}_n < \frac{c_s^H q_n}{q_s}$. We present the optimal (w^H, w^L) for each contract form under these two cases in the Tables B1 and B2 below.

Substituting (w^{H*}, w^{L*}) into the corresponding Π_n functions and then comparing all $\Pi_n^{(i)}, i \in (1,2,3,4)$, under each c_n interval, we obtain the following results: if $\frac{c_s^L q_n}{q_s} < c_n \le c_n^{(1)}$ (where $c_n^{(1)} = \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$), offering a $(H^I, L^{II}: w^L \le \frac{c_s^H q_n}{q_s})$ contract is most profitable for NBM; if $c_n^{(1)} < c_n \le \frac{c_s^H q_n}{q_s}$, it is best for the NBM to offer a $(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$ contract; if $\frac{c_s^H q_n}{q_s} < c_n \le \tilde{c}_n$, then a $(H^{II}, L^{II}: \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$ contract is optimal for the NBM.

c_n interval:		$\frac{c_s^L q_n}{q_s} < c_n \le c_n^{(1)}$	$c_n^{(1)} < c_n \le \frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s} < c_n < \tilde{c}_n$
$(H^I, L^{II}: w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	c_n	c_n	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	c_n	c_n	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$
$(H^{II}, L^{II}: w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n
	w^{L*}	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^{II}, L^{II}: \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$

Table B1 Parameter Setting 2: (i) $\frac{c_s^H q_n}{q_s} < \tilde{c}_n$

(Note: The global optimal (w^H, w^L) under each c_n interval is highlighted in gray; the same style is used in Table B2 and Table B3.)

c_n interval:		$\frac{c_s^L q_n}{q_s} < c_n \le c_n^{(1)}$	$c_n^{(1)} < c_n \le \tilde{c}_n$
$(H^I, L^{II}: w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	c_n	c_n
	w^{L*}	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	c_n	c_n
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$
$(H^{II}, L^{II}: w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^{II}, L^{II}: \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$

Table B2 Parameter Setting 2: (ii) $\tilde{c}_n < \frac{c_s^H q_n}{q_s}$

Parameter setting 3: $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$ **and** $q_n - vq_n - vq_s < 0$

When $q_n - vq_n - vq_s < 0$, $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are convex in w^L, thus the maximum exists at the boundary of w^L. Given $c_n > \frac{c_s^L q_n}{q_s}$, $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are both increasing in w^L. Consequently, both $\Pi_n^{(1)}$ and $\Pi_n^{(3)}$ are maximized at $w^L = \frac{c_s^H q_n}{q_s}$. In addition, both $\Pi_n^{(2)}$ and $\Pi_n^{(4)}$ are maximized at their respective interior optimal $w^L = c_n + \frac{v(c_s^H - c_s^L)}{1-v}$ under the condition $\frac{c_s^L q_n}{q_s} \le c_n < \bar{c}_n$. We list the optimal (w^H, w^L) for each contract form under two cases in Table B3.

Two cases:		if $\tilde{c}_n < \frac{c_s^H q_n}{q_s}$	if $\frac{c_s^H q_n}{q_s} < \tilde{c}_n$	
c_n interval:		$\frac{c_s^L q_n}{q_s} < c_n \le \tilde{c}_n$	$\frac{c_s^L q_n}{q_s} < c_n \le \frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s} < c_n < \tilde{c}_n$
$(H^I, L^{II} : w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	c_n	c_n	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	c_n	c_n	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$
$(H^{II}, L^{II} : w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n
	w^{L*}	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$

Table B3 Parameter Setting 3

We again substitute (w^{H*}, w^{L*}) into the respective $\Pi_n^{(i)}, i \in (1,2,3,4)$. The comparison among all $\Pi_n^{(i)*}$ under each c_n interval reveals the following result: if $\frac{c_s^L q_n}{q_s} < c_n \le \frac{c_s^H q_n}{q_s}$, it is most profitable for the NBM to offer a $(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$ contract; otherwise, i.e., $\frac{c_s^H q_n}{q_s} < c_n \le \tilde{c}_n$, then it is best for NBM to offer a $(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$ contract.

Summarizing the results from all three parameter settings above, we obtain that if $\frac{c_s^L q_n}{q_s} \le c_n < \bar{c}_n$, (H^*, L^*) is given by

$$(H^*, L^*) = \begin{cases} (H^I, L^{II}), & \text{if } c_n < \frac{c_s^H q_n}{q_s} \\ (H^{II}, L^{II}), & \text{if } c_n \ge \frac{c_s^H q_n}{q_s}. \end{cases}$$

(ii) Remark that if $c_n \ge \bar{c}_n$, then NBM prefers to offer a contract with shutdown, i.e., offering the first-best contract to H-type retailer and offering nothing to the L-type retailer. According to Proposition 1, the form of the H-type contract depends on the value of c_n: if $c_n < \frac{c_s^H q_n}{q_s}$, then a H^I contract will be offered; otherwise, a H^{II} contract will be offered. □

Proof. **Corollary 2.** The proof is embedded in the proof of Proposition 2 and main body of the paper, thus omitted here. □

Proof. **Proposition 3.** If $\frac{c_s^L q_n}{q_s} \le c_n < \bar{c}_n$, then a contract without shutdown is offered.

(i) Suppose the parameter q_n increases to $\hat{q}_n$. By substituting T^H in Table 1 into equation (5) and after some algebra, we can show that for any given contracts without shutdown, Π_n increases in q_n. Therefore, for any given (w^H, w^L), we show $\Pi_n(w^H, w^L|q_n) \le \Pi_n(w^H, w^L|\hat{q}_n)$ for any $q_n \le \hat{q}_n$. Let (w^{H*}, w^{L*}) and $(\hat{w}^{H*}, \hat{w}^{L*})$ be the optimal contracts of (w^H, w^L) for given q_n and $\hat{q}_n$, respectively. Since $(\hat{w}^{H*}, \hat{w}^{L*})$ is the optimal solution for NBM with parameter $\hat{q}_n$, $\Pi_n(w^{H*}, w^{L*}|\hat{q}_n) \le \Pi_n(\hat{w}^{H*}, \hat{w}^{L*}|\hat{q}_n)$. Hence, we show $\Pi_n(w^{H*}, w^{L*}|q_n) \le \Pi_n(\hat{w}^{H*}, \hat{w}^{L*}|\hat{q}_n)$.

Similarly, we can prove that the NBM's expected optimal profit Π_n^* decreases in q_s.

(ii) The L-type retailer's equilibrium profit is his reservation profit $\Pi_r^{III*}(c_s^L)$, which is increasing in q_s and independent from q_n.

(iii) When a contract without shutdown is offered, direct differentiation of Π_r^{H*} w.r.t. q_n yields the following results: if a $(H^I, L^{II} : w^L \leq \frac{c_s^H q_n}{q_s})$ contract is optimal, then $\frac{d\Pi_r^{H*}}{dq_n} = \frac{v(1-v)^2(c_n q_s - c_s^L q_n)\{c_n[(2q_n^2 - q_n q_s)(1-v) - vq_s^2] - c_s^L q_n(q_n + vq_n - 3vq_s)\}}{4(q_n-q_s)^2(q_n(1-v)-vq_s)^3}$; otherwise, i.e., any other contract form in Tables 5, 6, or 7 is optimal, then $\frac{d\Pi_r^{H*}}{dq_n} = \frac{v(c_s^H - c_s^L)(2c_n(1-v) - c_s^H + 3vc_s^H - c_s^L - vc_s^L)}{4(1-v)(q_n-q_s)^2}$. If a contract with shutdown is offered, then Π_r^{H*} is irrelevant to q_n. After some algebra, we can show that $\frac{d\Pi_r^{H*}}{dq_n} \geq 0$ under all possible parameter settings in Proposition 2.

In addition, we obtain numerical evidence that Π_r^{H*} may increase or decrease in q_s. Suppose $c_n = 0.493$, $q_n = 1.155$, $c_s^H = 0.346$, $c_s^L = 0.248$, $v = 0.594$. When q_s increases from 0.582 to 0.61 to 0.627, Π_r^H first increases from 0.0479 to 0.0511 then decreases to 0.0493.

(iv) It follows from part (ii) and (iii) that Π_r^* is increasing in q_n. Moreover, our numerical experiment shows that Π_r^* may increase or decrease in q_s. Suppose $c_n = 0.55$, $q_n = 1.125$, $c_s^H = 0.35$, $c_s^L = 0.26$, $v = 0.63$. When q_s increases from 0.532 to 0.55 to 0.559, Π_r first increases from 0.0348 to 0.0367 then decreases to 0.0358. □

Proof. **Proposition 4.** The proof for (i) and (ii) are similar to the proof of Proposition 3 (i)-(ii), thus it is omitted here.

(iii) If $\frac{c_s^L q_n}{q_s} \leq c_n < \bar{c}_n$ (i.e., a contract without shutdown is offered), direct differentiation of Π_r^{H*} w.r.t. c_n yields the following results: if a $(H^I, L^{II} : w^L \leq \frac{c_s^H q_n}{q_s})$ contract is optimal, then $\frac{d\Pi_r^{H*}}{dc_n} = \frac{-v(1-v)^2 q_n(c_n q_s - c_s^L q_n)}{2(q_n-q_s)(q_n(1-v)-vq_s)^2} \leq 0$; otherwise, i.e., any other contract form is optimal, then $\frac{d\Pi_r^{H*}}{dc_n} = \frac{-v(c_s^H - c_s^L)}{2(q_n-q_s)} \leq 0$. If a $(H^I, L^{II} : w^L \leq \frac{c_s^H q_n}{q_s})$ contract is offered, then Π_r^{H*} is irrelevant to c_s^H. Otherwise, i.e., any other without-shutdown contract form is optimal, $\frac{d\Pi_r^{H*}}{dc_s^H} = \frac{-v\left(2vq_s(c_s^H - c_s^L) + (1-v)(c_n q_s - c_s^H q_n)\right)}{2q_s(q_n-q_s)(1-v)}$, which can be shown to be negative with some algebra.

Furthermore, our numerical experiment below illustrates that Π_r^{H*} may increase or decrease in c_s^L. Suppose $c_n = 19.54$, $q_n = 59.6$, $c_s^H = 11.27$, $q_s = 19.07$, $v = 0.7$. When c_s^L increases from 4.8 to 5.4 to 6.1, Π_r^{H*} first increases from 2.037 to 2.232 then decreases to 2.198.

(iv) It follows from part (ii) and (iii) that Π_r^* is decreasing in c_n and c_s^H. Furthermore, our numerical experiment show that Π_r^* may increase or decrease in c_s^L. Use the same example above in (iii), when c_s^L increases from 4.8 to 5.4 to 6.1, Π_r^* first increases from 2.2264 to 2.2972 then decreases to 2.2004. □

Proof. **Proposition 5.** (i) If a $(H^I, L^{II} : w^L \leq \frac{c_s^H q_n}{q_s})$ contract is optimal, then $\frac{dV_n}{dc_s^H} = \frac{v(q_s - c_s^H)}{2q_s} \geq 0$. Otherwise, if any other without-shutdown contract form is offered, then with some algebra, we have $\frac{dV_n}{dc_s^H} = \frac{v\left((1-v)(c_s^H + q_n - q_s - c_n) - v(c_s^H - c_s^L)\right)}{2(1-v)(q_n-q_s)} \geq 0$. If a contract with shutdown is offered, then V_n becomes irrelevant to c_s^H. Thus, V_n is increasing (non-decreasing) in c_s^H overall.

Similarly, if a $(H^I, L^{II} : w^L \leq \frac{c_s^H q_n}{q_s})$ contract is optimal, then
$\frac{dV_n}{dc_s^L} = \frac{v[c_n q_n(1-v) - c_s^L(q_n - vq_s) - (q_n-q_s)((1-v)q_n - vq_s)]}{2(q_n-q_s)((1-v)q_n - vq_s)} < 0$. Otherwise, if any other without-shutdown contract form is offered, we have $\frac{dV_n}{dc_s^L} = \frac{v[v(c_s^H + q_n - q_s - c_n) + c_n - c_s^L - q_n + q_s]}{2(1-v)(q_n-q_s)} < 0$. Otherwise, if $\bar{c}_n < c_n \leq c_s^L + q_n - q_s$, $\frac{dV_n}{dc_s^L} = \frac{(1-v)(c_s^L + q_n - q_s - c_n)}{2(q_n-q_s)} \geq 0$; if $c_n > c_s^L + q_n - q_s$, then V_n becomes 0 and is irrelevant to c_s^L. Thus, V_n may increase or decrease in c_s^L.

(ii) $|V_r^L| = 0$, thus is independent from any parameter.

(iii) Our numerical experiments below demonstrate that $|V_r^H|$ may increase or decrease in c_s^H and c_s^L, respectively.

Example for c_s^H: Suppose $c_n = 14.49$, $q_n = 75.5$, $c_s^L = 3.64$, $q_s = 19.78$, $v = 0.81$. When c_s^H increases from 8 to 9.3 to 10.7, $|V_r^H|$ first increases from 0.9020 to 0.9697 then decreases to 0.9392.

Example for c_s^L: Suppose $c_n = 14.49$, $q_n = 75.5$, $c_s^H = 10.46$, $q_s = 19.78$, $v = 0.81$. When c_s^L increases from 1 to 2.4 to 3.5, $|V_r^H|$ first increases from 1.5650 to 1.7217 then decreases to 1.6028.

(iv) Since $|V_r| = |V_r^H|$, $|V_r|$ shares the same properties as those of $|V_r^H|$. □

Proof. **Proposition 6.** If a $(H^I, L^{II} : w^L \leq \frac{c_s^H q_n}{q_s})$ contract is optimal, then CS is irrelevant to c_s^H. If $(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \leq c_s^L + q_n - q_s)$ contract is optimal, then we have $\frac{dCS}{dc_s^H} = \frac{v[v(c_s^H + q_n - q_s - c_n) - (c_s^L + q_n - q_s - c_n)]}{4(1-v)(q_n - q_s)} \leq 0$. If a $(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \leq c_s^L + q_n - q_s)$ contract is offered, then $\frac{dCS}{dc_s^H} = \frac{v[c_s^H q_n - q_s(c_s^L + q_n - q_s) + v(q_s - c_s^H)(q_n - q_s)]}{4q_s(1-v)(q_n - q_s)} \leq 0$. Otherwise, if a contract with shutdown is offered, then CS behaves like the case under symmetric information, thus it decreases in c_s^H.

The proof for CS decreasing in c_n is similar to the proof above, thus it is omitted here.

Our numerical examples below show that CS may increase or decrease in q_s and c_s^L, respectively.

Example for q_s: Suppose $c_n = 2.84$, $q_n = 5.34$, $c_s^L = 1.21$, $c_s^H = 2.04$, $v = 0.252$. When q_s increases from 2.28 to 2.65 to 3.017, CS first decreases from 0.1462 to 0.1434 then increases to 0.1491.

Example for c_s^L: Suppose $c_n = 4.83$, $q_n = 11.55$, $c_s^H = 2.29$, $q_s = 4.78$, $v = 0.424$. When c_s^L increases from 0.66 to 1.33 to 1.99, CS first decreases from 0.4838 to 0.4564 then increases to 0.4879. □

Proof. **Proposition 7.** (i) We first solve the retailer's pricing problem. That is, given any NB retail price $\tilde{p}_n$ set by the NB manufacturer, the retailer decides whether to introduce SB products and the retail price $\tilde{p}_s$ for SB products. Note that given $\tilde{p}_n$ and $\tilde{p}_s$, the demands for the NB and SB products $(\tilde{Q}_n, \tilde{Q}_s)$ are determined according to equation (1). We analyze the retailer's local optimal decisions under Case I, II, III.

Case I: If $\tilde{p}_n \leq \frac{c_s q_n}{q_s}$, any $\tilde{p}_s$ higher than c_s satisfies the condition $\frac{\tilde{p}_s}{q_s} \geq \frac{\tilde{p}_n - \tilde{p}_s}{q_n - q_s}$, following which $\tilde{Q}_s = 0$ and $\tilde{\Pi}_r = 0$.

Case II: If, given any $\tilde{p}_n$, the retailer decides to sell his SB product such that there are two products in the market, the retailer's profit function is written as $\tilde{\Pi}_r^{II}(\tilde{p}_s) = (\tilde{p}_s - c_s)\left(\frac{\tilde{p}_n - \tilde{p}_s}{q_n - q_s} - \frac{\tilde{p}_s}{q_s}\right)$, which is concave in $\tilde{p}_s$. If $\frac{c_s q_n}{q_s} < \tilde{p}_n < \frac{c_s q_n + 2q_n^2 - 2q_n q_s}{2q_n - q_s}$, $\tilde{p}_s^*$ is the interior optimal solution $\frac{c_s q_n + \tilde{p}_n q_s}{2q_n}$, following which $\tilde{\Pi}_r^{II*}|_{\tilde{p}_s = \frac{c_s q_n + \tilde{p}_n q_s}{2q_n}} = \frac{(\tilde{p}_n q_s - c_s q_n)^2}{4q_n q_s (q_n - q_s)}$; if $\tilde{p}_n \geq \frac{c_s q_n}{q_s}$, then the interior optimal decision does not satisfy the condition $\frac{\tilde{p}_n - \tilde{p}_s}{q_n - q_s} > \frac{\tilde{p}_s}{q_s}$ such that $\tilde{p}_s^*$ has to take the boundary value $\frac{\tilde{p}_n q_s}{q_n}$, following which $\tilde{Q}_s = 0$ and $\tilde{\Pi}_r^{II*}|\tilde{p}_s = \frac{\tilde{p}_n q_s}{q_n} = 0$; if $\tilde{p}_n \geq \frac{c_s q_n + 2q_n^2 - 2q_n q_s}{2q_n - q_s}$, then the interior optimal decision does not satisfy the condition $\frac{\tilde{p}_n - \tilde{p}_s}{q_n - q_s} < 1$ such that the $\tilde{p}_s^*$ has to take the boundary value $\tilde{p}_n - q_n + q_s$ following which $\tilde{Q}_n = 0$ and $\tilde{\Pi}_r^{II*}|_{\tilde{p}_s = \tilde{p}_n - q_n + q_s} = \frac{(q_n - \tilde{p}_n)(\tilde{p}_n - c_s - q_n + q_s)}{q_s}$.

Case III: Given any $\tilde{p}_n$, if the retailer decides a retail price $\tilde{p}_s$ such that the demand of the NB product reduces to 0, the retailer's profit function becomes $\tilde{\Pi}_r^{III}(\tilde{p}_s) = (\tilde{p}_s - c_s)(1 - \frac{\tilde{p}_s}{q_s})$. If $\tilde{p}_n > \frac{c_s + 2q_n - q_s}{2}$, then $\tilde{p}_s^*$ is the interior optimal solution $\frac{c_s + q_s}{2}$ and $\tilde{\Pi}_r^{III*}|_{\tilde{p}_s = \frac{c_s + q_s}{2}} = \frac{(q_s - c_s)^2}{4q_s}$; if $\tilde{p}_n \leq \frac{c_s + 2q_n - q_s}{2}$, the interior optimal decision does not satisfy the condition $\frac{\tilde{p}_n-\tilde{p}_s}{q_n-q_s}>1$ such that $\tilde{p}_s^*$ has to take the boundary value $\tilde{p}_n-q_n+q_s$ following which $\tilde{\Pi}_r^{III*}|_{\tilde{p}_n-q_n+q_s}=\frac{(q_n-\tilde{p}_n)(\tilde{p}_n-c_s-q_n+q_s)}{q_s}$.

The retailer's best response to any given $\tilde{p}_n$ is summarized as follows. If $\tilde{p}_n\le\frac{c_sq_n}{q_s}$, then the retailer does not introduce SB; if $\frac{c_sq_n}{q_s}<\tilde{p}_n<\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$, the retailer sells SB product such that both NB and SB products have positive demands, and the optimal retail price for SB is $\tilde{p}_s^*=\frac{c_sq_n+\tilde{p}_nq_s}{2q_n}$; if $\tilde{p}_n\ge\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$, the SB product fully takes over the market and $\tilde{Q}_n=0$. Moreover, if $\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}\le\tilde{p}_n\le\frac{c_s+2q_n-q_s}{2}$, the retailer will set $\tilde{p}_s^*=\tilde{p}_n-q_n+q_s*$; if $\tilde{p}_n>\frac{c_s+2q_n-q_s}{2}$, the retailer will set $\tilde{p}_s^*=\frac{c_s+q_s}{2}$.

Based on the retailer best responses, we now proceed to analyze the NBM's optimal pricing decision. Under case I, $\tilde{p}_n$ should satisfy $\tilde{p}_n\le\frac{c_sq_n}{q_s}$ and the NBM's profit function writes as $\tilde{\Pi}_n(\tilde{p}_n)=(\tilde{p}_n-c_n)(1-\frac{\tilde{p}_n}{q_n})$, a concave function in $\tilde{p}_n$. Under case II, $\tilde{p}_n$ should satisfy $\frac{c_sq_n}{q_s}<\tilde{p}_n<\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$, and the NBM's profit function becomes $\tilde{\Pi}_n(\tilde{p}_n)=(\tilde{p}_n-c_n)(1-\frac{\tilde{p}_n-\tilde{p}_s}{q_n-q_s})$, which is concave in $\tilde{p}_n$. For any $\tilde{p}_n\ge\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$, $\tilde{\Pi}_n=0$. We derive the NBM's local optimal decisions by taking derivatives and checking the conditions of $\tilde{p}_n$, then compare the local optima across all possible parameter settings to find the global optimum. The NBM's optimal price and resulting demands for NB and SB are listed in the table below.

	$c_n<\frac{2c_sq_n-q_nq_s}{q_s}$	$\frac{2c_sq_n-q_nq_s}{q_s}\le c_n<\tilde{c}_n^{(1)}$	$\tilde{c}_n^{(1)}<c_n\le\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$	$c_n>\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$
$c_s<\frac{2q_nq_s-2q_s^2}{4q_n-3q_s}$	$\tilde{p}_n^*=\frac{q_n(c_s+2q_n-2q_s)+c_n(2q_n-q_s)}{4q_n-2q_s}$ $\tilde{Q}_n>0,\tilde{Q}_s>0$			$\tilde{p}_n^*=0$ $\tilde{Q}_n=0,\tilde{Q}_s>0$
$\frac{2q_nq_s-2q_s^2}{4q_n-3q_s}<c_s<\frac{q_s}{2}$	$\tilde{p}_n^*=\frac{c_sq_n}{q_s}$ $\tilde{Q}_n>0,\tilde{Q}_s=0$		$\tilde{p}_n^*=\frac{q_n(c_s+2q_n-2q_s)+c_n(2q_n-q_s)}{4q_n-2q_s}$ $\tilde{Q}_n>0,\tilde{Q}_s>0$	$\tilde{p}_n^*=0$ $\tilde{Q}_n=0,\tilde{Q}_s>0$
$\frac{q_s}{2}<c_s<q_s$	$\tilde{p}_n^*=\frac{c_n+q_n}{2}$ $\tilde{Q}_n>0,\tilde{Q}_s=0$	$\tilde{p}_n^*=\frac{c_sq_n}{q_s}$ $\tilde{Q}_n>0,\tilde{Q}_s=0$	$\tilde{p}_n^*=\frac{q_n(c_s+2q_n-2q_s)+c_n(2q_n-q_s)}{4q_n-2q_s}$ $\tilde{Q}_n>0,\tilde{Q}_s>0$	$\tilde{p}_n^*=0$ $\tilde{Q}_n=0,\tilde{Q}_s>0$

Table B4 NB manufacturer's optimal price and resulting market segmentation under the competition case (Note: $\tilde{c}_n^{(1)}\overset{def}{=}\frac{4c_sq_n^2-3c_sq_nq_s-2q_n^2q_s+2q_nq_s^2}{2q_nq_s-q_s^2}$)

In Table B4, $(\tilde{Q}_n>0,\tilde{Q}_s=0)$ denotes that $\tilde{\Pi}_n^*=\tilde{\Pi}_n^{I*}$ and $\tilde{\Pi}_r^*=0$; $(\tilde{Q}_n>0,\tilde{Q}_s>0)$ denotes that $\tilde{\Pi}_n^*=\tilde{\Pi}_n^{II*}$ and $\tilde{\Pi}_r^*=\tilde{\Pi}_r^{II*}$; $(\tilde{Q}_n=0,\tilde{Q}_s>0)$ denotes that $\tilde{\Pi}_n^*=0$ and $\tilde{\Pi}_r^*=\tilde{\Pi}_r^{III*}$. Note that $\tilde{c}_n^{(1)}<\frac{c_sq_n}{q_s}$ and $c_s+q_n-q_s<\frac{c_sq_n+2q_n^2-2q_nq_s}{2q_n-q_s}$. Recall from Proposition 1 that under the contracting case, SB enters the market if $c_n\ge\frac{c_sq_n}{q_s}$. Table B4 indicates that under the competition case, the threshold of introducing SB is lower than that under the contracting case. Therefore, we can conclude that it is more likely for the retailer to introduce SB under the competition case than under the contracting case.

(ii) We can easily obtain the equilibrium profits of the NBM and the retailer by substituting $(\tilde{p}_n^*,\tilde{p}_s^*)$ into the corresponding $\tilde{\Pi}_n$ and $\tilde{\Pi}_r$ functions. Under the contracting case, if $c_n<\frac{c_sq_n}{q_s},\hat{\Pi}_n^*=\hat{\Pi}_n^{I*}$; if

$\frac{c_s q_n}{q_s} \le c_n \le c_s + q_n - q_s, \hat{\Pi}_n^* = \hat{\Pi}_n^{I*}$; if $c_n > c_s + q_n - q_s, \hat{\Pi}_n^* = 0$. By comparing $\tilde{\Pi}_n^*$ with $\hat{\Pi}_n^*$ under all possible parameter settings, we find that if $c_n \le \frac{c_s q_n + 2q_n^2 - 2q_n q_s}{2q_n - q_s}$, $\tilde{\Pi}_n^* > \hat{\Pi}_n^*$; otherwise, $\tilde{\Pi}_n^* = \hat{\Pi}_n^* = 0$.

(iii) Similar to (ii), we compare $\tilde{\Pi}_r^*$ with $\hat{\Pi}_r^*$ under all possible parameter settings. The comparison result indicates that if $c_n \le \frac{c_s q_n + 2q_n^2 - 2q_n q_s}{2q_n - q_s}$, $\tilde{\Pi}_r^* < \hat{\Pi}_r^*$; otherwise, $\tilde{\Pi}_r^* = \hat{\Pi}_r^* = \hat{\Pi}_r^{III*}$.

(iv) Similar to (ii) and (iii), we compare the channel profit under the competition case (i.e., $\tilde{\Pi}_n^* + \tilde{\Pi}_r^*$) with the channel profit under the contracting case (i.e., $\hat{\Pi}_n^* + \hat{\Pi}_r^*$). We obtain that if $c_n \le \frac{c_s q_n + 2q_n^2 - 2q_n q_s}{2q_n - q_s}$, $\tilde{\Pi}_n^* + \tilde{\Pi}_r^* < \hat{\Pi}_n^* + \hat{\Pi}_r^*$; otherwise, $\tilde{\Pi}_n^* + \tilde{\Pi}_r^* = \hat{\Pi}_n^* + \hat{\Pi}_r^*$.