Online Appendix for "Structural Estimation of Attrition in a Last-Mile Delivery Platform: The Role of Driver Heterogeneity, Compensation, and Experience"

Lina Wang, Scott Webster, Elliot Rabinovich

Appendix A Preliminary Evidence

A.1 Semi-Parametric Survival Model Estimation

Table A.1 Cox Proportional Hazard Model Results

	(1)		(2)	
	Estimate	(Std. Err.)	Estimate	(Std. Err.)
Regular pay (week/$100)	-0.331***	(0.033)	-0.336***	(0.034)
Subsidy pay (week/$100)	-0.136***	(0.052)	-0.134***	(0.052)
Hours /10	0.072	(0.086)	0.082	(0.087)
Miles per stop	0.034***	(0.011)	0.034***	(0.011)
Age			-0.007	(0.006)
LL	-2467.163		-2466.450	
obs	15293		15293	
Number of drivers	396		396	

Notes: (1) Standard errors are in parentheses; (2) *, ** and *** denote statistical significance at 10 percent, 5 percent and 1 percent levels, respectively; (3) To avoid numerical overflows caused by large values, regular pay and subsidy pay are scaled down by a factor of 1/100 and hours is scaled down by a factor of 1/10; (4) Metro area and month fixed effects are controlled for.

A.2 Subsidy Pay and Earning Rates from Regular Pay During Tenure

Figure A.1 Decrease in weekly subsidy pay

(a) Means **(b)** Medians

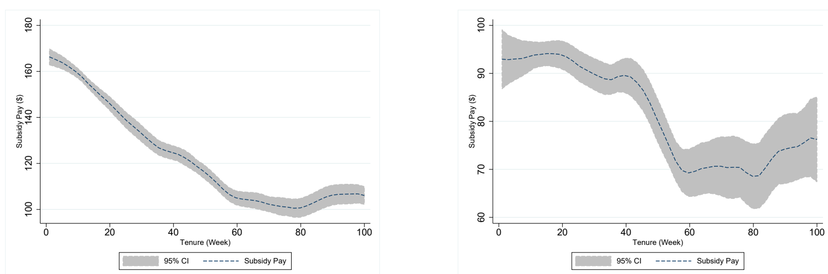
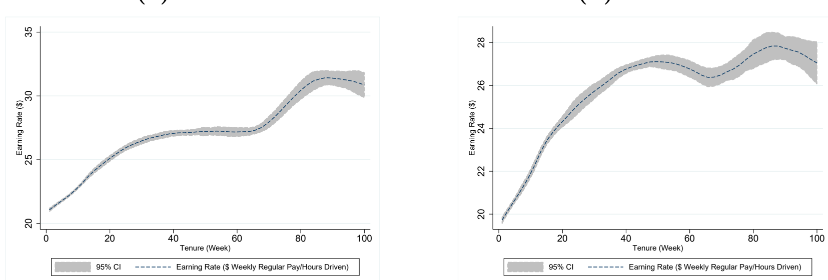


Figure A.2 Increase in earning rates from regular pay

(a) Means **(b)** Medians



A.3 Variability of Earning Rates From Regular Compensation Over Tenure, Across Metro Areas, and During Different Seasons

To evaluate the impact of tenure on the variability in earning rates from weekly regular payments, we adopt a two-stage estimation procedure (see Bavafa and Jónasson 2021). We first estimate the variability in earning rates as the deviation from the earning rates conditional on a set of covariates. We then take this deviation as a dependent variable and evaluate how it changes with tenure conditional on the same set of covariates used in the first stage. In particular, the two-stage procedure uses the following two regressions:

$$\frac{W_{ijt}}{H_{ijt}} = \sigma_{WH0} + \sigma_{WH1}T_{ijt} + \sigma_{WH2}T_{ijt}^2 + \sigma_{WH3}A_{ijt} + \sigma_{WH4}A_{ijt}^2 + \xi_j + \eta_t + u_{ijt}, \tag{A.1}$$

$$\ln(\hat{u}_{ijt}^2) = \sigma_{u0} + \sigma_{u1}T_{ijt} + \sigma_{u2}T_{ijt}^2 + \sigma_{u3}A_{ijt} + \sigma_{u4}A_{ijt}^2 + \xi_j + \eta_t + \epsilon_{ijt}. \quad \text{(A.2)}$$

In Eq. (A.1), $\frac{W_{ijt}}{H_{ijt}}$ denotes the earning rates ($/hour driven) as a function of the ratio between regular pay (W) and number of hours driven (H) for driver i in metro area j and week t. Earning rates are explained by a vector of variables that include tenure T_{ijt}, squared tenure T_{ijt}^2 (to capture the diminishing returns to experience), and age A_{ijt} and age squared A_{ijt}^2 of drivers to control for any effects age might have on earning rates. We also control for metro area fixed effects ξ_j and month fixed effects η_t (Orlando and January are used as the effects' baseline). The residual u_{ijt} denotes the deviation from the fitted earning rates. Eq. (A.2) includes the transformed residual, $\ln(\hat{u}_{ijt}^2)$ as a dependent variable and the same set of explanatory variables as in Eq. (A.1).

Columns (1) and (2) in Table A.2 present the estimation results based on the specifications in eqs. (A.1) and (A.2), respectively. The results show that average earning rates increase with drivers' tenure. The signs and magnitudes of the parameters for tenure and squared tenure in Column (1) indicate that the average learning curve is steep ($\sigma_{WH1} = 1.393$, $p < 0.01$) with diminishing returns as drivers gain more experience ($\sigma_{WH2} = -0.049$, $p < 0.01$). Moreover, as shown in Column (2), the variability in earning rates across drivers increases with drivers' tenure ($\sigma_{u1} = 0.041$, $p < 0.01$) but this effect marginally diminishes in tenure ($\sigma_{u2} = -0.003$, $p < 0.01$).

Table A.2 **Estimation results of experience on mean and variance of earning rates**

	(1) Average Earning Rates		(2) Variance Earning Rates	
Tenure (week/10)	1.393***	(0.308)	0.041***	(0.011)
Tenure (week/10)^2	-0.049***	(0.014)	-0.003***	(0.001)
Age	-0.016	(0.401)	0.019	(0.013)
Age^2	-0.000	(0.005)	0.000	(0.000)
R-squared	0.119		0.031	
Observations	15,293		15,293	

Notes: (1) Standard errors are in parentheses; (2) *, ** and *** denote statistical significance at 10 percent, 5 percent and 1 percent levels, respectively; (3) Estimates for metro area and month fixed effects are not reported.

Furthermore, we find that earning rates vary by metro areas and exhibit seasonality patterns. Figures A.3 and A.4 depict the estimated coefficients of the metro areas and months, respectively based on the specifications used to estimate Column (1) in Table A.2. Figure A.3 shows that drivers in Chicago have lower earning rates whereas drivers in Minneapolis—Saint Paul have higher earning rates, compared to drivers in Orlando as the baseline metro area. As to the seasonality patterns, Figure A.4 shows that earning rates among drivers are the highest in the second quarter and the lowest in the last quarter of the year.

Figure A.3 **Effect of branch on average earning rates**

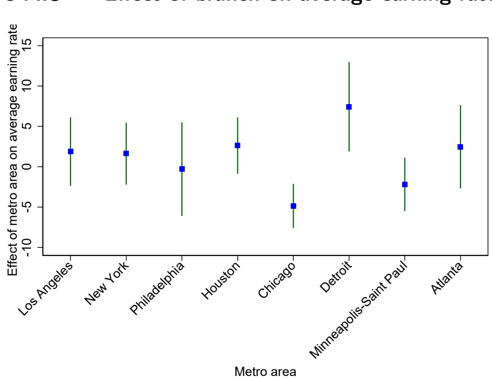
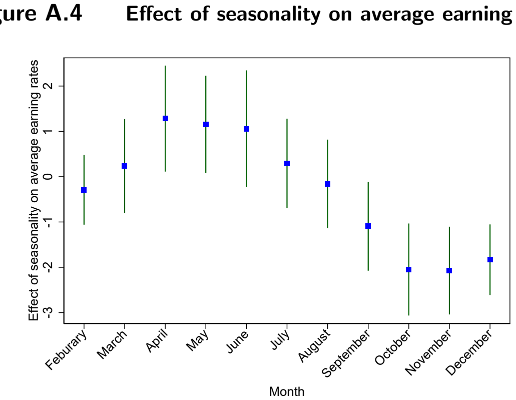


Figure A.4 **Effect of seasonality on average earning rates**



A.4 Change in Compensation and Driver Attrition

Table A.3 Logistics Regression Model Results with Quitting as Dependent Variable

	Estimate	(Std. Err.)
Percentage change in regular pay	-0.009*	(0.002)
Percentage change in subsidy pay	0.0003	(0.002)
Regular pay (week/$100)	-0.161***	(0.033)
Subsidy pay (week/$100)	-0.246***	(0.061)
Hours /10	-0.154	(0.087)
Miles per stop	0.047***	(0.014)
Tenure/10	-0.111***	(0.021)
obs	13,950	
LL	-1158.750	

Notes: (1) Standard errors are in parentheses; (2) *, ** and *** denote statistical significance at 10 percent, 5 percent and 1 percent levels, respectively; (3) To avoid numerical overflows caused by large values, regular pay and subsidy pay are scaled down by a factor of 1/100 and hours and tenure are scaled down by a factor of 1/10; (4) Observations with a percentage of compensation change greater than the 95th percentile are excluded from the analysis.

Appendix B Derivation of Conditional Choice Probabilities Estimator

Recall that we defined the conditional value function as $v_k(X_{ijt}) \equiv V_k(X_{ijt}) - \varepsilon_{ijkt}$. Then, we can rewrite Eq. (6) in Section 4 as

$$\begin{aligned}
V_1(X_{ijt}) &= u_1(X_{ijt}) + \varepsilon_{ij1t} + \max_{d_{t+1},\dots,d_T} E\left[\sum_{s=t+1}^{T} \beta^{s-t} \sum_{k=0}^{1} \mathbf{1}(d_s = k)(u_k(\tilde{X}_{ijs}) + \tilde{\varepsilon}_{ijks})\right] \\
&= u_1(X_{ijt}) + \varepsilon_{ij1t} + \beta E\left[\begin{array}{c} p_0(\tilde{X}_{ijt+1})\left[v_0(\tilde{X}_{ijt+1}) + E\left[\tilde{\varepsilon}_{ij0t+1}|d^*_{it+1}=0\right]\right] \\ +p_1(\tilde{X}_{ijt+1})\left[v_1(\tilde{X}_{ijt+1}) + E\left[\tilde{\varepsilon}_{ij1t+1}|d^*_{it+1}=1\right]\right] \end{array}\right] \\
&= u_1(X_{ijt}) + \varepsilon_{ij1t} + \beta E\left[\begin{array}{c} v_0(\tilde{X}_{ijt+1}) + p_0(\tilde{X}_{ijt+1}) E\left[\tilde{\varepsilon}_{ij0t+1}|d^*_{it+1}=0\right] \\ +p_1(\tilde{X}_{ijt+1})\left[v_1(\tilde{X}_{ijt+1}) - v_0(\tilde{X}_{ijt+1}) + E\left[\tilde{\varepsilon}_{ij1t+1}|d^*_{it+1}=1\right]\right] \end{array}\right] \\
&= u_1(X_{ijt}) + \varepsilon_{ij1t} + \beta E\left[\begin{array}{c} v_0(\tilde{X}_{ijt+1}) + p_0(\tilde{X}_{ijt+1}) E\left[\tilde{\varepsilon}_{ij0t+1}|d^*_{it+1}=0\right] \\ +p_1(\tilde{X}_{ijt+1})\left[\log\left(\frac{p_1(\tilde{X}_{ijt+1})}{p_0(\tilde{X}_{ijt+1})}\right) + E\left[\tilde{\varepsilon}_{ij1t+1}|d^*_{it+1}=1\right]\right] \end{array}\right].
\end{aligned} \tag{B.1}$$

The last expression uses the relationship shown in Eq. (10) in Section 4 that the difference between value functions $v_1 - v_0$ is equal to $\log(\frac{p_1}{p_0})$.

Next, we derive the conditional expectation of Gumbel random variables. Recall that $\tilde{\varepsilon}_{ijkt}$ are i.i.d. Gumbel random variables with location and scale parameters 0 and 1. The probability density and cumulative distribution functions of $\tilde{\varepsilon}_{ijkt}$ are $g(z) = e^{-z}e^{-e^{-z}}, z \in (-\infty, \infty)$ and $G(z) = e^{-e^{-z}}, z \in (-\infty, \infty)$. Therefore,

$$\begin{aligned}
E\left[\tilde{\varepsilon}_{ij1t+1}|X_{ijt+1}, d^*_{t+1}\right] &= \frac{1}{p_1}\int_{-\infty}^{\infty} \varepsilon_1 g(\varepsilon_1) G(\varepsilon_1 + v_1 - v_0) d\varepsilon_1 \\
&= \frac{1}{p_1}\int_{-\infty}^{\infty} \varepsilon_1 g(\varepsilon_1) G\left(\varepsilon_1 + \log\left(\frac{p_1}{1-p_1}\right)\right) d\varepsilon_1 \\
&= \frac{1}{p_1}\int_{-\infty}^{\infty} \varepsilon_1 e^{-\varepsilon_1} e^{-e^{-\varepsilon_1}} e^{-e^{-\left(\varepsilon_1 + \log\left(\frac{p_1}{1-p_1}\right)\right)}} d\varepsilon_1 \\
&= \frac{1}{p_1}\int_{-\infty}^{\infty} \varepsilon_1 e^{-\varepsilon_1} e^{-e^{-\varepsilon_1\left(1+e^{-\left(\log\left(\frac{p_1}{1-p_1}\right)\right)}\right)}} d\varepsilon_1
\end{aligned}$$

$$= \frac{1}{p_1}\int_{-\infty}^{\infty} \varepsilon_1 e^{-\varepsilon_1} e^{-e^{-\varepsilon_1\left(1+\left(\frac{p_1}{1-p_1}\right)\right)}} d\varepsilon_1$$
$$= \int_{-\infty}^{\infty} \varepsilon_1 \frac{e^{-\varepsilon_1}}{p_1} e^{-\frac{e^{-\varepsilon_1}}{p_1}} d\varepsilon_1.$$

Let $x = \frac{e^{-\varepsilon_1}}{p_1}$. Then $\varepsilon_1 = -\log(p_1 x)$, $dx = -\frac{e^{-\varepsilon_1}}{p_1} d\varepsilon_1$, $\varepsilon_1 = -\infty \Longrightarrow x = \infty$, $\varepsilon_1 = \infty \Longrightarrow x = 0$. Therefore, by change of variables

$$E\left[\tilde{\varepsilon}_{ij1t+1}|X_{ijt+1}, d^*_{t+1} = 1\right] = \int_{-\infty}^{0} \log(p_1 x) e^{-x} dx = -\int_{0}^{\infty} (\log(p_1) + \log(x)) e^{-x} dx = \gamma - \log p_1, \tag{B.2}$$

where γ is Euler constant (≈ 0.577), the mean of a standard Type I extreme value distribution (McFadden 1974).

Similarly,

$$\begin{aligned} E\left[\tilde{\varepsilon}_{ij0t+1}|\tilde{X}_{ijt+1}, d^*_{t+1} = 0\right] &= \frac{1}{p_0}\int_{-\infty}^{\infty} \varepsilon_0 g(\varepsilon_0) G(\varepsilon_0 + v_0 - v_1) d\varepsilon_0 \\ &= \frac{1}{p_1}\int_{-\infty}^{\infty} \varepsilon_0 g(\varepsilon_0) G\left(\varepsilon_0 + \log\left(\frac{p_0}{1-p_0}\right)\right) d\varepsilon_0 \\ &= \gamma - \log p_0. \end{aligned} \tag{B.3}$$

Note that, we can write $V_0(X_{ijt})$ introduced in Eq. (7) in Section 4 as

$$\begin{aligned} V_0(X_{ijt}) &= u_0(X_{ijt}) + \varepsilon_{ij0t} + \sum_{s=t+1}^{T} \beta^{s-t} E\left[u_0(X_{ijs}) + \tilde{\varepsilon}_{ij0s}\right] \\ &= \varepsilon_{ij0t} + E[\tilde{\varepsilon}_{ij0s}] \sum_{s=t+1}^{T} \beta^{s-t} \\ &= \varepsilon_{ij0t} + \gamma\left(\frac{\beta - \beta^{T-t+1}}{1-\beta}\right), \end{aligned}$$

and thus,

$$v_0(X_{ijt}) = V_0(X_{ijt}) - \varepsilon_{ij0t} = \gamma\left(\frac{\beta - \beta^{T-t+1}}{1-\beta}\right). \tag{B.4}$$

$$v_0(X_{ijt+1}) = \gamma\left(\frac{\beta - \beta^{T-t}}{1-\beta}\right). \tag{B.5}$$

(i.e., one less period of discounting in (B.4) compared to (B.5)).Next, we substitute (B.2), (B.3), (B.5) into (B.1) and integrate over possible states in week $t+1$.

$$\begin{aligned} V_1(X_{ijt}) &= u_1(X_{ijt}) + \varepsilon_{ij1t} + \beta E\left[\begin{matrix} v_0(\tilde{X}_{ijt+1}) + \left(1 - p_1(\tilde{X}_{ijt+1})\right)\left[\gamma - \log p_0\left(\tilde{X}_{ijt+1}\right)\right] \\ + p_1(\tilde{X}_{ijt+1})\left[\log\left(\frac{p_1(\tilde{X}_{ijt+1})}{p_0(\tilde{X}_{ijt+1})}\right) + \gamma - \log p_1(\tilde{X}_{ijt+1})\right] \end{matrix}\right] \\ &= u_1(X_{ijt}) + \varepsilon_{ij1t} + \beta E\left[\begin{matrix} v_0(\tilde{X}_{ijt+1}) + \gamma - \log p_0(\tilde{X}_{ijt+1}) \\ + p_1(\tilde{X}_{ijt+1})\left[\log\left(\frac{p_1(\tilde{X}_{ijt+1})}{p_0(\tilde{X}_{ijt+1})}\right) - \log\left(\frac{p_1(\tilde{X}_{ijt+1})}{p_0(\tilde{X}_{ijt+1})}\right)\right] \end{matrix}\right] \\ &= u_1(X_{ijt}) + \varepsilon_{ij1t} + \beta E\left[\gamma\left(\frac{\beta - \beta^{T-t}}{1-\beta}\right) + \gamma - \log(p_0(\tilde{X}_{ijt+1}))\right] \\ &= u_1(X_{ijt}) + \varepsilon_{ij1t} + \gamma\left(\frac{\beta - \beta^{T-t+1}}{1-\beta}\right) - \beta E[\log(p_0(\tilde{X}_{ijt+1}))] \end{aligned}$$

and thus

$$v_1(X_{ijt}) = V_1(X_{ijt}) - \varepsilon_{ij1t} = u_1(X_{ijt}) + \gamma\left(\frac{\beta - \beta^{T-t+1}}{1-\beta}\right) - \beta E[\log(p_0(\tilde{X}_{ijt+1}))]. \tag{B.6}$$

Finally, we take the difference between choice-specific conditional value function using Eq. (B.4) and Eq. (B.6) to obtain Eq. (10) as shown in Section 4

$$v_1(X_{ijt}) - v_0(X_{ijt}) = u_1(X_{ijt}) - \beta \int \log(p_0(x_{ijt+1}) f(X_{ijt+1}|X_{ijt}) dX_{ijt+1}. \tag{B.7}$$

Appendix C Details of First-Step Estimation

State transition probabilities follow an AR(1) process. The forecasting shocks to the one-period ahead state variables, denoted by $\varsigma_t^{(\cdot)}$, are assumed to be normally distributed and independent over time. The state variables include miles per stop and number of hours. When drivers forecast their future expected density of routes, D_{ijt}, they use the prior week's density of routes, $D_{ij,t-1}$, tenure T_{ijt}, metro area fixed effects ζ_j and month fixed effects ϕ_t. A similar specification is used for the forecast of hours, H_{ijt}.

$$D_{ijt} = \sigma_{D1} + \sigma_{D2} D_{ij,t-1} + \sigma_{D3} T_{ijt} + \zeta_j + \phi_t + \varsigma_t^D \tag{C.1}$$

$$H_{ijt} = \sigma_{H1} + \sigma_{H2} H_{ij,t-1} + \sigma_{H3} T_{ijt} + \zeta_j + \phi_t + \varsigma_t^H. \tag{C.2}$$

We assume that drivers forecast their regular pay $\log W_{ijt}$ based on the last week's $\log W_{ij,t-1}$, tenure T_{ijt}, squared tenure T^2_{ijt}, metro area fixed effects ζ_j and month fixed effects ϕ_t. Following a standard Mincerian wage equation (Mincer 1974), we use logged wage and introduce the squared term of tenure to capture the concave shape of the experience-productivity profile. The transition of subsidy pay ($\log I_{ijt}$) has a similar specification except for not having the squared term. The reason is that subsidy pay is expected to decrease in tenure monotonically.

$$\log W_{ijt} = \sigma_{W1} + \sigma_{W2} \log W_{ij,t-1} + \sigma_{W3} T_{ijt} + \sigma_{W4} T^2_{ijt} + \zeta_j + \phi_t + \varsigma_t^W \tag{C.3}$$

$$\log I_{ijt} = \sigma_{I1} + \sigma_{I2} \log I_{ij,t-1} + \sigma_{I3} T_{ijt} + \zeta_j + \phi_t + \varsigma_t^I. \tag{C.4}$$

Table C.1 shows the empirical results of the state transition probability functions using the specifications in eqs. (C.1) to (C.4). Table C.2 reports results from a flexible logit model to estimate the conditional choice probabilities of quitting in the first stage (see Section 5.1).

Table C.1 Transition probability

Parameter	Description	Estimate	(Std. Err.)
	Log regular pay		
Lag log regular pay	One week lagged of log regular pay	0.621***	(0.006)
Tenure (week/10)		0.020***	(0.005)
Tenure (week/10) ^{2}		-0.001***	(0.000)
Constant		2.445***	(0.055)
R^2			0.424
	Log subsidy pay		
Lag log subsidy pay	One week lagged of log subsidy pay	0.701***	(0.006)
Tenure (week/10)		-0.018***	(0.005)
Constant		1.299***	(0.055)
R^2			0.535
	Hours		
Lag hours (/10)	One week lagged of hours worked	0.593***	(0.007)
Tenure (week/10)		-0.006***	(0.002)
Constant		1.425 ***	(0.042)
R^2			0.436
	Mile per stop		
Lag mile per stop	One week lagged of mile per stop	0.701***	(0.006)
Tenure (week/10)		-0.001	(0.004)
Constant		0.775***	(0.065)
R^2			0.515

Notes: (1) Standard errors are in parentheses; (2) *, ** and *** denote statistical significance at 10 percent, 5 percent and 1 percent levels, respectively; (3) Metro area and month fixed effects are included for all transition equations and results are omitted here; (4) Hours and tenure are scaled down by a factor of 1/10.

Table C.2 Conditional choice probability

Variables	Estimate	(Std. Err.)
Constant	-2.296***	(0.389)
Regular pay	-0.516***	(0.100)
Regular pay^2	0.013***	(0.003)
Subsidy pay	-0.393**	(0.150)
Subsidy pay^2	0.040***	(0.011)
Hours	0.512*	(0.282)
Hours^2	-0.032	(0.068)
Miles per stop	0.274***	(0.055)
Miles per stop^2	-0.007***	(0.002)
Regular pay × Subsidy pay	0.032***	(0.010)
Regular pay × Hours	-0.010	(0.033)
Regular pay × Miles per stop	0.012**	(0.006)
Subsidy pay × Hours	-0.100*	(0.052)
Miles per stop × Hours	-0.088***	(0.025)
Miles per stop × Subsidy pay	-0.026	(0.018)
Tenure	-0.157**	(0.062)
Tenure^2	-0.001	(0.004)
Regular pay × Tenure	0.006	(0.008)
Subsidy pay × Tenure	0.035**	(0.014)
Hours× Tenure	0.001	(0.027)
Miles per stop × Tenure	-0.000	(0.005)
LL	-1194.702	

Notes: (1) Standard errors are in parentheses; (2) *, ** and *** denote statistical significance at 10 percent, 5 percent and 1 percent levels, respectively; (3) Metro area and month fixed effects are included for all transition equations and results are omitted here; (4) To avoid numerical overflows caused by large values, regular pay and subsidy pay are scaled down by a factor of 1/100 and hours and tenure are scaled down by a factor of 1/10; (5) Conditional choice probabilities are based on the probability of quitting (the terminal choice)

Appendix D Details of Second-Step Estimation

All the estimates of the state transition probability functions and conditional choice probability function are known at this point. Using these estimates, we can now simulate the value of the one-period ahead conditional choice probabilities. We present the estimation process first without considering unobserved heterogeneity and then while taking unobserved heterogeneity into account. *Step 1* Simulate the one period ahead state variables: $X_{ijt} \in (W_{ijt}, I_{ijt}, H_{ijt}, D_{ijt})$:

$$D_{ijt} = \sigma_{D1} + \sigma_{D2} D_{ij,t-1} + \sigma_{D3} T_{ijt} + \zeta_j + \phi_t + \varsigma_t^D$$

$$H_{ijt} = \sigma_{H1} + \sigma_{H2} H_{ij,t-1} + \sigma_{H3} T_{ijt} + \zeta_j + \phi_t + \varsigma_t^H$$

$$W_{ijt} = (W_{ij,t-1})^{\sigma_{W2}} \, e^{\sigma_{W1} + \sigma_{W3} T_{ijt} + \sigma_{W4} (T_{ijt})^2 + \zeta_j + \phi_t + \varsigma_t^W}$$

$$I_{ijt} = (I_{ij,t-1})^{\sigma_{I2}} \, e^{\sigma_{I1} + \sigma_{I3} T_{ijt} + \zeta_j + \phi_t + \varsigma_t^I},$$

where $(\tilde{\varsigma}_t^D, \tilde{\varsigma}_t^H, \tilde{\varsigma}_t^W, \tilde{\varsigma}_t^I)$ are the iid standard normal random variables reflecting the empirical distribution of their corresponding state variables.

Step 2 without unobserved heterogeneity: Calculate $\int \log\left[p_0\left(X_{ijt+1}\right)\right] f\left(X_{ijt+1} | X_{ijt}\right) dX_{ijt+1}$ by simulation similar to the approach used in Huang and Smith (2014), Murphy (2018), and Ransom (2022). We integrate the logged one-period ahead conditional choice probabilities over the empirical distribution of state variables using Monte Carlo methods:

$$\int \log p_0\left(\varsigma_{t+1}^D, \varsigma_{t+1}^H, \varsigma_{t+1}^W, \varsigma_{t+1}^I\right) dF\left(\varsigma^D, \varsigma^H, \varsigma^W, \varsigma^I\right) \approx \frac{1}{D} \sum_{d=1}^{D} \log p_0\left(\varsigma_d^D, \varsigma_d^H, \varsigma_d^W, \varsigma_d^I\right), \tag{D.1}$$

where $\left(\varsigma_{t+1}^D, \varsigma_{t+1}^H, \varsigma_{t+1}^W, \varsigma_{t+1}^I\right)$ are the vectors of shocks to the forecasting of their corresponding state in week $t+1$. We draw D draws from the standard normal distribution, plugging them into the conditional choice probabilities. We then take the average of the conditional choice probabilities value and multiply it by β. Next, we can take $p_0\left(X_{ijt+1}\right)$ as given and estimate the structural parameters using a simple logistic regression specification.

We now turn into the estimation that considers unobserved heterogeneity. In this case, Step 1 remains the same, but Step 2 is more involved which is discussed as follows:

Step 2 with unobserved heterogeneity: We let type s_i be a dummy variable which takes 0 $(s_i = r - 1 = 0)$ when a driver belongs to the first category, and 1 $(s_i = r - 1 = 1)$ when he belongs to the second category. Then, we let the initial guess of population probabilities be $\pi^1 = \{\pi_1^1, \pi_2^1\} = \{0.5, 0.5\}$ and the starting values of structural parameters $\theta = (\theta_1, \ldots, \theta_5)$ be the estimates obtained from the estimation without controlling for unobserved heterogeneity.

Step 3 Update the probability of a driver being a certain type q_{ir}:

$$q_{ir}^2 = \frac{\pi_r^1 \prod_{t=1}^{T} \mathcal{L}_{it}(d_{it} \mid X_{ijt}, \theta^1, \pi^1, s_i = r - 1)}{\sum_{r=1}^{2} \pi_r^1 \prod_{t=1}^{T} \mathcal{L}_{it}(d_{it} \mid X_{ijt}, \theta^1, \pi^1, s_i = r - 1)} \ \forall\ r. \tag{D.2}$$

Then, we average q_{ir} over the total number of drivers to update the population probabilities of types:

$$\pi_r^2 = \frac{\sum_{i=1}^{I} q_{ir}^2 (r | X_{ijt}, \theta^1, \pi^1)}{I} \ \forall\ r. \tag{D.3}$$

Step 4: Numerically update $\beta \int \log\left[p_0\left(X_{ijt+1}, s_i\right)\right] f\left(X_{ijt+1} | X_{ijt}\right) dX_{ijt+1}$ for each type r similar to the approach used in *Step 2 without unobserved heterogeneity*. Next, plug this value into the choice probability and update the structural parameters by maximizing:

$$\theta^2 = \arg\max \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{r=1}^{2} q_{ir}^2 \times log\left[\mathcal{L}_{it}(d_{it} \mid X_{ijt}, \theta^1, s_i = r-1)\right]. \tag{D.4}$$

Finally, use the new estimates of θ^2, go back to step 2 with unobserved heterogeneity and update the q_{ir} and continue to repeat steps 3-4 until the estimates of θ and π converge.

Appendix E Model Estimation Performance and Robustness Tests

Below, we evaluate the dynamic choice model's two-stage estimation performance by comparing the predicted value of quitting with its realized value observed in the data. To that end, we used the estimates of the structural parameters in Column (1) in Table 2 to simulate 1,000 times the predicted value of quitting. We then plotted the predicted quit hazard with the data-based quit hazard using an Epanechnikov kernel with a bandwidth of 20 weeks (see Figure E.1). Both the model-predicted and data-based quit hazard are decreasing in tenure. Although the hazard rates as a function of tenure are consistent across, the model predicts the quit hazard better in early stages of tenure than in later periods. This is likely driven by the smaller number of observations in the latter periods-most drivers do not stay at the platform for more than 50 weeks (as shown in Table 1, 254 out of 396 drivers quit within the first 50 weeks of their tenure).

We also present the results of a test we conducted to evaluate the robustness of our main results relative to the discount factor. For this test, we used two alternative values (0.90 and 0.80) to replace the discount factor of 0.9957 we used to generate originally these results. As shown in Table E.1, the signs and magnitudes of the parameter estimates remain largely consistent regardless of the value chosen for the discount factor.

Figure E.1 Model fit: quit hazard with Type

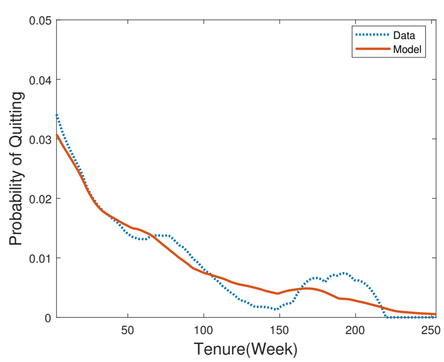


Table E.1 Structural estimation and alternate discount factors

	Discount factor=0.9 (1)		Discount factor=0.8 (2)	
	Estimate	(Std. Err.)	Estimate	(Std. Err.)
Constant	0.472*	(0.273)	0.647**	(0.245)
Miles per stop	-0.073***	(0.010)	-0.072***	(0.009)
Tenure (week/10)	0.033**	(0.016)	0.040**	(0.017)
Regular pay (week/$100)	0.126***	(0.023)	0.130***	(0.020)
Subsidy pay(week/$100)	0.108**	(0.023)	0.120**	(0.023)
Hours (/10)	0.026	(0.065)	0.056	(0.064)
Type	0.322*	(0.186)	0.379**	(0.188)
Type 1 (percent)	0.677		0.714	
Type 2 (percent)	0.323		0.286	
LL	-1230.599		-1231.007	
obs	15293		15293	
Number of drivers	396		396	

Next, we excluded non-statistically significant structural estimates in Table 2's Column (1) and re-estimated the model. As shown in Tables E.2 and E.3, the estimates obtained remain consistent with those in Tables 2 and 3. However, the evaluation of the alternative model's performance reveals that the model does not predict the value of quitting well, especially during the latter stages of tenure (Figure E.2). This underscores the importance of controlling for all state variables in predicting drivers' probability of quitting.

Another concern is that not all the estimates of conditional choice probabilities are statistically significant and using these estimates to simulate these probabilities may lead to overfitting. Conditional choice probabilities are inverted as a selection correction term in the second stage estimation to adjust for the fact

Table E.2 Structural estimation with Type and excluding tenure and hours as state variables

Variables	Estimate	(Std. Err.)
Constant	0.132	(0.354)
Regular pay (week/$100)	0.134***	(0.016)
Subsidy pay(week/$100)	0.112**	(0.046)
Miles per stop	-0.049***	(0.015)
Type	0.309**	(0.168)
Type 1	0.616	
Type 2	0.384	
LL	-1225.938	
obs	15293	
Number of drivers	396	

Table E.3 Summary statistics for the two types excluding tenure and hours as state variables

	Type 1	Type 2
Type	0.616	0.384
Regular pay (week)	807.688	832.602
Subsidy pay (week)	164.556	118.726
Hours	32.627	30.995
Miles per stop	3.463	3.292
Length of Tenure (week)	36.377	116.453

Figure E.2 Model fit of quit hazard excluding tenure and hours

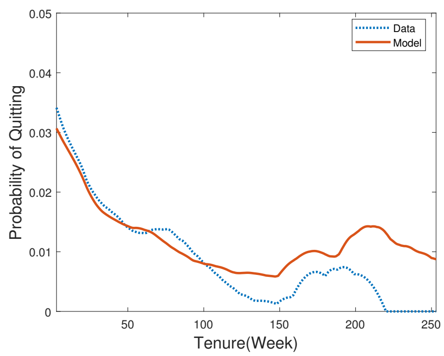


Table E.4 Structural estimation with Type and excluding non-significant estimates from simulated conditional choice probabilities

Variables	Estimate	(Std. Err.)
Constant	0.007	(0.365)
Hours (/10)	0.038	(0.079)
Miles per stop	-0.074***	(0.016)
Tenure (week/10)	0.014	(0.024)
Regular pay (week/$100)	0.124***	(0.029)
Subsidy pay(week/$100)	0.102**	(0.049)
Type	0.471***	(0.145)
Type 1	0.519	
Type 2	0.481	
LL	-1183.902	
obs	15293	
Number of drivers	396	

that choices made may not be optimal (Ellickson et al. 2012; De Groote and Verboven 2019). In the past, studies have predicted these probabilities by including all estimates, both significant and non-significant, obtained from flexible specifications (e.g., Arcidiacono and Miller 2011; Yoganarasimhan 2013; Chung et al. 2014). Although our analysis is consistent with this approach, we evaluated its robustness by excluding the non-significant parameters when simulating the conditional choice probabilities. Thus, we excluded all estimates of the flexible logit model that are statistically non-significant along with the non-significant estimates of metro area and month fixed effects. Table E.4 presents the results, consistent with those in Table 2.

We also performed a robustness check based exclusively on drivers who never experienced permanent changes in their route assignments. Table E.5 presents the results of the model estimation. These results are consistent with those in Table 2. In addition, we tested an alternative functional form in the utility equation by allowing for quadratic terms for regular and subsidy pay. The goal is to determine whether marginal effects for regular and subsidy pay vary across compensation levels. As shown in the results in Table E.6, the estimated coefficients for these quadratic terms are not statistically different from zero and the estimated coefficients for the other terms remain largely consistent with those in our original estimation. Therefore, we have no evidence to suggest that increases in regular or subsidy payments are marginally more/less effective for drivers with higher compensation levels. Finally, we performed a robustness check of our estimations by adding to the model specification in Column (1) of Table 2 a state variable corresponding to the average order weight delivered per week. The goal is to identify deviations in our results after considering the fact that time-consuming deliveries tend to involve heavier pieces. As Table E.7 shows, the results obtained from this alternative specification are consistent with those in Table 2.

Table E.5 Structural estimation results including only drivers with no permanent route variations

Variables	Estimate	(Std. Err.)
Constant	0.156	(0.301)
Hours (/10)	0.013	(0.076)
Miles per stop	-0.088***	(0.018)
Tenure (week/10)	0.027***	(0.013)
Regular pay (week/\$100)	0.134***	(0.023)
Subsidy pay(week/\$100)	0.100*	(0.053)
Type	0.310***	(0.095)
Type 1	0.597	
Type 2	0.403	
LL	-1059.787	
obs	13420	
Number of drivers	396	

Table E.6 Structural estimation results with squared terms

Variables	Estimate	(Std. Err.)
Constant	0.203	(0.202)
Hours (/10)	-0.133***	(0.063)
Miles per stop	-0.077***	(0.01)
Tenure (week/10)	0.02	(0.014)
Regular pay (week/\$100)	0.247***	(0.064)
Subsidy pay(week/\$100)	0.217***	(0.078)
Regular pay (week/\$100)^2	-0.006	(0.004)
Subsidy pay(week/\$100)^2	-0.018	(0.041)
Type	0.302**	(0.141)
Type 1	0.624	
Type 2	0.376	
LL	-1220.36	
obs	15293	
Number of drivers	396	

Table E.7 Structural estimation results including weight per piece

Variables	Estimate	(Std. Err.)
Constant	0.286**	0.194
Hours /10	0.013	0.061
Miles per stop	-0.069***	0.010
Tenure (week/10)	0.024***	0.009
Regular pay (week/\$100)	0.124***	0.022
Subsidy pay (week/\$100)	0.096***	0.022
Weight per piece /10	-0.210	0.209
Type	0.324*	0.179
Type 1 (percent)	0.597	
Type 2 (percent)	0.403	
LL	-1214.273	
obs	15293	
Number of drivers	396	

Appendix F Proofs of Propositions

Proof of Proposition 1. Let

$$q\left(X_{ijt}\right)=\frac{1}{\left(1+e^{v_0\left(X_{ijt}\right)-v_1\left(X_{ijt}\right)}\right)}.$$

Recall that

$$v_1\left(X_{ijt}\right)=u_1+\sum_{s=t+1}^{T}\beta^{s-t}E[v_1(X_{ijt+1})], \tag{F.1}$$

and that $v_1^{(1)}(=v_1-u_1)$ is defined as the conditional value function from period $t+1$ onward conditional a decision to continue in period t. Substituting it into (15) and implicitly differentiating with respect to x

$$\begin{aligned}\frac{\partial p_1\left(X_{ijt}\right)}{\partial x}=\frac{\partial q\left(X_{ijt}\right)}{\partial x}&=\frac{\partial q}{\partial(v_0-v_1)}\frac{\partial(v_0-v_1)}{\partial x}\\&=\frac{\partial q}{\partial(v_0-v_1)}\left(\frac{\partial v_0}{\partial x}-\frac{\partial v_1^{(1)}}{\partial x}-\frac{\partial u_1}{\partial x}\right).\end{aligned} \tag{F.2}$$

Note that

$$\frac{\partial q}{\partial(v_0-v_1)}=-\frac{e^{v_0-v_1}}{\left(1+e^{v_0-v_1}\right)^2}<0. \tag{F.3}$$

Recall that v_0 is the expected optimal utility in weeks t through T given a decision to quit in the current week t without the error term, i.e., $v_0(X_{ijt})=\gamma\left(\frac{\beta-\beta^{T-t}}{1-\beta}\right)$ (see (B.4)). Therefore,

$$\frac{dv_0}{dx}=0. \tag{F.4}$$

Substituting into the above to obtain part (i),

$$\frac{\partial p_1\left(X_{ijt}\right)}{\partial x}=\frac{e^{v_0\left(X_{ijt}\right)-v_1\left(X_{ijt}\right)}}{\left(1+e^{v_0\left(X_{ijt}\right)-v_1\left(X_{ijt}\right)}\right)^2}\left(\frac{\partial u_1}{\partial x}+\frac{\partial v_1^{(1)}}{\partial x}\right)=p_0\left(X_{ijt}\right)p_1\left(X_{ijt}\right)\left(\frac{\partial u_1}{\partial x}+\frac{\partial v_1^{(1)}}{\partial x}\right).$$

Suppose that $x=W_{ijt}$ so that the presentation of the following arguments is less abstract (the arguments similarly apply to other values of x). Recall that, φ_W is a function of W_{ijt}, tenure T_{ijt}, and the iid standard normal random variables $\tilde{\varsigma}^W$ capturing forecasting shocks to the one-period ahead state variable W_{ijt+1} (see Appendix C for details) . Since $u_0=0$, we concentrate on the change in u_1 as W_{ijt} increases. Suppose

$$\frac{\partial \varphi\left(X_{ijt}\right)}{\partial W_{ijt}}=\frac{\partial \varphi_{W}\left(W_{ijt},T_{ijt},\zeta_{t+1}^{w}\right)}{\partial W_{ijt}}\geq 0.$$

Therefore, any future state is increasing in the state variable W_{ijt} in period t, i.e.,

$$\frac{\partial W_{ijt+s}}{\partial W_{ijt}}\geq 0 \text{ for any } s\geq 1.$$

Then from

$$\frac{\partial u_{1}\left(X_{ijt+s}\right)}{\partial W_{ijt}}=\frac{\partial u_{1}\left(X_{ijt+s}\right)}{\partial W_{ijt+s}}\times\frac{\partial W_{ijt+s}}{\partial W_{ijt}}$$

it follows that $\frac{\partial u_{1}\left(X_{ijt+s}\right)}{\partial W_{ijt}}\geq 0$ if and only if $\frac{\partial u_{1}\left(X_{ijt+s}\right)}{\partial W_{ijt+s}}>0$.

Let us recap and summarize the implications of the above under the assumption $\frac{\partial \varphi}{\partial x}\geq 0$ for some state variable $x\in\{W_{ijt},I_{ijt},H_{ijt},D_{ijt}\}$. If $\frac{\partial u_1}{\partial x}<0$, then the deterministic utility u_1 *in any future state* is decreasing in state variable x. Thus, it is impossible for $v_1^{(1)}$ to be increasing in x, i.e., $\frac{\partial u_1}{\partial x}<0$ implies $\frac{\partial v_1^{(1)}}{\partial x}\leq 0$. Similarly, $\frac{\partial u_1}{\partial x}>0$ implies $\frac{\partial v_1^{(1)}}{\partial x}\geq 0$. Thus,

$$\frac{\partial p_1}{\partial x}=\frac{e^{v_0-v_1}}{\left(1+e^{v_0-v_1}\right)^2}\left(\frac{\partial u_1}{\partial x}+\frac{\partial v_1^{(1)}}{\partial x}\right)>0 \text{ if and only if } \frac{\partial u_1}{\partial x}>0. \quad \square$$

Proof of Proposition 2. Without loss of generality we set $i=1$ and $j=2$. From Proposition 2,

$$\frac{\partial p_1\left(X_t\right)}{\partial X_{it}}=\frac{e^{v_0(X_t)-v_1(X_t)}}{\left(1+e^{v_0(X_t)-v_1(X_t)}\right)^2}\left(\frac{\partial u_1}{\partial X_{it}}+\frac{\partial v_1^{(1)}}{\partial X_{it}}\right),\ i\ \in\{1,2\}, \tag{F.5}$$

we rewrite the linear state transition function as

$$\varphi_i\left(X_{it},t,\zeta_{it}\right)=\sigma_{0i}+\sigma_{1i}X_{it}+\sigma_{2i}t+\zeta_{it+1}. \tag{F.6}$$

By supposition,

$$\frac{\partial u_1}{\partial X_{1t}}=\theta_1>\theta_2=\frac{\partial u_1}{\partial X_{2t}}>0 \tag{F.7}$$

$$\frac{\partial \varphi_1}{\partial X_{1t}}=\sigma_{11}>\sigma_{12}=\frac{\partial \varphi_2}{\partial X_{2t}}. \tag{F.8}$$

We see from (F.5) and (F.7) that a proof of Proposition 3 is complete if we can show that

$$\frac{\partial u_1}{\partial X_{1t}}>\frac{\partial u_1}{\partial X_{2t}} \text{ implies } \frac{\partial v_1^{(1)}}{\partial X_{1t}}\geq\frac{\partial v_1^{(1)}}{\partial X_{2t}}. \tag{F.9}$$

The above holds for $t=T$ because $v_1^{(1)}\left(X_T\right)=v_0^{(1)}\left(X_T\right)=0$. We begin by showing that (F.9) holds for the case of $t=T-1$, i.e., there is one period remaining after the current period. We then show that the result continue to hold for $t<T-1$. For the remainder of the proof, we assume

$$\frac{\partial u_1}{\partial X_{1t}}>\frac{\partial u_1}{\partial X_{2t}}. \tag{F.10}$$

Suppose that $t=T-1$. Then

$$v_1^{(1)}\left(X_{T-1}\right)=\beta E\left[V\left(\tilde{X}_T\right)|X_{T-1},d_{T-1}=1\right]=\beta E\left[\max\{\tilde{\varepsilon}_{0T},u_1\left(\tilde{X}_t\right)+\tilde{\varepsilon}_{1T}\}|X_{T-1}\right]. \tag{F.11}$$

Recall that

$$u_1\left(\tilde{X}_T\right)=\theta_{0T}+\theta'\begin{bmatrix}\sigma_{01}+\sigma_{11}X_{1T-1}+\sigma_{21}\left(T-1\right)+\tilde{\zeta}_{1T}\\ \vdots\\ \sigma_{0n}+\sigma_{11}X_{nT-1}+\sigma_{2n}\left(T-1\right)+\tilde{\zeta}_{nT}\\ T\end{bmatrix}. \tag{F.12}$$

Let $\zeta_t=\left(\zeta_{1t},\dots\zeta_{nt}\right)$, $\varepsilon_t=\left(\varepsilon_{0t},\varepsilon_{1t}\right)$,

$$\varphi(X_t,\zeta_{t+1}) = (\varphi_1(X_{1t},t,\zeta_{it+1}),\ldots,\varphi_n(X_{nt},t,\zeta_{nt+1}),\varphi_{n+1}(t))$$
$$\Omega_0(\zeta,\varepsilon|X_{T-1}) = \{(\zeta,\varepsilon):\varepsilon_0 \geq u_1(\varphi(X_{T-1},\zeta))+\varepsilon_1\} \tag{F.13}$$
$$\Omega_1(\zeta,\varepsilon|X_{T-1}) = \{(\zeta,\varepsilon):\varepsilon_0 < u_1(\varphi(X_{T-1},\zeta))+\varepsilon_1\}$$

i.e., $\Omega_0(\zeta,\varepsilon|X_{T-1})$ is the set of realizations of random variables $\left(\tilde{\zeta}_T,\tilde{\varepsilon}_T\right)$ for which it is optimal to quit in period T, and $\Omega_1(\zeta,\varepsilon|X_{T-1})$ is the set of realizations of random variables $\left(\tilde{\zeta}_T,\tilde{\varepsilon}_T\right)$ for which it is optimal to continue in period T. With this notation, we can rewrite (F.11) as

$$v_1^{(1)}(X_{T-1}) = \beta\left(\int\limits_{\Omega_0(\zeta,\varepsilon|X_{T-1})}\varepsilon_0 dG(\zeta,\varepsilon) + \int\limits_{\Omega_1(\zeta,\varepsilon|X_{T-1})}(u_1(\varphi(X_{T-1},\zeta))+\varepsilon_1)\,dG(\zeta,\varepsilon)\right).$$

Let e_i denote an $n+1$ dimensional vector with the i^{th} element equal to 1 and 0 elsewhere, e.g., $e_2 = (0,1,0,\ldots,0)$. Let Δ be a positive value. We will next show that

$$v_1^{(1)}(X_{T-1}+\Delta e_2) - v_1^{(1)}(X_{T-1}) \leq v_1^{(1)}(X_{T-1}+\Delta e_1) - v_1^{(1)}(X_{T-1}) \text{ for any } \Delta > 0$$

and that this inequality implies $\frac{\partial v_1^{(1)}}{\partial X_{2T-1}} \leq \frac{\partial v_1^{(1)}}{\partial X_{1T-1}}$.

Note that

$$\begin{aligned} u_1(\varphi(X_{T-1}+\Delta e_2,\zeta)) =& u_1(\varphi(X_{T-1},\zeta)) + \left(\frac{\partial u_1}{\partial X_{2T}}\right)\left(\frac{\partial \varphi_2}{\partial X_{2T-1}}\right)\Delta = u_1(\varphi(X_{T-1},\zeta)) + \theta_2\sigma_{12}\Delta \\ &< u_1(\varphi(X_{T-1},\zeta)) + \theta_1\sigma_{11}\Delta \quad (see (F.7) \text{ and } (F.8)) \\ &= u_1(\varphi(X_{T-1}+\Delta e_1,\zeta)). \end{aligned} \tag{F.14}$$

Therefore,

$$\begin{aligned} & v_1^{(1)}(X_{T-1}+\Delta e_2) \\ =& \beta\left(\int\limits_{\Omega_0(\zeta,\varepsilon|X_{T-1}+\Delta e_2)}\varepsilon_0 dG(\zeta,\varepsilon) + \int\limits_{\Omega_1(\zeta,\varepsilon|X_{T-1}+\Delta e_2)}(u_1(\varphi(X_{T-1}+\Delta e_2,\zeta))+\varepsilon_1)\,dG(\zeta,\varepsilon)\right) \\ \leq& \beta\left(\int\limits_{\Omega_0(\zeta,\varepsilon|X_{T-1}+\Delta e_2)}\varepsilon_0 dG(\zeta,\varepsilon) + \int\limits_{\Omega_1(\zeta,\varepsilon|X_{T-1}+\Delta e_2)}(u_1(\varphi(X_{T-1}+\Delta e_1,\zeta))+\varepsilon_1)\,dG(\zeta,\varepsilon)\right) \\ \leq& \beta\left(\int\limits_{\Omega_0(\zeta,\varepsilon|X_{T-1}+\Delta e_1)}\varepsilon_0 dG(\zeta,\varepsilon) + \int\limits_{\Omega_1(\zeta,\varepsilon|X_{T-1}+\Delta e_1)}(u_1(\varphi(X_{T-1}+\Delta e_1,\zeta))+\varepsilon_1)\,dG(\zeta,\varepsilon)\right) \\ =& v_1^{(1)}(X_{T-1}+\Delta e_1). \end{aligned}$$

The first inequality follows from (F.14). The second inequality follows from the definition of $\Omega_0(\zeta,\varepsilon|X_{T-1}+\Delta e_1)$ and $\Omega_1(\zeta,\varepsilon|X_{T-1}+\Delta e_1)$ that are sets of realizations associated with each optimal decision (quit or continue) for a given state. From

$$v_1^{(1)}(X_{T-1}+\Delta e_2) \leq v_1^{(1)}(X_{T-1}+\Delta e_1) \tag{F.15}$$

for any $\Delta > 0$, it follows that

$$\lim_{\Delta\to 0}\frac{v_1^{(1)}(X_{T-1}+\Delta e_2) - v_1^{(1)}(X_{T-1})}{\Delta} = \frac{\partial v_1^{(1)}}{\partial X_{2T-1}} \leq \frac{\partial v_1^{(1)}}{\partial X_{1T-1}} = \lim_{\Delta\to 0}\frac{v_1^{(1)}(X_{T-1}+\Delta e_1) - v_1^{(1)}(X_{T-1})}{\Delta}. \tag{F.16}$$

Now suppose that $t = T-2$. Note that

$$\varphi\left(X_t+\Delta e_i,\zeta\right)=\varphi\left(X_t,\zeta\right)+\sigma_{1i}\Delta e_i \text{ (see (F.6) and (F.13)).} \tag{F.17}$$

Therefore,

$$\begin{aligned}
v_1^{(1)}\left(X_{T-2}+\Delta e_2\right) =&\beta\left(\begin{array}{l}\displaystyle\int\limits_{\Omega_0\left(\zeta,\varepsilon|X_{T-2}+\Delta e_2\right)}\left(\varepsilon_0+\beta E\left[\tilde{\varepsilon}_{0T}\right]\right)dG\left(\zeta,\varepsilon\right)\\
\displaystyle+\int\limits_{\Omega_1\left(\zeta,\varepsilon|X_{T-2}+\Delta e_2\right)}\left(\begin{array}{l}u_1\left(\varphi\left(X_{T-2}+\Delta e_2,\zeta\right)\right)+\varepsilon_1\\+v_1^{(1)}\left(\varphi\left(X_{T-2}+\Delta e_2,\zeta\right)\right)\end{array}\right)dG\left(\zeta,\varepsilon\right)\end{array}\right)\\
=&\beta\left(\begin{array}{l}\displaystyle\int\limits_{\Omega_0\left(\zeta,\varepsilon|X_{T-2}+\Delta e_2\right)}\left(\varepsilon_0+\beta\gamma\right)dG\left(\zeta,\varepsilon\right)\\
\displaystyle+\int\limits_{\Omega_1\left(\zeta,\varepsilon|X_{T-2}+\Delta e_2\right)}\left(\begin{array}{l}u_1\left(\varphi\left(X_{T-2}+\Delta e_2,\zeta\right)\right)+\varepsilon_1\\+v_1^{(1)}\left(\varphi\left(X_{T-2},\zeta\right)+\sigma_{12}\Delta e_2\right)\end{array}\right)dG\left(\zeta,\varepsilon\right)\end{array}\right)\text{ see (F.17)}\\
\leq&\beta\left(\begin{array}{l}\displaystyle\int\limits_{\Omega_0\left(\zeta,\varepsilon|X_{T-2}+\Delta e_2\right)}\left(\varepsilon_0+\beta\gamma\right)dG\left(\zeta,\varepsilon\right)\\
\displaystyle+\int\limits_{\Omega_1\left(\zeta,\varepsilon|X_{T-2}+\Delta e_2\right)}\left(\begin{array}{l}u_1\left(\varphi\left(X_{T-2}+\Delta e_1,\zeta\right)\right)+\varepsilon_1\\+v_1^{(1)}\left(\varphi\left(X_{T-2},\zeta\right)+\sigma_{11}\Delta e_1\right)\end{array}\right)dG\left(\zeta,\varepsilon\right)\end{array}\right)\\
\leq&\beta\left(\begin{array}{l}\displaystyle\int\limits_{\Omega_0\left(\zeta,\varepsilon|X_{T-2}+\Delta e_1\right)}\left(\varepsilon_0+\beta\gamma\right)dG\left(\zeta,\varepsilon\right)\\
\displaystyle+\int\limits_{\Omega_1\left(\zeta,\varepsilon|X_{T-2}+\Delta e_1\right)}\left(\begin{array}{l}u_1\left(\varphi\left(X_{T-2}+\Delta e_1,\zeta\right)\right)+\varepsilon_1\\+v_1^{(1)}\left(\varphi\left(X_{T-2},\zeta\right)+\sigma_{11}\Delta e_1\right)\end{array}\right)dG\left(\zeta,\varepsilon\right)\end{array}\right)\\
=&v_1^{(1)}\left(X_{T-2}+\Delta e_1\right).
\end{aligned} \tag{F.18}$$

for any $\Delta>0$. The first inequality follows from (F.8), (F.14), and (F.15). The second inequality follows from replacing the optimal decisions in period $T-1$ conditioned on state $X_{T-2}+\Delta e_2$ in period $T-2$ with optimal decisions conditioned on state $X_{T-2}+\Delta e_1$. Therefore,

$$\lim_{\Delta\to 0}\frac{v_1^{(1)}\left(X_{T-2}+\Delta e_2\right)-v_1^{(1)}\left(X_{T-2}\right)}{\Delta}=\frac{\partial v_1^{(1)}}{\partial X_{2T-2}}\leq\frac{\partial v_1^{(1)}}{\partial X_{1T-2}}=\lim_{\Delta\to 0}\frac{v_1^{(1)}\left(X_{T-2}+\Delta e_2\right)-v_1^{(1)}\left(X_{T-2}\right)}{\Delta}.$$

Thus, by induction, the result holds for $t=T-\tau$ for any $\tau\in\{1,\ldots,T-1\}$. □

Proof of Proposition 3. We rewrite Eq. (15) using our streamlined notation

$$p_1\left(X_t\right)=\frac{1}{\left(1+e^{v_0(X_t)-v_1(X_t)}\right)}$$

and implicitly differentiate with respect to state transition parameters σ_i

$$\begin{aligned}
\frac{\partial p_1\left(X_t\right)}{\partial\sigma_i}&=\frac{e^{v_0(X_t)-v_1(X_t)}}{\left(1+e^{v_0(X_t)-v_1(X_t)}\right)^2}\left(\frac{\partial u_1\left(X_t\right)}{\partial\sigma_i}+\frac{\partial v_1^{(1)}\left(X_t\right)}{\partial\sigma_i}\right)\\
&=\frac{e^{v_0(X_t)-v_1(X_t)}}{\left(1+e^{v_0(X_t)-v_1(X_t)}\right)^2}\frac{\partial v_1^{(1)}\left(X_t\right)}{\partial\sigma_i},
\end{aligned} \tag{F.19}$$

(i.e., u_1 drops out because it only depends on the current state, and not the future states that are influenced by σ_i). We define

$$\Omega_0(\zeta,\varepsilon|X_t) = \{(\zeta,\varepsilon) : \varepsilon_0 \geq u_1(\varphi(X_t,\zeta)) + \varepsilon_1 + v_1^{(1)}(\varphi(X_t,\zeta))\}$$
$$\Omega_1(\zeta,\varepsilon|X_t) = \{(\zeta,\varepsilon) : \varepsilon_0 < u_1(\varphi(X_t,\zeta)) + \varepsilon_1 + v_1^{(1)}(\varphi(X_t,\zeta))\}$$

i.e., $\Omega_0(\zeta,\varepsilon|X_t)$ is the set of realizations of random variables $\left(\tilde{\zeta}_{t+1},\tilde{\varepsilon}_{t+1}\right)$ for which it is optimal to quit in period $t+1$ and $\Omega_1(\zeta,\varepsilon|X_t)$ is the set of realizations of random variable $\left(\tilde{\zeta}_{t+1},\tilde{\varepsilon}_{t+1}\right)$ for which it is optimal to continue in period $t+1$. Then the conditional valuation function can be expressed as

$$v_1^{(1)}(X_t) = \beta \left(\begin{array}{l} \displaystyle\int\limits_{\Omega_0(\zeta,\varepsilon|X_t)} \left(\varepsilon_0 + \beta E\left[\sum_{s=2}^{T-t-1} \beta^{s-1}\tilde{\varepsilon}_{0t+S}\right]\right) dG(\zeta,\varepsilon) \\ + \displaystyle\int\limits_{\Omega_1(\zeta,\varepsilon|X_t)} \left(u_1(\varphi(X_t,\zeta)) + \varepsilon_1 + v_1^{(1)}(\varphi(X_t,\zeta))\right) dG(\zeta,\varepsilon) \end{array} \right).$$

From the definition of $\Omega_0(\zeta,\varepsilon|X_t)$ and $\Omega_1(\zeta,\varepsilon|X_t)$, it follows that the two integrands form a piecewise continuous function over realizations of $\left(\tilde{\zeta}_{t+1},\tilde{\varepsilon}_{t+1}\right)$. To clarify this point, we rewrite the above as

$$\beta \left(\int\limits_{\Omega_0(\zeta,\varepsilon|X_t)} h_1(\zeta,\varepsilon,\sigma_{ik})\, dG(\zeta,\varepsilon) + \int\limits_{\Omega_1(\zeta,\varepsilon|X_t)} h_2(\zeta,\varepsilon,\sigma_{ik})\, dG(\zeta,\varepsilon) \right),$$

where $h_1(\zeta,\varepsilon,\sigma_{ik}) = \varepsilon_0 + E\left[\sum_{s=2}^{T-t-1}\beta^{s-1}\tilde{\varepsilon}_{0t+S}\right] = u_1(\varphi(X_t,\zeta)) + \varepsilon_1 + v_1^{(1)}(\varphi(X_t,\zeta)) = h_2(\zeta,\varepsilon,\sigma_{ik})$ at all $(\zeta,\varepsilon) \in \{(\zeta,\varepsilon) : \varepsilon_0 = u_1(\varphi(X_t,\zeta)) + \varepsilon_1 + v_1(\varphi(X_t,\zeta))\}$. Therefore, the partial derivative of $v_1^{(1)}(X_t)$ with respect to σ_{ik} is obtained purely from the partial derivatives of the integrands, i.e.,

$$v_1^{(1)}(X_t) = \beta \left(\begin{array}{l} \displaystyle\int\limits_{\Omega_0(\zeta,\varepsilon|X_t)} \frac{\partial}{\partial\sigma_{ik}} \left(\varepsilon_0 + \beta E\left[\sum_{s=2}^{T-t-1} \beta^{s-1}\tilde{\varepsilon}_{0t+S}\right]\right) dG(\zeta,\varepsilon) \\ + \displaystyle\int\limits_{\Omega_1(\zeta,\varepsilon|X_t)} \frac{\partial}{\partial\sigma_{ik}} \left(u_1(\varphi(X_t,\zeta)) + \varepsilon_1 + v_1^{(1)}(\varphi(X_t,\zeta))\right) dG(\zeta,\varepsilon) \end{array} \right)$$
$$= \beta \left(\int\limits_{\Omega_1(\zeta,\varepsilon|X_t)} \left(\frac{\partial u_1(\varphi(X_t,\zeta))}{\partial\sigma_{ik}} + \frac{\partial v_1^{(1)}(\varphi(X_t,\zeta))}{\partial\sigma_{ik}} \right) dG(\zeta,\varepsilon) \right).$$

Note that

$$\frac{\partial u_1(\varphi(X_t,\zeta))}{\partial\sigma_{ik}} = \frac{\partial u_1(X_{t+1})}{\partial X_{it+1}} \frac{\varphi(X_t,\zeta)}{\partial\sigma_{ik}}$$
$$\frac{\partial v_1^{(1)}(\varphi(X_t,\zeta))}{\partial\sigma_{ik}} = \frac{\partial v_1^{(1)}(X_{t+1})}{\partial X_{it+1}} \frac{\varphi(X_t,\zeta)}{\partial\sigma_{ik}}.$$

Substituting the above into Eq. F.19

$$\frac{\partial p_1(X_t)}{\partial\sigma_{ik}} = \frac{e^{v_0(X_t)-v_1(X_t)}}{\left(1+e^{v_0(X_t)-v_1(X_t)}\right)^2} \beta \left(\int\limits_{\Omega_1(\zeta,\varepsilon|X_t)} \left(\frac{\partial u_1}{\partial X_{it+1}} + \frac{\partial v_1^{(1)}}{\partial X_{it+1}} \right) \frac{\partial\varphi(X_t,\zeta)}{\partial\sigma_{ik}} dG(\zeta,\varepsilon) \right).$$

Finally, as shown in the proof of Proposition 2

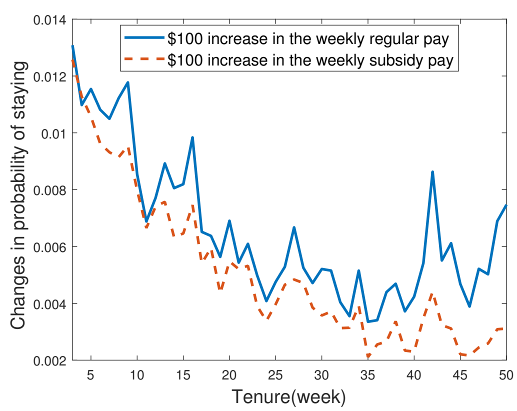
$$\frac{\partial u_1(X_{t+1})}{\partial X_{it+1}} > 0 \text{ implies } \frac{\partial v_1^{(1)}(X_{t+1})}{\partial X_{it+1}} \geq 0$$
$$\frac{\partial u_1(X_{t+1})}{\partial X_{it+1}} < 0 \text{ implies } \frac{\partial v_1^{(1)}(X_{t+1})}{\partial X_{it+1}} \leq 0.$$

Therefore, if $\frac{\partial\varphi_i}{\partial\sigma_{ik}} \geq 0$, then the sign of $\frac{\partial p_1(X_t)}{\partial\sigma_{ik}}$ matches the sign of $\frac{\partial u_1}{\partial X_{it+1}}$. □

Appendix G Compensation Improvement Effects on Retention

We evaluate compensation effects on retention by simulating a counterfactual intervention in which drivers' regular and subsidy payments increase on a weekly basis during drivers' tenure. The goal is to compare the effects on retention caused by increases across both forms of compensation and the evolution of these effects during tenure. To that end, we simulate a counterfactual intervention in which drivers' regular and subsidy payments increase by \$100 one week at a time during tenure (up until week 50). That is, both forms of compensation increase by \$100 starting in the third week of drivers' tenure, then in the fourth week, and so on and so forth.[1] The changes in the probability of staying in week t are simulated as $\frac{\sum_{i=1}^{N_t} p_1(X'_{ijt}) - \sum_{i=1}^{N_t} p_1(X_{ijt})}{\sum_{i=1}^{N_t} p_1(X_{ijt})}$, where N_t is the observed number of drivers' decisions in week t. X'_{ijt} is equal to the vector X_{ijt}, but with the regular (or subsidy) pay used in X'_{ijt} set to its baseline value in X_{ijt} plus \$100. Our approach is consistent with Kang et al. (2015)'s in that it considers interventions that are in effect for only one period (i.e., one week) at a time. This allows us to recover the counterfactual conditional choice probabilities without solving the full model via backward recursion while benefiting from the computational advantages of the conditional choice probabilities approach (Arcidiacono and Ellickson 2011).

Figure G.1 **Effects of \$100 increases in regular and subsidy compensations on retention**



The results we obtained are consistent with Propositions 1 and 2 (see Figure G.1). First, they show that retention is more sensitive to increases in regular pay than to increases in subsidy pay. While an increase in regular pay leads to an average increase of 0.69% per week in drivers' retention during tenure (with a standard deviation of 0.25%), the same increase in subsidy pay leads to a weekly average increase in retention rate of 0.51% (with a standard deviation of 0.27%). Thus, in making decisions to stay at TForce, drivers value regular pay more than subsidy pay. Second, the sensitivity of retention to higher pay decreases in tenure, and this phenomenon is more pronounced for subsidy payments. Thus, the impact of regular and subsidy pay on drivers' utility of staying decreases as tenure increases. Furthermore, relative to regular pay, subsidy compensation has a greater risk of becoming ineffective over time.

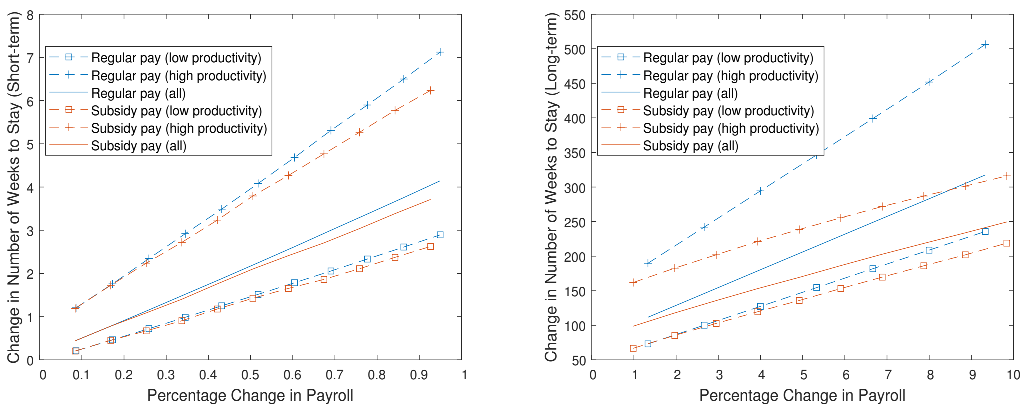
[1] All of our counterfactual analyses start in the third week because no driver in our sample left TForce during the first two weeks of tenure.

Appendix H Driver Productivity and Self-Selection Contingencies

We first analyze how effective increases in subsidy pay versus regular compensation are in promoting retention, contingent on productivity among drivers. Figure H.1(a) and Figure H.1(b) respectively plot the changes in the length of tenure in the short and long terms for high and low productivity drivers as well as for the entire driver pool. For purposes of comparison, both plots focus on compensation interventions taking place in week 3 of tenure and generating percentage increases in payroll costs of up to 1%, in Figure H.1(a), and up to 10%, in Figure H.1(b)).[2] The results in both figures show that holding percentage increases in payroll costs constant, regular compensation interventions are more effective at extending the length of tenure across all drivers, as well as high- and low-productivity drivers than subsidy pay interventions. They also show that regular compensation interventions are more effective at extending the length of tenure for high-productivity drivers. Hence, compared to subsidy pay, regular compensation not only is more effective at retaining drivers, but also contributes more to maintaining high workforce productivity.

Figure H.1 Regular and subsidy pay intervention effects on retention, contingent on productivity

(a) Short-term **(b)** Long-term



Notes: (1) The values in the Y axis of Figure H.1(a) correspond to the sum of changes in tenure length in week 3 across all drivers; (2) The values in Y axis of Figure H.1(b) correspond to the sum of changes in tenure length from week 3 to week 50 across all drivers.

We then examine the contingencies introduced by the weekly share of Type 2 drivers during tenure in relation to the effect of subsidy and regular pay on retention. We first estimate two functions corresponding to (1) the estimated probability of quitting assuming the proportion of Type 2 drivers stays the same as that in the previous period (removing self-selection) and (2) the estimated probability of quitting under the status quo (including both self-selection and learning). We use an Epanechnikov kernel with a bandwidth of 5 weeks to plot the probabilities of quitting. Please refer respectively to the dotted blue line and red solid line in Figure H.2(a). The plots show that the values for the estimated quitting probability decrease fairly consistently up until week 35

[2]We set these limits to generate conservative estimates in the counterfactual outputs. The limit to the percentage increase in payroll costs in the short-term analysis is lower than the limit in the long-term analysis to ensure increases in weekly pay are consistent in magnitude across both analyses. Recall that while the short-term analysis spans only two periods (the intervention week and one-period ahead), the long-term analysis spans 48 periods (from week 3 to week 50).

and level off after that period. Therefore, we analyze the effects of self-selection and learning on retention as a function of their contribution to the reduction in the probability of quitting in each period up until week 35. The higher this contribution, the greater the effect on retention.

To perform this analysis, we decompose the change in the probability of quitting estimated under the status quo for every period (red line in Figure H.2(a)) into the fractions attributed to self-selection versus learning. To estimate the fraction due to self-selection, we calculate the inverse ratio of the change in the status-quo probability of quitting at every period to the amount of such change due to self-selection. The difference between 1 and this ratio corresponds to the fraction attributed to learning.

Each decrease in the status-quo probability of quitting corresponds to the difference between the status-quo probability of quitting in the current period (i.e., solid red line in period t) and its four-period lagged value (i.e., solid red line in period $t-4$).[3] The amount of such decrease due to self-selection is equal to the difference between the status-quo probability of quitting in the current period (i.e., red solid line in period t) and the estimated probability of quitting assuming the proportion of Type 2 drivers stays the same as that four periods prior, i.e., removing self-selection based on the dotted blue line at $t-4$.

Figure H.2 **The role of self-selection in the reductions in the probability of quitting**

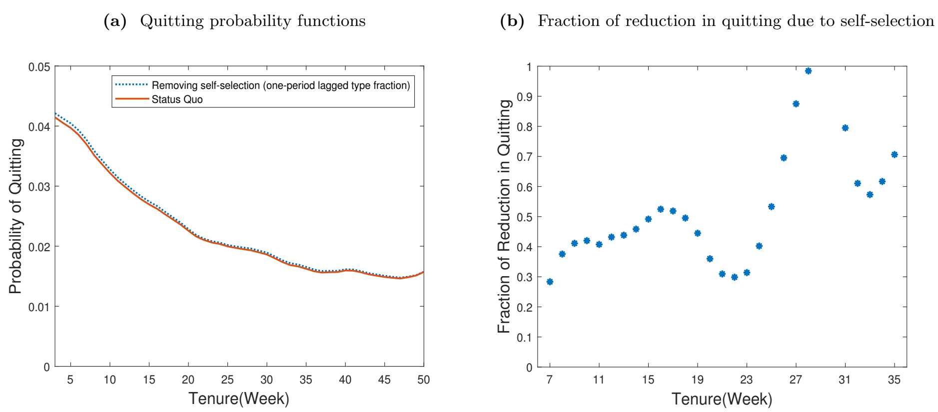


Figure H.2(b) plots the values of the fraction attributed to self-selection for the decreases in the probability of quitting across all periods up until week 35. We find that, on average, self-selection accounts for a smaller fraction in the decreases in the probability of quitting earlier in tenure. On average, self-selection accounts for an average of 41% up until week 23, while on-the-job learning accounts for 59%. Nevertheless, self-selection becomes more important after week 23, as it accounts, on average, for 68% of the decreases in the probability of quitting, while subsidy-supported learning accounts for only 32%.

[3] Recall from Section 8.1 that the proportion of Type 1 drivers decreases, on average, by only a one percentage point per week. Therefore, to bring out the reduction in quitting due to self-selection, we use a four-period lagged value (lagged by a month) as opposed to using a one-period lagged value (lagged by one week).