
E-Companion to:

Paid Priority in Service Systems: Theory and Experiments

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Appendix

A. Additional Tables

Table EC.1 Cost profiles in the All-Human Study.

	Sequence									
Profile	1	2	3	4	5	6	7	8	9	10
1	H	L	H	H	H	L	L	H	L	H
2	H	H	L	L	L	L	H	L	H	H
3	H	L	L	L	L	L	H	H	H	L
4	H	L	L	L	L	L	L	H	L	L
5	L	L	H	L	L	L	L	L	L	H

Table EC.2 Equilibrium paths and summary statistics in equilibrium for the All-Human Study.

	Base Model ($\gamma = 0$):	Priority			Regular		
Profile	PBE Path (1: Pri., 0: Reg.)	Length	Low	High	Length	Low	High
1	1 1 1 1 1 0 1 1 0 1	8	2	6	2	2	0
2	1 1 1 1 1 0 1 0 1 1	8	3	5	2	2	0
3	1 1 1 1 1 0 1 1 1 0	8	4	4	2	2	0
4	1 1 1 1 1 0 1 1 0 0	7	5	2	3	3	0
5	1 1 1 1 1 0 1 0 1 1	8	6	2	2	2	0
	Average:	7.8	4	3.8	2.2	2.2	0
	Comp. Model ($\gamma = 1$):	Priority			Regular		
	PBE Path (1: Pri., 0: Reg.)	Length	Low	High	Length	Low	High
1	1 0 1 1 1 0 0 1 0 1	6	0	6	4	4	0
2	1 1 0 0 0 0 1 1 1 1	6	1	5	4	4	0
3	1 0 0 0 1 0 1 1 1 0	5	1	4	5	5	0
4	1 0 0 0 1 0 1 1 0 1	5	3	2	5	5	0
5	0 0 1 0 1 0 1 1 0 1	5	3	2	5	5	0
	Average:	5.4	1.6	3.8	4.6	4.6	0

B. Details of Simulation for One-Human Study

In the simulations, we considered the same sets of parameters tested in the One-Human Study, i.e., $N = 10$, compensation fraction $\gamma \in \{0, 1\}$, priority fee p between \$0.50 and \$20 in increments of \$0.50, and waiting costs for each customer (human or computerized) drawn IID from \$1 or \$2 per service with equal probability. In order to obtain estimates with negligible standard errors, we conducted 5 million replications for each combination of priority fee and compensation fraction. Given the huge sample size, for purposes of comparison we treat sample averages as essentially equal to the true means. For each replication, we randomly generated the vector of 10 waiting costs. As mentioned, these waiting costs were IID across customers within a replication; however, the waiting-cost vectors for a given replication were coupled across different values of the priority fee and compensation fraction; that is, the vector used in the j-th replication for $p = \$0.50$ and $\gamma = 0$ was also used in the j-th replication for all other combinations of p and γ. We also used the same 5 million randomly drawn waiting-cost vectors to compute equilibrium and FCFS performance measures.

Table EC.3 Observed paths and summary statistics in the All-Human Study.

	Base Model ($\gamma = 0$):	Priority			Regular		
Profile	Observed (1: Pri., 0: Reg.)	Length	Low	High	Length	Low	High
1	1 0 0 1 1 1 1 1 1 0	7	3	4	3	1	2
2	1 1 1 1 1 0 1 1 0 0	7	4	3	3	1	2
3	1 1 0 1 1 0 1 0 1 1	7	4	3	3	2	1
4	1 1 1 0 1 1 1 0 1 1	8	7	1	2	1	1
5	1 1 1 0 0 1 1 0 0 1	6	4	2	4	4	0
	Average:	7	4.4	2.6	3	1.8	1.2
	Comp. Model ($\gamma = 1$):	Priority			Regular		
Profile	Observed (1: Pri., 0: Reg.)	Length	Low	High	Length	Low	High
1	1 1 0 1 0 1 0 0 0 1	5	2	3	5	2	3
2	1 1 1 1 0 1 0 0 0 1	6	3	3	4	2	2
3	1 1 0 1 1 0 0 1 0 1	6	4	2	4	2	2
4	1 1 0 1 1 0 0 1 0 1	6	4	2	4	4	0
5	1 1 1 0 1 0 0 1 1 0	6	5	1	4	3	1
1	0 1 1 1 1 1 0 1 0 1	7	2	5	3	2	1
2	0 0 1 1 1 1 0 1 0 1	6	5	1	4	0	4
3	1 1 0 0 1 1 0 1 1 0	6	3	3	4	3	1
4	1 1 0 1 0 0 1 1 0 1	6	4	2	4	4	0
5	1 1 1 0 0 0 1 1 1 0	6	5	1	4	3	1
	Average:	6	3.7	2.3	4	2.5	1.5

We refer to a particular combination of replication number (and the associated waiting-cost vector), priority fee, and compensation fraction (e.g., fifth replication, $p = 4.50$, $\gamma = 1$) as an *instance*. The priority queue length facing the i-th customer in a given instance was determined from the simulated actions of the $i-1$ earlier customers in the instance, and given the priority fee and the customer's waiting cost and sequence number, the equilibrium decision was determined by comparing the priority queue length with the customer's PBE threshold. We then calculated the predicted probability of priority purchase using the logit models (4) from Tables 6 and 7 for $\gamma = 0$ and 1, respectively. The customer's decision (regular or priority) was then realized from a Bernoulli distribution with the calculated probability of priority purchase.

C. Proofs of Lemma 1 and Theorem 1

Proof of Lemma 1. We take a sample path approach. Consider a particular vector of realized waiting costs $(c_1, \ldots, c_N)$, a focal customer i, and $k \in \{0, \ldots, i-2\}$. Under the threshold strategy $\bar{x}_{j-1}(c_j)$, customer $j \in \{i+1, \ldots, N\}$ purchases priority if and only if $x_{j-1} \le \bar{x}_{j-1}(c_j)$. Given the fixed (but arbitrary) threshold strategies on the sample path, L_i^k is no longer random: we use ℓ_i^k to denote its realization corresponding to the realized waiting costs $(c_1, \ldots, c_N)$. Denote by x_j^k the length of the priority queue that is observed by customer $j+1$, given that $x_{i-1} = k$, customer i chooses the regular queue, and each customer $j \in \{i+1, \ldots, N\}$ uses the threshold strategy $\bar{x}_{j-1}(c_j)$. We proceed by cases.

Case 1: $\bar{x}_{j-1}(c_j) \ne x_{j-1}^k$ for all $j \in \{i+1, \ldots, N\}$. In this case, if customer i chooses the regular queue, then all customers $j \in \{i+1, \ldots, N\}$ will take the same actions whether $x_{i-1} = k$ or $x_{i-1} = k+1$. To see this, consider customer $i+1$. We have $x_i^{k+1} = x_i^k + 1$. Because $x_i^k \ne \bar{x}_i(c_{i+1})$, we either have $\bar{x}_i(c_{i+1}) \ge x_i^{k+1} = x_i^k + 1 > x_i^k$, or $\bar{x}_i(c_{i+1}) < x_i^k < x_i^k + 1 = x_i^{k+1}$. Either way, customer $i+1$ will make the same decision with $x_i = x_{i-1} = k$ as with $x_i = x_{i-1} = k+1$, and by induction, so will customers $j \in \{i+2, \ldots, N\}$.

Hence, after choosing the regular queue, customer i will wait through the same number of services whether $x_{i-1} = k$ or $x_{i-1} = k+1$. Denoting by α the number of priority purchases among customers $j \in \{i+1, \ldots, N\}$, we then have

$$\ell_i^k = i + \alpha = \ell_i^{k+1}. \tag{EC.1}$$

Case 2: $\bar{x}_{j-1}(c_j) = x_{j-1}^k$ for some $j \in \{i+1, \ldots, N\}$. In this case, define j' by

$$j' := \min\left\{ j \in \{i+1, \ldots, N\} : \bar{x}_{j-1}(c_j) = x_{j-1}^k \right\}.$$

By the same argument as in Case 1, if customer i chooses the regular queue, then whether $x_{i-1} = k$ or $x_{i-1} = k+1$, customers $j \in \{i+1, \ldots, j'-1\}$ will take the same actions in either case because $\bar{x}_{j-1}(c_j) \neq x_{j-1}^k$ for all $j \in \{i+1, \ldots, j'-1\}$ (if $j' = i+1$, then this interval of customers is vacuous and thus trivially there is no difference in this empty set of customers between the cases with $x_{i-1} = k$ and $x_{i-1} = k+1$). By the definition of j', we have $x_{j'-1}^k = \bar{x}_{j'-1}(c_j)$, so if $x_{i-1} = k$, then customer j' will purchase priority because the priority queue length will be exactly at her threshold. Also, because all customers $j \in \{i+1, \ldots, j'-1\}$ take the same actions with $x_{i-1} = k+1$ as with $x_{i-1} = k$, we have $x_{j'-1}^{k+1} = x_{j'-1}^k + 1 = \bar{x}_{j'-1}(c_j) + 1$. So, if $x_{i-1} = k+1$, then customer j' will *not* purchase priority because her threshold will be exceeded by one. Consequently, we have $x_{j'}^{k+1} = x_{j'-1}^k + 1 = x_{j'}^k$, meaning that if customer i chooses the regular queue, then $x_{j'}$ is the same whether $x_{i-1} = k$ or $x_{i-1} = k+1$.

Therefore, all customers $j \in \{j'+1, \ldots, N\}$ take the same action whether $x_{i-1} = k$ or $x_{i-1} = k+1$ because $x_{j'}^k = x_{j'}^{k+1}$ implies that $x_{j-1}^k = x_{j-1}^{k+1}$ for all $j \in \{j'+1, \ldots, N\}$ (if $j' = N$, then again this empty interval of customers has no effect on ℓ_i^k or ℓ_i^{k+1}). So, conditional on customer i choosing the regular queue, the total number of customers to purchase priority among customers $j \in \{i+1, \ldots, j'-1, j'+1, \ldots, N\}$ is the same whether $x_{i-1} = k$ or $x_{i-1} = k+1$. Denoting this number by β, we can write

$$\ell_i^k = i + \beta + 1 = \ell_i^{k+1} + 1, \tag{EC.2}$$

where the difference of 1 between ℓ_i^k and ℓ_i^{k+1} is due to customer j' purchasing priority if $x_{i-1} = k$ (because in this case $x_{j'-1} = \bar{x}_{j'-1}(c_{j'})$)—and accordingly being served before customer i—but choosing the regular queue if $x_{i-1} = k+1$ (because in this case $x_{j'-1} = \bar{x}_{j'-1}(c_{j'}) + 1$).

Equations (EC.1) and (EC.2) imply the bounds $0 \leq \ell_i^k - \ell_i^{k+1} \leq 1$. Taking expectation over the waiting costs (and by extension, over the other customers' cost-dependent thresholds) yields the lemma. ☐

Proof of Theorem 1. The proof is by a double induction. Consider a customer $i \in \{1, \ldots, N-1\}$, and suppose that all customers $j \in \{i+1, \ldots, N\}$ use some cost-dependent threshold strategies $\bar{x}_{j-1}(C_j)$. That is, customer j purchases priority if and only if $x_{j-1} \leq \bar{x}_{j-1}(C_j)$.

For a given waiting-cost realization c_i, consider customer i's optimal strategy as a function of x_{i-1}. Given the cost-dependent threshold strategies $\bar{x}_{j-1}(C_j)$ for customers $j \in \{i+1, \ldots, N\}$, let $0 \leq k \leq i-1$ be the smallest integer such that, if $x_{i-1} = k$, then it is optimal for customer i to stay in the regular queue. We note that, by definition, it is optimal for customer i to purchase priority if $x_{i-1} < k$. If it is never optimal for customer i to choose the regular queue, then the optimal strategy for customer i is the threshold strategy $\bar{x}_{i-1} = i-1$, and by convention we say that $k = i$ in this case. Similarly, if $k = i-1$, then the optimal strategy for customer i is the threshold strategy $\bar{x}_{i-1} = i-2$. The remainder of the argument establishes that a threshold strategy is also optimal if $k \leq i-2$.

As in Lemma 1, let L_i^k denote the random number of services (including her own) that customer i will wait through if $x_{i-1} = k$ and she chooses the regular queue; the exact value of L_i^k will depend on the thresholds used by the later customers, which in turn depend on their waiting-cost realizations. Let $\bar{\boldsymbol{x}}_{\boldsymbol{i,j-1}}$ be a vector of cost-dependent threshold strategies $(\bar{x}_i(C_{i+1}), \ldots, \bar{x}_{j-1}(C_j))$ for customers $i+1, \ldots, j$. For the case in which customers $i+1, \ldots, N$ use the cost-dependent threshold strategies $\bar{\boldsymbol{x}}_{\boldsymbol{i,N-1}}$, we represent customer i's net utilities from choosing the regular or priority queue by $U_{R,i}(x_{i-1}; \bar{\boldsymbol{x}}_{\boldsymbol{i,N-1}})$ and $U_{P,i}(x_{i-1})$, respectively. Note that the utility from the regular queue is a random variable because the strategies of the later customers depend on their realized waiting costs. Taking expectation over the remaining customers' waiting costs (the earlier customers' waiting costs are irrelevant because their decisions have already been observed), the assumption that the regular queue is optimal for customer i if $x_{i-1} = k$ implies

$$\mathsf{E}[U_{R,i}(k\,;\bar{\boldsymbol{x}}_{\boldsymbol{i,N-1}})] = V - c_i \mathsf{E}[L_i^k] > V - p - c_i(k+1) = U_{P,i}(k). \tag{EC.3}$$

By Lemma 1, we then have

$$\begin{aligned} \mathsf{E}[U_{R,i}(k+1\,;\bar{\boldsymbol{x}}_{\boldsymbol{i,N-1}})] = V - c_i \mathsf{E}[L_i^{k+1}] &\geq V - c_i \mathsf{E}[L_i^k] \\ &> V - p - c_i(k+1) \\ &> V - p - c_i(k+2) \\ &= U_{P,i}(k+1), \end{aligned} \tag{EC.4}$$

where the inequality on the second line holds by equation (EC.3). We conclude that if it is optimal for customer i to choose the regular queue when $x_{i-1} = k$, then it is also optimal for her to choose the regular queue when $x_{i-1} = k+1$, and therefore by induction for any $k \leq x_{i-1} \leq i-1$. Because by the definition of k it is optimal for customer i to join the priority queue if $x_{i-1} < k$, we conclude that customer i's optimal strategy for waiting-cost realization c_i is the threshold strategy $\bar{x}_{i-1}^*(c_i) = k-1$. The above derivation holds for any realization of the waiting cost, so the overall optimal strategy for customer i is a cost-dependent threshold strategy $\bar{x}_{i-1}^*(C_i)$.

The outer induction hypothesis is verified in equilibrium for customer $N-1$ by equation (1): customer N optimally uses the cost-dependent threshold strategy $\bar{x}_{N-1}^*(C_N) = \lfloor N-1-p/C_N \rfloor$. The above then implies that it is also optimal for customers $i \in \{1, \ldots, N-1\}$ to use cost-dependent threshold strategies. ☐

D. Supplementary Result and Proof for Theorem 2

LEMMA EC.1. *Consider a customer $i \in \{1, \ldots, N-1\}$, and suppose that each customer $j \in \{i+1, \ldots, N\}$ uses a cost-dependent threshold strategy $\bar{x}_{j-1}(C_j)$. Given these strategies, let L_i^k (L_{i+1}^k) be the random variable for the number of services (including her own) that customer i ($i+1$) will wait through if $x_{i-1} = k$ ($x_i = k$) and customer i ($i+1$) chooses the regular queue. For $k \in \{0, \ldots, i-1\}$, we have*

$$0 \leq \mathsf{E}[L_{i+1}^k] - \mathsf{E}[L_i^k] \leq 1.$$

Proof. Consider a particular vector of realized waiting costs $(c_1, \ldots, c_N)$, and again let ℓ_i^k (ℓ_{i+1}^k) denote the realization of L_i^k (L_{i+1}^k) for these waiting costs and the corresponding thresholds. If $x_{i-1} = k$ and at least one of customers i and $i+1$ chooses the regular queue, then we will have $x_{i+1} \in \{k, k+1\}$.

Case 1: $\bar{\boldsymbol{x}}_{\boldsymbol{i}}(\boldsymbol{c}) < \boldsymbol{k}$. In this case, if $x_{i-1} = k$ and customer i chooses the regular queue, then we have $x_i = k$, and customer $i+1$ will not purchase priority because her threshold is exceeded. Let the number of priority purchases among customers $i+2, \ldots, N$ be denoted by α in this case. We have $\ell_i^k = i + \alpha$. If $x_i = k$, and if customer $i+1$ chooses the regular queue, then the number of priority purchases among customers $i+2, \ldots, N$ will also be α, so we have $\ell_{i+1}^k = i + 1 + \alpha$, and therefore

$$\ell_{i+1}^k = \ell_i^k + 1. \tag{EC.5}$$

Case 2: $\bar{\boldsymbol{x}}_{\boldsymbol{i}}(\boldsymbol{c}) \geq \boldsymbol{k}$. In this case, if $x_{i-1} = k$ and customer i chooses the regular queue, then we again have $x_i = k$, but now customer $i+1$'s strategy will prescribe priority for her because her threshold is at least k. Denote by $x_{j,m}^k$ the length of the priority queue that is observed by customer $j+1$, given that $x_m = k$, customer m chooses the regular queue, and each customer $j \in \{m+1, \ldots, N\}$ uses the threshold strategy $\bar{x}_{j-1}(c_j)$. Suppose first that $\bar{x}_j(c_{j+1}) \neq x_{j,i+1}^k$ for all $j \in \{i+2, \ldots, N\}$. By arguments analogous to Case 1 of the proof of Lemma 1, in this case the number of priority purchases among customers $i+2, \ldots, N$ will be the same with $x_{i+1} = k$ and with $x_{i+1} = k+1$. Denoting this number by α, and for ℓ_{i+1}^k letting customer $i+1$ contemplate choosing the regular queue even though the strategy $\bar{x}_i(c_i)$ prescribes priority, we have

$$\ell_i^k = i + 1 + \alpha = \ell_{i+1}^k, \tag{EC.6}$$

where $\ell_i^k = i + 1 + \alpha$ because customer i anticipates that customer $i+1$ will purchase priority, and then there will be an additional α priority purchases among customers $i+2, \ldots, N$.

If instead $\bar{x}_j(c_{j+1}) = x_{j,i+1}^k$ for at least one $j \in \{i+2, \ldots, N\}$, then an analogous argument to that in Case 2 of the proof of Lemma 1 implies that there will be one less priority purchase among customers $i+2, \ldots, N$ with $x_{i+1} = k+1$ than with $x_{i+1} = k$. Let these numbers be denoted $\alpha - 1$ and α, respectively. We then have $\ell_i^k = i + 1 + (\alpha - 1) = i + \alpha$ and $\ell_{i+1}^k = i + 1 + \alpha$, which implies

$$\ell_{i+1}^k = \ell_i^k + 1. \tag{EC.7}$$

Combining equations (EC.5), (EC.6), and (EC.7) gives

$$0 \leq \ell_{i+1}^k - \ell_i^k \leq 1,$$

and taking expectation over the waiting costs completes the proof. □

Proof of Theorem 2. For a given constant c, suppose that the equilibrium threshold for customer i is $\bar{x}_{i-1}^*(c) \geq k$, so if $x_{i-1} = k$, then in equilibrium customer i will purchase priority. We must then have

$$U_{P,i}(k) = V - p - c(k+1) \geq V - c\mathsf{E}[L_i^k] = \mathsf{E}[U_{R,i}(k\,;\, \bar{\boldsymbol{x}}^*_{\boldsymbol{i},\boldsymbol{N-1}})]. \tag{EC.8}$$

Lemma EC.1 and equation (EC.8) then imply that

$$\begin{aligned} U_{P,i+1}(k) = V - p - c(k+1) &\geq V - c\mathsf{E}[L_i^k] \\ &\geq V - c\mathsf{E}[L_{i+1}^k] \\ &= \mathsf{E}[U_{R,i+1}(k\,;\, \bar{\boldsymbol{x}}^*_{\boldsymbol{i+1},\boldsymbol{N-1}})], \end{aligned} \tag{EC.9}$$

where customer $i+1$'s comparisons are made assuming the same waiting-cost realization c. Thus, for a given k, if in equilibrium customer i purchases priority upon observing $x_{i-1}=k$, then customer $i+1$ must also purchase priority if she observes $x_i=k$. We conclude that customer $i+1$'s equilibrium threshold is at least as large as that for customer i, which in turn implies that $\bar{x}_i^*(c)\le \bar{x}_j^*(c)$ for $i<j$.

Finally, consider a given customer i and two waiting-cost realizations c and c', with $c<c'$. Suppose that $\bar{x}_{i-1}^*(c)\ge k$. Upon observing $x_{i-1}=k$, then, customer i with waiting-cost realization c will purchase priority, which implies $\mathsf{E}[U_{R,i}(k\,;\,\bar{\boldsymbol{x}}^*_{\boldsymbol{i,N-1}}\,;c)]-U_{P,i}(k;c)\le 0$. We then have

$$\begin{aligned}\mathsf{E}[U_{R,i}(k\,;\,\bar{\boldsymbol{x}}^*_{\boldsymbol{i,N-1}}\,;c')]-U_{P,i}(k;c') &= p-c'\big(\mathsf{E}[L_i^k]-(k+1)\big)\\ &< p-c\big(\mathsf{E}[L_i^k]-(k+1)\big)\\ &= \mathsf{E}[U_{R,i}(k\,;\,\bar{\boldsymbol{x}}^*_{\boldsymbol{i,N-1}}\,;c)]-U_{P,i}(k;c)\\ &\le 0.\end{aligned}$$

Therefore, for customer i, for any priority queue length such that with waiting cost c she will purchase priority, she will also purchase priority with waiting cost $c'>c$ for the same queue length. We conclude that the corresponding thresholds must satisfy $\bar{x}_{i-1}^*(c)\le\bar{x}_{i-1}^*(c')$. ☐

E. Proofs of Lemma 2 and Theorem 3

First, it is important to note that Lemma 1 applies to the compensation model as well as the base model because it holds for any cost-dependent threshold strategies for customers $j\in\{i+1,\dots,N\}$, independent of how these thresholds were determined. Theorem 3 also depends on Lemma 2.

Proof of Lemma 2. Under the cost-dependent threshold strategies $\bar{\boldsymbol{x}}_{\boldsymbol{i,N-1}}$, let A^k denote the random number of priority purchases among customers $j\in\{i+1,\dots N\}$ if $x_{i-1}=k$ and customer i chooses the regular queue. By equation (3), we have

$$g_i^\gamma(k)=\gamma\frac{p(k+A^k)}{N-(k+A^k)}\qquad\text{and}\qquad g_i^\gamma(k+1)=\gamma\frac{p(k+1+A^{k+1})}{N-(k+1+A^{k+1})}.$$

Let α^k denote a realization of the random variable A^k for a given vector of realized waiting costs. It follows from the proof of Lemma 1, for any vector $(c_1,\dots,c_N)$ of waiting-cost realizations, we have $\alpha^{k+1}\ge\alpha^k-1$, which implies

$$\gamma\frac{p(k+\alpha^k)}{N-(k+\alpha^k)}\le\gamma\frac{p(k+1+\alpha^{k+1})}{N-(k+1+\alpha^{k+1})}.$$

Taking expectation over the waiting costs gives $\mathsf{E}[g_i^\gamma(k)]\le\mathsf{E}[g_i^\gamma(k+1)]$, as desired. ☐

Proof of Theorem 3. The proof uses a similar approach to that of Theorem 1. Consider a customer $i\in\{1,\dots,N-1\}$, and suppose that all customers $j\in\{i+1,\dots,N\}$ use some cost-dependent threshold strategies $\bar{x}_{j-1}(C_j)$. Fix a waiting-cost realization c_i for customer i. Given the cost-dependent threshold strategies $\bar{x}_{j-1}(C_j)$ for customers $j\in\{i+1,\dots,N\}$, let $0\le k\le i-1$ be the smallest integer such that, if $x_{i-1}=k$, then it is optimal for customer i to stay in the regular queue. We note that, by definition, it is optimal for customer i to purchase priority if $x_{i-1}<k$. The cases with $k=i$ and $k=i-1$ trivially imply a threshold strategy, as in the proof of Theorem 1. We proceed to the case with $k\le i-2$.

As in Theorem 1, let L_i^k denote the random number of services (including her own) that customer i will wait through if $x_{i-1}=k$ and she chooses the regular queue. Taking expectation over the remaining customers' waiting costs, the assumption that the regular queue is optimal for customer i if $x_{i-1}=k$ implies

$$\mathsf{E}[U_{R,i}(k\,;\bar{\boldsymbol{x}}_{\boldsymbol{i},\boldsymbol{N-1}})] = V + \mathsf{E}[g_i^\gamma(k)] - c_i\mathsf{E}[L_i^k] > V - p - c_i(k+1) = U_{P,i}(k). \tag{EC.10}$$

Lemmas 1 and 2 then imply

$$\begin{aligned}\mathsf{E}[U_{R,i}(k+1\,;\bar{\boldsymbol{x}}_{\boldsymbol{i},\boldsymbol{N-1}})] = V + \mathsf{E}[g_i^\gamma(k+1)] - c_i\mathsf{E}[L_i^{k+1}] &\geq V + \mathsf{E}[g_i^\gamma(k)] - c_i\mathsf{E}[L_i^k]\\ &> V - p - c_i(k+1)\\ &> V - p - c_i(k+2)\\ &= U_{P,i}(k+1),\end{aligned} \tag{EC.11}$$

where the inequality on the second line holds by equation (EC.10).

The same logic as in Theorem 1—with the outer induction hypothesis verified for $i=N-1$ by equation (4) (or its analog for $\gamma<1$)—then implies that it is optimal for all customers to use cost-dependent threshold strategies. □

F. Supplementary Result and Proof for Theorem 4

As with Lemma 1, we note that Lemma EC.1 also applies to the compensation model because it does not depend on how the threshold strategies are determined. We also need an additional lemma for Theorem 4.

LEMMA EC.2. *Consider a customer $i\in\{1,\dots,N-1\}$, and suppose that each customer $j\in\{i+1,\dots,N\}$ uses a cost-dependent threshold strategy $\bar{x}_{j-1}(C_j)$. Under these strategies for the other customers, let A_i^k (A_{i+1}^k) denote the random number of priority purchases among customers $j\in\{i+2,\dots N\}$ if customer i ($i+1$) observes $x_{i-1}=k$ ($x_i=k$) and chooses the regular queue. Also, let $g_i^\gamma(k)$ ($g_{i+1}^\gamma(k)$) be the compensation that customer i ($i+1$) receives by choosing the regular queue after observing $x_{i-1}=k$ ($x_i=k$), for compensation fraction γ. For $k\in\{0,\dots,i-1\}$, we have*

$$\mathsf{E}[g_{i+1}^\gamma(k)] \leq \mathsf{E}[g_i^\gamma(k)].$$

Proof. Let α_i^k (α_{i+1}^k) denote a realization of the random variable A_i^k (A_{i+1}^k) for a given vector of realized waiting costs (and note that we are using A_i^k and A_{i+1}^k to both cover the same customers $i+2,\dots,N$, different from A^k in the proof of Lemma 2). From the proof of Lemma EC.1, for any vector $(c_1,\dots,c_N)$ of waiting-cost realizations, we have $\alpha_i^k\geq\alpha_{i+1}^k-1$.

Case 1: $\bar{\boldsymbol{x}}_{\boldsymbol{i}}(\boldsymbol{c_{i+1}})<\boldsymbol{k}$. In this case, customer $i+1$ will not purchase priority if $x_i=k$, we will have $\alpha_i^k=\alpha_{i+1}^k$, and both customers will receive the same compensation in the respective scenario, i.e., we have

$$g_{i+1}^\gamma(k) = \gamma\frac{p(k+\alpha_{i+1}^k)}{N-(k+\alpha_{i+1}^k)} = \gamma\frac{p(k+\alpha_i^k)}{N-(k+\alpha_i^k)} = g_i^\gamma(k).$$

Case 2: $\bar{\boldsymbol{x}}_{\boldsymbol{i}}(\boldsymbol{c_{i+1}})\geq\boldsymbol{k}$. In this case, customer $i+1$ will purchase priority upon observing $x_i=k$. By arguments in the proof of Lemma EC.1, we will either have $\alpha_{i+1}^k=\alpha_i^k$, or $\alpha_{i+1}^k=\alpha_i^k+1$. If $\alpha_i^k=\alpha_{i+1}^k$, then because customer $i+1$'s strategy prescribes priority if $x_i=k$, customer i will receive one more customer's worth of compensation from choosing regular with $x_{i-1}=k$ than would customer $i+1$ from choosing regular with $x_i=k$, so we have

$$g_{i+1}^{\gamma}(k)=\frac{p(k+\alpha_{i+1}^k)}{N-(k+\alpha_{i+1}^k)}<\frac{p(k+1+\alpha_{i+1}^k)}{N-(k+1+\alpha_{i+1}^k)}=\frac{p(k+1+\alpha_i^k)}{N-(k+1+\alpha_i^k)}=g_i^{\gamma}(k).$$

If instead $\alpha_{i+1}^k=\alpha_i^k+1$, then we have

$$g_{i+1}^{\gamma}(k)=\frac{p(k+\alpha_{i+1}^k)}{N-(k+\alpha_{i+1}^k)}=\frac{p(k+1+\alpha_i^k)}{N-(k+1+\alpha_i^k)}=g_i^{\gamma}(k),$$

where the last equality holds because for customer i's calculations, customer $i+1$ will purchase priority if $x_i=k$ by the assumption of this case, so after customer i there will be α_i^k+1 priority purchases in total.

We conclude that for any waiting-cost realizations and their corresponding thresholds, we have $g_{i+1}^{\gamma}(k)\le g_i^{\gamma}(k)$. Taking expectation over the waiting costs completes the proof. □

Proof of Theorem 4. For a given constant c and compensation fraction γ, suppose that the equilibrium threshold for customer $i+1$ is $\bar{x}_i^*(c)<k\le i-1$, so if $x_i=k$, then in equilibrium customer $i+1$ will *not* purchase priority. We must then have

$$U_{P,i+1}(k)=V-p-c(k+1)<V+\mathsf{E}[g_{i+1}^{\gamma}(k)]-c\mathsf{E}[L_{i+1}^k]=\mathsf{E}[U_{R,i+1}(k\,;\,\boldsymbol{\bar{x}^*_{i+1,N-1}})]. \tag{EC.12}$$

Lemmas EC.1 and EC.2 and equation (EC.12) then imply that

$$\begin{aligned}U_{P,i}(k)=V-p-c(k+1)&<V+\mathsf{E}[g_{i+1}^{\gamma}(k)]-c\mathsf{E}[L_{i+1}^k]\\&\le V+\mathsf{E}[g_i^{\gamma}(k)]-c\mathsf{E}[L_i^k]\\&=\mathsf{E}[U_{R,i}(k\,;\,\boldsymbol{\bar{x}^*_{i,N-1}})],\end{aligned}$$

where customer $i+1$'s comparisons are made assuming the same waiting-cost realization c. Thus, if in equilibrium customer $i+1$ chooses the regular queue upon observing $x_i=k$, then it must also be that customer i chooses the regular queue if she observes $x_{i-1}=k$. Put another way, there does not exist a queue length k such that customer i will purchase priority if $x_{i-1}=k$ but customer $i+1$ will choose the regular queue if $x_i=k$, for the same waiting-cost realization. We conclude that customer $i+1$'s equilibrium threshold is at least as large as that for customer i, which in turn implies that $\bar{x}_i^*(c)\le\bar{x}_j^*(c)$ for $i<j$.

Finally, consider a given customer i and two waiting-cost realizations c and c', with $c<c'$. Suppose that $\bar{x}_i^*(c)\ge k$. Upon observing $x_{i-1}=k$, then, customer i with waiting-cost realization c will purchase priority, which implies $\mathsf{E}[U_{R,i}(k\,;\,\boldsymbol{\bar{x}^*_{i,N-1}}\,;c)]-U_{P,i}(k;c)\le 0$. We then have

$$\begin{aligned}\mathsf{E}[U_{R,i}(k\,;\,\boldsymbol{\bar{x}^*_{i,N-1}}\,;c')]-U_{P,i}(k;c')&=p+\mathsf{E}[g_i^{\gamma}(k)]-c'\big(\mathsf{E}[L_i^k]-(k+1)\big)\\&<p+\mathsf{E}[g_i^{\gamma}(k)]-c\big(\mathsf{E}[L_i^k]-(k+1)\big)\\&=\mathsf{E}[U_{R,i}(k\,;\,\boldsymbol{\bar{x}^*_{i,N-1}}\,;c)]-U_{P,i}(k;c)\\&\le 0.\end{aligned}$$

Therefore, for customer i, for any priority queue length such that with waiting cost c she will purchase priority, she will also purchase priority with waiting cost $c'>c$ for the same queue length. We conclude that the equilibrium threshold functions must satisfy $\bar{x}_{i-1}^*(c)\le\bar{x}_{i-1}^*(c')$. □

G. Supplementary Result and Proof for Theorem 5

Lemma EC.3. *Take* $i \in \{1, \ldots, N-1\}$*, and for customers* $i+1, \ldots, N$*, consider two vectors of cost-dependent threshold functions,* $\boldsymbol{\bar{x}_{i,N}}$ *and* $\boldsymbol{\bar{x}'_{i,N}}$*, with elements* $\bar{x}_j(C_j)$ *and* $\bar{x}'_j(C_j)$*, respectively. For a sample path of realizations* $(c_{i+1}, \ldots, c_N)$*, suppose that* $\bar{x}'_j(c_j) \le \bar{x}_j(c_j)$ *for all* $j \in \{i+1, \ldots, N\}$*. Let* α_i^k $(\tilde{\alpha}_i^k)$ *be the number of priority purchases among customers* $i+1, \ldots, N$ *if* $x_{i-1} = k$*, customer* i *chooses the regular queue, and the thresholds are* $\boldsymbol{\bar{x}_{i,N}}(c_{i+1}, \ldots, c_N)$ $(\boldsymbol{\bar{x}'_{i,N}}(c_{i+1}, \ldots, c_N))$*. For* $k \in \{0, \ldots, i-1\}$*, we have*

$$\tilde{\alpha}_i^k \le \alpha_i^k.$$

Proof. Consider the thresholds $\boldsymbol{\bar{x}_{i,N}}(c_{i+1}, \ldots, c_N)$. Let x_{j-1}^k be the priority queue length observed by customer j if $x_{i-1} = k$ and customer i chooses the regular queue, given these thresholds. For some $j' \in \{i+1, \ldots, N\}$, consider also the vector of thresholds obtained from $\boldsymbol{\bar{x}_{i,N}}(c_{i+1}, \ldots, c_N)$ by reducing by 1 the threshold of customer j', from $\bar{x}_{j'-1}(c_{j'})$ to $\bar{x}_{j'-1}(c_{j'}) - 1$ (the other thresholds are the same as in the original vector). Under these modified thresholds, let $x_{j-1}^{k(-)}$ be the priority queue length observed by customer j if $x_{i-1} = k$ and customer i chooses the regular queue.

For each customer $j \in \{i+1, \ldots, j'-1\}$, we have $x_{j-1}^k = x_{j-1}^{k(-)}$, so these customers will take the same actions either way, and there will be the same number of priority purchases among these customers for either vector of thresholds. We thus have $x_{j'-1}^k = x_{j'-1}^{k(-)}$.

For customer j', then, if $x_{j'-1}^{k(-)} = x_{j'-1}^k \ne \bar{x}_{j'-1}(c_{j'})$, then either $\bar{x}_{j'-1}(c_{j'}) - 1 < \bar{x}_{j'-1}(c_{j'}) < x_{j'-1}^k = x_{j'-1}^{k(-)}$, or $x_{j'-1}^{k(-)} = x_{j'-1}^k \le \bar{x}_{j'-1}(c_{j'}) - 1 < \bar{x}_{j'-1}(c_{j'})$. In either case, customer j' takes the same action for either vector of thresholds. In this case, we will also have $x_{j-1}^k = x_{j-1}^{k(-)}$ for $j \in \{j'+1, \ldots, N\}$, so these customers also will take the same actions under either vector of thresholds. Thus, we have

$$\alpha_i^k = \tilde{\alpha}_i^k. \tag{EC.13}$$

If instead $x_{j'-1}^{k(-)} = x_{j'-1}^k = \bar{x}_{j'-1}(c_{j'})$, then customer j' purchases priority with her original threshold $\bar{x}_{j'-1}(c_{j'})$, but not with her modified threshold $\bar{x}_{j'-1}(c_{j'}) - 1$. There are two cases.

Case 1: $\boldsymbol{x_{j-1}^{k(-)} \ne \bar{x}_{j-1}(c_j)}$ **for all** $\boldsymbol{j \in \{j'+1, \ldots, N\}}$**.** In this case, because of customer i's different action, we have $x_{j'}^k = x_{j'}^{k(-)} + 1$. So, similar to the above for customer j', by the hypothesis of this case, for customer $j'+1$, we either have $x_{j'}^{k(-)} < x_{j'}^k = x_{j'}^{k(-)} + 1 \le \bar{x}_{j'}(c_{j'+1})$, or $\bar{x}_{j'}(c_{j'+1}) < x_{j'}^{k(-)} < x_{j'}^{k(-)} + 1 = x_{j'}^k$. Hence, customer $j'+1$ will take the same action under both the original threshold vector and that with the threshold for customer j' decreased by 1. By induction, we then have $x_{j-1}^k = x_{j-1}^{k(-)} + 1$ for all $j \in \{j'+2, \ldots, N\}$. Therefore, customers $j'+2, \ldots, N$ will also take the same actions under either vector by the same reasoning as for customer $j'+1$. In total, then, there is one less priority purchase among customers $j \in \{i+1, \ldots, N\}$ when customer j' has a decreased threshold, so we have

$$\alpha_i^k = \tilde{\alpha}_i^k + 1. \tag{EC.14}$$

Case 2: $\boldsymbol{x_{j''-1}^{k(-)} = \bar{x}_{j''-1}(c_{j''})}$ **for some** $\boldsymbol{j'' \in \{j'+1, \ldots, N\}}$**.** We have $x_{j-1}^k = x_{j-1}^{k(-)} + 1$ for $j \in \{j'+1, \ldots, j''\}$ by the same reasoning as in Case 1 because of customer i's different actions under the two threshold vectors. Customers $j \in \{j'+1, \ldots, j''-1\}$ will thus take the same actions under either the original or the modified threshold vectors, also by arguments in Case 1. For customer j'', we have $\bar{x}_{j''-1}(c_{j''}) = x^{k(-)}_{j''-1} < x^{k}_{j''-1}$, so customer j'' will purchase priority for the modified threshold vector (when customer j' has her threshold reduced by 1), but not for the original vector. Summarizing, other than customers j' and j'', all customers $j \in \{i+1,\dots,N\}$ will take the same action under either threshold vector. Under the original vector, customer j' will purchase priority but customer j'' will choose the regular queue, while under the modified vector, customer j' will choose the regular queue but customer j'' will purchase priority. In either case, there is exactly one priority purchase among these two customers (and no change at all for the other customers), so we conclude that in this case

$$\alpha_i^k = \tilde{\alpha}_i^k. \tag{EC.15}$$

Combining equations (EC.13), (EC.14), and (EC.15) gives $\tilde{\alpha}_i^k \le \alpha_i^k$. By induction, we can successively reduce the thresholds customer by customer and in increments of 1 until we reach $\bar{\boldsymbol{x}}'_{\boldsymbol{i+1,N}}(c_{i+1},\dots,c_N)$. Because $\tilde{\alpha}_i^k \le \alpha_i^k$ at every step of this process, we have the desired result. □

Proof of Theorem 5. Consider a customer $i \in \{1,\dots,N-1\}$, and suppose that $\bar{x}^*_{j-1,\gamma}(c_j) \le \bar{x}^*_{j-1}(c_j)$ for $j \in \{i+1,\dots,N\}$ and all c_j in the support of C_j. For customers $i+1,\dots,N$, consider a given sample path of waiting costs $(c_{i+1},\dots,c_N)$. In the base model (compensation model with compensation fraction γ), let α_i^k ($\alpha_{i,\gamma}^k$) be the number of priority purchases among customers $i+1,\dots,N$, under the equilibrium thresholds for the waiting-cost sample path $(c_{i+1},\dots,c_N)$ if $x_{i-1} = k$ and customer i chooses the regular queue. For $k \in \{0,\dots,i-1\}$, Lemma EC.3 and our hypothesis that $\bar{x}^*_{j-1,\gamma}(c_j) \le \bar{x}^*_{j-1}(c_j)$ together imply that $\alpha_{i,\gamma}^k \le \alpha_i^k$, i.e., the number of priority purchases after customer i will be weakly less with compensation than without. Let A_i^k ($A_{i,\gamma}^k$) be the random variable for the number of priority purchases after customer i in the base model (compensation model). Because $\alpha_{i,\gamma}^k \le \alpha_i^k$ on every sample path, taking expectation over the waiting costs yields

$$\mathsf{E}[A_{i,\gamma}^k] \le \mathsf{E}[A_i^k]. \tag{EC.16}$$

Moreover, since the number of services L_i^k ($L_{i,\gamma}^k$) that customer i must wait through if $x_{i-1} = k$ and she chooses the regular queue in the base model (compensation model) is equal to i plus the number of priority purchases after her, equation (EC.16) also implies

$$\mathsf{E}[L_{i,\gamma}^k] = i + \mathsf{E}[A_{i,\gamma}^k] \le i + \mathsf{E}[A_i^k] = \mathsf{E}[L_i^k]. \tag{EC.17}$$

Let $U_{P,i}(x_{i-1};C_i)$ ($U_{P,i,\gamma}(x_{i-1};C_i)$) be the utility from purchasing priority in the base model (compensation model), and similarly $U_{R,i}(x_{i-1};C_i)$ ($U_{R,i,\gamma}(x_{i-1};C_i)$) for the utility from the regular queue. Because priority customers are not compensated even in the compensation model, we have $U_{P,i,\gamma}(x_{i-1};C_i) = U_{P,i}(x_{i-1};C_i)$. Suppose that for waiting-cost realization c_i, if $x_{i-1} = k$, then in equilibrium in the base model, customer i chooses the regular queue. In this case, we must have $U_{P,i}(x_{i-1};c_i) < \mathsf{E}[U_{R,i}(x_{i-1};c_i)]$. In the compensation model, customer i's compensation in the regular queue is $g_i^\gamma(k)$, which is random but nonnegative. We have

$$\begin{aligned}
U_{P,i,\gamma}(x_{i-1};c_i) &= U_{P,i}(x_{i-1};c_i) \\
&< \mathsf{E}[U_{R,i}(x_{i-1};c_i)] \\
&= V - c_i \mathsf{E}[L_i^k] \\
&\le V - c_i \mathsf{E}[L_{i,\gamma}^k] + \mathsf{E}[g_i^\gamma(k)] = \mathsf{E}[U_{R,i,\gamma}(x_{i-1};c_i)].
\end{aligned}$$

Therefore, for any priority queue length k such that customer i will choose the regular queue in the base model, she will also choose the regular queue in the compensation model with the same waiting-cost realization, under any compensation fraction $0 < \gamma \le 1$. Under our hypothesis that $\bar{x}^*_{j-1,\gamma}(c_j) \le \bar{x}^*_{j-1}(c_j)$ for customers $j \in \{i+1, \ldots, N\}$ and all c_j in the support of C_j, this implies that also $\bar{x}^*_{i-1,\gamma}(c_i) \le \bar{x}^*_{i-1}(c_i)$ for customer i and all c_i in the support of C_i. For $\gamma = 1$, the induction hypothesis is verified for $i = N-1$ by comparing equations (1) and (4) under our assumption that $p \le \underline{c}(N-1)$. For $\gamma < 1$, the comparison requires some algebra, but it follows by the same assumption, completing the proof of the first part of the theorem.

The second part of the theorem, that a customer's threshold decreases in the compensation fraction for fixed strategies of the customers after her, follows by a related but simpler argument, which we merely sketch here for brevity. For fixed strategies of the later customers and two compensation fractions $\gamma < \gamma'$, we have $L^k_{i,\gamma} = L^k_{i,\gamma'}$. We also have $g^\gamma_i(k) \le g^{\gamma'}_i(k)$. These two relations make the regular queue more attractive as the compensation fraction increases, so the optimal threshold decreases in the compensation fraction. $\square$

H. Algorithms to Compute PBE Threshold Functions

Here, we give algorithms to calculate the PBE threshold functions for an arbitrary continuous waiting-cost distribution in both models. The analogous algorithms for discrete distributions are obtained in the natural way. The conditions in the indicator functions in the last lines of both algorithms are equivalent to $U_{P,i}(k) \le \mathsf{E}[U_{R,i}(k)]$ under the respective models. Finally, note that for fixed thresholds, L_i^k is deterministic and can be calculated easily by iteratively recording the decisions prescribed for each customer given their thresholds and determining the number of priority purchases after customer i. For each customer i, the resulting threshold function is an increasing step function in the waiting-cost realization c_i.

Algorithm 1: Compute PBE cost-dependent thresholds for base model

```
Result: Vector $\bar{\boldsymbol{x}}^*$ of threshold functions
for $i = N, N-1, \ldots, 1$ do
    for $(\bar{x}_i^m, \ldots, \bar{x}_{N-1}^m) \in \{ \times_{j=i}^{N-1} \{-1, 0, 1, \ldots, j\}\}$ do
        $\pi_m \leftarrow \prod_{k=i+1}^{N} \int \mathbb{1}\{\bar{x}_{k-1}^*(c) = \bar{x}_{k-1}^m\} \mathrm{d}F(c)$ // PBE probability of threshold vector $m$
    end
    $\bar{x}_{i-1}^*(c_i) \leftarrow -1$ for $c_i$ in support of $C_i$;
    for $k \in \{0, 1, \ldots, i-1\}$ do
        $\lambda_i^k \leftarrow \sum_m \pi_m L_i^k((\bar{x}_i^m, \ldots, \bar{x}_{N-1}^m))$ // Expected services to wait through
        $\bar{x}_{i-1}^*(c_i) \leftarrow \bar{x}_{i-1}^*(c_i) + \mathbb{1}\{c_i \ge p/(\lambda_i^k - (k+1))\}$ for $c_i$ in support of $C_i$ // If priority is preferred at current $k$, increment previous threshold
    end
end
```

Algorithm 2: Compute PBE cost-dependent thresholds for compensation model

```
Result: Vector $\bar{\boldsymbol{x}}^*$ of threshold functions
for $i = N, N-1, \ldots, 1$ do
    for $(\bar{x}_i^m, \ldots, \bar{x}_{N-1}^m) \in \{ \times_{j=i}^{N-1} \{-1, 0, 1, \ldots, j\}\}$ do
        $\pi_m \leftarrow \prod_{k=i+1}^{N} \int \mathbb{1}\{\bar{x}_{k-1}^*(c) = \bar{x}_{k-1}^m\} \mathrm{d}F(c)$ // PBE probability of threshold vector $m$
    end
    $\bar{x}_{i-1}^*(c_i) \leftarrow -1$ for $c_i$ in support of $C_i$;
    for $k \in \{0, 1, \ldots, i-1\}$ do
        $\lambda_i^k \leftarrow \sum_m \pi_m L_i^k((\bar{x}_i^m, \ldots, \bar{x}_{N-1}^m))$ // Expected services to wait through
        $\rho_i^k \leftarrow \sum_m \pi_m g_i^\gamma(k)$ // Expected compensation
        $\bar{x}_{i-1}^*(c_i) \leftarrow \bar{x}_{i-1}^*(c_i) + \mathbb{1}\{c_i \ge (p + \rho_i^k)/(\lambda_i^k - (k+1))\}$ for $c_i$ in support of $C_i$ // If priority is preferred at current $k$, increment previous threshold
    end
end
```

I. Performance Measure Definitions

In this appendix, we formally define each of the performance measures considered in Sections 6 and 7. As in Appendix H, the exposition is for a continuous waiting-cost distribution, and the corresponding quantities for a discrete distribution can be obtained in the natural way.

The starting point for calculating performance measures in equilibrium is the output from Algorithm 1 or 2, namely a vector $\bar{\boldsymbol{x}}^*$ of PBE threshold functions. Let L_i^* be the random variable for the number of services (including her own) that customer i waits through in the PBE. Furthermore, for waiting-cost realization vector $(c_1,\dots,c_N)\in\operatorname{supp}(C_1,\dots,C_N)$, let $\ell_i^*(c_1,\dots,c_N)$ be the realization for L_i^* associated with the threshold vector $\big(\bar{x}_0^*(c_1),\dots,\bar{x}_N^*(c_N)\big)$. Also, let A^* be the random variable for the total number of priority purchases in the PBE, and let $\alpha^*(c_1,\dots,c_N)$ be the realization for A^* associated with the threshold vector $\big(\bar{x}_0^*(c_1),\dots,\bar{x}_N^*(c_N)\big)$. For each $(c_1,\dots,c_N)$, both ℓ_i^* and α^* can be determined by simple bookkeeping.

Recalling that $\gamma=0$ in the base model and $\gamma>0$ in the compensation model, we have the following definitions that apply to both models.

Definition EC.1 (Aggregate Waiting Cost). The expected aggregate waiting cost C_{Agg} is

$$C_{\text{Agg}}=\mathsf{E}\Big[\sum_{i=1}^{N}C_iL_i^*\Big]=\underset{\times_{j=1}^{N}\operatorname{supp}(C_j)}{\int\cdots\int}\Big(\sum_{i=1}^{N}c_i\ell_i^*(c_1,\dots,c_N)\Big)\mathrm{d}F(c_1)\cdots\mathrm{d}F(c_N).$$

Definition EC.2 (Customer Surplus). The expected customer surplus S is

$$S=VN-C_{\text{Agg}}-p\gamma\mathsf{E}[A^*]=VN-C_{\text{Agg}}-p\gamma\underset{\times_{j=1}^{N}\operatorname{supp}(C_j)}{\int\cdots\int}\alpha^*(c_1,\dots,c_N)\mathrm{d}F(c_1)\cdots\mathrm{d}F(c_N).$$

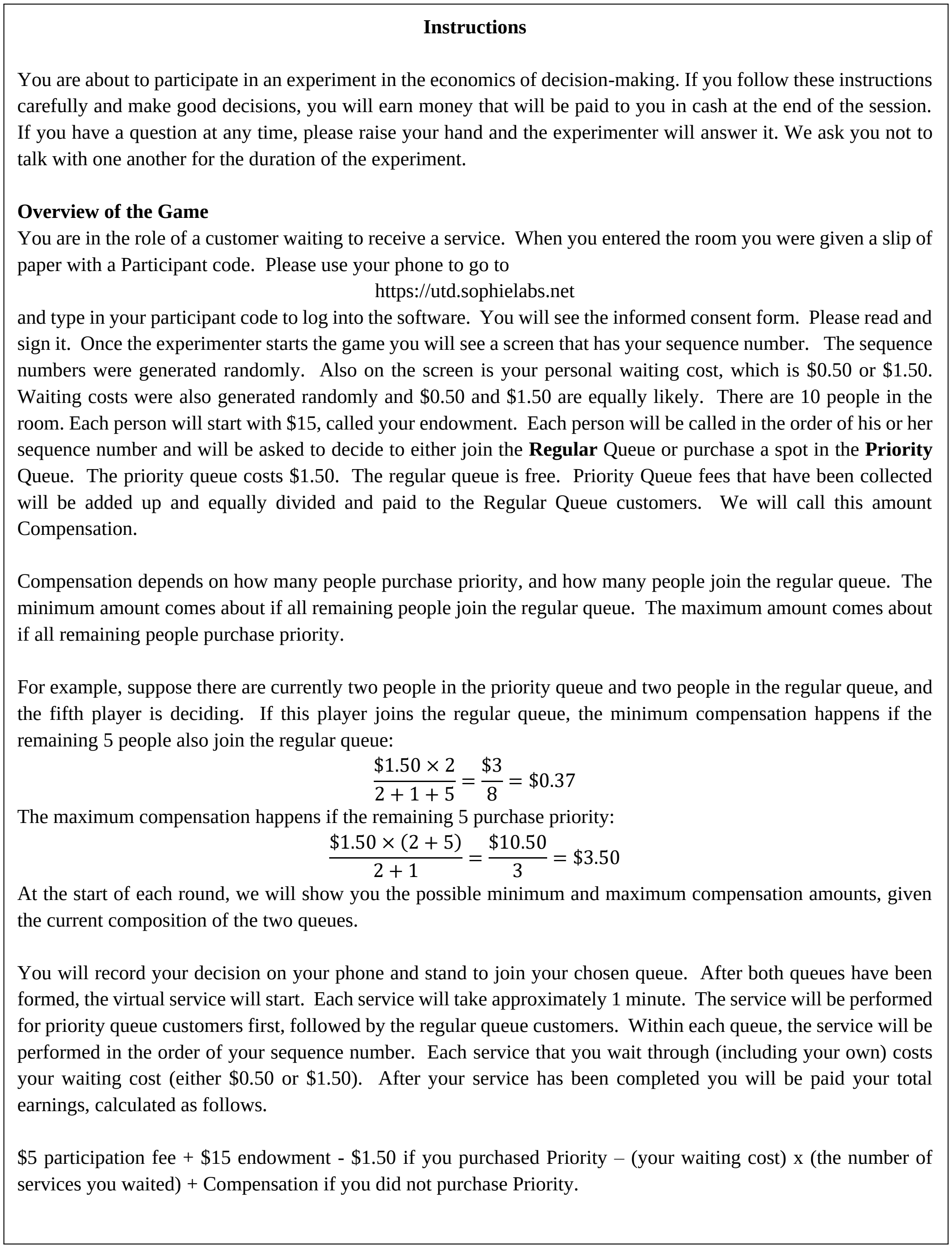
Definition EC.3 (Provider Net Revenue). The expected provider net revenue Z from priority purchases (i.e., after subtracting compensation payments) is

$$Z=p(1-\gamma)\mathsf{E}[A^*]=p(1-\gamma)\underset{\times_{j=1}^{N}\operatorname{supp}(C_j)}{\int\cdots\int}\alpha^*(c_1,\dots,c_N)\mathrm{d}F(c_1)\cdots\mathrm{d}F(c_N).$$

The versions of these measures used in Section 7 for the experiments and logit simulations are the analog of the above for sample averages: for each instance of the game, we compute the measures based on the waiting-cost realization vector and the path of play, and we then take the average across the instances. For fair comparisons, when computing equilibrium and FCFS measures in Section 7, we also use sample averages, computed with the same set of waiting-cost realization vectors as in the experiment or simulation.

J. Laboratory Instructions

The instructions below are for the compensation treatment ($\gamma = 1$) in the All-Human Study (for the sessions where minimum and maximum compensation were displayed). The instructions for the other treatments are similar to these; they are omitted due to space constraints but are available from the authors upon request.



Instructions

You are about to participate in an experiment in the economics of decision-making. If you follow these instructions carefully and make good decisions, you will earn money that will be paid to you in cash at the end of the session. If you have a question at any time, please raise your hand and the experimenter will answer it. We ask you not to talk with one another for the duration of the experiment.

Overview of the Game

You are in the role of a customer waiting to receive a service. When you entered the room you were given a slip of paper with a Participant code. Please use your phone to go to

https://utd.sophielabs.net

and type in your participant code to log into the software. You will see the informed consent form. Please read and sign it. Once the experimenter starts the game you will see a screen that has your sequence number. The sequence numbers were generated randomly. Also on the screen is your personal waiting cost, which is $0.50 or $1.50. Waiting costs were also generated randomly and $0.50 and $1.50 are equally likely. There are 10 people in the room. Each person will start with $15, called your endowment. Each person will be called in the order of his or her sequence number and will be asked to decide to either join the **Regular** Queue or purchase a spot in the **Priority** Queue. The priority queue costs $1.50. The regular queue is free. Priority Queue fees that have been collected will be added up and equally divided and paid to the Regular Queue customers. We will call this amount Compensation.

Compensation depends on how many people purchase priority, and how many people join the regular queue. The minimum amount comes about if all remaining people join the regular queue. The maximum amount comes about if all remaining people purchase priority.

For example, suppose there are currently two people in the priority queue and two people in the regular queue, and the fifth player is deciding. If this player joins the regular queue, the minimum compensation happens if the remaining 5 people also join the regular queue:

$$\frac{\$1.50 \times 2}{2+1+5} = \frac{\$3}{8} = \$0.37$$

The maximum compensation happens if the remaining 5 purchase priority:

$$\frac{\$1.50 \times (2+5)}{2+1} = \frac{\$10.50}{3} = \$3.50$$

At the start of each round, we will show you the possible minimum and maximum compensation amounts, given the current composition of the two queues.

You will record your decision on your phone and stand to join your chosen queue. After both queues have been formed, the virtual service will start. Each service will take approximately 1 minute. The service will be performed for priority queue customers first, followed by the regular queue customers. Within each queue, the service will be performed in the order of your sequence number. Each service that you wait through (including your own) costs your waiting cost (either $0.50 or $1.50). After your service has been completed you will be paid your total earnings, calculated as follows.

$5 participation fee + $15 endowment - $1.50 if you purchased Priority – (your waiting cost) x (the number of services you waited) + Compensation if you did not purchase Priority.

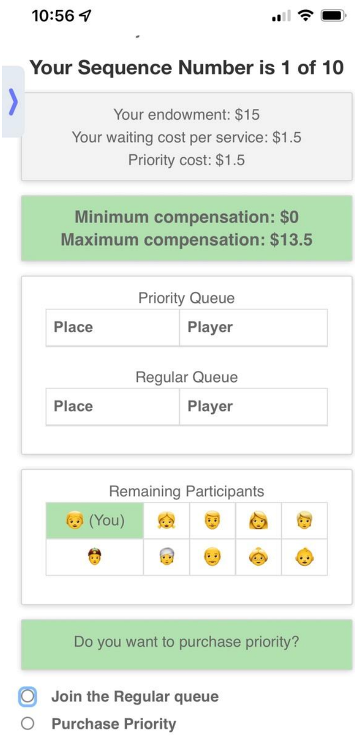
Figure EC.1 **Instructions for compensation treatment in the All-Human Study (page 1)**

Example:
Suppose your sequence number is 7 and your waiting cost is $1.50. When your turn to make the decision comes, you observe that there are 6 people in front of you, 3 in the priority queue and 3 in the regular queue. Suppose you decided to join the regular queue. Suppose that, of the 3 remaining people behind you, 2 joined the priority queue. This means that once the service starts, there will be 3+2 = 5 people in the priority queue, and 5 people (including you) in the regular queue. Out of those 5 people in the regular queue, 3 are in front of you. This means that you will wait for 5+3+1 = 9 services. Your total waiting cost will be 9 x $1.50 = $13.50. Your Compensation will be ($1.50 x 5)/5 = $1.5. Your total earnings will be: $5 + $15 - $13.50 + $1.50 = $8.00

Now suppose that you chose to pay $1.50 and join the priority queue. In this case, your total waiting cost will be $1.50 x (3+1) = $6 because you only have to wait for the 3 Priority people in front of you, and your own service. Your total earnings will be: $5 + $15 -$1.50 - $6 = $12.50

How you will be paid
As soon as your service is completed, you will be paid your earnings in cash and in private. You will remain in the room until everyone has been served. Everybody will leave the session at the same time.

Decision Screen:



Your Sequence Number is 1 of 10

Your endowment: $15
Your waiting cost per service: $1.5
Priority cost: $1.5

Minimum compensation: $0
Maximum compensation: $13.5

Priority Queue

Place	Player

Regular Queue

Place	Player

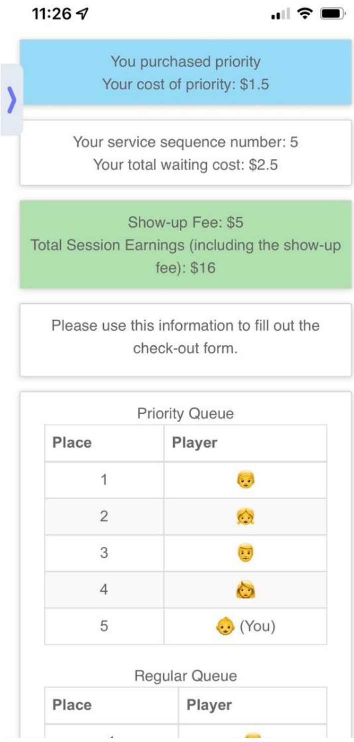
Remaining Participants

👦 (You)	👧	👨	👩	👱
👳	👵	👨	👵	👶

Do you want to purchase priority?

○ Join the Regular queue
○ Purchase Priority

Final Screen:



You purchased priority
Your cost of priority: $1.5

Your service sequence number: 5
Your total waiting cost: $2.5

Show-up Fee: $5
Total Session Earnings (including the show-up fee): $16

Please use this information to fill out the check-out form.

Priority Queue

Place	Player
1	👦
2	👧
3	👨
4	👩
5	👶 (You)

Regular Queue

Place	Player

Figure EC.2 Instructions for compensation treatment in the All-Human Study (page 2)