Online Appendix to
"Green Disposable Packaging and Communication: The Implications of Bring-Your-Own-Container"

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In this online appendix, we present the main model's analysis and proofs, and leave the analysis and proofs of the model with discount pricing and extensions to the online supplement (available in an unabridged memo at https://ssrn.com/abstract=3888378).

OA1 Benchmark Case Without BYOC

In this section, we provide the analysis about the benchmark case in the absence of BYOC. Note that this model can be applied to the following two scenarios: (i) There is no consumer considering engaging in BYOC; (ii) the firm does not accept BYOC. Under both scenarios, everyone in the market uses the firm-offered disposable packaging to buy and consume the product. As a result, we can obtain the equilibrium outcome for this benchmark case as shown in Lemma OA.1.

Lemma OA.1. *Without BYOC, there exist thresholds* $\mathcal{A}^{\circ} = \min(\frac{v-c_{\mathbf{b}}}{v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{g}}}, \frac{v-c_{\mathbf{b}}}{(1-\beta)(v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{b}})})$ *and* $\mathcal{C}^{\circ} = \beta(v+(\mu q_{\mathbf{g}}+\mu e)) + (1-\beta)c_{\mathbf{b}}$ *such that the firm's optimal strategy* ($p^{\circ}, k^{\circ}, m^{\circ}$) *is as follows.*

- *If* $\alpha \leq \mathcal{A}^{\circ}$, *then* $p^{\circ} = v$, $k^{\circ} = \mathbf{b}$ *and* $m^{\circ} = \mathbf{n}$;
- *If* $\alpha > \mathcal{A}^{\circ}$, *then*
  - *If* $c_{\mathbf{g}} \leq \mathcal{C}^{\circ}$, *then* $p^{\circ} = v + (\mu q_{\mathbf{g}} + \mu e)$, $k^{\circ} = \mathbf{g}$ *and* $m^{\circ} = \mathbf{ps}$;
  - *If* $c_{\mathbf{g}} > \mathcal{C}^{\circ}$, *then* $p^{\circ} = v + (\mu q_{\mathbf{g}} + \mu e)$, $k^{\circ} = \mathbf{b}$ *and* $m^{\circ} = \mathbf{ps}$.

First, Lemma OA.1 shows that the silent green strategy is always dominated by the other three strategies, since it causes a higher cost but does not have any positive impact on the revenue. As for the remaining three strategies, when the size of eco-conscious consumers is small (i.e., $\alpha \leq \mathcal{A}^{\circ}$), the firm does not need to make green claims (i.e., $k^{\circ} = \mathbf{b}$ and $m^{\circ} = \mathbf{n}$). When the size of eco-conscious consumers is large (i.e., $\alpha > \mathcal{A}^{\circ}$), the firm should make green claims ($m^{\circ} = \mathbf{ps}$) to charge a higher price and extract more surplus from eco-conscious consumers. Furthermore, if the cost of green packaging is large (i.e., $c_{\mathbf{g}} > \mathcal{C}^{\circ}$), the firm chooses to use the greenwashing strategy (i.e., $k^{\circ} = \mathbf{b}$ and $m^{\circ} = \mathbf{ps}$) to save the cost. Otherwise, it should offer the green packaging (i.e., $k^{\circ} = \mathbf{g}$ and $m^{\circ} = \mathbf{ps}$) to consumers.

OA2 Equilibrium Outcomes

Lemma OA.2. *For the base model where the firm accepts BYOC, there exist thresholds* A_1, A_2, C_1, C_2, H_1, H_2, E_1 *such that:*

- *If* $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$,
  - *If* $\alpha \leq A_1$, *then* $p^b = v$, $k^b = \mathbf{b}$, $m^b = \mathbf{n}$.
  - *If* $\alpha > A_1$, *then*
    - *If* $c_{\mathbf{g}} \leq C_1$, *then* $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$, $k^b = \mathbf{g}$, $m^b = \mathbf{ps}$.
    - *If* $c_{\mathbf{g}} > C_1$ *and* $h \leq H_1$, *then* $p^b = p_1$, $k^b = \mathbf{b}$, $m^b = \mathbf{n}$.
    - *If* $c_{\mathbf{g}} > C_1$ *and* $h > H_1$, *then* $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$, $k^b = \mathbf{b}$, $m^b = \mathbf{ps}$.
- *If* $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$,
  - *If* $\alpha \leq A_2$, *then* $p^b = v$, $k^b = \mathbf{b}$, $m^b = \mathbf{n}$.
  - *If* $\alpha > A_2$, *then*
    - *If* $c_{\mathbf{g}} \leq C_2$, *then* $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$, $k^b = \mathbf{g}$, $m^b = \mathbf{ps}$.
    - *If* $c_{\mathbf{g}} > C_2$ *and* $h \leq H_2$,
      - *If* $e \leq E_1$ *then* $p^b = p_1$, $k^b = \mathbf{b}$, $m^b = \mathbf{n}$.
      - *If* $e > E_1$, *then* $p^b = p_2$, $k^b = \mathbf{g}$, $m^b = \mathbf{ps}$.
    - *If* $c_{\mathbf{g}} > C_2$ *and* $h > H_2$, *then* $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$, $k^b = \mathbf{b}$, $m^b = \mathbf{ps}$.

where $A_1 = \frac{v - c_{\mathbf{b}}}{y_1 - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)}$, $A_2 = \frac{v - c_{\mathbf{b}}}{y_2 - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)}$

$C_1 = v + (\mu q_{\mathbf{g}} + \mu e) - y_3$, $C_2 = \frac{1}{1 - \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}}\left[v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}s - y_4\right]$,

$y_1 = \max(y_3, v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}})$,

$y_2 = \max(y_4, v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s))$,

$y_3 = \max(\frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}, (1 - \beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}))$,

$y_4 = \max\left(\frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}, \frac{(p_2 - s)(v + (q_{\mathbf{r}} + \mu e) - p_2)}{h}, (1 - \beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}} + \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{b}} - s))\right)$

$H_1 = \min\left\{\arg\min_{h \geq 0}\left[y_3 - (1 - \beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}})\right]\right\}$,

$H_2 = \min\left\{\arg\min_{h \geq 0}\left[y_4 - (1 - \beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}} + \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{b}} - s))\right]\right\}$,

$E_1 = \max\left\{\arg\min_{e \geq 0}\left[(p_2 - s)(v + (q_{\mathbf{r}} + \mu e) - p_2) - (p_1 - s)(v + q_{\mathbf{r}} - p_1)\right]^+\right\}$,

$p_1 = \max(v, \min(\frac{v + q_{\mathbf{r}} + s}{2}, v + q_{\mathbf{r}}))$, $p_2 = \max(v + (\mu q_{\mathbf{g}} + \mu e), \min(\frac{v + (q_{\mathbf{r}} + \mu e) + s}{2}, v + (q_{\mathbf{r}} + \mu e)))$.

Lemma OA.3. *There exists a threshold* $\mathcal{S}^{vc}$ *such that the firm accepts BYOC if and only if* $s \leq \mathcal{S}^{vc}$. *Under this condition,*

- *If* $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$*:*
  - *If* $f < F_1$ *and* $c_{\mathbf{g}} < \min(C_3, C_4)$, *then* $p^{vc} = v + (q_{\mathbf{g}} + e)$, $k = \mathbf{g}$, $m = \mathbf{pc}$.
  - *Otherwise, the firm never adopts the certification, and therefore the equilibrium outcome is given in Lemma OA.2.*
- *If* $\mu q_{\mathbf{g}} < q_{\mathbf{r}} \leq q_{\mathbf{g}}$
  - *If* $f < F_1$ *and* $c_{\mathbf{g}} < \min(C_3, C_5, C_6)$, *then* $p^{vc} = v + (q_{\mathbf{g}} + e)$, $k = \mathbf{g}$, $m = \mathbf{pc}$.

- *Otherwise, the firm never adopts the certification, and therefore the equilibrium outcome is given in Lemma OA.2.*

- *If* $q_{\mathbf{r}} > q_{\mathbf{g}}$*:*
  - *If* $f < F_2$ *and* $c_{\mathbf{g}} < \min(C_7, C_8, C_9, C_{10})$*, then* $p^{vc} = v + (q_{\mathbf{g}} + e)$*,* $k = \mathbf{g}$*,* $m = \mathbf{pc}$*.*
  - *Else if* $f < F_3$*, then* $p^{vc} = p_3$*,* $k = \mathbf{g}$*,* $m = \mathbf{pc}$*.*
  - *Otherwise, the firm never adopts the certification, and therefore the equilibrium outcome is given in Lemma OA.2.*

where $F_1 = [\alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}}) - \pi^b]^+$*,*
$C_3 = \frac{\alpha(v+(q_{\mathbf{g}}+e))-(v-c_{\mathbf{b}})-\alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s)}{\alpha}$*,* $C_4 = v + (q_{\mathbf{g}} + e) - y_3$*,*
$C_5 = v + (q_{\mathbf{g}} + e) - y_4$*,* $C_6 = s + \frac{h(1-\mu)(q_{\mathbf{g}}+e)}{q_{\mathbf{r}}-\mu q_{\mathbf{g}}}$*,*
$F_2 = [\alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s)) - \pi^b]^+$*,*
$C_7 = \frac{\alpha(v+(q_{\mathbf{g}}+e)-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}s)-(v-c_{\mathbf{b}}+\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s))}{\alpha(1-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h})}$*,* $C_8 = \frac{v+(q_{\mathbf{g}}+e)-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}s-y_4}{1-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}}$*,*
$C_9 = s + \frac{h(q_{\mathbf{g}}+e)}{q_{\mathbf{g}}}$*,* $C_{10} = \frac{(v+(q_{\mathbf{g}}+e)-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}s)-\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h}}{1-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}}$*,*
$F_3 = [\alpha\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} - \pi^b]^+$*,*
$p_3 = \max(v + (q_{\mathbf{g}} + e), \min(\frac{v+(q_{\mathbf{r}}+e)+s}{2}, v + (q_{\mathbf{r}} + e)))$*,* y_2, y_3, y_4 *are defined in Lemma OA.2, and* π^b *is the optimal expected profit in the base model.*

Lemma OA.4. *There exists a threshold* $\mathcal{S}^{mc}$ *such that the firm accepts BYOC if and only if* $s \leq \mathcal{S}^{mc}$*. Under this condition,*

- *If* $q_{\mathbf{r}} \leq q_{\mathbf{g}}$
  - *If* $f < F_4$ *and* $c_{\mathbf{g}} < \min(C_3, C_{11})$*, then* $p^{mc} = v + (q_{\mathbf{g}} + e)$*,* $k^{mc} = \mathbf{g}$*,* $m^{mc} = \mathbf{pc}$*.*
  - *Otherwise,*
    - *If* $\alpha > A_3$*, then* $p^{mc} = p_1$*,* $k^{mc} = \mathbf{b}$*,* $m^{mc} = \mathbf{n}$*.*
    - *If* $\alpha < A_3$*, then* $p^{mc} = v$*,* $k^{mc} = \mathbf{b}$*,* $m^{mc} = \mathbf{n}$*.*
- *If* $q_{\mathbf{r}} > q_{\mathbf{g}}$
  - *If* $f < F_5$ *and* $c_{\mathbf{g}} < \min(C_7, C_{10}, C_{12})$*, then* $p^{mc} = v + (q_{\mathbf{g}} + e)$*,* $k^{mc} = \mathbf{g}$*,* $m^{mc} = \mathbf{pc}$*.*
  - *Else if* $f < F_6$*, then* $p^{mc} = p_3$*,* $k^{mc} = \mathbf{g}$*,* $m^{mc} = \mathbf{pc}$*.*
  - *Otherwise,*
    - *If* $\alpha > A_3$*, then* $p^{mc} = p_1$*,* $k^{mc} = \mathbf{b}$*,* $m^{mc} = \mathbf{n}$*.*
    - *If* $\alpha < A_3$*,* $p^{mc} = v$*,* $k^{mc} = \mathbf{b}$*,* $m^{mc} = \mathbf{n}$*.*

where $F_4 = [\alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}}) - y_5]^+$*,* $y_5 = \max(v - c_{\mathbf{b}} + \alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s), \alpha\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h})$*,*
$C_{11} = v + (q_{\mathbf{g}} + e) - \frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}$*,* $A_3 = \frac{v-c_{\mathbf{b}}}{[\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}-\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s)]^+}$*,*

$F_5 = [\alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}} - q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s)) - \max(\alpha\frac{(p_3 - s)(v + (q_{\mathbf{r}} + e) - s)}{h}, y_5)]^+$,
$C_{12} = \frac{v + (q_{\mathbf{g}} + e) - \frac{(q_{\mathbf{r}} - q_{\mathbf{g}})}{h} s - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}}{1 - \frac{(q_{\mathbf{r}} - q_{\mathbf{g}})}{h}}$, $F_6 = [\alpha\frac{(p_3 - s)(v + (q_{\mathbf{r}} + e) - p_3)}{h} - y_5]^+$,
and p_1 is defined in Lemma OA.2, p_3, C_3, C_7, C_{10} are defined in Lemma OA.3.

OA3 Parameter Values in the Numerical Study

The parameter values used in our numerical calculations are chosen with the intention of representing typical market settings with BYOC consumers. Here, we present evidence to support the selection of certain key parameter values.

- α: According to IBM (2020), 40% consumers are considered as purpose-driven consumers and think that sustainability is very important for them. In addition, 50% of consumers rank sustainability as a top 5 value driver as their purchase criteria (Simon-Kucher & Partners, 2021). Specifically, as for the consumer goods, 25% of consumers think that sustainability is extremely important for them in their purchase decision (Simon-Kucher & Partners, 2021). It is also reported that 55% of U.S. consumers are extremely or very concerned about the environmental impact of product packaging (McKinsey & Company, 2020). Therefore, we choose α to be around 0.4.

- $q_{\mathbf{b}}$, $q_{\mathbf{g}}$ and $q_{\mathbf{r}}$: If $q_{\mathbf{b}}$ is normalized to 0, we can think of $q_{\mathbf{g}}$ (resp., $q_{\mathbf{r}}$) as the relative difference between the environmental impacts of the brown disposable packaging and the green disposable packaging (resp., reusable packging). Thus, $q_k = \frac{i_{\mathbf{b}}}{i_k} - 1,\ k \in \{\mathbf{g}, \mathbf{r}\}$. Based on the real data of environmental impact we collected (i.e.,Foteinis (2020); Greenwood et al. (2021); Changwichan and Gheewala (2020); Costa Coffee (2022)), an upper bound of 3 for $q_{\mathbf{g}}$ is appropriate, while an upper bound of 7 for $q_{\mathbf{r}}$ is appropriate.

In addition, while we choose the other parameter values, we tend to make sure that BYOC consumers in equilibrium account for less than 40%. In particular, among all the numerical results, the proportion of BYOC consumers among the total population ($\alpha\frac{\min((q_{\mathbf{r}} - q_p)^+, (v + (q_{\mathbf{r}} + e_p) - p)^+, h)}{h}$ in the model without discount pricing and $\alpha\frac{\min((\xi_w w + (q_{\mathbf{r}} - q_p))^+, (v + (q_{\mathbf{r}} + e_p) + \xi_w w - p)^+, h)}{h} + (1 - \alpha)\frac{\min(w, (v + w - p)^+, h')}{h'}$ in the model with discount pricing) ranges from 5.71% to 22.67%; this aligns with real-world observations: For example, according to the public polling Hubbub and Starbucks commissioned in 2019, 17% of people use their own reusable cup every time they buy a hot drink (Packaging European, 2021). Kearney (2020) find that 20% of survey respondents have brought reusable container to restaurants in the past 12 months. Moreover, Simon-Kucher & Partners (2021) find that the fraction of consumers who have significantly changed their purchasing behavior to be sustainable varies across different countries, from 6% in Japan to 44% in Brazil.

OA4 Proofs for the Main Model

We make the following assumption to rule out unrealistic or uninteresting cases: (A1) $v > c_{\mathbf{b}} - (\mu q_{\mathbf{g}} + \mu e)$. Note that (A1) rules out the case where all strategies have a negative profit in the benchmark case without BYOC.

Proof of Lemma OA.1: When there is no BYOC, the firm has four potential strategies to choose from:

($\circ$.1) Silent brown strategy ($k = \mathbf{b}, m = \mathbf{n}$). Under this strategy, both types of consumers have the same utility: $u_c = u_u = v - p$. Then, the firm sets price at $p = v$ and the corresponding profit is $\pi(\circ.1) = v - c_{\mathbf{b}}$.

($\circ$.2) Vocal green strategy ($k = \mathbf{g}, m = \mathbf{ps}$). Under this strategy, we know $u_c = v + (\mu q_{\mathbf{g}} + \mu e) - p$ and $u_u = v - p$. Then, the firm will set price at $p = v + (\mu q_{\mathbf{g}} + \mu e)$ and the profit is $\pi(\circ.2) = \alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}})$. Note that if the firm sets price at $p = v$, it is always dominated by the silent brown strategy.

($\circ$.3) Greenwashing strategy ($k = \mathbf{b}, m = \mathbf{ps}$). Similar to vocal green strategy, the firm sets price at $p = v + (\mu q_{\mathbf{g}} + \mu e)$ and the profit is $\pi(\circ.3) = (1 - \beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}})$.

($\circ$.4) Silent green strategy ($k = \mathbf{g}, m = \mathbf{n}$). Under this strategy, we know $u_c = u_u = v - p$. The firm sets price at $p = v$ and the profit is $\pi = v - c_{\mathbf{g}}$, which is dominated by the silent brown strategy.

Let $\mathcal{C}^\circ = \beta(v + (\mu q_{\mathbf{g}} + \mu e)) + (1 - \beta)c_{\mathbf{b}}$ and $\mathcal{A}^\circ = \min(\frac{v - c_{\mathbf{b}}}{\max(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}, 0)}, \frac{v - c_{\mathbf{b}}}{(1-\beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}})})$. Then, we can conclude that:

- If $\alpha \leq \mathcal{A}^\circ$, the firm should choose the silent brown strategy and set price at $p = v$.
- If $\alpha > \mathcal{A}^\circ$,
  - If $c_{\mathbf{g}} \leq \mathcal{C}^\circ$, the firm should choose the vocal green strategy and set price at $p = v + (\mu q_{\mathbf{g}} + \mu e)$.
  - If $c_{\mathbf{g}} > \mathcal{C}^\circ$, the firm should choose the greenwashing strategy and set price at $p = v + (\mu q_{\mathbf{g}} + \mu e)$.

Then, we can arrive at Lemma OA.1. Note that the optimal expected profit in this benchmark case can be expressed as $\pi^\circ = \max(\pi(\circ.1), \pi(\circ.2), \pi(\circ.3))$.

$\square$

Proof of Lemma OA.2: If the firm allows BYOC, the possible optimal strategy is as follows:

- Silent brown strategy: under this strategy, a fraction $\frac{q_{\mathbf{r}}}{h}$ of eco-conscious consumers prefers the BYOC option. The firm has the following two pricing options:

  (b.1) Sell to all consumers at price $p = v$. Then, the corresponding profit is $\pi(b.1) = (1 - \alpha\frac{q_{\mathbf{r}}}{h})(v - c_{\mathbf{b}}) + \alpha\frac{q_{\mathbf{r}}}{h}(v - s) = v - c_{\mathbf{b}} + \alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)$

  (b.2) Sell only to eco-conscious consumers who use their own reusable packaging. Then, the firm's profit maximizing problem is:

$$\max\ (p - s)\alpha\frac{v + q_{\mathbf{r}} - p}{h}$$
$$s.t.\ v - p \leq 0 \Rightarrow p \geq v$$
$$v + q_{\mathbf{r}} - p \geq 0 \Rightarrow p \leq v + q_{\mathbf{r}}$$
$$v + q_{\mathbf{r}} - p \leq h \Rightarrow p > v + q_{\mathbf{r}} - h$$

  Then, the optimal price is $p = p_1 = \max(v, \min(\frac{v + q_{\mathbf{r}} + s}{2}, v + q_{\mathbf{r}}))$ and the corresponding profit is $\pi(b.2) = (p_1 - s)\alpha\frac{v + q_{\mathbf{r}} - p_1}{h}$.

Note that $\pi(b.2) > \pi(b.1)$ only if $v - q_{\mathbf{r}} < s < v + q_{\mathbf{r}}$.

- Vocal green strategy: note that eco-conscious consumers might engage in BYOC behavior only if $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$. Then, if $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, it is the same as the benchmark case without BYOC. If $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, the firm has two pricing options:
  - (b.3) Sell to all eco-conscious consumers at price $p = v + (\mu q_{\mathbf{g}} + \mu e)$. Then, a fraction $\frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}$ of eco-conscious consumers will choose to engage in BYOC, and the corresponding profit is $\pi(b.3) = \pi = \alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s))$
  - (b.4) Sell only to eco-conscious consumers who choose to use their own reusable packaging. Then, the firm's profit maximizing problem is:

$$\begin{aligned} \max\ & (p-s)\alpha \frac{v + (q_{\mathbf{r}} + \mu e) - p}{h} \\ s.t.\ & v + (\mu q_{\mathbf{g}} + \mu e) - p \leq 0 \Rightarrow p \geq v + (\mu q_{\mathbf{g}} + \mu e) \\ & v + (q_{\mathbf{r}} + \mu e) - p \geq 0 \Rightarrow p \leq v + (q_{\mathbf{r}} + \mu e) \\ & v + (q_{\mathbf{r}} + \mu e) - p \leq h \Rightarrow p \geq v + (q_{\mathbf{r}} + \mu e) - h \end{aligned}$$

    Then, the optimal price is $p = p_2 = \max(v + (\mu q_{\mathbf{g}} + \mu e), \min(\frac{v + (q_{\mathbf{r}} + \mu e) + s}{2}, v + (q_{\mathbf{r}} + \mu e)))$ and the corresponding profit is $\pi(b.4) = (p_2 - s)\alpha \frac{v + (q_{\mathbf{r}} + \mu e) - p_2}{h}$.

  Note that $\pi(b.4) > \pi(b.3)$ only if $v + (2\mu q_{\mathbf{g}} + \mu e) - q_{\mathbf{r}} < s < v + (q_{\mathbf{r}} + \mu e)$

- Greenwashing strategy: note that eco-conscious consumers might engage in BYOC behavior only if $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$. Then, if $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, it is the same as the benchmark case. If $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, the firm has two pricing options:
  - (b.5) Sell to all eco-conscious consumers at price $p = v + (\mu q_{\mathbf{g}} + \mu e)$. Then, the corresponding profit is $\pi(b.5) = (1 - \beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}} + \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{b}} - s))$.
  - (b.6) Sell only to eco-conscious consumers who use their own reusable packaging. This case is always dominated by the vocal green strategy, since it does not help the firm save packaging cost but results in additional loss once greenwashing is detected.

- Silent green strategy. Note that this case is always dominated by silent brown strategy since the silent green strategy gains the same revenue as the silent brown strategy but incurs a higher packaging cost (i.e., $c_{\mathbf{g}} > c_{\mathbf{b}}$).

Next, we summarize the equilibrium outcomes.

First, if $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, the firm needs to choose from the following options: b.1, b.2, ∘.2, ∘.3. Recall that $\pi(b.2) = \alpha \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}$.

- If $h < \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{\max(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}, (1-\beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}))}$, then strategies ∘.2 and ∘.3 are dominated by strategy b.2.
  - If $\alpha > \frac{v - c_{\mathbf{b}}}{\frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h} - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)}$, then $k^b = \mathbf{b}$, $m^b = \mathbf{n}$, $p^b = \max(v, \frac{v + q_{\mathbf{r}} + s}{2})$.
  - Else, $k^b = \mathbf{b}$, $m^b = \mathbf{n}$, $p^b = v$.
- If $h \geq \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{\max(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}, (1-\beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}))}$, then strategy b.2 is dominated by strategies ∘.2 or ∘.3.
  - If $\alpha \leq \frac{v - c_{\mathbf{b}}}{\max(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}, (1-\beta)(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}})) - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)}$, then $k^b = \mathbf{b}$, $m^b = \mathbf{n}$, $p^b = v$.

- If $\alpha < \frac{v-c_{\mathbf{b}}}{\max(v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{g}},(1-\beta)(v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{b}}))-\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s)}$, then
  * If $c_{\mathbf{g}} \leq (1-\beta)c_{\mathbf{b}} + \beta(v+(\mu q_{\mathbf{g}}+\mu e))$, then $k^b = \mathbf{g}$, $m^b = \mathbf{ps}$, $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$.
  * If $c_{\mathbf{g}} > (1-\beta)c_{\mathbf{b}} + \beta(v+(\mu q_{\mathbf{g}}+\mu e))$, then $k^b = \mathbf{b}$, $m^b = \mathbf{ps}$, $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$.

Second, if $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, the firm needs to compare strategy b.1, b.2, b.3, b.4, and b.5.

Recall that $\pi(b.4) = \alpha\frac{(p_2-s)(v+(q_{\mathbf{r}}+\mu e)-p_2)}{h}$. We let $y_4 = \max\left(\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}, \frac{(p_2-s)(v+(q_{\mathbf{r}}+\mu e)-p_2)}{h}, (1-\beta)(v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{b}}+\frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}(c_{\mathbf{b}}-s))\right)$ and $y_2 = \max(y_4, v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{g}}+\frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}(c_{\mathbf{g}}-s))$.

- If $\alpha \leq \frac{v-c_{\mathbf{b}}}{y_2-\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s)}$, then $k^b = \mathbf{b}$, $m^b = \mathbf{n}$, $p^b = v$.
- Else,
  - If $c_{\mathbf{g}} \leq \frac{v+\mu q_{\mathbf{g}}-\frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}s-y_4}{1-\frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}}$ then $k^b = \mathbf{g}$, $m^b = \mathbf{ps}$, $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$.
  - Else,
    * If $h > \frac{(1-\beta)(v+(\mu q_{\mathbf{g}}+\mu e))-c_{\mathbf{b}}}{\max((p_1-s)(v+q_{\mathbf{r}}-p_1),(p_2-s)(v+(q_{\mathbf{r}}+\mu e)-p_2))-(1-\beta)(q_{\mathbf{r}}-\mu q_{\mathbf{g}})(c_{\mathbf{b}}-s)}$, then $k^b = \mathbf{b}$, $m^b = \mathbf{ps}$, $p^b = v + (\mu q_{\mathbf{g}} + \mu e)$.
    * Else,
      - If $(p_2-s)(v+(q_{\mathbf{r}}+\mu e)-p_2) > (p_1-s)(v+q_{\mathbf{r}}-p_1)$, then $k^b = \mathbf{g}$, $m^b = \mathbf{ps}$, $p^b = p_2$.
      - Else, $k^b = \mathbf{b}$, $m^b = \mathbf{n}$, $p^b = p_1$.

Summarizing the above discussion, we can arrive at Lemma OA.2. Note that the optimal expected profit in the base model can be expressed as:

$$\pi^b = \begin{cases} \max(\pi(b.1), \pi(b.2), \pi(\circ.2), \pi(\circ.3)), & \text{if } q_{\mathbf{r}} \leq \mu q_{\mathbf{g}} \\ \max(\pi(b.1), \pi(b.2), \pi(b.3), \pi(b.4), \pi(b.5)), & \text{if } q_{\mathbf{r}} > \mu q_{\mathbf{g}}. \end{cases}$$

where $\pi(\circ.2)$ and $\pi(\circ.3)$ are defined in the proof of Lemma OA.1. □

Proof of Lemma OA.3: If the firm adopts certification, there are two cases:

- if $q_{\mathbf{r}} \leq q_{\mathbf{g}}$, then no consumer chooses to engage in BYOC. And the pricing strategy is as follows:

  (vc.0) Set price at $p = v + (q_{\mathbf{g}} + e)$ to sell to all eco-conscious consumers and the corresponding profit is $\pi(vc.0) = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}}) - f$. Note this this is also the profit for certified green strategy when BYOC is not allowed.

- if $q_{\mathbf{r}} > q_{\mathbf{g}}$, then a fraction $\frac{q_{\mathbf{r}}-q_{\mathbf{g}}}{h}$ of eco-conscious consumers prefer to use their own reusable packaging. Then, the firm has two possible pricing options:

  (vc.1) Sell to all eco-conscious consumers at price $v + (q_{\mathbf{g}} + e)$. Then, the corresponding profit is $\pi(vc.1) = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s)) - f$

  (vc.2) Sell only to those eco-conscious consumers who use their own reusable packaging. Under this case, the firm's profit maximizing problem is:

$$\begin{aligned} &\max\ (p-s)\alpha\frac{v+(q_{\mathbf{r}}+e)-p}{h} - f \\ &s.t.\ v + (q_{\mathbf{g}} + e) - p \leq 0 \Rightarrow p \geq v + (q_{\mathbf{g}} + e) \\ &\quad v + (q_{\mathbf{r}} + e) - p \geq 0 \Rightarrow p \leq v + (q_{\mathbf{r}} + e) \\ &\quad v + (q_{\mathbf{r}} + e) - p \leq h \Rightarrow p \geq v + (q_{\mathbf{r}} + e) - h \end{aligned}$$

Then, the optimal price is $p = p_3 = \max(v+(q_{\mathbf{g}}+e), \min(\frac{v+(q_{\mathbf{r}}+e)+s}{2}, v+(q_{\mathbf{r}}+e)))$ and the corresponding profit is $\pi(vc.2) = \alpha\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} - f$.

When the third-party certification is available, we denote π_0 as the optimal expected profit if the firm does not allow BYOC, and π_{vc} as the optimal expected profit if the firm allows BYOC. Then, $\pi_0 = \max(\pi^\circ, \pi(vc.0))$. Furthermore, $\pi_{vc} = \max(\pi^b, \pi(vc.0))$ if $q_{\mathbf{r}} \le q_{\mathbf{g}}$; and $\pi_{vc} = \max(\pi^b, \pi(vc.1), \pi(vc.2))$ if $q_{\mathbf{r}} > q_{\mathbf{g}}$. Since π_{vc} is weakly decreasing in s and π_0 is independent of s, we can find a threshold $\mathcal{S}^{vc}$ such that $\pi_{vc} \ge \pi_0$ if and only if $s < \mathcal{S}^{vc}$.

Next, we analyze the possible optimal strategy when $s \le \mathcal{S}^{vc}$ such that the firm allows BYOC.

- If $q_{\mathbf{r}} \le \mu q_{\mathbf{g}}$, the firm will choose the optimal strategy from the following options:
  - certified green strategy (vc.1): $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = v + (q_{\mathbf{g}} + e)$. The corresponding profit is $\pi = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}}) - f$.
  - keeping the choice same as the base model. Recall that the optimal expected profit in the base model is $\pi^b = \max(\pi(b.1), \pi(b.2), \pi(\circ.2), \pi(\circ.3))$.
- If $\mu q_{\mathbf{g}} < q_{\mathbf{r}} \le q_{\mathbf{g}}$, the firm will choose the optimal strategy from the following options:
  - certified green strategy (vc.1): $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = v + (q_{\mathbf{g}} + e)$. The corresponding profit is $\pi = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}}) - f$.
  - keeping the choice same as the base model. Recall that the optimal expected profit in the base model is $\pi^b = \max(\pi(b.1), \pi(b.2), \pi(b.3), \pi(b.4), \pi(b.5))$.
- If $q_{\mathbf{r}} > q_{\mathbf{g}}$, the firm will choose the optimal strategy from the following options:
  - certified green strategy (vc.1): $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = v + (q_{\mathbf{g}} + e)$. The corresponding profit is $\pi = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s)) - f$.
  - certified green strategy (vc.2): $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = p_3 = \max(v+(q_{\mathbf{g}}+e), \min(\frac{v+(q_{\mathbf{r}}+e)+s}{2}, v+(q_{\mathbf{r}}+e)))$. The corresponding profit is $\pi = \alpha\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} - f$.
  - keeping the choice same as the base model. Recall that the optimal expected profit in the base model is $\pi^b = \max(\pi(b.1), \pi(b.2), \pi(b.3), \pi(b.4), \pi(b.5))$.

Comparing the above cases, the results are summarized in Lemma OA.3. Since the profit of vocal green strategy is strictly decreasing in f and π^b is independent of f, we first use f as the condition; furthermore, given the constraint of $f \ge 0$, we also add the $c_{\mathbf{g}}$ condition in Lemma OA.3 to ensure the f thresholds (i.e., F_1 and F_2) are greater than 0. □

Proof of Lemma OA.4: Under the regulation, both vocal green strategy and greenwashing strategy are not available. Let π_0' denotes the optimal expected profit when the firm does not allow BYOC, and π_{mc} denotes the optimal exptected profit when the firm allows BYOC. Then, $\pi_0' = \max(\pi(\circ.1), \pi(vc.0))$. Besides, $\pi_{mc} = \max(\pi(b.1), \pi(b.2), \pi(vc.0))$ if $q_{\mathbf{r}} \le q_{\mathbf{g}}$, and $\pi_{mc} = \max(\pi(b.1), \pi(b.2), \pi(vc.1), \pi(vc.2))$ if $q_{\mathbf{r}} > q_{\mathbf{g}}$. Since π_{mc} is weakly decreasing in s and π_0' is independent of s, we can find a threshold $\mathcal{S}^{mc}$ such that $\pi_{mc} \ge \pi_0'$ if and only if $s \le \mathcal{S}^{mc}$.

When $s \le S^{mc}$, the firm compares the silent brown strategy and certified green strategy.

- If $q_{\mathbf{r}} \le q_{\mathbf{g}}$, the firm will choose the optimal strategy from the following options:

- $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = v + (q_{\mathbf{g}} + e)$. The corresponding profit is $\pi = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}}) - f$.
- $k = \mathbf{b}$, $m = \mathbf{n}$, $p = v$. The corresponding profit is $\pi = v - c_{\mathbf{b}} + \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)$
- $k = \mathbf{b}$, $m = \mathbf{n}$, $p = p_1 = \max(v, \min(\frac{v+q_{\mathbf{r}}+s}{2}, v+q_{\mathbf{r}}))$. The corresponding profit is $\pi = (p_1 - s)\alpha\frac{v+q_{\mathbf{r}}-p_1}{h}$

- If $q_{\mathbf{r}} > q_{\mathbf{g}}$, the firm will chooses the optimal strategy from the following options:
  - $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = v + (q_{\mathbf{g}} + e)$. The corresponding profit is $\pi = \alpha(v + (q_{\mathbf{g}} + e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}) - f$.
  - $k = \mathbf{g}$, $m = \mathbf{pc}$, $p = p_3 = \max(v + (q_{\mathbf{g}} + e), \min(\frac{v+(q_{\mathbf{r}}+e)+s}{2}, v + (q_{\mathbf{r}} + e)))$. The corresponding profit is $\pi = \alpha\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} - f$.
  - $k = \mathbf{b}$, $m = \mathbf{n}$, $p = v$. The corresponding profit is $\pi = v - c_{\mathbf{b}} + \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)$
  - $k = \mathbf{b}$, $m = \mathbf{n}$, $p = p_1 = \max(v, \min(\frac{v+q_{\mathbf{r}}+s}{2}, v+q_{\mathbf{r}}))$. The corresponding profit is $\pi = \alpha\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}$.

Comparing the above cases, the results are summarized in Lemma OA.4. □

Proof of Proposition 1: When the firm does not allows BYOC, according to the proof of Lemma OA.1, the profit $\pi^\circ = \max(\pi(\circ.1), \pi(\circ.2), \pi(\circ.3))$ is independent of s. When the firm allows BYOC, according to the proof of Lemma OA.2, if $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, its profit is $\pi^b = \max(\pi(b.1), \pi(b.2), \pi(\circ.2), \pi(\circ.3))$; if $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, its profit is $\pi^b = \max(\pi(b.1), \pi(b.2), \pi(b.3), \pi(b.4), \pi(b.5))$. Then, π^b is weakly decreasing in s. Therefore, there exists a threshold $\mathcal{S}_1^b$ such that $\pi^b \geq \pi^\circ$ if and only if $s \leq \mathcal{S}_1^b$. We can define $Z_1(s) = [\pi^b - \pi^\circ]^+$, then we have

$$\mathcal{S}_1^b = \min\{\arg\min_s Z_1(s)\}$$

In this way, we arrive at the first part of Proposition 1. Furthermore, it is easy to find that $c_{\mathbf{b}} \leq \mathcal{S}_1^b \leq v + q_{\mathbf{r}}$ always holds. Specifically, if $s < c_{\mathbf{b}}$, it always helps the firm save the packaging cost and the firm should always allows BYOC; if $s > v + q_{\mathbf{r}}$, the firm always obtain a negative profit when allowing BYOC.

Note that π^b is also weakly decreasing in h while π° is independent of h, then Z_1 is weakly decreasing in h. Hence, we know $\frac{d\mathcal{S}_1^b}{dh} = -\frac{\partial Z_1}{\partial h} / \frac{\partial Z_1}{\partial s} \leq 0$, then $\mathcal{S}_1^b$ is weakly decreasing in h.

Next, we prove Proposition 1(i).

In the case without BYOC, if the firm does not engage in greenwashing, there are only two cases:

(i) The firm adopts silent brown strategy. According to the proof of Lemma OA.1, we have $\pi(\circ.1) = v - c_{\mathbf{b}} > (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}) = \pi(\circ.3)$. When the firm allows BYOC,
  - If $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, the profit of greenwashing strategy is still $\pi(\circ.3)$, which is smaller than silent brown strategy.
  - If $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, according to the proof of Lemma OA.2, the profit of greenwashing strategy is $\pi(b.5)$. Furthermore, we know $\pi(b.5) = (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}} + \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}(c_{\mathbf{b}} - s)) < (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}) + \alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s) < v - c_{\mathbf{b}} + \alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s) = \pi(b.1) \leq \max(\pi(b.1), \pi(b.2))$. Then, the profit of greenwashing strategy is still smaller than silent brown strategy.

  Therefore, the firm never changes to use the greenwashing strategy.

(ii) The firm adopts vocal green strategy. According to the proof of Lemma OA.1, we have $\pi(\circ.2) = \alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}) > (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}) = \pi(\circ.3)$. When the firm allows BYOC,

– If $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, the profit of greenwashing strategy is still $\pi(\circ.3)$, which is smaller than vocal green strategy.

– If $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, according to the proof of Lemma OA.2, we know $\pi(b.5) < (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}) + \alpha\frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s) < \alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}) + \alpha\frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s) = \pi(b.3) \leq \max(\pi(b.3), \pi(b.4))$. Then, the profit of greenwashing strategy is still smaller than vocal green strategy.

Then, the firm never changes to use the greenwashing strategy.

To sum up, if the firm does not engage in greenwashing when there is no BYOC, it will not engage in greenwashing when it allows BYOC. And we can conclude Proposition 1(i).

Finally, we prove Proposition 1(ii).

In the case without BYOC, if the firm chooses to engage in greenwashing strategy, then we must have $\pi(\circ.3) = (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}) > \max(v - c_{\mathbf{b}}, \alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}})) = \max(\pi(\circ.1), \pi(\circ.2))$.

When the firm allows BYOC, according to the proof of Lemma OA.2,

- If $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$: Recall that the firm is willing to accept BYOC if and only if $\max(\pi(b.1), \pi(b.2), \pi(\circ.3)) \geq \pi(\circ.3) \Leftrightarrow s \leq \mathcal{S}_1^b$, then $\mathcal{S}_1^b = +\infty$ since $\max(\pi(b.1), \pi(b.2), \pi(\circ.3)) \geq \pi(\circ.3)$ always holds. Under this condition, the firm will stop greenwashing if and only if it changes to silent brown strategy, i.e., $\max(\pi(b.1), \pi(b.2)) \geq \pi(\circ.3)$. Note that $\max(\pi(b.1), \pi(b.2))$ is slightly decreasing in s, while $\pi(\circ.3)$ is independent of s. Then, there exists a threshold $\mathcal{S}_2^b$ such that $\max(\pi(b.1), \pi(b.2)) \geq \pi(\circ.3)$ if and only if $s \geq \mathcal{S}_2^b$. Specifically, we can define $Z_2(s) = [\max(\pi(b.1), \pi(b.2)) - \pi(\circ.3)]^+$ if $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, then

$$\mathcal{S}_2^b = \min\{\arg\min_s Z_2(s)\} \quad \text{if} \quad q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$$

Therefore, $\mathcal{S}_2^b \leq \mathcal{S}_1^b$ always holds. Furthermore, similar to the proof about $Z_1(s)$, we can find that $\frac{d\mathcal{S}_2^b}{dh} = -\frac{\partial Z_2}{\partial h} / \frac{\partial Z_2}{\partial s} \leq 0$. Therefore, $\mathcal{S}_2^b$ is weakly decreasing in h.

- If $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$, then there are some eco-conscious consumers engaging in BYOC for any strategies. Under this case, the firm is willing to accept BYOC if and only if $\max(\pi(b.1), \pi(b.2), \pi(b.3), \pi(b.4), \pi(b.5)) \geq \pi(\circ.3) \Leftrightarrow s \leq \mathcal{S}_1^b$.

  Recall that (b.1) and (b.2) are silent brown strategy, (b.3) and (b.4) are vocal green strategy, and (b.5) is greenwashing strategy. Furthermore, $\max(\pi(b.1), \pi(b.3)) - \pi(b.5) = \max(v - c_{\mathbf{b}} - (1-\beta)\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{b}}) + \frac{\alpha}{h}[\beta q_{\mathbf{r}} + (1-\beta)\mu q_{\mathbf{g}}](c_{\mathbf{b}} - s), \alpha[\beta(v + (\mu q_{\mathbf{g}} + \mu e)) - c_{\mathbf{g}} + (1-\beta)c_{\mathbf{b}}] + \frac{\alpha(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}[c_{\mathbf{g}} - s - (1-\beta)(c_{\mathbf{b}} - s)])$ is strictly decreasing in s. Then, we define

$$S_1 = \min\{\arg\min_s[\max(\pi(b.1), \pi(b.3)) - \pi(\circ.3)]^+\}$$

$$S_2 = \min\{\arg\min_s[\max(\pi(b.2), \pi(b.4)) - \pi(\circ.3)]^+\}$$

$$S_3 = \min\{\arg\min_s[\pi(b.5) - \pi(\circ.3)]^+\}$$

$$S_4 = \min\{\arg\min_s[\max(\pi(b.1), \pi(b.3)) - \pi(b.5)]^+\}$$

  Therefore, $\mathcal{S}_1^b = \max(S_1, S_2, S_3)$. Note that the firm will change to use silent brown strategy or vocal green strategy if $\max(\pi(b.1), \pi(b.3)) \geq \max(\pi(b.5), \pi(\circ.3)) \Leftrightarrow s \leq \min(S_1, S_4)$.

Let $\mathcal{S}_2^b = \min(S_1, S_4)$. Therefore, the firm will stop greenwashing if $s \le \mathcal{S}_2^b$. Furthermore, we know $\mathcal{S}_2^b = \min(S_1, S_4) \le \max(S_1, S_2, S_3) = \mathcal{S}_1^b$.

We can also define $Z_2(s) = [\max(\pi(b.1), \pi(b.3)) - \max(\pi(b.5), \pi(\circ.3))]^+$ if $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$. And we can rewrite $\mathcal{S}_2^b$ in the following way: $\mathcal{S}_2^b = \min\{\arg\min_s Z_2(s)\}$ if $q_{\mathbf{r}} \le \mu q_{\mathbf{g}}$. Then, it is easy to find that Z_2 is weakly decreasing in h. As a result, we know $\frac{d\mathcal{S}_2^b}{dh} = -\frac{\partial Z_2}{\partial h} / \frac{\partial Z_2}{\partial s} \le 0$, i.e., $\mathcal{S}_2^b$ is weakly decreasing in h when $q_{\mathbf{r}} > \mu q_{\mathbf{g}}$. Therefore, we can conclude Proposition 1(ii). Furthermore, we know $\mathcal{S}_2^b$ is weakly decreasing in h.

□

Proof of Proposition 2: First, in the benchmark case without BYOC, if the firm chooses to offer brown disposable packaging, including silent brown strategy and greenwashing strategy, we know that $\varepsilon^\circ = q_{\mathbf{b}} = 0 \le \varepsilon^b$ always holds. Then, we can prove the result in Proposition 2(i).

Therefore, $\varepsilon^b < \varepsilon^\circ$ can hold only if the firm chooses vocal green strategy in the benchmark case without BYOC. According to Lemma OA.1, in the benchmark case without BYOC, the firm chooses vocal green strategy if and only if $\alpha > \mathcal{A}^\circ = \frac{v - c_{\mathbf{b}}}{v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}}$ and $c_{\mathbf{g}} < \mathcal{C}^\circ = \beta(v + (\mu q_{\mathbf{g}} + \mu e)) + (1 - \beta)c_{\mathbf{b}}$. Then, we have $\varepsilon^\circ = q_{\mathbf{g}}$.

When the firm allows BYOC, according to Proposition 1, given that the vocal green strategy chosen in the benchmark case without BYOC, the firm never changes to use greenwashing strategy, therefore, the firm will choose to either keep the vocal green strategy or change to use silent brown strategy.

- If $q_{\mathbf{r}} \le \mu q_{\mathbf{g}}$, according to the proof of Lemma OA.2, there are three possible strategies: $b.1$, $b.2$ and $\circ.2$. Note that if the firm still uses the vocal green strategy (i.e., strategy $\circ.2$), there is no BYOC and the average environmental quality has no change, i.e., $\varepsilon^b = q_{\mathbf{g}} = \varepsilon^\circ$. However, once the firm changes to use strategy $b.1$ or $b.2$, we find that $\varepsilon^b \le q_{\mathbf{r}} < q_{\mathbf{g}} = \varepsilon^\circ$ always hold. Therefore, we just need to show the condition such that the firm changes to use silent brown strategy, i.e., $\max(\pi(b.1), \pi(b.2)) > \pi(\circ.2)$.
  - If $c_{\mathbf{g}} > v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}$, which means strategy $\circ.2$ is dominated by strategy $b.2$, then the firm always changes to use silent brown strategy. Under this case, we do not need a condition over α, then we can say that the average environmental quality decrease if $\alpha < \mathcal{A}_2^b$, where $\mathcal{A}_2^b = +\infty > \mathcal{A}^\circ$.
  - Else, the firm will change to use the silent brown strategy if and only if $\pi(b.1) > \pi(\circ.2) \Leftrightarrow \alpha < \mathcal{A}_2^b$, where $\mathcal{A}_2^b = \frac{v - c_{\mathbf{b}}}{v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}} - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)} > \mathcal{A}^\circ$.

  Specifically,
  - If $v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h} \le \mathcal{C}^\circ$, then given the precondition $c_{\mathbf{g}} < \mathcal{C}^\circ$ and $\alpha > \mathcal{A}^\circ$, we know $\mathcal{A}_2^b = \frac{v - c_{\mathbf{b}}}{v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}} - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)} > \mathcal{A}^\circ$ and it is a non-empty set.
  - If $v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h} < \mathcal{C}^\circ$, then
    * If $c_{\mathbf{g}} \le v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}$, we know $\mathcal{A}_2^b = \frac{v - c_{\mathbf{b}}}{v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}} - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s)} > \mathcal{A}^\circ$ and it is a non-empty set.
    * If $v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h} < c_{\mathbf{g}} < \mathcal{C}^\circ$, then we know $\mathcal{A}_2^b = +\infty > \mathcal{A}^\circ$ and it is a non-empty set.

  In other words, the relationship between $v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(p_1 - s)(v + q_{\mathbf{r}} - p_1)}{h}$ and $\mathcal{C}^\circ$ has no impact. As long as $\mathcal{A}_2^b > \mathcal{A}^\circ$, it is a non-empty set. Based on the above analysis, we find $\mathcal{A}_2^b > \mathcal{A}^\circ$ always holds. In this way, we can conclude Proposition 2(ii.3).

- If $\mu q_{\mathbf{g}} < q_{\mathbf{r}} < q_{\mathbf{g}}$, then the firm chooses the optimal strategy from the following options: $b.1$, $b.2$, $b.3$ and $b.4$. Since there are always some consumers using their own reusable packaging under any strategies, we have $\varepsilon^b < q_{\mathbf{g}} = \varepsilon^\circ$. (Note that the reason why some consumers choose to engage in BYOC when the firm offers green disposable packaging is that they are unsure about whether the firm actually offers the green disposable and $q_{\mathbf{r}} > q_p = \mu q_{\mathbf{g}}$.)

  In this way, we can conclude Proposition 2(ii.1).

- If $q_{\mathbf{r}} \geq q_{\mathbf{g}}$, then the firm chooses the optimal strategy from the following options: $b.1$, $b.2$, $b.3$ and $b.4$. If the firm chooses strategy $b.2$ or $b.4$, i.e., selling only to consumers engaging in BYOC, then $\varepsilon^b = q_{\mathbf{r}} > \varepsilon^\circ$. If the firm chooses strategy $b.3$, then $\varepsilon^b = \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h} q_{\mathbf{r}} + (1 - \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}) q_{\mathbf{g}} > q_{\mathbf{g}}$. Therefore, we can find that the average environmental quality could decreases only if the firm changes to use strategy $b.1$.

  Note that the condition for $\pi(b.1) > \max(\pi(b.2), \pi(b.3), \pi(b.4))$ is that $\alpha < A_2$, where A_2 is defined in Lemma OA.2. Under this condition, we know $\varepsilon^b = \alpha \frac{q_{\mathbf{r}}}{h} q_{\mathbf{r}}$. Then, $\varepsilon^b < q_{\mathbf{g}} = \varepsilon^b$ if $\alpha < \frac{h}{q_{\mathbf{r}}} \cdot \frac{q_{\mathbf{g}}}{q_{\mathbf{r}}}$. Therefore, we can define $\mathcal{A}_1^b = \max(\min(\frac{hq_{\mathbf{g}}}{q_{\mathbf{r}}^2}, A_2), \mathcal{A}^\circ)$. The average environmental quality will decrease if and only if $\alpha < \mathcal{A}_1^b$.

  Furthermore, $\mathcal{A}_1^b > \mathcal{A}^\circ$ if and only if $A_2 > \mathcal{A}^\circ$ and $\frac{hq_{\mathbf{g}}}{q_{\mathbf{r}}^2} > \mathcal{A}^\circ$. Note that $A_2 > \mathcal{A}^\circ$ is equivalent to $y_2 - \frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s) < v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}}$, where y_2 is defined in Lemma OA.2 and $y_2 = \max(\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}, \frac{(p_2-s)(v+(q_{\mathbf{r}}+\mu e)-p_2)}{h}, v + (\mu q_{\mathbf{g}} + \mu e) - c_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}(c_{\mathbf{g}} - s))$. Then, we find $A_2 > \mathcal{A}^\circ$ holds if and only if $c_{\mathbf{g}} < \tilde{C}^b$, where $\tilde{C}^b = \min(v + (\mu q_{\mathbf{g}} + \mu e) - \max(\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}, \frac{(p_2-s)(v+(q_{\mathbf{r}}+\mu e)-p_2)}{h}), s + \frac{q_{\mathbf{r}}}{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}(c_{\mathbf{b}} - s))$. Similarly, $\frac{hq_{\mathbf{g}}}{q_{\mathbf{r}}^2} > \mathcal{A}^\circ = \frac{v-c_{\mathbf{b}}}{v+(\mu q_{\mathbf{g}}+\mu e)-c_{\mathbf{g}}}$ if and only if $c_{\mathbf{g}} < v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(v-c_{\mathbf{b}})q_{\mathbf{r}}^2}{hq_{\mathbf{g}}}$. Therefore, $\mathcal{A}_1^b > \mathcal{A}^\circ$ if and only if $c_{\mathbf{g}} < \mathcal{C}^b = \min(\tilde{C}^b, v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(v-c_{\mathbf{b}})q_{\mathbf{r}}^2}{hq_{\mathbf{g}}})$.

  In this way, we can conclude Proposition 2(ii.2).

To sum up, the average environmental quality will decrease if and only if either of the following condition holds:

(i) $\mu q_{\mathbf{g}} < q_{\mathbf{r}} < q_{\mathbf{g}}$

(ii) $q_{\mathbf{r}} > q_{\mathbf{g}}$, $\alpha < \mathcal{A}_1^b$ and $c_{\mathbf{g}} < \mathcal{C}^b$.

(iii) $q_{\mathbf{r}} < \mu q_{\mathbf{g}}$ and $\alpha < \mathcal{A}_2^b$.

Then, we can arrive at Proposition 2(ii).

□

Proof of Lemma 1: Note that when the firm adopts certification, its profit is strictly decreasing in f; if the firm does not adopt certification, its profit is independent of f. Therefore, $f < \mathcal{F}^{vc}$ is enough to be a sufficient and necessary condition to let the firm adopt the certified green strategy, where

$$\mathcal{F}^{vc} = \min\{\arg\min_f [\max(\pi(vc.1), \pi(vc.2)) - \pi^b]^+\}$$

However, given the constraint that $f \geq 0$, we also try to provide the additional condition such that $0 < f < \mathcal{F}^{vc}$ is a non-empty set. Specifically, according to Lemma OA.3,

(1) If $q_{\mathbf{r}} \le \mu q_{\mathbf{g}}$, $\mathcal{F}^{vc} = F_1$, which is strictly decreasing in $c_{\mathbf{g}}$. Then, $\mathcal{F}^{vc} > 0 \Leftrightarrow c_{\mathbf{g}} < \mathcal{C}_1^{vc} = \min(C_3, C_4)$, where F_1, C_3, C_4 are defined in Lemma OA.3.

(2) If $\mu q_{\mathbf{g}} < q_{\mathbf{r}} \le q_{\mathbf{g}}$, $\mathcal{F}^{vc} = F_1$, which is strictly decreasing in $c_{\mathbf{g}}$. Then, $\mathcal{F}^{vc} > 0 \Leftrightarrow c_{\mathbf{g}} < \mathcal{C}_1^{vc} = \min(C_3, C_5, C_6)$, where F_2, C_3, C_5, C_6 are defined in Lemma OA.3.

(3) If $q_{\mathbf{r}} > q_{\mathbf{g}}$: $\mathcal{F}^{vc} = \max(F_2, F_3)$, where F_2 is strictly decreasing in $c_{\mathbf{g}}$. Furthermore,

(3.1) if $\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} \le \max(y_4, v-c_{\mathbf{b}}+\alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s))$, i.e., $\pi(vc.2)|_{f=0} < \max(\pi(b.1), \pi(b.2), \pi(b.4), \pi(b.5))$, then $F_3 = 0$ always hold. Under this case, $\mathcal{F}^{vc} = F_2 > 0 \Leftrightarrow c_{\mathbf{g}} < \mathcal{C}_1^{vc} = \min(C_7, C_8, C_9)$. (Note that p_3 is defined in Lemma OA.3, and y_4 is defined in Lemma OA.2, both of which are independent of $c_{\mathbf{g}}$.)

(3.2) if $\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} > \max(y_4, v-c_{\mathbf{b}}+\alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}}-s))$, then $\mathcal{F}^{vc} = \max(F_2, F_3)$. Note that F_3 might not be decreasing in $c_{\mathbf{g}}$.

For case (1), (2), (3.1), we find $\mathcal{F}^{vc} > 0$ if and only if $c_{\mathbf{g}} < \mathcal{C}_1^{vc}$. Therefore, we add the condition $c_{\mathbf{g}} < \mathcal{C}_1^{vc}$ such that that $0 \le f < \mathcal{F}^{vc}$ is a non-empty set.

For case (3.2), we let $\mathcal{C}_1^{vc} = +\infty$, in other words, we don't provide the sufficient and necessary condition to ensure $\mathcal{F}^{vc} > 0$. However, we can find that for case (3.2), a sufficient condition such that $\mathcal{F}^{vc} > 0$ is $F_2 > 0 \Leftrightarrow c_{\mathbf{g}} < \min(C_7, C_8, C_9)$, where C_7, C_8, C_9 are defined in Lemma OA.3.

□

Proof of Proposition 3: According to Proposition 2(ii), when BYOC hurts the environment, we know the firm chooses vocal green strategy in the benchmark case without BYOC, and we have $\varepsilon^b < \varepsilon^\circ = q_{\mathbf{g}}$. Then, when the firm adopts certification (the condition to adopt certification is shown in Lemma 1),

- If $q_{\mathbf{r}} \le q_{\mathbf{g}}$, then no consumer chooses to engage in BYOC and we know the average environmental quality is $\varepsilon^{vc} = q_{\mathbf{g}}$.
- If $q_{\mathbf{r}} > q_{\mathbf{g}}$, then some consumers choose to engage in BYOC and we know $\varepsilon^{vc} > q_{\mathbf{g}}$.

Therefore, $\varepsilon^{vc} \ge q_{\mathbf{g}} = \varepsilon^\circ > \varepsilon^b$ always holds. In other words, adopting third-party certification could eliminate the negative impact of BYOC. □

Proof of Proposition 4: If $q_{\mathbf{r}} \le q_{\mathbf{g}}$, then all consumers would choose the firm's green disposable packaging when it adopts certification. Therefore, $\varepsilon^{vc} = q_{\mathbf{g}} \ge \varepsilon^b$ always hold, and we can arrive at Proposition 4 (i).

Next, we analyze the case when $q_{\mathbf{r}} > q_{\mathbf{g}}$.

- When it is profitable for the firm to adopt the certification (i.e., $c_{\mathbf{g}} < \mathcal{C}_1^{vc}$ and $f < \mathcal{F}_1^{vc}$), according to the proof of Lemma OA.3, there are two pricing strategies: (vc.1) and (vc.2). Note that the average environmental quality of strategy (vc.2) is $q_{\mathbf{r}}$, which is always greater than ε^b. Then, the average environmental quality could decrease only if the firm chooses strategy (vc.1). Specifically, it is profitable for the firm to adopt strategy (vc.1) if $\pi(vc.1) > \pi(vc.2)$, which is equivalent to $c_{\mathbf{g}} < C_{10}$, where $C_{10} = \frac{(v+(q_{\mathbf{g}}+e)-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}s)-\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h}}{1-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}}$ is defined in Lemma OA.3. Under this condition, $\varepsilon^{vc} = (1-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h})q_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}q_{\mathbf{r}} = q_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}(q_{\mathbf{r}}-q_{\mathbf{g}}) > q_{\mathbf{g}}$.

- Given the necessary condition that the firm chooses (vc.1) and $\varepsilon^{vc} = q_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}(q_{\mathbf{r}} - q_{\mathbf{g}})$, we compare ε^{b} with it. According to the proof of Lemma OA.2, in the base model, there are five possible options: (b.1), (b.2), (b.3), (b.4) and (b.5), where (b.3) and (b.4) are vocal green strategy.
  - If the firm chooses strategy (b.4), then $\varepsilon^{b} = q_{\mathbf{r}} > \varepsilon^{vc}$.
  - If the firm chooses strategy (b.3), then $\varepsilon^{b} = (1-\frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h})q_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}q_{\mathbf{r}} > (1-\frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h})q_{\mathbf{g}} + \frac{(q_{\mathbf{r}}-q_{\mathbf{g}})}{h}q_{\mathbf{r}} = \varepsilon^{vc}$.

  Therefore, if the firm chooses vocal green strategy in the base model, the average environmental quality always decrease when the firm changes to adopt the certification (i.e., $\varepsilon^{vc} < \varepsilon^{b}$). Here, we provide a sufficient condition such that the firm chooses vocal green strategy in the base model: $\pi(b.3) > \max(\pi(b.1), \pi(b.2), \pi(b.5))$. Note that $\pi(b.3) > \max(\pi(b.1), \pi(b.2), \pi(b.5))$ if and only if $c_{\mathbf{g}} < C_{13}$, where $C_{13} = \frac{1}{\alpha(1-(q_{\mathbf{r}}-\mu q_{\mathbf{g}})/h)}[\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}s) - \max(\pi(b.1), \pi(b.2), \pi(b.5))]$, and $\pi(b.1)$, $\pi(b.2)$, $\pi(b.5))$ are defined in the proof of Lemma OA.2 and are independent of $c_{\mathbf{g}}$.

Therefore, the sufficient condition for $\varepsilon^{vc} < \varepsilon^{b}$ is that (i) $q_{\mathbf{r}} > q_{\mathbf{g}}$ and (ii) the firm chooses vocal green strategy in the base model and chooses certified green strategy (vc.1) in the voluntary certification model, i.e., $c_{\mathbf{g}} < \tilde{\mathcal{C}}_2^{vc} = \min(C_{10}, C_{13})$, where C_{10} is defined in Lemma OA.3, and $C_{13} = \frac{1}{\alpha(1-(q_{\mathbf{r}}-\mu q_{\mathbf{g}})/h)}[\alpha(v + (\mu q_{\mathbf{g}} + \mu e) - \frac{(q_{\mathbf{r}}-\mu q_{\mathbf{g}})}{h}s) - \max(\pi(b.1), \pi(b.2), \pi(b.5))]$.

Furthermore, we define $\mathcal{C}_2^{vc} = \min(\tilde{\mathcal{C}}_2^{vc}, \mathcal{C}_1^{vc})$ such that $\mathcal{C}_2^{vc} \leq \mathcal{C}_1^{vc}$. Then, we arrive at Proposition 4(ii).

In addition, recall that the average environmental quality in the benchmark case without BYOC $\varepsilon^{\circ} \leq q_{\mathbf{g}}$ always holds. Then, under the condition shown in Proposition 4(ii), we know that $\varepsilon^{b} > \varepsilon^{vc} > q_{\mathbf{g}} \geq \varepsilon^{\circ}$ always holds. In other words, BYOC has a positive environmental implications in the base model under this case.

$\square$

Proof of Lemma 2: Similar to the proof of Lemma 1, we can find that it is profitable for the firm to adopt certification if and only if $f < \mathcal{F}^{mc}$. Note that $\mathcal{F}^{mc} = \max(\pi(vc.1), \pi(vc.2))|_{f=0} - \max(\pi(b.1), \pi(b.2)) \geq \max(\pi(vc.1), \pi(vc.2))|_{f=0} - \pi^{b} = \mathcal{F}^{vc}$.

In particular, according to Lemma OA.4,

(1) If $q_{\mathbf{r}} \leq \mu q_{\mathbf{g}}$, $\mathcal{F}^{mc} = F_4$, and $\mathcal{C}^{mc} = \min(C_3, C_{11}) \geq \min(C_3, C_4) = \mathcal{C}_1^{vc}$, since $C_{11} \geq C_4$.

(2) If $\mu q_{\mathbf{g}} < q_{\mathbf{r}} < q_{\mathbf{g}}$, $\mathcal{F}^{mc} = F_4$ and $\mathcal{C}^{mc} = \min(C_3, C_{11}) > \min(C_3, C_5, C_6) = \mathcal{C}_1^{vc}$, since $C_{11} \geq C_5$.

(3) If $q_{\mathbf{r}} > q_{\mathbf{g}}$, then $\mathcal{F}^{mc} = \max(F_5, F_6)$.

  (3.1) if $\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} \leq \max(\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}, v - c_{\mathbf{b}} + \alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s))$, i.e., $\pi(vc.2)|_{f=0} < \max(\pi(b.1), \pi(b.2))$, then $F_6 = 0$ always hold. Under this case, $\mathcal{F}^{mc} > 0$ if and only if $F_5 > 0 \Leftrightarrow c_{\mathbf{g}} < \mathcal{C}^{mc} = \min(C_7, C_{12})$. Then, we find $\mathcal{C}^{mc} = \min(C_7, C_{12}) \geq \min(C_7, C_8, C_9) = \mathcal{C}_1^{vc}$, since $C_{12} \geq C_8$.

  (3.2) If $\frac{(p_3-s)(v+(q_{\mathbf{r}}+e)-p_3)}{h} > \max(\frac{(p_1-s)(v+q_{\mathbf{r}}-p_1)}{h}, v - c_{\mathbf{b}} + \alpha\frac{q_{\mathbf{r}}}{h}(c_{\mathbf{b}} - s))$, then we let $\mathcal{C}^{mc} = +\infty$ and we know $\mathcal{C}^{mc} \geq \mathcal{C}_1^{vc}$

For case (1), (2), (3.1), we find $\mathcal{F}^{mc} > 0$ if and only if $c_{\mathbf{g}} < \mathcal{C}^{mc}$. Therefore, we add the condition $c_{\mathbf{g}} < \mathcal{C}^{mc}$ to satisfy the constraint that $f \geq 0$.

For case (3.2), we let $\mathcal{C}^{mc} = +\infty$, in other words, we don't provide the sufficient and necessary condition to ensure $\mathcal{F}^{mc} > 0$. However, we can find that for case (3.2), a sufficient condition such that $\mathcal{F}^{mc} > 0$ is $F_5 > 0 \Leftrightarrow c_{\mathbf{g}} < \min(C_7, C_{12})$.

To sum up, we can conclude that $\mathcal{F}^{mc} \geq \mathcal{F}^{vc}$ and $\mathcal{C}^{mc} \geq \mathcal{C}_1^{vc}$.

Furthermore, we find that $\mathcal{F}^{mc} > \mathcal{F}^{vc}$ if and only if $\pi^b > \max(\pi(b.1), \pi(b.2))$, which is equivalent to $\max(\pi(b.3), \pi(b.4), \pi(b.5)) > \max(\pi(b.1), \pi(b.2))$.

□

Proof of Proposition 5: The proof is similar to the proof of Proposition 4.

First, if $q_{\mathbf{r}} \leq q_{\mathbf{g}}$ the average environmental quality always increases (i.e., $\varepsilon^{mc} = q_{\mathbf{g}} \geq \varepsilon^{vc}$). Then, we arrive at Proposition 5 (i).

Second, if $q_{\mathbf{r}} > q_{\mathbf{g}}$, when the firm chooses to adopt certification (i.e., $c_{\mathbf{g}} < \mathcal{C}^{mc}$ and $f < \mathcal{F}^{mc}$), there are two possible strategies: (vc.1) and (vc.2). Then, we find that the average environmental quality decreases (i.e., $\varepsilon^{mc} < \varepsilon^{vc}$) only if the firm uses strategy (vc.1). Note that the firm chooses strategy (vc.1) if $\pi(vc.1) > \pi(vc.2) \Leftrightarrow c_{\mathbf{g}} < C_{10}$, where C_{10} is defined in Lemma OA.3. Under this condition, $\varepsilon^{mc} = q_{\mathbf{g}} + \frac{(q_{\mathbf{r}} - q_{\mathbf{g}})}{h}(q_{\mathbf{r}} - q_{\mathbf{g}}) > q_{\mathbf{g}}$.

Next, we analyze the case when there is no regulation, and the average environmental quality is ε^{vc}. If $f > \mathcal{F}^{vc}$, the firm will not adopt the certification and choose the same strategy as in the base model. Therefore, similar to the proof of Proposition 4, we can find that the optimal strategy is vocal green strategy (b.3 or b.4) if $\max(\pi(b.3)) > \max(\pi(b.1), \pi(b.2), \pi(b.5))$, i.e., $c_{\mathbf{g}} < C_{13}$, where C_{13} is defined in the proof of Proposition 4. Under this condition, $\varepsilon^{vc} \in \{q_{\mathbf{r}}, q_{\mathbf{g}} + \frac{(q_{\mathbf{r}} - \mu q_{\mathbf{g}})}{h}(q_{\mathbf{r}} - q_{\mathbf{g}})\} > q_{\mathbf{g}} + \frac{(q_{\mathbf{r}} - q_{\mathbf{g}})}{h}(q_{\mathbf{r}} - q_{\mathbf{g}}) = \varepsilon^{mc}$ always holds.

Note that when the firm chooses vocal green strategy in the voluntary certification model, it is also optimal for the firm to choose the same vocal green strategy in the base model without certification. Furthermore, recall that $\varepsilon^{\circ} \leq q_{\mathbf{g}}$, then we also know $\varepsilon^{vc} = \varepsilon^b > \varepsilon^{mc} > \varepsilon^{\circ}$ under this condition. That is, BYOC has positive environmental implications in the base model.

Therefore, given $q_{\mathbf{r}} > q_{\mathbf{g}}$, a sufficient condition such that the average environmental quality decreases is that the firm chooses vocal green strategy (b.3 or b.4) when there is no regulation and change to use certified green strategy (vc.1) when the regulation is introduced. That is: $f > \mathcal{F}^{vc}$ and $c_{\mathbf{g}} < \mathcal{C}_2^{vc}$, where $\mathcal{C}_2^{vc} = \min(C_{10}, C_{13}, \mathcal{C}_1^{vc})$ is defined in the proof of Proposition 4.

Note that given condition $c_{\mathbf{g}} < \mathcal{C}_2^{vc} \leq C_{13}$, we know $\max(\pi(b.3), \pi(b.4)) > \max(\pi(b.1), \pi(b.2))$. Then, we find

$$\begin{aligned}
\mathcal{F}^{vc} &= \max(\pi(vc.1), \pi(vc.2))|_{f=0} - \max(\pi(b.3), \pi(b.4)) \\
&< \max(\pi(vc.1), \pi(vc.2))|_{f=0} - \max(\pi(b.1), \pi(b.2)) \\
&= \mathcal{F}^{mc}
\end{aligned}$$

always holds.

To sum up, if $q_{\mathbf{r}} > q_{\mathbf{g}}$, when the firm adopts certification, the average environmental quality decreases if $f > \mathcal{F}^{vc}$ and $c_{\mathbf{g}} < \mathcal{C}_2^{vc}$. Then, we can conclude Proposition 5 (ii). Under this condition, we know $\varepsilon^{vc} = \varepsilon^b > \varepsilon^{mc} > \varepsilon^{\circ}$.

□

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