*Manufacturing & Service Operations Management*
manuscript MSOM-19-387 - Online Supplement

# Reference Pricing for Healthcare Services

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# Materials for Online Appendix

## Appendix A: Notations

| | |
|---|---|
| $n$ | number of providers |
| $m$ | population size of the patients |
| $c$ | treatment cost |
| $\gamma$ | patients' price sensitivity |
| $a_j$, $\eta_{ij}$ | the non-idiosyncratic, idiosyncratic payoff for patient $i$ when visiting provider $j$ |
| $o_j$ | out-of-pocket cost for the patient obtaining care at provider $j$ |
| $u_0$ | value of outside option: patient utility when not seeking treatment |
| $\overline{p}_j$ | maximum allowable price that provider $j$ can set under the fixed payment model |
| $p_j$ | price of the treatment charged by provider $j$ |
| $P$ | vector of provider prices |
| $S_j(P)$ | probability that provider $j$ is chosen by a randomly selected patient when providers price at $P$ |
| $V_j(P)$ | utility that provider $j$ gains when offering a procedure when providers price at $P$ |
| $W(P)$ | cost of the insurer when providers price at $P$ |
| $p^*$ | reference price |
| $f$ | fixed amount that patients pay under the fixed payment model |
| $\lambda$ | portion of the payment that the patient is responsible for under variable payment |

**Table 3 Key Notations**

## Appendix B: Input Parameters for Numerical Experiments

We calibrate the base-case parameters according to the knee and hip replacement surgery, for which CalPERS implemented reference pricing in 2011, using the medical and health economics literature. We normalize the size of the population of patients seeking treatment to $m = 1$. While our analytical results were obtained in the case of two providers, in the base-case scenario of the numerical study we consider $n = 10$ providers.

We set the fixed payment $f$ at \$3000. CalPERS normally imposes a 20% co-insurance ($\lambda = 0.2$) subject to a \$3000 maximum out-of-pocket. The amount a patient must pay as co-insurance would meet this maximum as soon as the procedure price exceeds \$15,000, which is virtually always the case for a hip and knee replacement surgery (Fronstin and Roebuck 2014). We set the reference price ($p^*$) at \$30,000 in the base case, consistent with the CalPERS experiment. The maximum allowable prices under the fixed-payment scheme ($\overline{p}_j$) take values within [\$20,000;\$50,000]. These values are consistent with the prices before implementation of reference pricing for CalPERS (Robinson and Brown 2013). We assign specific values of $\overline{p}_j$ so the reference price is at the $67^{th}$ percentile of the distribution of prices $\overline{p}_j, j = 1, \ldots, n$. The cost of treatment may be affected by many different factors. The Healthcare Bluebook estimates the *fair* price of knee replacement at about \$27,000, including anesthesia, postoperative care, implant or device, and hospital stay (Healthcare Bluebook 2019). Using this estimated price and considering a profit margin of 20% for the provider, we use $c = \$22{,}400$ as a base-case value. We use $u_0 = 0$ as the value of the outside option, but did further experiments evaluating the sensitivity of the outcome to $u_0$ (not presented here) and observed that the discussions in this section remain qualitatively the same.

We model the non-price attribute of provider $j$ as $a_j = \alpha + \sigma_j \beta$, where $\sigma_j \in \{-n/2, \ldots, n/2\} \setminus \{0\}$ if $n$ is even; and $\sigma_j \in \{-(n-1)/2, \ldots, (n-1)/2]$, if $n$ is odd. Parameter $\alpha$ represents the average non-price attribute and parameter $\beta$ is a measure of the level of differentiation among providers; we use $\alpha = 1000$, $\beta = 2000$ as base values. We tested the model for various values of $\gamma$ and $\delta$ and set $\gamma = 1.5$, $\delta = 1/\gamma$ for the base scenario.

| Variable | Base case | Sensitivity Analysis |
|---|---|---|
| $n$ | 10 | {2,5,10,20,50,100} |
| $m$ | 1 | |
| $f$ | 0.3 ($\times 10^4$) | $[0.2, 0.4](\times 10^4)$ |
| $\lambda$ | 0.2 | [0.1,1] |
| $p^*$ | 3 ($\times 10^4$) | $[2.24, 4](\times 10^4)$ |
| $\overline{p}$ | [2.5311, 2.6522, 2.7015, 2.7263, 2.8557, 2.9443, 3.3365, 4.2555, 4.6873, 4.8056] ($\times 10^4$) | |
| $c$ | 2.24 ($\times 10^4$) | $[0.5, p^*](\times 10^4)$ |
| $\alpha$ | 0.1 ($\times 10^4$) | |
| $\beta$ | 0.2 ($\times 10^4$) | $[0.05, 0.4](\times 10^4)$ |
| $u_0$ | 0 | [0,5] |
| $\gamma$ | 1.5 | [0.5,3] |
| $\Omega = (\omega_1, \omega_2)$ | $\{(0,0);(1/\gamma,1);(1/\gamma,0),(1/(2\gamma),0.5)\}$ | |
| $\tilde{c}$ | 0.15($\times 10^4$) | |

**Table 4 Parameter values for the numerical experiments**

The calibration of parameters $\alpha$, $\beta$, and $\gamma$ ensures that the observable characteristics of outcomes under reference pricing (e.g., fraction of providers who choose to be value-based) match the CalPERS experiment, so our numerical findings on the agents' utilities is meaningful. Finally, in the CalPERS example, the patient fixed payment under reference pricing varied within [\$0, \$3,000], where \$3,000 was the maximum yearly out-of-pocket value per patient per year. Hence, we set $\tilde{c} = \$1,500$.

## Appendix C: Extension – Reference Pricing with Variable Payment

### C.1. Equilibrium analysis

In the reference pricing system with variable cost share (RV) payment system, a reference price $p^*$ is set for the procedure. Providers select their prices.which determines whether they are "value-based" or "non-value-based". Finally, patients select a provider. All patients pay a variable payment for the portion of the payment below the reference price level and, if they choose a non-value-based provider, they also pay the portion of the price above the reference price. The patient out-of-pocket is thus $o_j = \lambda \min\{p_j, p^*\} + (p_j - p^*)^+ = \lambda p_j + (1-\lambda)(p_j - p^*)^+$ for $j = 1, 2$. Similar to the variable payment analysis, as a preliminary, we formalize a condition that guarantees the existence of a pure Nash equilibrium. In the following assumption, the notations are similar to those defined immediately preceding Assumption 1.

**Assumption 5.** $\overline{S}_j^{RV} \leq 50\%$ *for* $j = 1, 2$.

The following result explains how a given set of value-based and non-value-based providers jointly determine their prices at the Nash equilibrium.

**Theorem 5.** *Consider as given the set of non-value-based providers,* $\mathcal{N}$*. At equilibrium, the provider prices are the unique solution of the system of equations:*

$$\begin{cases} 1 - \gamma(p_j - c)(1 - S_j^{RV}(P)) = 0 & \forall j \in \mathcal{N} \\ 1 - \gamma\lambda(\hat{p}_j - c)(1 - S_j^{RV}(\hat{p}_j, p_{-j})) = 0 & \forall j \notin \mathcal{N} \\ p_j = \min\{\hat{p}_j, p^*\} & \forall j \notin \mathcal{N} \end{cases} \tag{11}$$

Proofs of the results in Appendices C.1 and C.2 are similar to the proofs of the analogous results in Sections 4.3 and 5. Detailed derivations are available upon request. We notice that value-based providers may price below the reference under RV, in contrast with reference pricing. Hence, RV helps mitigate shadow pricing.

The ensuing result helps determine the set of non-value-based providers.

**Proposition 11.** *If provider 1 is non-value-based, then provider 2 is non-value-based.*

Proposition 11 confirms the notion that providers with better non-price attributes may choose to be non-value-based, just like under reference pricing. As a result, we can use an algorithm similar to Algorithm 1 to identify the set of non-value-based providers as well as the prices in equilibrium.

Finding analytically the optimal reference price under RV is not tractable. We studied numerically the effect of varying the reference price on the insurer's objective (results omitted due to space constraints, but available upon request).

### C.2. Comparison of variable payment and RV

We now obtain some properties of the equilibrium under RV in the case of two providers.

**Lemma 5.** *As the reference price increases, the number of non-value-based providers decreases or remains constant under RV.*

Increasing the reference price motivates providers to become value-based as they can achieve improving margins and attract a higher market share.

**Proposition 12.** *Suppose that the insurer selects the parameters of the payment model such that there is at least one value-based provider in the system under RV. (i) If one provider is value-based and the other is non-value-based under RV, there exists $\lambda_j^* \in (0,1]$ such that provider $j$ prices higher under the variable payment system iff $\lambda < \lambda_j^*$ for $j = 1,2$. (ii) If both providers are value-based under RV, they price higher than or the same as under the variable payment system.*

Prices tend to be higher under RV than under the variable payment if $\lambda$ is high. The cost share has a more significant impact on patient out-of-pocket under the variable payment system as it is implemented on the total price of a procedure, which forces providers to limit prices.

**Proposition 13.** *The patient's out-of-pocket comparison between the variable payment system and RV is the same as the price comparison as stated in Proposition 12. Moreover, if $o_j^{RV} < o_j^{VP}$ for $j = 1,2$, the expected patient population utility is higher under RV than under a variable payment system at equilibrium iff the outside option is weak (i.e., $u_0$ smaller than a threshold $\hat{u}_0 > 0$). If $o_j^{RV} \geq o_j^{VP}$ for $j = 1,2$, the expected patient population utility is higher under RV than under a variable payment system at equilibrium iff the outside option is strong.*

When the outside option is strong, the patient benefit is aligned with the out-of-pocket. Since more patients choose not to seek treatment and take advantage of the outside option, the total patient utility improves. When the outside option is weaker, patients benefit more under variable payment than RV when they incur higher out-of-pocket under RV.

**Proposition 14.** *Suppose there is at least one value-based provider in the insurer's network under RV. If (i) one provider is value-based and the other is non-value-based, or (ii) both providers are value-based (either both pricing at $p^*$ or one prices at $p^*$ and the other prices below $p^*$) and the outside option is strong, then there exists $\hat{\lambda}_j \in (0,1]$ such that provider $j$ gains higher utility under the variable payment system than under RV iff $\lambda < \hat{\lambda}_j$. If both providers are value-based and price below $p^*$, the providers' utilities are the same under RV and the variable payment system.*

Provider utilities tend to be higher under RV than under the variable payment if $\lambda$ is high because prices are higher under RV then and the effect of market share is less strong. If both providers are value-based and price below $p^*$, the variable payment system is equivalent to RV.

## Appendix D: Proofs

This section provides the sketch of the proofs. Detailed derivations are available upon request.

**Proof of Proposition 1.** Under a fixed payment, when visiting provider $j$, the patient out-of-pocket $o_j = f$ is independent of $p_j$. Therefore, each provider's market share is also independent of $p_j$. It follows that the provider profit is monotonically increasing in $p_j$, hence the optimal price is $p_j = \overline{p}_j$. □

**Proof of Theorem 1.** Aksoy-Pierson et al. (2013, Lemma 4.1) show that under Assumption 1, in the VP payment system, for $\lambda \in (0,1]$, the best response price of provider $j \in N$ to any competing provider prices does not exceed $\overline{p}_j^{VP}$. Where $N$ is the set of all providers. Therefore providers only become worse off if they price above $\overline{p}_j^{VP}$. This property constructs a closed action set for providers' prices.

We have $\frac{\partial S_j^{VP}}{\partial p_j} = -\lambda\gamma S_j^{VP}(P)(1-S_j^{VP}(P))$ and therefore, $\frac{\partial V_j^{VP}}{\partial p_j} = mS_j^{VP}(P)\left(1-\gamma\lambda(p_j-c)(1-S_j^{VP}(P))\right)$, and

$$\frac{\partial^2 V_j^{VP}}{\partial p_j^2} = -m\gamma\lambda S_j^{VP}(P)(1-S_j^{VP}(P))\left(2+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)\right),$$

Where

$$2+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1) = \frac{[2-\gamma\lambda(p_j-c)](e^{u_0}+\sum_{k\neq j} e^{a_k-\gamma\lambda p_k}) + [2+\gamma\lambda(p_j-c)]e^{a_j-\gamma\lambda p_j}}{e^{u_0}+\sum_{k=1}^n e^{a_k-\gamma\lambda p_k}},$$

and the denominator is positive. The partial derivative with respect to $p_j$ of the numerator is $-\gamma\lambda(e^{u_0} + \sum_{k\neq j} e^{a_k-\gamma\lambda p_k}) - \gamma\lambda(1+\gamma\lambda(p_j-c))e^{a_j-\gamma\lambda p_j} \leq 0$. Therefore, $2+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)$ can change sign at most once, from positive to negative, as $p_j$ increases. Hence, $\frac{\partial^2 V_j^{VP}}{\partial p_j^2}$ can only change sign at most once, from negative to positive, as $p_j$ gets larger. Moreover, $\lim_{p_j\to\infty} V_j^{VP}(P) \to 0$ as a result of a disappearing market share. Thus, the provider utility is quasi-concave in $p_j$ and provider $j$ finds its optimal price (assuming all other prices are fixed) by solving for the first order condition

$$1-\gamma\lambda(p_j-c)(1-S_j^{VP}(P)) = 0. \tag{12}$$

The Debreu-Fan-Glicksberg theorem states that, if for all players (providers) the set of actions is a non-empty, convex, and compact set, and the utility of players is a continuous function which is quasi-concave on the action set, then there exists a Nash Equilibrium for the game (Debreu 1952, Fan 1952, Glicksberg 1952). Therefore, there exists a Nash equilibrium.

Based on Vives (1999), to show uniqueness we show that $\left|\frac{\partial^2 V_j^{VP}}{\partial p_j^2}\right| > \sum_{\substack{i\neq j\\ i\in N}} \left|\frac{\partial^2 V_j^{VP}}{\partial p_j \partial p_i}\right|, \quad \forall j \in N$, where

$$\left|\frac{\partial^2 V_j^{VP}}{\partial p_j^2}\right| = m\gamma\lambda S_j^{VP}(P)[1-S_j^{VP}(P)]|2+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)|,$$

and

$$\sum_{\substack{i\neq j\\ i\in N}}\left|\frac{\partial^2 V_j^{VP}}{\partial p_j \partial p_i}\right| = \sum_{\substack{i\neq j\\ i\in N}} m\gamma\lambda S_j^{VP}(P)S_i^{VP}(P)|1+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)|$$
$$= m\gamma S_j^{VP}(P)|1+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)|\sum_{i\neq j} S_i^{VP}(P)$$
$$< m\gamma S_j^{VP}(P)|1+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)|(1-S_j^{VP}(P)),$$

where the last inequality follows from observing that $S_j^{VP}(P)+\sum_{i\neq j} S_i^{VP}(P)<1$. Thus, to prove uniqueness, it remains to show that

$$|2+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)| > |1+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1)|,$$

which holds as long as $1+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1) > -1/2$. We can show that

$$1+\gamma\lambda(p_j-c)(2S_j^{VP}(P)-1) = \gamma\lambda(p_j-c)-1 = \frac{1}{1-S_j^{VP}(P)} - 1 \geq 0.$$

Observing that $\sum_{\substack{i\neq j\\ i\in N}} S_i^{VP}(P) \leq 1-S_j^{VP}(P)$, the uniqueness of the Nash equilibrium follows. Hence, the unique Nash equilibrium is the solution to the system of FOC equations. □

**Proof of Proposition 2.** We can show that $S_j^{RP}(P)$ is continuous in $p_j$. Its partial derivative with respect to $p_j$ is continuous everywhere except at $p^*$, and we have

$$\frac{\partial S_j^{RP}}{\partial p_j} = \begin{cases} 0 & \text{if } p_j < p^* \\ -\gamma(1-S_j^{RP}(P))S_j^{RP}(P) & \text{if } p_j > p^*. \end{cases} \tag{13}$$

Using (13), we have

$$\frac{\partial V_j^{RP}}{\partial p_j} = mS_j^{RP}(P) + m(p_j-c)\frac{\partial S_j^{RP}}{\partial p_j} = \begin{cases} mS_j^{RP}(P) > 0 & \text{if } p_j < p^* \\ mS_j^{RP}(P)[1-\gamma(p_j-c)(1-S_j^{RP}(P))] & \text{if } p_j > p^* \end{cases}. \tag{14}$$

Hence, provider $j$'s utility is monotonically increasing in $p_j$ over the domain $p_j \leq p^*$. □

**Proof of Theorem 2.** Using (14), we obtain that for $p_j > p^*$,

$$\frac{\partial^2 V_j^{RP}}{\partial p_j^2} = -m\gamma S_j^{RP}(P)[1-S_j^{RP}(P)]\left[2+\gamma(p_j-c)(2S_j^{RP}(P)-1)\right]. \tag{15}$$

Following the same steps illustrated in the proof of Theorem 1, we have

$$2+\gamma(p_j-c)(2S_j^{RP}(P)-1) = 2+\gamma(p_j-c)\frac{e^{a_j-\gamma(p_j-p^*+\tilde{c})} - e^{u_0} - \sum_{k\neq j} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}{e^{u_0}+\sum_{k=1}^n e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}$$
$$= \frac{[2-\gamma(p_j-c)](e^{u_0}+\sum_{k\neq j} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}) + [2+\gamma(p_j-c)]e^{a_j-\gamma(p_j-p^*+\tilde{c})}}{e^{u_0}+\sum_{k=1}^n e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}.$$

The denominator of the expression above is positive. The partial derivative with respect to $p_j$ of the numerator is

$$-\gamma(e^{u_0}+\sum_{k\neq j} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}) - \gamma(1+\gamma(p_j-c))e^{a_j-\gamma(p_j-p^*+\tilde{c})} \leq 0.$$

Therefore, $2+\gamma(p_j-c)(2S_j^{RP}(P)-1)$ can change sign at most once, from positive to negative, as $p_j$ increases. Hence, $\frac{\partial^2 V_j^{RP}}{\partial p_j^2}$ can change sign at most once, from negative to positive, as $p_j$ gets larger. Moreover, as $p_j$

approaches infinity, the provider utility $V_j^{RP}$ approaches zero as a result of a disappearing market share. Hence, the provider profit $V_j^{RP}$ is quasi-concave in $p_j$ on the domain $p_j > p^*$. In particular, a non-value-based provider $j$ finds its optimal price (assuming all other prices are fixed) by solving for the first order condition:

$$1 - \gamma(p_j - c)(1 - S_j^{RP}(P)) = 0, \tag{16}$$

under Assumption 2 and based on Aksoy-Pierson et al. (2013, Lemma 4.1). The Debreu-Fan-Glicksberg theorem states that, if for all players (providers) the set of actions is a non-empty, convex, and compact (under Assumption 2 and based on Aksoy-Pierson et al. (2013, Lemma 4.1)) set, and the utility of players is a continuous function which is quasi-concave on the action set, then there exists a Nash Equilibrium for the game (Debreu 1952, Fan 1952, Glicksberg 1952). Therefore, there exists a Nash equilibrium.

Based on Vives (1999), to show uniqueness we show that

$$\left|\frac{\partial^2 V_j^{RP}}{\partial p_j^2}\right| > \sum_{\substack{i\neq j\\ i\in N}} \left|\frac{\partial^2 V_j^{RP}}{\partial p_j \partial p_i}\right|, \quad \forall j \in \mathcal{N},$$

where, if $p_j > p^*$, from (15)

$$\left|\frac{\partial^2 V_j^{RP}}{\partial p_j^2}\right| = m\gamma S_j^{RP}(P)[1 - S_j^{RP}(P)]|2 + \gamma(p_j - c)(2S_j^{RP}(P) - 1)|.$$

Moreover, using (14) and (13), we obtain that for $i \neq j,\ i, j \in \mathcal{N}$,

$$\sum_{\substack{i\neq j\\ i\in N}} \left|\frac{\partial^2 V_j}{\partial p_j \partial p_i}\right| = \sum_{\substack{i\neq j\\ i\in N}} m\gamma S_j^{RP}(P) S_i^{RP}(P)|1 + \gamma(p_j - c)(2S_j^{RP}(P) - 1)|.$$

Observing that

$$\sum_{\substack{i\neq j\\ i\in N}} S_i^{RP}(P) \le 1 - S_j^{RP}(P),$$

the uniqueness of the Nash equilibrium thus follows. Hence, the unique Nash equilibrium is the solution to the system of FOC equations. □

**Proof of Proposition 3.** We start by showing a technical lemma.

**Lemma 6.** *Provider $j$ is value-based if and only if* $1 - \gamma(p^* - c)(1 - S_j^{RP}(p_j = p^*, P_{-j})) \le 0$.

From the proof of Proposition 2 we know that a provider's utility is monotonically increasing to the left of $p^*$, with a slope discontinuity at $p^*$. Moreover, it follows from the proof of Theorem 2 that using (14), we observe that for $p_j > p^*$, the sign of $\partial V_j^{RP}/\partial p_j$ is given by the sign of $\psi(P) \equiv 1 - \gamma(p_j - c)(1 - S_j^{RP}(P))$. Taking the derivative of this expression with respect to $p_j$ (on the domain when $p_j > p^*$), using (13), we find

$$\begin{aligned}
\frac{\partial \psi}{\partial p_j} &= -\gamma(1 - S_j^{RP}(P)) + \gamma(p_j - c)\frac{\partial S_j^{RP}}{\partial p_j} \\
&= -\gamma(1 - S_j^{RP}(P)) - \gamma^2(p_j - c)[1 - S_j^{RP}(P)]S_j^{RP}(P) \\
&= -\gamma(1 - S_j^{RP}(P))[1 + \gamma(p_j - c)S_j^{RP}(P)] \\
&\le 0,
\end{aligned}$$

where the last inequality follows from the fact that provider $j$ must price above its cost to make a profit. Thus $\psi$ is monotonically decreasing in $p_j$. Hence it takes the value 0 at most once, and if it does, it goes from being positive to being negative on the domain $p_j > p^*$. As a result, $\frac{\partial V_j^{RP}}{\partial p_j}$ also takes the value 0 at most once, and if it does, it goes from being positive to being negative on the domain $p_j > p^*$. furthermore, it is easy to observe that $\lim_{p_j \to \infty} V_j^{RP}(P) = 0$, and thus $V_j^{RP}$ is not monotonically increasing in $p_j$ on the domain $p_j > p^*$. $V_j^{RP}$ is a unimodal function of $p_j$: it increases in $p_j$ on the domain $p_j < p^*$, and it is either monotonically decreasing on the domain $p_j > p^*$ (when its derivative at $(p^*)^+$ is non-positive) or it is increasing and then decreasing on the domain $p_j > p^*$ (when its derivative at $(p^*)^+$ is positive), with a derivative equal to zero at exactly one point $p_j > p^*$. Either provider $j$ maximizes its profit by selecting $p_j = p^*$ (i.e., it is value-based); or provider $j$ maximizes its profit by selecting $p_j > p^*$ (i.e., it is non-value-based). Since $\frac{\partial V_j^{RP}}{\partial p_j}\Big|_{p_j=(p^*)^+} = mS_j^{RP}(p_j = p^*, P_{-j})\left[1-\gamma(p^*-c)\left(1-S_j^{RP}(p_j=p^*, P_{-j})\right)\right]$, the result follows. □

We now prove the result of the Proposition. Assume that provider $j$ is non-value-based. We want to show that any provider ranked $k > j$ is also non-value-based. By induction, it suffices to show that provider $j+1$ is non-value-based. We proceed by contradiction: suppose that provider $j+1$ is value-based, that is, $p_{j+1} = p^*$. Since provider $j+1$ is value-based, we have $1-\gamma(p^*-c)(1-S_{j+1}^{RP}(p_{j+1} = p^*, P_{-j})) = 1-\gamma(p^*-c)\frac{e^{u_0}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}{e^{u_0}+e^{a_{j+1}}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}$. This expression has the same sign as $\delta \equiv e^{u_0} + e^{a_{j+1}} + \sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+} - \gamma(p^*-c)\left(e^{u_0}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}\right)$. We obtain

$$\delta \geq e^{u_0} + e^{a_{j+1}} + \sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+} - \gamma(p^*-c)\left(e^{u_0}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}\right).$$

Provider $j$ is non-value-based, so $p_j > p^*$ and thus

$$\delta > e^{u_0} + e^{a_{j+1}} + \sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+} - \gamma(p_j-c)\left(e^{u_0}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}\right).$$

Provider $j$ selects its price $p_j$ such that $1-\gamma(p_j-c)(1-S_j^{RP}(P)) = 0$, that is, $\frac{e^{u_0}+\sum_k e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}{e^{u_0}+\sum_{k\neq j} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}} = \gamma(p_j-c)$. Therefore,

$$\delta > e^{u_0} + e^{a_{j+1}} + \sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+} - \frac{e^{u_0}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}{e^{u_0}+\sum_{k\neq j} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}\left(e^{u_0}+\sum_{k} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}\right).$$

Note that the ratio in the expression above can be written as $\frac{e^{u_0}+\sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}{e^{u_0}+\sum_{k\neq j} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}} = \frac{e^{a_j-\gamma(p_j-p^*+\tilde{c})^+}+e^{u_0}+\sum_{k\neq j,j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}}{e^{a_{j+1}}+e^{u_0}+\sum_{k\neq j,j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+}} < 1$, which follows from $a_j - \gamma(p_j-p^*)^+ < a_j \leq a_{j+1}$. Therefore, we have $\delta > e^{u_0} + e^{a_{j+1}} + \sum_{k\neq j+1} e^{a_k-\gamma(p_k-p^*+\tilde{c})^+} - e^{u_0} - \sum_k e^{a_k-\gamma(p_k-p^*+\tilde{c})^+} = 0$. Hence, $1-\gamma(p^*-c)(1-S_{j+1}^{RP}(p_{j+1}=p^*, P_{-j})) > 0$, which contradicts Lemma 6 given that provider $j+1$ is value-based. Therefore, provider $j+1$ is non-value-based. □

**Proof of Lemma 1.** We denote the two providers as providers 1 and 2 such that $a_1 < a_2$ (without loss of generality). The proof proceeds in 3 steps. First, we determine how changes in $p^*$ affect provider 1 when 2 is value-based. Second, we determine how changes in $p^*$ affect provider 1 when 2 is non-value-based. Third, we determine how changes in $p^*$ affect provider 2.

If provider 2 is value-based, based on Proposition 3, provider 1 can only be value-based since $a_1 < a_2$. In this interval both providers remain value-based as the reference price increases.

Second, consider the case when 2 is non-value-based. Using Lemma 6, provider 1 chooses to be value-based iff $e^{a_1-\gamma\tilde{c}} - (\gamma(p^*-c)-1)\left(e^{u_0}+e^{a_2-\gamma(p_2^{RP}(p^*)-p^*+\tilde{c})}\right) \leq 0$. Since we can show that $\partial p_2^{RP}(p^*)/\partial p^* > 0$, the left-hand side of this inequality is monotonically decreasing in $p^*$, and hence, it can change sign at most once, from positive to negative, as $p^*$ increases, while provider 2 remains non-value-based. Since $a_1 < a_2$, from Proposition 3 provider 2 remains non-value-based at least until after provider 1 changes status to value-based as $p^*$ increases.

Third, if provider 1 is non-value-based, provider 2 remains non-value-based. When $p^* > p_{11}^*$, which solves for $p^*$ in the equation $p^* = \frac{1}{\gamma}\left(1+\frac{e^{a_j-\gamma\tilde{c}}}{e^{u_0}+e^{a_{-j}-\gamma((p_{-j}-p^*)^+ +\tilde{c})}}\right)+c$ for $j=1,2$, provider 1 becomes value-based. Then from Lemma 6, provider 2 is value-based iff $1-\gamma(p^*-c)(1-S_2^{RP}(p_2,p_1)) \leq 0$ or iff $p^* \geq p_{12}^*$. For the same reason as above, provider 2 can only change status from non-value-based to value-based as $p^*$ increases. □

**Proof of Proposition 4.** From Lemma 1, as the reference price increases non-value-based providers have incentives to become value-based. We focus on the case where at least one value-based provider is present. Since the providers are ordered so that $a_1 < a_2$, from Proposition 3, provider 1 is value-based and as $p^*$ increases provider 2 has incentives to become value-based. Provider 1 is value-based and provider 2 is non-value-based when $p_{11}^* \leq p^* \leq p_{22}^*$, while both providers are value-based when $p^* \geq p_{22}^*$.

When the insurer is greedy $\omega_1 = \omega_2 = 0$, then it only considers its own cost which is increasing in $p^*$. Thus the optimal $p^*$, is $p_{11}^*$.

Taking the derivative of insurer objective (6) with respect to $p^*$, we can show that when both providers are value-based (i.e., for $p^* \geq p_{22}^*$) $\frac{\partial \Pi^{RP}}{\partial p^*} \leq 0$, since $\frac{\partial o_j^{RP}}{\partial p^*} = 0$ for $j=1,2$. On this interval the optimal reference price is $p_{22}^*$.

When provider 1 is value-based and provider 2 is non-value-based (i.e., for $p_{11}^* \leq p^* \leq p_{22}^*$), with a public non profit insurer ($\omega_1 = \frac{1}{\gamma}$ and $\omega_2 = 1$), there are two scenarios to consider.

**1.** If $(u_0 - a_2 + \frac{c}{\delta})e^{u_0} - (a_2-a_1)e^{a_1} < 0$: $\frac{\partial \Pi^{RP}}{\partial p^*} > 0$. Thus the optimal $p^*$ ($\hat{p}^*$), is $p_{22}^*$.

**2.** If $(u_0 - a_2 + \frac{c}{\delta})e^{u_0} - (a_2-a_1)e^{a_1} \geq 0$: $\frac{\partial \Pi^{RP}}{\partial p^*} < 0$. Thus the optimal $p^*$ is $p_{11}^*$. The overall optimal reference price then is $\hat{p}^* = \underset{p^*}{Argmax}\left\{\Pi^{RP}(p^*=p_{11}^*), \Pi^{RP}(p^*=p_{22}^*)\right\}$.

For general $\omega_1$ and $\omega_2$, the optimal reference price is only tractable when $\gamma\omega_1 \leq \omega_2$ and $p_{22}^* \leq \omega_1(a_2 - \gamma\tilde{c} - u_0)$. In which case, we can show that the insurer objective function can only change sign from positive to negative resulting in an optimal solution either on boundaries or an intermediate point in $(p_{11}^*, p_{22}^*)$. This intermediate point is the solution to the first order condition as stated in Proposition 4. □

**Proof of Lemma 2.** (a) Taking the derivative of provider 2's FOC with respect to $\lambda$ and using (4) and (12) we have $-\left(p_2^{VP}+\lambda\frac{\partial p_2^{VP}}{\partial\lambda}\right)+c\,(1-S_2^{VP}(P))+\left(p_1^{VP}+\lambda\frac{\partial p_1^{VP}}{\partial\lambda}\right)\frac{S_1^{VP}(P)S_2^{VP}(P)}{1-S_2^{VP}(P)}=0$. Similarly, the derivative with respect to $\lambda$ of provider 1's FOC can be calculated and replaced in the previous expression. Since $p_2^{VP} \geq c$, after simplifications, it follows that $\partial p_2^{VP}/\partial\lambda \leq 0$.

(b) Suppose that provider 2 is value-based. Then, by Proposition 2, provider 2 prices at $p^*$, and so the price is continuously increasing in $p^*$ while the provider remains value-based (based on Lemma 1). Now suppose that provider 2 is non-value-based. If provider 1 is value-based, we established in the proof of Lemma 1 that in this case $\partial p_2^{RP}/\partial p^* > 0$. We thus now focus on the case when provider 1 is also non-value-based. Taking the derivative of provider 2's FOC and its market share with respect to $p^*$, and using $\gamma(p_2 - c) = 1/(1 - S_2^{RP}(P))$ at equilibrium, it follows that $\frac{\partial p_2^{RP}}{\partial p^*} = \frac{S_2^{RP}(P)\left(1 - S_2^{RP}(P) + S_1^{RP}(P)\left(\frac{\partial p_1^{RP}}{\partial p^*} - 1\right)\right)}{1 - S_2^{RP}(P)}$. Hence, $\partial p_2^{RP}/\partial p^* > 0$. Finally, from the proof of Lemma 1, the price of a provider remains continuous as the provider transitions from non-value-based to value-based. □

**Proof of Lemma 3.** (a) Under VP, the patient out-of-pocket from visiting provider 2 is $o_2^{VP} = \lambda p_2$. Using the proof of Lemma 2, we can show that $\frac{\partial o_2^{VP}}{\partial \lambda} \geq 0$.

(b) Under RP, the patient out-of-pocket from visiting a value-based provider is $\tilde{c}$. The patient out-of-pocket from visiting non-value-based provider 2 is $o_2^{RP} = p_2 - p^* + \tilde{c}$. If provider 1 is value-based, it is easy to see that $\frac{\partial p_2}{\partial p^*} < 1$, and thus $\frac{\partial o_2^{RP}}{\partial p^*} \leq 0$. If provider 1 is non-value-based, we can show that $\frac{\partial o_2^{RP}}{\partial p^*} \leq 0$.

**Proof of Proposition 5.** Lemma 2(b) establishes that the provider prices under VP are decreasing in $\lambda$. The prices under RP are indifferent to $\lambda$. Moreover, as $\lambda$ approaches zero, $p_j^{VP}$ must become infinitely large so the FOC has a solution. Hence, it suffices to show that, as $\lambda$ approaches 1, each of the provider prices under VP, approaches a value lower than its RP counterparts. We examine three cases, depending on whether providers 1 and/or 2 are value-based or not.

**Case 1: providers** 1 **and** 2 **are value-based.** Then $p_1^{RP} = p_2^{RP} = p^*$. Using the expression for value-based providers in Lemma 6, assuming $\tilde{c} \leq c$ and assuming without loss of generality that $p_2^{VP} \geq p_1^{VP}$, $\gamma(p_2^{VP} - c) = \frac{e^{u_0} + e^{a_1 - \gamma p_1^{VP}} + e^{a_2 - \gamma p_2^{VP}}}{e^{u_0} + e^{a_1 - \gamma p_1^{VP}}} < \frac{e^{u_0} + e^{a_1 - \gamma \tilde{c}} + e^{a_2 - \gamma \tilde{c}}}{e^{u_0} + e^{a_1 - \gamma \tilde{c}}} \leq \gamma(p^* - c)$, It follows that $\gamma(p_1^{VP} - c) \leq \gamma(p_2^{VP} - c) < \gamma(p^* - c)$, and thus $p_1^{VP} \leq p_2^{VP} < p^*$.

**Case 2: providers** 1 **and** 2 **are non-value-based.** First, suppose that $p_2^{VP} < p_2^{RP}$ is already established. Let $p = p_1^{RP}$ be the equilibrium price under RP solving the FOC equation $e^{a_1 - \gamma(p - p^* + \tilde{c})} - (\gamma(p - c) - 1)\left(e^{u_0} + e^{a_2 - \gamma(p_2^{RP} - p^* + \tilde{c})}\right) = 0$. We can show that the left hand side above is decreasing in $p$. Therefore, to show that $p_1^{VP} < p_1^{RP}$, it suffices to show that this left-hand-side evaluated at $p = p_1^{VP}$ is positive. This result follows by using providers 1's expression for price at equilibrium under VP and under the assumption that $p_2^{VP} < p_2^{RP}$ and knowing that $\tilde{c} < p^*$. To complete the proof, it remains to show that $p_2^{VP} < p_2^{RP}$. We first show a preliminary lemma.

**Lemma 7.** *Let* $g(p_2) = e^{a_2 - \gamma p_2} - (\gamma(p_2 - c) - 1)\left(e^{u_0 - \gamma(p^* - \tilde{c})} + e^{a_1 - \gamma p_1^{RP}(p_2)}\right)$ *and* $f(p_2) = e^{a_2 - \gamma p_2} - (\gamma(p_2 - c) - 1)\left(e^{u_0} + e^{a_1 - \gamma p_1^{VP}(p_2)}\right)$, *where* $p_1^{RP}(p_2)$ *is provider* 1*'s best response price to provider* 2*'s price* $p_2$ *under RP and* $p_1^{VP}(p_2)$ *is provider* 1*'s best response price to provider* 2*'s price* $p_2$ *under VP. Then* $f(.)$ *and* $g(.)$ *are decreasing at* $p_2$ *solving for* $f(p_2) = 0$ *and* $g(p_2) = 0$ *respectively and for* $p_2 > c + 1/\gamma$. *Moreover, we have* $g(p_2) > f(p_2)$ $\forall p_2 > c + 1/\gamma$.

**Proof of Lemma 7.** If provider $i$ is value-based, its price is $p^*$ and the monotonicity result is clear. We thus focus on the case when provider $i$ is non-value-based. We have $g'(p_2) = -\gamma e^{a_2 - \gamma p_2} - \gamma\left(e^{u_0 - \gamma(p^* - \tilde{c})} + e^{a_1 - \gamma p_1^{RP}(p_2)}\right) + (\gamma(p_2 - c) - 1)\gamma \frac{\partial p_1^{RP}}{\partial p_2} e^{a_1 - \gamma p_1^{RP}(p_2)}$. We can show that $\frac{\partial p_1^{RP}}{\partial p_2} > 0$ using the FOC equation for provider 1. Moreover, when $g(p_2) = 0$, $\gamma(p_2 - c) - 1 = \frac{e^{a_2 - \gamma p_2}}{e^{u_0 - \gamma(p^* - \tilde{c})} + e^{a_1 - \gamma p_1^{RP}(p_2)}}$ resulting in a unique solution for $p_2$. Hence $g'(p_2)|_{p_2 = g^{-1}(0)} < 0$. Similarly, we calculate the expression for $f'(p_2)$ and show that $\frac{\partial p_1^{VP}}{\partial p_2} > 0$. Thus, $f(.)$ is decreasing at $p_2$ that solves $f(p_2) = 0$. With some algebra we can show that $g(p_2) - f(p_2) \geq 0$. That is, that $p_1^{RP}(p_2) > p_1^{VP}(p_2)$.

We now return to the proof of Proposition 12. From Lemma 7, $g(p_2^{VP}) > f(p_2^{VP}) = 0$. Because $g(.)$ is decreasing at the unique value of price that sets $g(.)$ to zero, $p_2^{VP} < p_2^{RP}$.

**Case 3: provider 2 is non-value-based, provider 1 is value-based.** Since 1 is value-based, from Lemma 6, we have $e^{a_1-\gamma\tilde{c}} + (1-\gamma(p^*-c))\left(e^{u_0} + e^{a_2-\gamma(p_2^{RP}(p^*)-p^*+\tilde{c})}\right) \leq 0$. If $p^*$ is also chosen under the variable payment for provider 1, $p_2^{RP}(p^*) > p_2^{VP}(p^*)$ and $f(p_1 = p^*) < g(p_1 = p^*) \leq 0$. Since $f(p_1 = p^*) < 0$ and $f(.)$ is decreasing at the price that sets $f(.)$ to zero, we can conclude $p_1^{VP}$ which makes $f(p_1^{VP}) = 0$ is such that $p_1^{VP} < p^*$. □

**Proof of Proposition 6.** Under the reference pricing scheme, if the provider is value-based, the patient out-of-pocket is $\tilde{c}$ and assuming $\tilde{c} \leq \lambda c$, is always less than that of the variable payment regardless of the value of $\lambda$. Let's suppose that provider 2 is non-value-based. We first consider the case when provider 1 is also non-value-based. Let $\tilde{g}(o_2) = e^{a_2-\gamma o_2} - (\gamma(o_2 + p^* - \tilde{c} - c) - 1)\left(e^{u_0} + e^{a_1-\gamma o_1^{RP}(o_2)}\right)$ and $\tilde{f}(o_2) = e^{a_2-\gamma o_2} - (\gamma(o_2 - \lambda c) - 1)\left(e^{u_0} + e^{a_1-\gamma o_1^{VP}(o_2)}\right)$, where $o_1^{RP}(o_2) = p_1^{RP}(o_2 + p^* - \tilde{c}) - p^* + \tilde{c}$ and $o_1^{VP}(o_2) = \lambda p_1^{VP}(o_2/\lambda)$. Observe that $\tilde{g}(o_2) = e^{\gamma(p^*-\tilde{c})} g(o_2 + p^* - \tilde{c})$ and $\tilde{f}(o_2)$ is similar to $f(o_2/\lambda)$ after modifying $f(.)$ with a co-insurance rate $\lambda$ not necessarily equal to 1. By Lemma 7, $\tilde{g}(.)$ and $\tilde{f}(.)$ are decreasing at the value of out-of-pocket that sets these functions equal to zero for $o_2 > c + 1/\gamma - p^* + \tilde{c}$ and $o_2 > \lambda(c + 1/\gamma)$, respectively. Following the same steps as Proposition 5 we can show that Because $\tilde{f}(.)$ is decreasing at the unique value of out-of-pocket that sets it equal to zero, and assuming that $\tilde{c} \leq \lambda c$, $o_2^{RP} < o_2^{VP}$. When provider 1 is value-based, $o_1^{RP}(o_2) = \tilde{c}$. Then, $\tilde{g}(.)$ is decreasing at the value of out-of-pocket that sets it equal to zero and we have $o_1^{RP}(o_2) = \tilde{c} < o_1^{VP}(o_2)$. The rest of the proof for the case with 1 non-value-based is valid.

We now focus on the expected patient population utility, $E[U] = m\left(\sum_{j=1,2}(a_j - \gamma o_j)S_j + u_0 S_0\right)$ (we omit the dependence on price vector $P$ and we denote $u_j \equiv a_j - \gamma o_j$ for clarity of exposition). Since $o_j^{RP} < o_j^{VP}$, we have $u_j^{RP} > u_j^{VP}$. Thus, $S_0^{VP} > S_0^{RP}$. Using these inequalities and with some algebra we next show that $\sum_{j=1,2} u_j^{RP} S_j^{RP} > \sum_{j=1,2} u_j^{VP} S_j^{VP}$. Therefore, $E[U^{RP}] > E[U^{VP}] \Leftrightarrow u_0 < \frac{\sum_{j=1,2}(a_j-\gamma o_j^{RP})S_j^{RP} - \sum_{j=1,2}(a_j-\gamma o_j^{VP})S_j^{VP}}{S_0^{VP} - S_0^{RP}}$, where the above threshold on $u_0$ is positive. □

**Proof of Lemma 4.** (a) Using Theorem 1, under VP, the provider profit is $V_j^{VP} = m(p_j^{VP} - c)\left(1 - \frac{1}{\gamma\lambda(p_j^{VP}-c)}\right)$. Taking the derivative of this expression with respect to $\lambda$ and with some algebra we can show that $\mathrm{V}_j^{VP}/\partial\lambda < 0$.

(b) Under RP, the provider profit is $V_j^{RP}(P) = m(p_j^{RP} - c)\frac{e^{a_j-\gamma(p_j^{RP}-p^*+\tilde{c})^+}}{e^{u_0}+e^{a_j-\gamma(p_j^{RP}-p^*+\tilde{c})^+}+e^{a_{-j}-\gamma(p_{-j}^{RP}-p^*+\tilde{c})^+}}$ for $j = 1,2$. If both providers are value-based, provider profit is increasing in $p^*$.

If provider 1 is value-based and provider 2 is non-value-based, then observing that, from the proof of Lemma 1, $p_2^{RP}$ increasing in $p^*$, $p_2^{RP} - p^*$ is decreasing in $p^*$, it follows that $V_2^{RP}(P)$ is increasing in $p^*$.

If both providers are non-value-based, from Lemma 2, $p_2^{RP}$ is increasing in $p^*$ and $S_2^{RP}(P)$ is increasing in $p^*$ as well. Therefore, the provider profit is increasing in $p^*$.

**Proof of Proposition 7.** As shown in the proof of Proposition 5, as $\lambda$ approaches zero, the price under VP becomes arbitrarily large. Notice that as $\lambda$ approaches zero, $S_j^{VP}$ remains within $[0,1]$, so $\lambda p_j^{VP}$ remains bounded and $S_j^{VP}$ does not approach zero for $j=1,2$. Therefore, for a small enough value of $\lambda$, the provider profit under VP exceeds that under RP ($V_j^{RP} < V_j^{VP}$). Lemma 4(b) shows that the provider profit under VP decreases in $\lambda$ (under RP, the provider profit is independent of $\lambda$). Hence, it suffices to show that, as $\lambda$ approaches 1, the provider profit under VP at equilibrium is lower than that under RP. As $\lambda$ approaches 1, the provider profit under VP approaches $m(p_j^{VP}-c-1/\gamma)$. When provider $j$ is non-value-based, from Theorem 2, and Proposition 5, $p_j^{RP} > p_j^{VP}$ when $\lambda$ approaches 1. Therefore, $V_j^{RP} > V_j^{VP}$. □

**Proof of Proposition 8** The insurer's expected cost under the variable payment is $W^{VP} = m(1-\lambda)\sum_{k=i,j} p_k^{VP} S_k^{VP}(P)$. We then have,

$$\frac{1}{m}\frac{\partial W^{VP}}{\partial \lambda} = \sum_{k=i,j}\left[\frac{\gamma\lambda(p_k^{VP}-c)-1}{\gamma\lambda(p_k^{VP}-c)}\left((1-\lambda)\frac{\partial p_k^{VP}}{\partial\lambda} - p_k^{VP}\right) + \frac{(1-\lambda)p_k^{VP}}{\gamma\lambda(p_k^{VP}-c)}\left(\frac{\partial p_k^{VP}}{\partial\lambda}\frac{1}{(p_k^{VP}-c)} + \frac{1}{\lambda}\right)\right].$$

which can be shown to be decreasing in $\lambda$. As $\lambda$ approaches 1, the insurer share of the cost approaches zero, and thus becomes lower than the insurer payment under RP. As $\lambda$ approaches zero, $S_k^{VP} = 1 - 1/(\gamma\lambda(p_k^{VP}-c))$ remains within $[0,1]$, so $\lambda p_k^{VP}$ remains bounded. Therefore, the insurer cost, becomes arbitrarily large. Hence, for a small enough co-insurance rate $\lambda$, the insurer cost under VP is larger than that under RP. Thus follows, with the value of the co-insurance rate yielding equal insurer costs under the two payment models. □

**Proof of Theorem 3.** After straightforward calculations, we obtain

$$\frac{\partial^2 V_i}{\partial p_i^2} = -m\gamma S_i S_j (2+\gamma(1-2S_j)(p_i-c)) \ (<0 \text{ under Assumption 3}); \ \frac{\partial^2 V_j}{\partial p_j^2} = -m\gamma S_j(1-S_j)(2-\gamma(1-2S_j)(p_j-c)).$$

For $p_2$ fixed, we find the best response $p_1^*(p_2)$ using the FOC. For $p_1 \in [c, p_2]$: if $p_2 < c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma e^{a_2-\gamma\tilde{c}})$, then $p_1^* = p_2$; else, $p_1^* \le p_2$ is the unique solution to the equation

$$e^{u_0} + e^{a_1-\gamma\tilde{c}} = (\gamma(p_1-c)-1)e^{a_2-\gamma(p_2-p_1+\tilde{c})}. \tag{17}$$

For $p_1 \in (p_2, \infty)$, similar to the proof of Theorem 2, $V_1$ is quasi-concave and tends to zero as $p_1$ becomes large. Hence, if $p_2 > c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma(e^{u_0} + e^{a_2-\gamma\tilde{c}}))$, then $p_1^* = p_2$; else, $p_1^* > p_2$ is the unique solution to the equation

$$(e^{u_0} + e^{a_2-\gamma\tilde{c}})(\gamma(p_1-c)-1) = e^{a_1-\gamma(p_1-p_2+\tilde{c})}. \tag{18}$$

Finally, since $c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma e^{a_2-\gamma\tilde{c}}) > c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma(e^{u_0} + e^{a_2-\gamma\tilde{c}}))$,

- if $p_2 \le c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma(e^{u_0} + e^{a_2-\gamma\tilde{c}}))$, then $p_1^* = \bar{p}$ ($> p_2$) where $\bar{p}$ solves (18);
- if $c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma(e^{u_0} + e^{a_2-\gamma\tilde{c}})) < p_2 < c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma e^{a_2-\gamma\tilde{c}})$, then $p_1^* = p_2$;
- else (i.e., if $p_2 \ge c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma e^{a_2-\gamma\tilde{c}})$), then $p_1^* = \tilde{p}$ ($\le p_2$) where $\tilde{p}$ solves (17).

$p_2^*(p_1)$ is obtained similarly by symmetry. Since the Nash equilibrium is the intersection of the best responses, we find that when $c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma(e^{u_0} + e^{a_1-\gamma\tilde{c}})) \le c + (e^{u_0} + e^{a_1-\gamma\tilde{c}} + e^{a_2-\gamma\tilde{c}})/(\gamma e^{a_2-\gamma\tilde{c}})$, the Nash equilibrium is $p_1 = p_2$. Otherwise, the unique Nash equilibrium is $p_1 = \tilde{p}(p_2),\ p_2 = \bar{p}(p_1)$. □

**Proof of Proposition 9.** In case (i), the endogenous price equilibrium $p_1 = p_2$ is above $p_{22}^*(> p^*)$. Moreover, $\gamma(p_2^{\text{exo}} - c) - 1 < \gamma(p_{22}^* - c) - 1$. Hence, we have $p_1^{\text{exo}} = p^* < p_2^{\text{exo}} < p_{22}^* < p_1^{\text{endo}} = p_2^{\text{endo}}$. In case (ii), $p^* \ge p_{22}^*$ implies that $p^*$ is above the lower end of the interval of possible Nash equilibria under endogenous RP. Hence, if $p^*$ is above the upper limit of the interval, all endogenous equilibria lie below the exogenous equilibrium. Otherwise, the endogenous price may be either below or above the exogenous price. In case (iii), we have $\gamma(p_2^{\text{endo}}-c)-1=\frac{e^{a_2-\gamma(p_2^{\text{endo}}-p_1^{\text{endo}}+\tilde{c})}}{e^{u_0}+e^{a_1-\gamma\tilde{c}}}$, $\gamma(p_2^{\text{exo}}-c)-1=\frac{e^{a_2-\gamma(p_2^{\text{exo}}-p^*+\tilde{c})}}{e^{u_0}+e^{a_1-\gamma\tilde{c}}}$. Hence, $p_2^{\text{exo}}>p_2^{\text{endo}}$ iff $p_2^{\text{exo}}-p^*<p_2^{\text{endo}}-p_1^{\text{endo}}$. In case (iv), the endogenous prices satisfy $p_1^{\text{endo}}<p_2^{\text{endo}}$ and $\gamma(p_2^{\text{endo}}-c)-1=\frac{e^{a_2-\gamma(p_2-p_1)}}{e^{u_0}+e^{a_1-\gamma\tilde{c}}}<\frac{e^{a_2-\gamma\tilde{c}}}{e^{u_0}+e^{a_1-\gamma\tilde{c}}}=\gamma(p_{22}^*-c)-1$, hence, $p_1^{\text{endo}}<p_2^{\text{endo}}<p_{22}^*$. Since, $p_{22}^*<p^*$, the result follows. □

**Proof of Proposition 10** The result follows immediately from comparing expressions of the insurer's objective pairwise. □

**Proof of Theorem 4.** a) We can show that

$$\frac{\partial V_j^{HRP}}{\partial p_j}=\begin{cases} m\left(\zeta S_j^L(P)+(1-\zeta)S_j^H(P)\right)>0 & \text{if } p_j<p^* \\ m\zeta S_j^L(P)[1-\gamma_L(p_j-c)(1-S_j^L(P))]+m(1-\zeta)S_j^H(P)[1-\gamma_H(p_j-c)(1-S_j^H(P))] & \text{if } p_j>p^* \end{cases}. \tag{19}$$

Hence, provider $j$'s utility is monotonically increasing in $p_j$ over the domain $p_j\le p^*$.

b) Following the same steps as those illustrated in the proof of Theorem 2, we can show that a unique Nash equilibrium exits where a non-value-based provider $j$ finds its optimal price by solving the first order condition:

$$\zeta S_j^L(P)\left(1-\gamma_L(p_j-c)(1-S_j^L(P))\right)+(1-\zeta)S_j^H(P)\left(1-\gamma_H(p_j-c)(1-S_j^H(P))\right)=0 \tag{20}$$

□