Online Supplement for "Coping with Spatial Mismatch: Subsidy Design for Electric Vehicle and Charging Markets"

Section A: Model Extensions

In this section, we check the robustness of our previous results and generate additional insights by relaxing a few assumptions in our base model.

A.1 Heterogeneous Charging Cycles

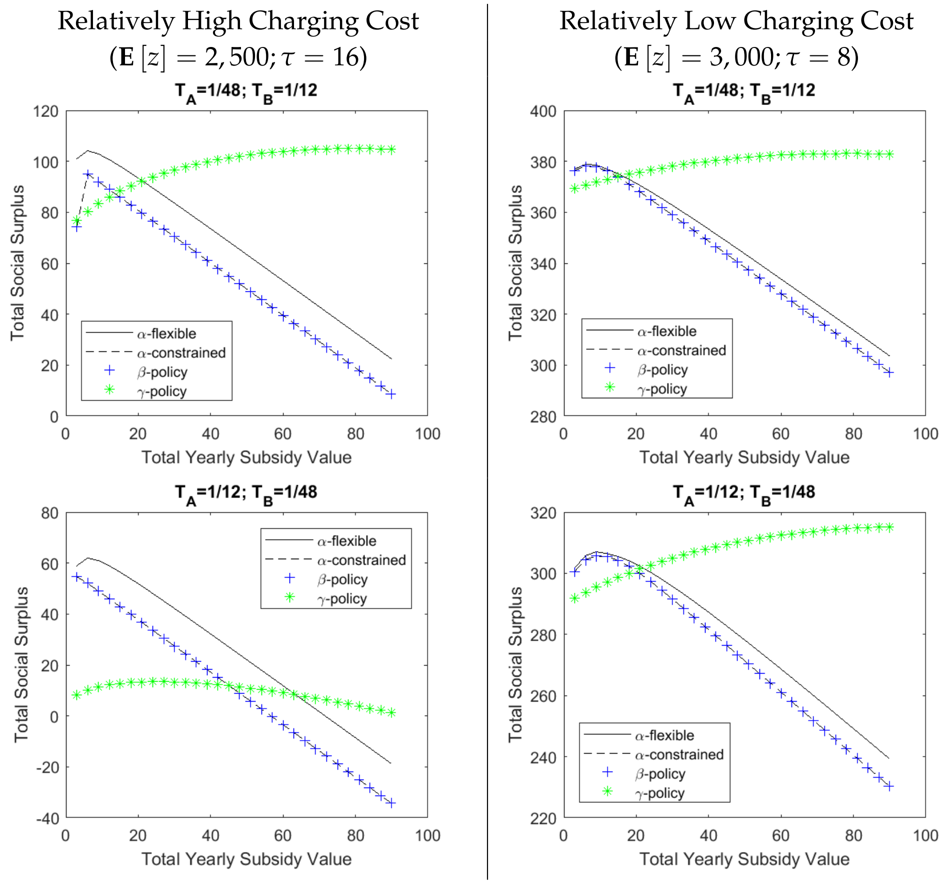
This extension examines the impacts of driver heterogeneity due to different intensities of EV usage and the availability of private charging piles. In particular, we assume that drivers residing at point $\mathcal{B}$ need to charge EVs with public piles more (or less) frequently because they drive longer distances on average (or they are able to access private piles at home). Let T_i denote the average charging cycle of drivers residing at point i. We assume that the distribution of the charging cycle is independent of the utility of driving an EV. The following proposition indicates that driver heterogeneity in the charging cycle will amplify or mitigate the spatial mismatch, depending on the relative sizes of T_A and T_B.

Proposition S.1. *Define* $r_0 = m_B/m_A$. *We have* $\partial r_0/\partial T_A < 0$ *if and only if* $u^e_A/T_A > 1/a$; $\partial r_0/\partial T_B < 0$ *if and only if* $u^e_B/T_B < 1/a$.

In other words, if and only if the annual charging cost is higher (lower) than the average annual EV driving utility for drivers residing in downtown (suburbs), a longer charging cycle for these drivers means weakened spatial mismatch. This is because, when the charging cost is relatively high (low) for drivers residing at one point, the adoption rate will be low (high), and thus an increase in charging cycles reduces drivers' charging costs (i.e., spurs EV adoptions) more (less) than it reduces the charging demand at that point. Note that the spatial mismatch (measured by r_0) will be weakened if the charging demand at point $\mathcal{B}$ is weakened because SPs will have a bigger incentive to build piles downtown.

We explore this revised model with numerical examples in Figure S1. Here we focus on the objective of social surplus maximizing because our previous analyses have shown that the comparative effectiveness of different policies is largely consistent for the two possible government objectives. We use the same default parameter values as those in Section 6. To see the impact of the charging cycle on the performance of

Figure S1: Policy Comparison with Heterogeneous Charging Cycles


Relatively High Charging Cost ($\mathbf{E}[z] = 2,500; \tau = 16$)

Relatively Low Charging Cost ($\mathbf{E}[z] = 3,000; \tau = 8$)

subsidy policies, we set $\{T_A = 1/48; T_B = 1/12\}$ in the top row and $\{T_A = 1/12; T_B = 1/48\}$ in the bottom row; in addition, we set $\{\mathbf{E}[z] = 2,500; \tau = 16\}$ (i.e., relatively high charging cost compared to the mean utility of EV driving) in the left column and $\{\mathbf{E}[z] = 3,000; \tau = 8\}$ (i.e., relatively low charging cost) in the right column. According to Proposition S.1, the examples in the top left and bottom right panels illustrate stronger spatial mismatch, and we can see that subsidizing EV purchases is a more effective policy when the spatial mismatch is stronger. Note that this result does not contradict our previous results, which are primarily related to the cost factors in the charging market rather than the degree of spatial mismatch.

A.2 Disjoint Suburbs

In this subsection, we model the suburbs as two disjoint points: $\mathcal{B}$ and $\mathcal{C}$. Drivers residing at a suburban point will choose between downtown and their residential point for EV charging but will not travel to the other point because the distance is too far. Without loss of generality, we assume that $\mathcal{B}$ and $\mathcal{C}$ are symmetric in the sense that all the cost parameters are the same. All the other assumptions in the base model remain.

According to Theorem 1, the pile utilization rate in equilibrium in the downtown areas or suburban areas is the same as the utilization rate in the base model. This is because the problem facing the SPs has a similar structure, except that there are three points. The condition for equilibrium is independent of the number of points. In addition, the expected charging cost, the location choice probability for drivers from a suburban point, and the marginal types are still given by Equations (2) - (5). As a result, we have $u_B^e = u_C^e$, $\phi_B = \phi_C$, $z_B^* = z_C^*$, and thus

$$\lambda_i = \begin{cases} \frac{1-\eta}{2T}\phi_A\left[1 - H(z_B^*)\right] + \frac{1-\eta}{2T}\phi_A\left[1 - H(z_C^*)\right] + \frac{\eta}{T}\left[1 - H(z_A^*)\right] & \\ \qquad = \frac{1-\eta}{T}\phi_A\left[1 - H(z_B^*)\right] + \frac{\eta}{T}\left[1 - H(z_A^*)\right] & i = \mathcal{A}; \\ \frac{1-\eta}{2T}\phi_i\left[1 - H(z_i^*)\right] & i = \mathcal{B}, \mathcal{C}. \end{cases} \tag{S-1}$$

Accordingly, given the same subsidy policy ω, we can see that λ_A is exactly the same as λ_A given by Equation (6) and that $\lambda_B + \lambda_C$ is exactly the same as λ_B given by Equation (6). Therefore, the government objective is exactly the same as that in the base model, and the results are exactly the same, too.

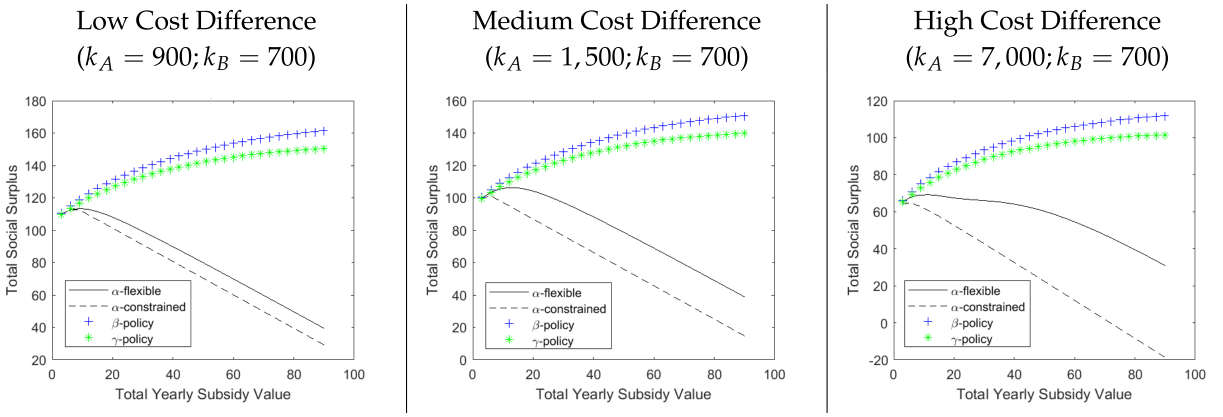
A.3 Unregulated Charging Fees

Here, we relax the assumption of a regulated, flat charging fee and allow SPs to determine the charging fees. We consider a game of three stages. In Stage 1, SPs and potential EV buyers first form a rational expectation of the pile utilization at each point; then, each SP determines the number of piles m_A and m_B, respectively, to build at point $\mathcal{A}$ and point $\mathcal{B}$, and each potential EV buyer decides whether to buy an EV, given their rational expectations. Due to the homogeneity among SPs, their decisions are identical. In Stage 2, given the rational expectations of the pile utilizations, each SP determines the charging service fee at each point (f_i) for their own piles. Due to the homogeneity among the SPs, their prices at one point will be identical in equilibrium (i.e., the SPs have no incentive to deviate from the equilibrium price, knowing that deviating away from the equilibrium price will affect drivers' choices and thus change the pile utilizations). In Stage 3, given the charging service fees and the rationally expected pile utilizations, the EV drivers minimize the cost of each charging by deciding on the charging location first and then choosing the SP; then the rational expectations are fulfilled. We solve this game through backward induction and obtain the following result.

Proposition S.2. *With unregulated charging fees, the pile utilizations are not affected by the β-policy or the γ-policy, and the β-policy dominates the γ-policy.*

This is because, under the β-policy, SPs will always reduce the charging fee in competition as the subsidy value increases. However, the β-policy can effectively reduce EV drivers' charging cost (due to the

Figure S2: Policy Comparison with Unregulated Charging Fees



Low Cost Difference ($k_A = 900; k_B = 700$)

Medium Cost Difference ($k_A = 1,500; k_B = 700$)

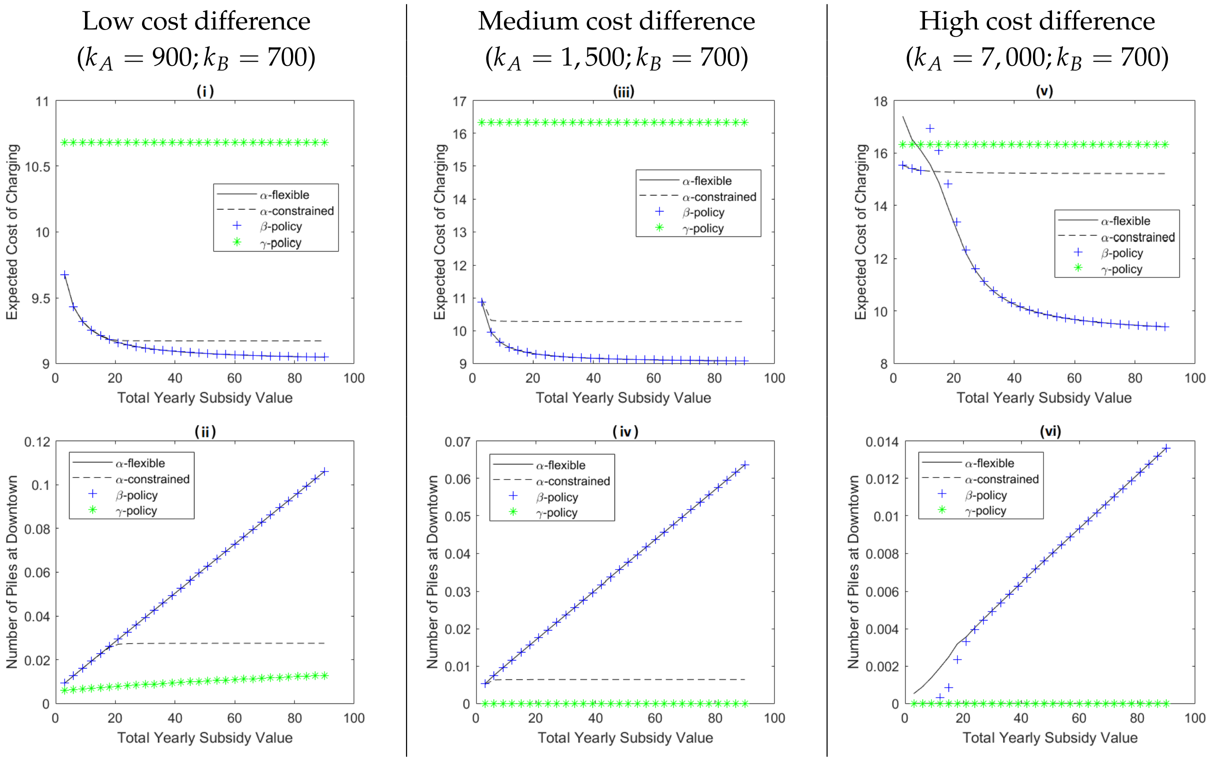
High Cost Difference ($k_A = 7,000; k_B = 700$)

reduction in charging fee) and thus promote EV adoption. This also offers an indirect incentive for SPs to build more piles. As a result, the β-policy in this case works in the same way as the γ-policy. Given that the government subsidy has a higher fiscal efficiency under the β-policy, it dominates the γ-policy.

Regarding the α-policy, there will be two-fold effects on the charging cost and EV adoption because the subsidy can reduce both pile utilizations and charging fees (see the detailed reasoning in the proof of Proposition S.2). However, the aggregate effect could be weaker than that of the β-policy or the γ-policy. Compared to the case with regulated charging fees, SPs in this case are less willing to build more piles under the α-policy because the subsidy value partly goes to EV drivers (through fee reductions); in turn, EV drivers will still expect high utilizations of piles, high search cost, and high charging fees, which suppress their willingness to adopt EVs and again provides a weak incentive for SPs to build charging piles. In Figure S2, we use numerical examples to illustrate the comparison among different policies. We can see that the α-policy is almost always dominated by the β-policy.

In sum, when the charging service fees are unregulated, SPs compete not only on charging service fees, but also on the availability of charging piles. As a result, SPs will enter an equilibrium wherein each SP will set the fees high enough so that the availability of charging piles will not increase significantly when drivers completely abandon the SP. The high service fees will motive SPs to build a sufficient number of charging piles as long as there is enough charging demand. In this case, the β-policy motivates SPs to reduce the charging service fee in the competition, which becomes an incentive for drivers to purchase EVs and an indirect incentive for SPs to build piles. The β-policy has a higher fiscal efficiency than the γ-policy, although they have similar effects. Furthermore, the β-policy is more effective than the α-policy in promoting EV adoptions. Hence, if the charging fees are unregulated, the β-policy turns out to be the best choice.

Figure S3: Policy Comparison (Part I)



Low cost difference ($k_A = 900; k_B = 700$)

Medium cost difference ($k_A = 1,500; k_B = 700$)

High cost difference ($k_A = 7,000; k_B = 700$)

Section B: Additional Figures

We supplement our numerical analysis in Section 6 by showing how different subsidies affect the expected cost of each charging activity and the number of piles in the downtown areas in Figure S3.

Section C: Technical Proofs

Proof of Lemma 1: Given that $H(z) = 1 - \exp(-az)$, we have

$$\begin{aligned}\int_{z_i^*}^{+\infty} z dH(z) &= a\int_{z_i^*}^{+\infty} ze^{-az}dz = -a\int_{z_i^*}^{+\infty} \frac{de^{-az}}{da}dz = -a\frac{d}{da}\left(\int_{z_i^*}^{+\infty} e^{-az}dz\right) \\ &= a\frac{d}{da}\left(\frac{1}{a}\int_{z_i^*}^{+\infty} de^{-az}\right) = a\frac{d}{da}\left(-\frac{e^{-az_i^*}}{a}\right) = a\cdot\frac{e^{-az_i^*}+az_i^*e^{-az_i^*}}{a^2} = e^{-az_i^*}\left(\frac{1}{a}+z_i^*\right).\end{aligned}$$

Hence, we have

$$\sum_{i=A,B}\eta_i\int_{z_i^*}^{+\infty}(z-z_i^*)\,dH(z) = \sum_{i=A,B}\eta_i e^{-az_i^*}\left(\frac{1}{a}+z_i^*\right) - \sum_{i=A,B}\eta_i z_i^* e^{-az_i^*} = \sum_{i=A,B}\frac{\lambda_i T}{a}.$$

Then we know

$$
\begin{aligned}
U_G(\omega) = \theta T \sum_{i=A,B} \lambda_i + \sum_{i=A,B} \eta_i \int_{z_i^*}^{+\infty} (z - z_i^*)\, dH(z) \\
+ \left[\sum_{i=A,B} \alpha_i m_i n + \beta \sum_{i=A,B} \lambda_i + \sum_{i=A,B} f_i \lambda_i - \sum_{i=A,B} k_i m_i n \right] - (1+\delta) S(\omega) \\
= \left(\theta + \frac{1}{a}\right) T \sum_{i=A,B} \lambda_i + \left[\sum_{i=A,B} f_i \lambda_i - \sum_{i=A,B} k_i m_i n - \frac{\gamma}{\mu} T \sum_{i=A,B} \lambda_i \right] - \delta S(\omega) \\
= \left(\theta + \frac{1}{a}\right) T \sum_{i=A,B} \lambda_i + \left[\sum_{i=A,B} f_i \lambda_i - \sum_{i=A,B} \frac{k_i m_i n}{\lambda_i} \cdot \lambda_i - \frac{\gamma}{\mu} T \sum_{i=A,B} \lambda_i \right] - \delta S(\omega) \\
= \sum_{i=A,B} \lambda_i \left[\left(\theta + \frac{1}{a}\right) T + f_i - \frac{k_i m_i n}{\lambda_i} - \frac{\gamma}{\mu} T \right] - \delta S(\omega). \ \square
\end{aligned}
$$

Proof of Proposition 1: According to condition (iii) of Definition 1, if $m_i = 0$ in equilibrium, we must have $\phi_i = 0$ and/or $z_i^* = \infty$. In either case, we must have $\rho_i = \xi_i = 1$. Given this condition, we know that it is optimal for the SP to set $m_i = 0$ only if $\beta + f_i \leq (k_i - \alpha_i)\tau$. The result follows. □

Proof of Theorem 1: We rewrite the SP's problem in Definition 1 as follows:

$$
\max_{m_A, m_B} \ \Pi_S(\xi_A, \xi_B) = \sum_i \left[\alpha_i - k_i + \frac{(\beta + f_i)\, \xi_i}{\tau} \right] m_i.
$$

To obtain finite $m_A > 0$ and $m_B > 0$ in equilibrium, we must have $\alpha_i + (\beta + f_i)\,\xi_i/\tau = k_i$ for $i = \mathcal{A}, \mathcal{B}$, which is equivalent to $\xi_i = (k_i - \alpha_i)\,\tau / (\beta + f_i)$. Given this condition, $\phi_i(\xi_A, \xi_B)$ and $z_i^*(\xi_A, \xi_B)$ are uniquely determined according to (4) and (5). Therefore, $m_i(\xi_A, \xi_B)$ is also uniquely determined according to condition (iii) of Definition 1.

Because we can always use the above procedure to construct an equilibrium, we know that the RE equilibrium always exits and it is unique. □

Proof of Proposition 2: We consider the decision m_i for an SP. Suppose that other SPs decide to build M_i piles in total at point i and the charging arrival rate is λ_i at point i. Thus, the focal SP's problem is to maximize $\Pi_S = (\alpha_i - k_i)\, m_i + (\beta + f)\, \lambda_i m_i / (M_i + m_i)$. Accordingly, we can solve out the focal SP's best response as $\hat{m}_i = \sqrt{\frac{(\beta+f)\lambda_i M_i}{k_i - \alpha_i}} - M_i$. Due to the symmetry among the SPs, we know that $M_i = (n-1)\,\hat{m}_i$ and thus $n\hat{m}_i = \frac{(\beta+f)\lambda_i}{k_i - \alpha_i} \cdot \frac{n-1}{n}$. Therefore, regardless of the best response $\hat{\lambda}_i$ of the EV buyers, we will always have

$$
\rho_i = \frac{\hat{\lambda}_i \tau}{n \hat{m}_i} = \frac{(k_i - \alpha_i)\,\tau}{\beta + f} \cdot \frac{n}{n-1}.
$$

As n goes to infinity, ρ_i converges to the value given by Equation (11). □

Proof of Proposition 3: Define $y_i := f + s_i f / (f - k_i \tau)$. According to (2), (3), (5), and the result of Theorem 1, we know that $z_A^* = p + \frac{1}{T}(c_A \tau + y_A)$ and $z_B^* = p - \frac{1}{T}\ln\left(\sum_{i=A,B} \exp(-y_i - c_i \tau)\right)$. We have

$$\frac{\partial z_A^*}{\partial y_A} = \frac{1}{T} > 0 \text{ and } \frac{\partial z_B^*}{\partial y_i} = \frac{1}{T} \cdot \frac{\exp(-y_i - c_i \tau)}{\sum_{i'=A,B} \exp(-y_{i'} - c_{i'} \tau)} > 0 \text{ for any } i.$$

Thus, according to the chain rule, we have the following:

$$\begin{aligned} \frac{\partial z_A^*}{\partial f} &= \frac{\partial z_A^*}{\partial y_A} \cdot \frac{\partial y_A}{\partial f} = \frac{\partial z_A^*}{\partial y_A} \cdot \left[1 - \frac{k_A \tau}{(f - k_A \tau)^2}\right]; \\ \frac{\partial z_B^*}{\partial f} &= \sum_{i=A,B} \frac{\partial z_B^*}{\partial y_i} \cdot \frac{\partial y_i}{\partial f} = \frac{1}{T} \cdot \sum_{i=A,B} \phi_i \cdot \left[1 - \frac{k_i \tau}{(f_i - k_i \tau)^2}\right]. \end{aligned}$$

The result follows. □

Proof of Theorem 2: According to the definition of the RE equilibrium (i.e., Equation (10)), we know that $\frac{m_A}{m_B} = \frac{\lambda_A}{\lambda_B} \cdot \frac{\xi_B}{\xi_A}$. Therefore, we know that $m_A < m_B$ if and only if $\lambda_A / \lambda_B < \xi_A / \xi_B$. According to Theorem 1, we know that $\xi_A / \xi_B = (k_A - \alpha_A) / (k_B - \alpha_B)$. Hence, $m_A < m_B$ if and only if $G(\Delta) < (k_A - \alpha_A) / (k_B - \alpha_B)$. According to Equations (2) - (6), we have

$$\begin{aligned} \frac{\lambda_A}{\lambda_B} &= \frac{\phi_A}{\phi_B} + \frac{\eta}{1 - \eta}\left(1 + \frac{\phi_A}{\phi_B}\right) e^{a\left(z_B^* - z_A^*\right)} = \left(1 + \frac{\phi_A}{\phi_B}\right)\left[1 + \frac{\eta}{1 - \eta} e^{\frac{a}{T}\left(u_B^e - u_A^e\right)}\right] - 1 \\ &= \left(1 + e^{\Delta}\right)\left[1 + \frac{\eta}{1 - \eta}\left(\frac{1}{1 + e^{-\Delta}}\right)^{\frac{a}{T}}\right] - 1 = G(\Delta). \end{aligned}$$

It is easy to see that both $\left(1 + e^{\Delta}\right)$ and $\left(1 + e^{-\Delta}\right)^{-a/T}$ are increasing in Δ. Hence, $G(\Delta)$ is a monotone, increasing function of Δ. In addition, the ratio $\eta / (1 - \eta)$ is increasing in η, so $G(\Delta)$ is increasing in η. The result follows. □

Proof of Lemma 2: According to Theorem 1 and 2, we know that G, Δ, and ξ_B / ξ_A are all independent of γ. □

Proof of Proposition 4: Note that the social surplus under the γ-policy can be written as

$$U_G = \sum_{i=A,B}\left[\theta T + \frac{T}{a} - \frac{T\gamma}{\mu} - \frac{\delta T \gamma}{\mu}\right] \lambda_i = \lambda_B \sum_{i=A,B}\left[\theta T + \frac{T}{a} - \frac{T\gamma}{\mu} - \frac{\delta T \gamma}{\mu}\right] \frac{\lambda_i}{\lambda_B}.$$

According to Theorem 2, we know that λ_A/λ_B is independent of γ. Taking the first order derivative with respect to γ, we obtain

$$\begin{aligned}\frac{\partial U_G}{\partial \gamma} =& \frac{\partial \lambda_B}{\partial \gamma}\sum_{i=A,B}\left[\theta T + \frac{T}{a} - \frac{T\gamma}{\mu} - \frac{\delta T\gamma}{\mu}\right]\frac{\lambda_i}{\lambda_B} - \lambda_B\sum_{i=A,B}\left(1+\delta\right)\frac{T}{\mu}\cdot\frac{\lambda_i}{\lambda_B}\\ =& a\lambda_B\sum_{i=A,B}\left[\theta T + \frac{T}{a} - \frac{T\gamma}{\mu} - \frac{\delta T\gamma}{\mu}\right]\frac{\lambda_i}{\lambda_B} - \left(1+\delta\right)\frac{T}{\mu}\cdot\sum_{i=A,B}\lambda_i \le \sum_{i=A,B}\left[a\theta T + T - \left(1+\delta\right)\frac{T}{\mu}\right]\lambda_i.\end{aligned}$$

wherein the inequality is due to $-\gamma \le 0$. Therefore, if $a\theta \le (1+\delta)/\mu - 1$, we must have $\partial U_G/\partial\gamma \le 0$. □

Proof of Lemma 3: According to Theorem 1, we know that the ratio ξ_A/ξ_B is independent of β. Given $f + \beta > k_i\tau$ (i.e., positive m_i) and $s_A k_A \ge s_B k_B$, we have

$$\frac{\partial \Delta}{\partial \beta} = \frac{s_A k_A \tau}{\left(\beta + f - k_A\tau\right)^2} - \frac{s_B k_B \tau}{\left(\beta + f - k_B\tau\right)^2} > 0.$$

As a result, the maximum value of Δ is given by $\lim_{\beta\to\infty}\Delta = (c_B - c_A)\tau + s_B - s_A$. In order to have $\Delta > G^{-1}(k_A/k_B)$, we must have $(c_B - c_A)\tau + s_B - s_A > G^{-1}(k_A/k_B)$. The result follows. □

Proof of Proposition 5: Note that the social surplus under the β-policy can be written as

$$U_G = \sum_{i=A,B}\left[\theta T + \frac{T}{a} - \beta - \delta\beta\right]\lambda_i = W(\beta)\sum_{i=A,B}\lambda_i.$$

Taking the first order derivative with respect to β, we obtain

$$\frac{\partial U_G}{\partial \beta} = -\left(1+\delta\right)\sum_i \lambda_i + W(\beta)\sum_i \frac{\partial \lambda_i}{\partial \beta} = -\left(1+\delta\right)\sum_i \lambda_i + \frac{1}{\lambda_A}\cdot\frac{\partial \lambda_A}{\partial \beta}\cdot U_G - W(\beta)\frac{\lambda_B^2}{\lambda_A}G'(\Delta)\frac{\partial \Delta}{\partial \beta},$$

wherein the second equation is due to $\lambda_B = \lambda_A/G$ and thus $\frac{\partial \lambda_B}{\partial \beta} = \frac{\lambda_B}{\lambda_A}\cdot\frac{\partial \lambda_A}{\partial \beta} - \frac{\lambda_B^2}{\lambda_A}G'(\Delta)\frac{\partial \Delta}{\partial \beta}$. Define $F_i = f_i + c_i\tau + s_i(1-\rho_i)^{-1}$. Because $\frac{k_i s_i}{(\beta+f-k_i\tau)^2} \le \frac{k_A s_A}{(\beta+f-k_A\tau)^2}$ given that $s_B k_B \le s_A k_A$, we have

$$\begin{aligned}\frac{\partial \lambda_A}{\partial \beta} =& \lambda_B\phi_A\frac{\partial \Delta}{\partial \beta} + \frac{1-\eta}{T}\phi_A e^{-az_B^*}\left[\sum_i \frac{\phi_i k_i s_i \tau a}{T\left(\beta + f - k_i\tau\right)^2}\right] + \frac{\eta}{T}e^{-az_A^*}\cdot\frac{k_A s_A \tau a}{T\left(\beta + f - k_A\tau\right)^2}\\ &\le \lambda_A\left[\frac{k_A s_A \tau a}{T\left(\beta + f - k_A\tau\right)^2}\right] + \lambda_B\phi_A\frac{\partial \Delta}{\partial \beta}.\end{aligned}$$

Therefore,

$$\frac{\partial U_G}{\partial \beta} \le -\left(1+\delta\right)\sum_i \lambda_i + \frac{k_A s_A \tau a U_G}{T\left(\beta + f - k_A\tau\right)^2} + \frac{\lambda_B}{\lambda_A}\frac{\partial \Delta}{\partial \beta}\phi_A U_G - W(\beta)\frac{\lambda_B^2}{\lambda_A}G'(\Delta)\frac{\partial \Delta}{\partial \beta}.$$

Note that

$$G'(\Delta) = \left(\frac{a}{T} + e^{\Delta}\right) \frac{\eta}{1-\eta} \left(1 + e^{-\Delta}\right)^{-a/T} + e^{\Delta};$$

$$\begin{aligned} G'(\Delta)\,\lambda_B &= \frac{\frac{a}{T} + e^{\Delta}}{1+e^{\Delta}} \cdot \frac{\eta}{T} \cdot \left(1 + e^{-\Delta}\right)^{-a/T} e^{-az_B^*} + \frac{1-\eta}{T} \cdot \frac{e^{\Delta}}{1+e^{\Delta}} \cdot e^{-az_B^*} \\ &\geq \frac{e^{\Delta}}{1+e^{\Delta}} \cdot \frac{\eta}{T} \cdot \frac{e^{-az_A^*}}{e^{-az_B^*}} \cdot e^{-az_B^*} + \frac{1-\eta}{T} \cdot \frac{e^{\Delta}}{1+e^{\Delta}} \cdot e^{-az_B^*} = \phi_A \left(\frac{\eta}{T} \cdot e^{-az_A^*} + \frac{1-\eta}{T} \cdot e^{-az_B^*}\right) = \phi_A \sum_i \lambda_i. \end{aligned}$$

As a result,

$$\begin{aligned} \frac{\partial U_G}{\partial \beta} &\leq -\left(1+\delta\right) \sum_i \lambda_i + \frac{k_A s_A \tau a U_G}{T\left(\beta + f - k_A \tau\right)^2} + \frac{\lambda_B}{\lambda_A} \frac{\partial \Delta}{\partial \beta} \phi_A U_G - W(\beta) \frac{\lambda_B}{\lambda_A} \frac{\partial \Delta}{\partial \beta} \phi_A \sum_i \lambda_i \\ &= -\left(1+\delta\right) \sum_i \lambda_i + \frac{k_A s_A \tau a W(\beta)}{T\left(\beta + f - k_A \tau\right)^2} \sum_i \lambda_i = \left[\frac{k_A s_A \tau a W(\beta)}{T\left(\beta + f - k_A \tau\right)^2} - \left(1+\delta\right)\right] \sum_i \lambda_i. \end{aligned}$$

Because

$$\frac{k_A s_A \tau a W(\beta)}{T\left(\beta + f - k_A \tau\right)^2} \leq \frac{k_A s_A \tau a}{T\left(f - k_A \tau\right)^2} \left(\theta T + \frac{T}{a}\right),$$

we know that $\partial U_G / \partial \beta \leq 0$ for any β if $\frac{k_A s_A \tau}{\left(f - k_A \tau\right)^2} \leq \frac{1+\delta}{1+a\theta}$. The result follows. □

Proof of Proposition 6: By setting $\alpha_i = \beta k_i / \left(f + \beta\right)$, the pile utilization rate at point i under the α-policy is

$$\xi_i = \frac{\left(k_i - \alpha_i\right)\tau}{f} = \frac{\left(f k_i\right)\tau}{\left(f+\beta\right) f} = \frac{k_i \tau}{f + \beta},$$

which is the pile utilization rate at point i under the β-policy. The annual subsidy equals

$$S\left(\alpha_i = \beta' k_i / \left(f + \beta'\right), \beta = \gamma = 0\right) = \sum_i \frac{\beta' k_i m_i}{f + \beta'} = \sum_i \frac{\beta' \xi_i m_i}{\tau} = \beta' \sum_i \lambda_i = S\left(\beta = \beta', \alpha_i = \gamma = 0\right).$$

Hence, the social surplus under the α-policy is the same as that under the β-policy, given the same pile utilizations and annual subsidy.

Note that the constraint of $\alpha_i = \beta k_i / \left(f + \beta\right)$ for $i = \mathcal{A}, \mathcal{B}$ and any $\beta \geq 0$ is equivalent to $\alpha_A / \alpha_B = k_A / k_B$. Therefore, the β-policy is equivalent to the α-policy with the constraint of $\alpha_A / \alpha_B = k_A / k_B$. It is clear that the unconstrained problem has a higher optimal social surplus than the constrained problem does. The result thus follows. □

Proof of Proposition 7: If the constrained α-policy is used, we have $\alpha_A = \alpha_B = \alpha$. Define $r(\alpha) = m_B / m_A = \xi_A / \xi_B / G = \left(k_A - \alpha\right) / \left(k_B - \alpha\right) / G$. To alleviate the mismatch, we should at least have $r'(0) <$

0. Taking the first order derivative, we get

$$\frac{\partial r}{\partial \alpha} = \frac{\frac{k_A - k_B}{(k_B-\alpha)^2} G - \frac{k_A-\alpha}{k_B-\alpha} \cdot \frac{\partial \Delta}{\partial \alpha} \cdot \frac{\partial G}{\partial \Delta}}{G^2}.$$

Hence, the constrained α-policy is able to alleviate the mismatch if and only if

$$\frac{1}{k_B} - \frac{1}{k_A} < \left(\frac{1}{G} \cdot \frac{\partial \Delta}{\partial \alpha} \cdot \frac{\partial G}{\partial \Delta}\right)|_{\alpha=0} = \left[\frac{f s_A \tau}{(f-k_A\tau)^2} - \frac{f s_B \tau}{(f-k_B\tau)^2}\right]\left(\frac{1}{G} \cdot \frac{\partial G}{\partial \Delta}\right)|_{\alpha=0}.$$

When k_A is large enough such that $k_A\tau$ approaches f, the right hand side of the above condition goes to infinity and thus the condition holds. In contrast, if $k_A < \sqrt{\frac{s_A}{s_B}} k_B + \frac{f}{\tau}\left(1 - \sqrt{\frac{s_A}{s_B}}\right)$, then the right hand side of the above condition is negative and thus the condition can never hold. The result thus follows. □

Proof of Theorem 3: We first focus on the marginal impact of subsidies on the GSS, respectively, under the γ-policy and β-policy, given the same amount of increase in subsidy value. Suppose that the current policy is $\omega := (\beta, \gamma)$. Note that the GSS given any subsidy policy can be written as $\sum_i \lambda_i \left[\theta T + \frac{T}{a}\right]$ according to Lemma 1 and the fact that the SPs break even in equilibrium. Regarding the subsidy value, we have $\partial S(\omega)/\partial\gamma = (T/\mu)\sum_i \lambda_i + (T\gamma/\mu + \beta)\sum_i \partial\lambda_i/\partial\gamma$ and $\partial S(\omega)/\partial\beta = \sum_i \lambda_i + (T\gamma/\mu+\beta)\sum_i \partial\lambda_i/\partial\beta$. For the same increment of subsidy (i.e., $(T/\mu)\sum_i \lambda_i d\gamma + (T\gamma/\mu+\beta)\sum_i d\lambda_i = \sum_i \lambda_i d\beta + (T\gamma/\mu+\beta)\sum_i d\lambda_i$), the ratio of change in γ over the change in β (i.e., $d\gamma/d\beta$) is μ/T. Hence, we need to compare $(\mu/T)\,\partial GSS(\omega)/\partial\gamma$ against $\partial GSS(\omega)/\partial\beta$.

We have

$$\frac{\partial GSS(\omega)}{(T/\mu)\,\partial\gamma} = \sum_i \left[\theta T + \frac{T}{a}\right]\frac{\mu}{T}\frac{\partial\lambda_i}{\partial\gamma} \text{ and } \frac{\partial GSS(\omega)}{\partial\beta} = \sum_i \left[\theta T + \frac{T}{a}\right]\frac{\partial\lambda_i}{\partial\beta}.$$

Next, we have

$$\frac{\partial\lambda_i}{(T/\mu)\,\partial\gamma} = \frac{a\mu}{T}\lambda_i;$$

$$\frac{\partial\lambda_A}{\partial\beta} = \lambda_B\phi_A\frac{\partial\Delta}{\partial\beta} + \frac{1-\eta}{T}\phi_A e^{-az_B^*}\left[\sum_i \frac{\phi_i k_i s_i \tau a}{T(\beta+f-k_i\tau)^2}\right] + \frac{\eta}{T}e^{-az_A^*}\cdot\frac{k_A s_A \tau a}{T(\beta+f-k_A\tau)^2};$$

$$\frac{\partial\lambda_B}{\partial\beta} = \lambda_B\left[\sum_i \frac{\phi_i k_i s_i \tau a}{T(\beta+f-k_i\tau)^2} - \phi_A\frac{\partial\Delta}{\partial\beta}\right].$$

Given $k_A s_A \geq k_B s_B$, we have $\frac{k_B s_B \tau}{(\beta+f-k_B\tau)^2} \leq \frac{k_A s_A \tau}{(\beta+f-k_A\tau)^2}$ and thus $\frac{\partial\Delta}{\partial\beta} \geq 0$. Because $\frac{\partial\lambda_A}{\partial\beta} \geq \lambda_A\left[\frac{k_B s_B \tau a}{T(\beta+f-k_B\tau)^2}\right] +$

$\lambda_B \phi_A \frac{\partial \Delta}{\partial \beta}$ and $\frac{\partial \lambda_B}{\partial \beta} \geq \lambda_B \left[\frac{k_B s_B \tau a}{T(\beta+f-k_B\tau)^2} \right] - \lambda_B \phi_A \frac{\partial \Delta}{\partial \beta}$, we will have

$$\frac{\partial GSS\left(\omega\right)}{\partial \beta} \geq \sum_i \left[\theta T + \frac{T}{a} \right] \left[\frac{a \lambda_i k_B s_B \tau}{T\left(\beta + f - k_B \tau\right)^2} \right].$$

Hence, the sufficient condition for $\frac{\partial U_G(\omega)}{\partial \beta} \geq \frac{\partial U_G(\omega)}{(T/\mu)\partial \gamma}$ is

$$\frac{a k_B s_B \tau}{T\left(\beta + f - k_B \tau\right)^2} \geq \frac{a \mu}{T} \text{ or } \frac{k_B s_B \tau}{\left(\beta + f - k_B \tau\right)^2} \geq \mu.$$

Next, we compare the γ-policy against the α-policy, given any policy mix $\omega = (\alpha_A, \alpha_B, \gamma)$. Given that α_A and α_B are balanced, we know that the marginal impacts of α_A and α_B are identical. Therefore, it is marginally optimal (indifferent) to set

$$\frac{f k_A d\alpha_A}{\left(k_A - \alpha_A\right)^2} = \frac{f k_B d\alpha_B}{\left(k_B - \alpha_B\right)^2} = dC.$$

Accordingly, the change in subsidy value under the α-flexible policy is $dS = \sum \frac{\lambda_i f k_i d\alpha_i}{(k_i-\alpha_i)^2} + \sum \left(\frac{f\alpha_i}{k_i-\alpha_i} + \frac{T}{\mu}\gamma \right) d\lambda_i = \sum \lambda_i dC + \sum \left(\frac{f\alpha_i}{k_i-\alpha_i} + \frac{T}{\mu}\gamma \right) d\lambda_i$, and the change in subsidy value under the γ-policy is $dS = \frac{T}{\mu} \sum \lambda_i d\gamma + \sum \left(\frac{f\alpha_i}{k_i-\alpha_i} + \frac{T}{\mu}\gamma \right) d\lambda_i$. Therefore, for the same amount of change in subsidy value, there is the ratio of $dC/d\gamma = T/\mu$.

For the marginal change $(d\alpha_A, d\alpha_B)$ at balanced (α_A, α_B), we have

$$\begin{aligned} d\lambda_A =& \lambda_B \phi_A d\Delta + \frac{1-\eta}{T} \phi_A e^{-az_B^*} \sum \phi_i \frac{a f s_i \tau d\alpha_i}{T\left[f - \left(k_i - \alpha_i\right)\tau\right]^2} + \frac{\eta}{T} e^{-az_A^*} \frac{a f s_A \tau d\alpha_A}{T\left[f - \left(k_A - \alpha_A\right)\tau\right]^2}; \\ d\lambda_B =& \lambda_B \left(\sum \phi_i \frac{a f s_i \tau d\alpha_i}{T\left[f - \left(k_i - \alpha_i\right)\tau\right]^2} - \phi_A d\Delta \right). \end{aligned}$$

Now suppose $\frac{f s_i \tau}{(f-(k_i-\alpha_i)\tau)^2} \leq \frac{\mu k_i f}{(k_i-\alpha_i)^2}$ for both i. Then $d\lambda_A \leq \lambda_A \frac{a\mu}{T} dC + \lambda_B \phi_A d\Delta$ and $d\lambda_B \leq \lambda_B \frac{a\mu}{T} dC - \lambda_B \phi_A d\Delta$. Let $dGSS\left(\alpha\right)$ and $dGSS\left(\gamma\right)$ denote respectively the marginal change in the GSS under the α-flexible policy and the γ-policy. We then have:

$$\begin{aligned} dGSS\left(\alpha\right) \leq& \sum_i \left[\theta T + \frac{T}{a} \right] \lambda_i \frac{a\mu}{T} dC + \left[\theta T + \frac{T}{a} \right] \left(\lambda_B \phi_A d\Delta - \lambda_B \phi_A d\Delta \right) \\ =& \sum_i \left[\theta T + \frac{T}{a} \right] \lambda_i \frac{a\mu}{\mu} d\gamma = dGSS\left(\gamma\right). \end{aligned}$$

Accordingly, the sufficient condition for $dGSS\left(\alpha\right) \leq dGSS\left(\gamma\right)$ is $\frac{f s_i \tau}{(f-(k_i-\alpha_i)\tau)^2} \leq \frac{\mu k_i f}{(k_i-\alpha_i)^2}$ for both i, or equivalently, $\max_i \frac{(k_i-\alpha_i)^2 s_i \tau}{k_i[f-(k_i-\alpha_i)\tau]^2} \leq \mu$.

We then focus on the marginal impact of subsidies on the scale of EV adoption (i.e., $SA = T\sum_i \lambda_i$), respectively, under the γ-policy and β-policy, given the same amount of increase in subsidy value. According to the previous analysis, we know that we need to compare $(\mu/T)\,\partial\left(T\sum_i \lambda_i\right)/\partial\gamma$ against $\partial\left(T\sum_i \lambda_i\right)/\partial\beta$. We have

$$\begin{aligned}\frac{\mu}{T}\cdot\frac{\partial\left(T\sum_i \lambda_i\right)}{\partial\gamma} &=\mu\sum_i\frac{\partial\lambda_i}{\partial\gamma}=a\mu\sum_i\lambda_i;\\ \frac{\partial\left(T\sum_i \lambda_i\right)}{\partial\beta} &=T\sum_i\frac{\partial\lambda_i}{\partial\beta}\geq\frac{ak_Bs_B\tau}{\left(\beta+f-k_B\tau\right)^2}\sum_i\lambda_i.\end{aligned}$$

Hence, the sufficient condition for the β-policy to dominate the γ-policy is still $k_Bs_B\tau \geq \mu\left(\beta+f-k_B\tau\right)^2$.

Finally, we compare the γ-policy against the α-policy for the subsidy minimizing problem. From previous analysis, we know $dSA(\alpha) \leq \sum_i T\lambda_i a d\gamma = dSA(\gamma)$ if $\frac{fs_i\tau}{(f-(k_i-\alpha_i)\tau)^2} \leq \frac{\mu k_i f}{(k_i-\alpha_i)^2}$. The results follow.$\square$

Proof of Corollary 1: First, note that

$$\frac{\left(k_i-\alpha_i\right)^2 s_i\tau}{k_i\left[f-\left(k_i-\alpha_i\right)\tau\right]^2}\leq\frac{s_i\tau k_i^2}{\left(f-k_i\tau\right)^2k_i}\leq\frac{s_A\tau k_A}{\left(f-k_A\tau\right)^2},$$

given that $s_Ak_A \geq s_Bk_B$ and $k_A \geq k_B$. Hence, $\frac{s_A\tau k_A}{(f-k_A\tau)^2} \leq \mu$ is a sufficent condition for $\max_i \frac{(k_i-\alpha_i)^2 s_i\tau}{k_i[f-(k_i-\alpha_i)\tau]^2} \leq \mu$. Hence, if $\frac{s_A\tau k_A}{(f-k_A\tau)^2} \leq \mu$, the γ-policy is always marginally better than the α-flexible and the β-policy.

Next, we have $\frac{k_Bs_B\tau}{(\beta+f-k_B\tau)^2} \geq \frac{k_Bs_B\tau}{\left(\bar{\beta}+f-k_B\tau\right)^2}$ for $\beta \in \left[0,\bar{\beta}\right]$. Hence, if $\frac{k_Bs_B\tau}{\left(\bar{\beta}+f-k_B\tau\right)^2} \geq \mu$, then the β-policy is always marginally better than the γ-policy, given any policy mix (β,γ) for $\beta \in \left[0,\bar{\beta}\right]$. In other words, the γ-policy is always worse than the β-policy for $\beta \in \left[0,\bar{\beta}\right]$ if the condition holds. $\square$

Proof of Corollary 2: Suppose $k_A \to \infty$. Then we must have $\xi_A \to 1$, $u_A^e \to \infty$, $\phi_A \to 0$, and $\lambda_A \to 0$. Thus, we have for any policy mix (α_B, β):

$$U_G\left(\alpha_B,\beta\right)\to\tilde{U_G}=\lambda_B\left(\theta T+\frac{T}{a}\right)-\left(1+\delta\right)\left[\frac{\alpha_B\lambda_B\left(\beta+f\right)}{k_B-\alpha_B}+\beta\lambda_B\right].$$

Taking the first order derivatives:

$$\begin{aligned}\frac{\partial\tilde{U_G}}{\partial\alpha_B}&=\left(\theta T+\frac{T}{a}\right)\frac{\partial\lambda_B}{\partial\alpha_B}-\left(1+\delta\right)\left[\frac{\alpha_B\left(\beta+f\right)}{k_B-\alpha_B}+\beta\right]\frac{\partial\lambda_B}{\partial\alpha_B}-\left(1+\delta\right)\lambda_B\frac{\left(\beta+f\right)k_B}{\left(k_B-\alpha_B\right)^2};\\ \frac{\partial\tilde{U_G}}{\partial\beta}&=\left(\theta T+\frac{T}{a}\right)\frac{\partial\lambda_B}{\partial\beta}-\left(1+\delta\right)\left[\frac{\alpha_B\left(\beta+f\right)}{k_B-\alpha_B}+\beta\right]\frac{\partial\lambda_B}{\partial\beta}-\left(1+\delta\right)\lambda_B\frac{k_B}{k_B-\alpha_B}.\end{aligned}$$

Note that

$$\frac{\partial \lambda_B}{\partial \xi_B} \cdot \frac{\partial \xi_B}{\partial \alpha_B} = -\frac{\partial \lambda_B}{\partial \xi_B} \cdot \frac{\tau}{\beta + f} \text{ and } \frac{\partial \lambda_B}{\partial \xi_B} \cdot \frac{\partial \xi_B}{\partial \beta} = -\frac{\partial \lambda_B}{\partial \xi_B} \cdot \frac{(k_B - \alpha_B)\,\tau}{(\beta + f)^2},$$

so $(\beta + f)\,\partial\lambda_B/\partial\beta = (k_B - \alpha_B)\,\partial\lambda_B/\partial\alpha_B$. Furthermore, for the same amount of increase in subsidy value, we have

$$\left[\frac{\alpha_B(\beta + f)}{k_B - \alpha_B} + \beta\right] d\lambda_B + \lambda_B \frac{(\beta + f)\,k_B}{(k_B - \alpha_B)^2} d\alpha_B = \left[\frac{\alpha_B(\beta + f)}{k_B - \alpha_B} + \beta\right] d\lambda_B + \lambda_B \frac{k_B}{k_B - \alpha_B} d\beta$$

and thus $d\alpha_B/d\beta = (k_B - \alpha_B)\,/\,(\beta + f)$. Therefore, we should compare $\partial\tilde{U_G}/\left(\partial\alpha_B\,(\beta + f)\,/\,(k_B - \alpha_B)\right)$ against $\partial\tilde{U_G}/\partial\beta$:

$$\frac{\partial\tilde{U_G}/\partial\alpha_B}{\partial\tilde{U_G}/\partial\beta} \cdot \frac{k_B - \alpha_B}{\beta + f} = \frac{\partial\lambda_B/\partial\alpha_B}{\partial\lambda_B/\partial\beta} \cdot \frac{k_B - \alpha_B}{\beta + f} = \frac{\beta + f}{k_B - \alpha_B} \cdot \frac{k_B - \alpha_B}{\beta + f} = 1,$$

which means that the α-flexible policy and the β-policy are equivalent. The proof and conclusion are the same for the subsidy minimizing problem.

Next, we compare the β-policy against the γ-policy. According to Theorem 3, for the same amount of increase in subsidy value, we should have $d\gamma/d\beta = \mu/T$. Given $k_A \to \infty$, we have

$$\begin{aligned}\frac{\partial GSS(\omega)}{(T/\mu)\,\partial\gamma} &\to \left[\theta T + \frac{T}{a}\right] \frac{\partial\lambda_B}{(T/\mu)\,\partial\gamma} = \left[\theta T + \frac{T}{a}\right] \frac{a\mu}{T}\lambda_B;\\ \frac{\partial GSS(\omega)}{\partial\beta} &\to \left[\theta T + \frac{T}{a}\right] \frac{\partial\lambda_B}{\partial\beta} = \left[\theta T + \frac{T}{a}\right] \frac{k_B s_B \tau a \lambda_B}{T\,(\beta + f - k_B\tau)^2}.\end{aligned}$$

Hence, given $k_B s_B \tau < \mu\,(f - k_B\tau)^2$, there exists k_A^* such that, if $k_A > k_A^*$, the γ-policy is always better than the β-policy and thus the α-flexible policy. The proof and conclusion are the same for the subsidy minimizing problem.

When $k_A \to k_B$, we should have $k_A s_A \tau < \mu\,(f - k_A\tau)^2$ given that $s_A \le s_B$ and $k_B s_B \tau < \mu\,(f - k_B\tau)^2$. According to Corollary 1, we know that γ-policy is better than the other two policies. □

Proof of Proposition 8: Given any policy mix (β, γ), we take the first order derivative of U_G with respect to β and get

$$\frac{\partial U_G}{\partial\beta} = \sum_i \lambda_i\,[-1 - \delta] + \sum_i \left[\theta T + \frac{T}{a} - (1 + \delta)\,\beta - T\gamma\frac{1 + \delta}{\mu}\right] \frac{\partial\lambda_i}{\partial\beta}.$$

It is easy to verify that $\partial^2\lambda_i/(\partial\beta\partial\gamma) = a\partial\lambda_i/\partial\beta$ for all i. Hence, $\frac{\partial^2 U_G}{\partial\beta\partial\gamma} = a\frac{\partial U_G}{\partial\beta} - \frac{1+\delta}{\mu} T \sum_i \frac{\partial\lambda_i}{\partial\beta}$. Clearly, we have $\partial U_G/\partial\beta = 0$ at $\beta = \beta^*(\gamma)$. In addition, recall that $\frac{\partial\lambda_A}{\partial\beta} \ge \lambda_A \left[\frac{k_B s_B \tau a}{T(\beta + f - k_B\tau)^2}\right] + \lambda_B \phi_A \frac{\partial\Delta}{\partial\beta}$ and $\frac{\partial\lambda_B}{\partial\beta} \ge$

$\lambda_B \left[\frac{k_B s_B \tau a}{T(\beta+f-k_B\tau)^2}\right] - \lambda_B \phi_A \frac{\partial \Delta}{\partial \beta}$. Hence, $\sum_i \frac{\partial \lambda_i}{\partial \beta} \geq \sum_i \lambda_i \left[\frac{k_i s_i \tau a}{T(\beta+f-k_i\tau)^2}\right] > 0$. As a result, $\partial^2 U_G / \left(\partial\beta\partial\gamma\right)|_{\beta=\beta^*(\gamma)} = -\left(1+\delta\right)\left(T/\mu\right)\sum_i \partial\lambda_i/\partial\beta < 0$. □

Proof of Proposition 9: A sufficient condition for the failure of the γ-policy to achieve the EV adoption target D (i.e., $T\sum_{i=A,B}\lambda_i \geq D$) within budget constraint $S \leq B$ is that

$$\frac{\partial\left(T\sum_{i=A,B}\lambda_i\right)}{\partial S} = \frac{\partial\left(T\sum_{i=A,B}\lambda_i\right)}{\partial\gamma}\cdot\frac{\partial\gamma}{\partial S} \leq \frac{D-\Lambda_0}{B}.$$

According to the proof of Proposition 4, we know $\partial S/\partial\gamma = \sum_{i=A,B}\lambda_i\left(1+a\gamma\right)T/\mu$ and $\partial\left(T\sum_{i=A,B}\lambda_i\right)/\partial\gamma = \sum_{i=A,B}\lambda_i aT$. Hence, the sufficient condition holds if $a\mu B \leq D-\Lambda_0$.

Similarly, a sufficient condition for the failure of the β-policy to achieve the EV adoption target D (i.e., $T\sum_{i=A,B}\lambda_i \geq D$) within budget constraint $S \leq B$ is that

$$\frac{\partial\left(T\sum_{i=A,B}\lambda_i\right)}{\partial S} = \frac{\partial\left(T\sum_{i=A,B}\lambda_i\right)}{\partial\beta}\cdot\frac{\partial\beta}{\partial S} \leq \frac{D-\Lambda_0}{B} \text{ or } \frac{\partial\left(T\sum_{i=A,B}\lambda_i\right)}{\partial\beta} - \frac{D-\Lambda_0}{B}\cdot\frac{\partial S}{\partial\beta} \leq 0.$$

Define $W(\beta) = T - \beta\left(D-\Lambda_0\right)/B$. Then the condition becomes $\partial\left[W(\beta)\sum_{i=A,B}\lambda_i\right]/\partial\beta \leq 0$. We first consider $\beta \in \left[0, BT/\left(D-\Lambda_0\right)\right]$ and thus $W(\beta) \geq 0$. From the proof of Proposition 5, we know

$$\frac{\partial\left[W(\beta)\sum_{i=A,B}\lambda_i\right]}{\partial\beta} \leq \left[\frac{k_A s_A \tau a W(\beta)}{T\left(\beta+f-k_A\tau\right)^2} - \frac{D-\Lambda_0}{B}\right]\sum_i \lambda_i \leq \left[\frac{k_A s_A \tau a}{\left(f-k_A\tau\right)^2} - \frac{D-\Lambda_0}{B}\right]\sum_i \lambda_i.$$

Hence, the sufficient condition is guaranteed by $k_A s_A \tau a/\left(f-k_A\tau\right)^2 \leq \left(D-\Lambda_0\right)/B$. Next, suppose $\beta > BT/\left(D-\Lambda_0\right)$, which suggests that $T\sum_i\lambda_i < \beta\sum_i\lambda_i\left(D-\Lambda_0\right)/B$ (i.e., the number of EVs in equilibriumis fewer than the linearly targeted incremental number given subsidy value $\beta\sum_i\lambda_i$, or, the β-policy to achieve the target). The result follows. □

Proof of Proposition S.1: First, the rational expectations of ρ_A and ρ_B are not affected by the driver heterogeneity given the structure of the SP's problem. As a result, $\rho_i = \xi_i$ is still given by Equation (11). Given pile utilization rates ρ_A and ρ_B, the expected cost of each charging at a point ($u_i^e\left(\rho_A,\rho_B\right)$) and the probability of choosing a point ($\phi_i\left(\rho_A,\rho_B\right)$) are the same as those given by Equation (2), (3), and (4).

The marginal type residing at point i is $z_i^*\left(\xi_A,\xi_B\right) = p - \gamma + u_i^e\left(\xi_A,\xi_B\right)/T_i$, and the arrival rate of charging activities at point i is

$$\lambda_i = \begin{cases} \frac{1-\eta}{T_B}\phi_A\left[1-H(z_B^*)\right] + \frac{\eta}{T_A}\left[1-H(z_A^*)\right] & i = \mathcal{A}; \\ \frac{1-\eta}{T_B}\phi_B\left[1-H(z_B^*)\right] & i = \mathcal{B}. \end{cases}$$

Accordingly,

$$G_0 = \frac{\lambda_A}{\lambda_B} = \left(1 + \frac{\phi_A}{\phi_B}\right)\left[1 + \frac{\eta}{1-\eta} \cdot \frac{T_B \exp\left(au_B^e/T_B\right)}{T_A \exp\left(au_A^e/T_A\right)}\right] - 1,$$

and $r_0 = m_B/m_A = (\xi_A/\xi_B)/G_0$. Define $y_i = T_i \exp\left(au_i^e/T_i\right)$ for $i = \mathcal{A}, \mathcal{B}$. It is easy to check that $\partial y_i/\partial T_i > 0$ if and only if $u_i^e/T_i < 1/a$. Therefore, $\partial G_0/\partial T_A > 0$ and $\partial r_0/\partial T_A < 0$ if and only if $u_A^e/T_A > 1/a$; $\partial G_0/\partial T_B > 0$ and $\partial r_0/\partial T_B < 0$ if and only if $u_B^e/T_B < 1/a$. □

Proof of Proposition S.2: In Stage 3, suppose an SP's charging fee at point i is f_i', which could be different from the fee f_i in a competitive equilibrium. If f_i' is too high, such that the total expected cost of charging is always higher than with other SPs, then drivers will never visit the focal SP's charging piles and the pile utilization is zero. Otherwise, depending on whether it is $f_i' > f_i$ or $f_i' < f_i$, fewer or more drivers will choose this SP until the pile utilization there is lower or higher, such that the total expected cost of charging is equal to that with other SPs. Denote as ρ_i' the focal SP's pile utilization at point i given charging service fee f_i' and ρ_i the pile utilization for other SPs. In equilibrium, a driver should be indifferent between visiting this SP and other SPs. If a driver visits this SP, he or she will incur the search cost s_i; with probability $1-\rho_i'$, the driver will find a vacant pile and incur the cost of $f_i' + c_i\tau$; with probability ρ_i', the driver will continue the search and incur the total expected cost of $f_i + c_i\tau + s_i(1-\rho_i)^{-1}$. Therefore, in equilibrium, we should have $s_i + (1-\rho_i')(f_i' + c_i\tau) + \rho_i'\left[f_i + c_i\tau + s_i(1-\rho_i)^{-1}\right] = f_i + c_i\tau + s_i(1-\rho_i)^{-1}$, or

$$\rho_i'(f_i') = 1 - s_i\left[f_i - f_i' + s_i(1-\rho_i)^{-1}\right]^{-1}. \tag{S-2}$$

In Stage 2, suppose the equilibrium charging service fee at point i is $f_i(\xi_i)$, given rationally expected pile utilization ξ_i. Given $f_i(\xi_i)$ and rational expectations, each SP maximizes the total profit by optimizing the charging service fee and the optimal fee equals $f_i(\xi_i)$ in equilibrium. Applying the first order condition, we should have

$$\frac{\partial}{\partial f}\left[(\beta + f)\rho_i'(f)\right]|_{f=f_i(\xi_i)} = 0, \tag{S-3}$$

wherein $\rho_i'(f)$ is given by Equation (S-2) with ρ_i being replaced by ξ_i (because we only consider a marginal change in the fee). Accordingly, we have

$$f_i(\xi_i) = \frac{s_i\xi_i}{(1-\xi_i)^2} - \beta. \tag{S-4}$$

In Stage 1, SPs determine m_i and the EV buyers determine z_i^* given rationally expected pile utilization ξ_i and equilibrium charging fee $f_i(\xi_i)$. Based on Theorem 1, we can obtain the rationally expected pile utilization at point i by solving

$$\xi_i = \frac{(k_i - \alpha_i)\,\tau}{\beta + f_i(\xi_i)}. \tag{S-5}$$

Denote the solution as ξ_i^*. Combining (S-4) and (S-5), it is easy to demonstrate that

$$\xi_i^* = \left[1 + \sqrt{s_i\,(k_i - \alpha_i)^{-1}\,\tau^{-1}}\right]^{-1}. \tag{S-6}$$

Therefore, ξ_i^* is not affected by the β-policy in this case. We thus claim that the β-policy is no longer effective in motivating SPs to build more charging piles because SPs will always reduce the charging fee in competition as the subsidy value increases under the β-policy. However, the β-policy can effectively reduce EV drivers' charging cost (due to the reduction in charging fee) and thus promote EV adoption. This also offers an indirect incentive for SPs to build more piles. As a result, the β-policy in this case works in the same way as the γ-policy, the effect of which is not affected by the fee deregulation.

Given any policy mix (β, γ), for the same increment of subsidy (i.e., $(T/\mu)\sum_i \lambda_i d\gamma + (T\gamma/\mu + \beta)\sum_i d\lambda_i = \sum_i \lambda_i d\beta + (T\gamma/\mu + \beta)\sum_i d\lambda_i$), the ratio of change in γ over the change in β (i.e., $d\gamma/d\beta$) is μ/T. Hence, we need to compare the impact of $d\gamma$ against $d\beta = (T/\mu)\,d\gamma$. We know that ξ_i^* is independent of both β and γ, so the impact of β and γ are both through $z_i^* = p - \gamma + u_i^e/T$. Under the β-policy, $dz_i^* = -d\beta/T = -(T/\mu)\,d\gamma/T = -d\gamma/\mu \leq -d\gamma$; under the γ-policy, $dz_i^* = -d\gamma$. Therefore, the increase in λ_i is always larger under the β-policy, given a higher fiscal efficiency. The result follows. $\square$