Online Appendix for *Smart Charging of Electric Vehicles: An Innovative Business Model for Utility Firms*

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A. Charging Schedule for Non-simultaneous Arrivals when Customers Choose Completion Times upon Arrival

In this section, we consider non-simultaneous arrivals where arrival times $\{s_1, \ldots, s_N\}$ are exogenously given, but each customer chooses their completion time upon their arrival. As in Section 6.1, the utility firm divides time interval $[0, T]$ into L periods and at time $t = 0$ sets a period-specific menu $\{(p_\ell^{(i)}, \tau_\ell^{(i)}) : i = 1, \ldots, I\}$ for the customers arriving in period ℓ. Because the customers' delay-sensitivity is uncertain from the utility firm's perspective and only revealed through their choices upon arrival, optimizing the charging schedule requires solving a stochastic dynamic control problem, which is beyond the scope of this paper. Instead, we provide a heuristic method that generates a feasible charging schedule under a given time-varying menu, $\{(p_\ell^{(i)}, \tau_\ell^{(i)}) : i = 1, \ldots, I, \ell = 1, \ldots, L\}$. Using the same logic as in Section 6.1, we can see that the menu satisfies the properties in Propositions 2 and 3 in each period. Below, we describe our heuristic, which is based on the generalized juice-filling policy in Section 6.2.

First, we solve an auxiliary scheduling problem as follows. At each arrival time s_n, we replace the n-th customer by I customers all arriving at s_n, indexed as n_i, $i = 1, \ldots, I$. Customer n_i belongs to class i and demands $\beta^{(i)}$ units of energy. Customer n_i's vehicle can be charged at a maximum speed of $\overline{a}\beta^{(i)}$. Because the given menu is incentive compatible, customer n_i will choose $(p_\ell^{(i)}, \tau_\ell^{(i)})$, where ℓ is the period index such that $s_n \in [t_\ell, t_{\ell+1})$. Therefore, the utility firm faces a total of NI customers with known arrival and completion times, which allows us to apply the generalized juice-filling policy in Section 6.2 to find a charging schedule for these NI customers. Let $a_{n_i}(t)$ denote the charging schedule for customer n_i.

Next, we construct a charging schedule for each customer upon their arrival. If customer n arriving at $s_n \in [t_\ell, t_{\ell+1})$ is in class i, we set the charging schedule $a_n(t)$ to be $a_{n_i}(t)/\beta^{(i)}$. This charging schedule is feasible because, by construction, $a_n(t)$ charges one unit of energy during $[s_n, s_n + \tau_\ell^{(i)}]$ at maximum speed of $\overline{a}$.

The above heuristic prepares charging schedules for customers with different levels of delay-sensitivity at each arrival time. Note that if there is a large number of customers arriving together, the charging schedules of customers in the same class can be combined, resulting in block charging schedules that are easier to demonstrate, as shown in Figure 3.

B. Details of the Parameter Estimation for Numerical Analysis

We first estimate the generation cost and emissions intensity of the conventional sources in the PJM Interconnection by using the Emissions and Generation Resource Integrated Database (eGRID) of the Environmental Protection Agency (EPA 2019). eGRID provides detailed information for each power plant, including the heat rate (the amount of thermal energy needed to generate one unit of electrical energy, measured in MMBTU/kWh), nameplate capacity (the theoretical maximum power output of a generator in MW), and emissions rate (the amount of CO_2-equivalent emissions for each unit of electricity generated in lb/MWh). Multiplying the heat rate with the cost of the fuel source of a power plant (given by EIA 2017 in \$/MMBTU) gives the plant's cost of generating electricity (in \$/kWh).

We convert the nameplate capacity to effective capacity for each plant by multiplying the plant's nameplate capacity with the equivalent availability factor (provided by NERC 2019), which accounts for the unavailability due to planned outages for maintenance. We then sort the power plants in the increasing order of their generation costs, which gives a supply curve but does not take into account unplanned outages. To account for them, we calibrate the supply curve by multiplying the capacity with a daily scaling factor $\alpha_m \leq 1$ for each day m. We find the factor α_m by minimizing the sum of squared differences between the actual electricity prices and the estimated electricity prices (based on the adjusted supply curve and the net demand). To find the net electricity demand that should be satisfied by conventional sources, we subtract the hourly generation of renewable sources from the electricity demand. Figure 2 illustrates the resulting calibrated supply curve.

In the numerical setup, we also experimented using a finer division of periods (e.g., 10 or 15 arrival times) for arrival times, and the results are found to be similar. We choose to present the results of the simpler model for the ease of illustration (see Figure 3) as well as for computational efficiency. The computation is still demanding: For each of the 31 days in August 2016, we optimize the charging schedule under the objectives of a public and a private utility firm and compare the results with the current practice of as soon as possible (ASAP) charging policy. The computational procedure takes approximately two hours to converge to a set of optimal completion times (with optimal charging schedule) for each day.

C. Proofs

Proof of Proposition 1: The utility firm's second-stage problem (7)-(11) can be expressed in the standard form of a continuous-time optimal control problem (Bertsekas 2017, Section 7.1) as

$$\min_{\{a_n(t),\, n=1,\ldots,N\}} \; h\big(x_1(\tau_1),\ldots,x_N(\tau_N)\big) + \int_0^T \widetilde{c}\Big(\sum_{n=1}^{N} a_n(t) + d(t)\Big)\mathrm{d}t - \int_0^T \widetilde{c}(d(t))\mathrm{d}t, \tag{A.1}$$

$$\text{s.t.} \quad \frac{\mathrm{d}x_n(t)}{\mathrm{d}t} = a_n(t) \qquad \forall n = 1,\ldots,N, \tag{A.2}$$

$$0 \leq a_n(t) \leq \overline{a} \qquad \forall n = 1,\ldots,N, \tag{A.3}$$

where $x_n(t) = \int_0^t a_n(s)\,\mathrm{d}s$ is the state variable, representing the amount of electricity charged to vehicle n by time t, $a_n(t)$ is the control variable, and

$$h\big(x_1(\tau_1),\ldots,x_N(\tau_N)\big) = \begin{cases} 0, & \text{if } x_n(\tau_n) = 1, \quad \forall n \in \{1,\ldots,N\}, \\ \infty, & \text{otherwise,} \end{cases}$$

represents the terminal conditions in (8). The cost function $c(q(t)|(t))$ in (7) is substituted by (5)-(6), as reflected in (A.1). The last term in (A.1), $\int_0^T \widetilde{c}(d(t))\mathrm{d}t$, is independent of the control variables and can be omitted from the optimization below. A unique feature of this problem is that the terminal condition imposed on $x_n(t)$ through $h(x_1(\tau_1),\ldots,x_N(\tau_N))$ applies at different times τ_n. Accordingly, the number of control variables decreases over time.

We first show that the control policy found through the juice-filling policy is an optimal solution to the problem (A.1)-(A.3) for the special case with $N = 1$ vehicle, using Pontryagin's minimum principle (Bertsekas 2017, Section 7.3). In particular, the Hamiltonian is

$$H(x, a, p) = \widetilde{c}(a + d) + pa,$$

where the adjoint equation $p(t)$ (not to be confused with charging price $p^{(i)}$) is given by

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = -\frac{\partial H\left(x^*(t), a^*(t), p(t)\right)}{\partial x} = 0,$$

where $x^*(t)$ is the state trajectory corresponding to the optimal control $a^*(t)$. This implies that $p(t)$ is a constant for $t \in [0, T]$, i.e., $p(t) = p$ for some p. We next minimize the Hamiltonian, subject to the terminal condition:

$$\min_{0 \le a \le \overline{a}} \left\{H(x, a, p) = \widetilde{c}(a + d) + pa\right\}$$

$$s.t. \int_0^T a(t)\mathrm{d}t = 1. \tag{A.4}$$

Because $\widetilde{c}(\cdot)$ is convex, $H(x, a, p)$ is also convex in a for a given x and p, and the first-order condition is given as $\frac{\mathrm{d}H(x,a,p)}{\mathrm{d}a} = \widetilde{c}^{\,\prime}(a+d)+p = 0$, where $\widetilde{c}^{\,\prime}(\cdot)$ is the derivative of the cost function. Taking the constraint that $0 \le a(t) \le \overline{a}$ into account, the optimal action is to set $a^*(t) = \min\left((z - d(t))^+, \overline{a}\right)$, where $\widetilde{c}^{\,\prime}(z) = -p$. Note that z is constant because p is a constant. To find the value of z, we substitute $a^*(t)$ into the terminal condition (A.4) so that z is implicitly given by

$$\int_0^T \min\left((z - d(t))^+, \overline{a}\right) \mathrm{d}t = 1.$$

This control policy is the same as the one found by the juice-filling policy for the special case with one electric vehicle (i.e., when $N = 1$), where z is given in Step 2 and $L(t) = d(t)$. Therefore, the juice-filling policy finds an optimal solution to the problem (A.1)-(A.3).

We next prove that the juice-filling policy is optimal for $N > 1$. We index electric vehicles with n such that $\tau_N \le \cdots \le \tau_1$. Let $a_n^*(t)$ for $n \in \{1, \ldots, N\}$ be an optimal solution to the charging problem (A.1)-(A.3). Using the juice-filling policy, we construct another solution $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$, which we show is also optimal. We next describe the construction of this solution in the interval $t \in [0, \tau_N]$, where τ_N is the earliest completion time. In essence, we find $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ for N vehicles with simultaneous arrival (i.e., $t = 0$) and completion times (i.e., $t = \tau_N$) such that each vehicle is charged by time τ_N to the same level as that under the optimal policy, i.e., $\hat{x}_n(\tau_N) = x_n^*(\tau_N) = \int_0^{\tau_N} a_n^*(t)\mathrm{d}t$ for $n \in \{1, \ldots, N\}$. Then, we show that the charging policy $\hat{a}_n(t)$ achieves the same cost as the optimal policy $a_n^*(t)$.

The charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ in the interval $t \in [0, \tau_N]$ can be constructed by applying the juice-filling policy with two modifications: Let $T = \tau_N$ and in Step 2, find z_n such that $\int_0^{\tau_N} \min\left\{(z_n - L(t))^+, \overline{a}\right\} \mathrm{d}t = x_n^*(\tau_N)$. We refer to this policy as the modified juice-filling policy. Note that, for vehicle $n = N$, the modified juice-filling policy is equivalent to the juice-filling policy described in Section 4 because $x_N^*(\tau_N) = 1$. That is, the charging schedule of vehicle N, i.e., $\hat{a}_N(t)$, is the same as that found through the juice-filling policy.

We next verify that the resulting charging cost in $[0, \tau_N]$ under the modified juice-filling policy is the same as that under the juice-filling policy for charging one electric vehicle. Specifically, consider the juice-filling policy for one vehicle, where the vehicle's completion time is τ_N, its maximum charging speed is $\overline{a}N$, and the vehicle needs to be charged with $\sum\limits_{n=1}^{N} x_n^*(\tau_N)$ units of energy. Following similar steps as above, we can show that there exists z such that

$$\int_0^{\tau_N} \min\left((z - d(t))^+, \overline{a}N\right) \mathrm{d}t = \sum_{n=1}^{N} x_n^*(\tau_N). \tag{A.5}$$

Moreover, it is straightforward to verify that z in (A.5) is the same as z_N of the modified juice-filling policy such that the two policies have the same cost. Given the optimality of the juice-filling policy for one vehicle, charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, I\}$ is an optimal policy for $t \in [0, \tau_N]$. That is, $\int_0^{\tau_N} \widetilde{c}\left(\sum_{n=1}^N \hat{a}_n(t) + d(t)\right) \mathrm{d}t = \int_0^{\tau_N} \widetilde{c}\left(\sum_{n=1}^N a_n^*(t) + d(t)\right) \mathrm{d}t$.

Notice that in constructing $\hat{a}_N(t)$ for $t \in [0, \tau_N]$, we have used the juice-filling policy, where $\int_0^{\tau_N} \min\left\{(z_N - d(t))^+, \overline{a}\right\} \mathrm{d}t = 1$ so that $\hat{a}_N(t) = \min\left\{(z_N - d(t))^+, \overline{a}\right\}$ for $t \in [0, \tau_N]$.

We then update $L(t)$ with $L(t) + \hat{a}_N(t)$ and proceed to the time interval $t \in [0, \tau_{N-1}]$, where τ_{N-1} is the completion time of vehicle $N - 1$. By the same arguments as above, we can construct $\hat{a}_n(t)$ for all $n \in \{1, \ldots, N-1\}$ in $t \in [0, \tau_{N-1}]$ and show that it achieves the same cost as the optimal policy. This procedure can be repeated for all vehicles and time intervals. Therefore, the control policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ found through the juice-filling policy identifies an optimal solution to the charging problem (A.1)-(A.3). ∎

Proof of Lemma 1: (i) For given $(\tau_1, \ldots, \tau_N)$, let $\{a_1^*(t), \ldots, a_N^*(t)\}$ be an optimal control. Given that $\{b_1, \ldots, b_N\}$ is a permutation of $\{1, \ldots, N\}$, the control $\{a_{b_1}^*(t), \ldots, a_{b_N}^*(t)\}$ must be feasible under completion time $(\tau_{b_1}, \ldots, \tau_{b_N})$ because $\int_0^{\tau_{b_n}} a_{b_n}^*(t)\, \mathrm{d}t = 1$, $\forall n = 1, \ldots, N$. Furthermore, this feasible control yields the same objective value in (7) as the control $\{a_1^*(t), \ldots, a_N^*(t)\}$ because $\sum\limits_{n=1}^{N} a_{b_n}^*(t) = \sum\limits_{n=1}^{N} a_n^*(t)$. If, however, there exists another control $\{\widetilde{a}_{b_1}^*(t), \ldots, \widetilde{a}_{b_N}^*(t)\}$ that yields a strictly lower objective value in (7) than $\{a_{b_1}^*(t), \ldots, a_{b_N}^*(t)\}$, then $\{\widetilde{a}_1^*(t), \ldots, \widetilde{a}_N^*(t)\}$ must be feasible under $(\tau_1, \ldots, \tau_N)$ and yield a strictly lower objective value in (7) than $\{a_1^*(t), \ldots, a_N^*(t)\}$, leading to a contradiction. Hence, we have $C(\tau_{b_1}, \ldots, \tau_{b_N}) = C(\tau_1, \ldots, \tau_N)$.

(ii) The optimal control for (7)-(11) under any given $(\tau_1, \ldots, \tau_N)$ remains a feasible control under $(\hat{\tau}_1, \ldots, \hat{\tau}_N) \geq (\tau_1, \ldots, \tau_N)$. It follows that $C(\tau_1, \ldots, \tau_N)$ decreases in τ_n for any n.

To show the continuity, first note that, by construction of the juice-filling policy, the fill-up-to level z_n is continuous in $(\tau_1, \ldots, \tau_N)$. Consider the cost $\widetilde{c}\big(d(t) + q^*(t)\big)$ as function of time. The area under this cost function is completely determined by $(\tau_1, \ldots, \tau_N)$ and $(z_1, \ldots, z_N)$. Because $(z_1, \ldots, z_N)$ is continuous in $(\tau_1, \ldots, \tau_N)$, and $\widetilde{c}(\cdot)$ is convex and hence continuous, the area under the curve, $\int_0^T \widetilde{c}\big(d(t) + q^*(t)\big)\mathrm{d}t$, is also continuous in $(\tau_1, \ldots, \tau_N)$. Therefore, $C(\tau_1, \cdots, \tau_N) = \int_0^T \tilde{c}(d(t) + q^*(t))\mathrm{d}t - \int_0^T \tilde{c}(d(t))\mathrm{d}t$ is also continuous in $(\tau_1, \cdots, \tau_N)$. ∎

Proof of Lemma 2: The objective function in (15) can be written as

$$\begin{aligned}
&\mathbb{E}_{\{\theta_1, \ldots, \theta_N\}}\left[\sum_{n=1}^{N} \theta_n \delta(\tau_n) + C(\tau_1, \ldots, \tau_N)\right] \\
&\qquad = \sum_{i_1=1}^{I} \sum_{i_2=1}^{I} \cdots \sum_{i_N=1}^{I} \left\{ \prod_{n=1}^{N} \beta^{(i_n)} \left[\sum_{n=1}^{N} \theta^{(i_n)} \delta\big(\tau^{(i_n)}\big) + C\big(\tau^{(i_1)}, \tau^{(i_2)}, \cdots, \tau^{(i_N)}\big) \right] \right\}.
\end{aligned} \tag{A.6}$$

The objective function in (A.6) is continuous in $(\tau^{(1)}, \dots, \tau^{(N)})$ because of the continuity of $\delta(\tau)$ and the continuity of $C(\tau_1, \dots, \tau_N)$ from Lemma 1. Minimizing the continuous function in (A.6) over the compact set $\{\tau^{(i)} \in [\underline{w}, T] : i = 1, \dots, I\}$ ensures the existence of an optimal solution by Weierstrass Theorem (Sundaram 1996, p. 90). To complete the existence proof, we note that there always exists prices low enough to ensure that the individual rationality constraints are satisfied.

For any solution with $\tau^{(j)} < \tau^{(k)}$ for $j < k$, we can strictly reduce the objective value by swapping $\tau^{(j)}$ and $\tau^{(k)}$. This swap does not affect the charging cost $C(\tau^{(i_1)}, \tau^{(i_2)}, \cdots, \tau^{(i_N)})$ due to Lemma 1(i), but it reduces the inconvenience cost $\sum_{n=1}^{N} \theta^{(i_n)} \delta(\tau^{(i_n)})$ because

$$\theta^{(j)}\delta(\tau^{(k)}) + \theta^{(k)}\delta(\tau^{(j)}) < \theta^{(j)}\delta(\tau^{(j)}) + \theta^{(k)}\delta(\tau^{(k)}),$$

or equivalently, $\theta^{(j)}\left(\delta(\tau^{(k)}) - \delta(\tau^{(j)})\right) < \theta^{(k)}\left(\delta(\tau^{(k)}) - \delta(\tau^{(j)})\right)$. This inequality follows from $\theta^{(j)} < \theta^{(k)}$ and $\delta(\tau)$ strictly increases in τ. Therefore, it is not possible to have $\tau^{(j)} < \tau^{(k)}$ for $j < k$ in any optimal solution. Hence, the optimal completion times are such that $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$. ∎

Proof of Proposition 2: (i) For simplicity, let $\delta^{(i)} \doteq \delta(\tau^{(i)})$ for $i = 1, \dots, I$. Because the disutility $\delta(\tau)$ increases in τ and $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, we have $\delta^{(1)} \geq \delta^{(2)} \geq \cdots \geq \delta^{(I)}$.

First, we set $p^{(I)} \leq u_0 - \theta^{(I)}\delta^{(I)} - \underline{u}$, so that a customer with $\theta^{(I)}$ receives at least the reservation utility if the customer chooses completion time $\tau^{(I)}$. Then, for $i = I-1, I-2, \dots, 1$, we sequentially set $p^{(i)}$, such that the following $2(I-i)$ incentive compatibility constraints are satisfied:

$$\theta^{(j)}\delta^{(j)} + p^{(j)} \leq \theta^{(j)}\delta^{(i)} + p^{(i)}, \qquad \theta^{(i)}\delta^{(i)} + p^{(i)} \leq \theta^{(i)}\delta^{(j)} + p^{(j)}, \qquad \forall j = i+1, \dots, I. \tag{A.7}$$

At the end of the above procedure, all of $2 + 4 + \cdots + 2(I-1) = I(I-1)$ incentive compatibility constraints for all I customer classes are satisfied.

The constraints in (A.7) can be equivalently written as

$$p^{(i)} \in \big[\max_{j>i}\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)})\},\ \min_{j>i}\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)})\}\big]. \tag{A.8}$$

The lower (upper) bound ensures that a customer from class i strictly prefers not to choose any lower (higher) class than their class.

We next simplify the bounds for $p^{(i)}$ in (A.8) to obtain the bounds given in Proposition 2. The incentive compatibility for any two $\theta^{(j)}$ and $\theta^{(k)}$ customers requires $\theta^{(k)}\delta^{(k)} + p^{(k)} \leq \theta^{(k)}\delta^{(j)} + p^{(j)}$ and $\theta^{(j)}\delta^{(j)} + p^{(j)} \leq \theta^{(j)}\delta^{(k)} + p^{(k)}$, which are equivalent to

$$\theta^{(k)}(\delta^{(k)} - \delta^{(j)}) \leq p^{(j)} - p^{(k)} \leq \theta^{(j)}(\delta^{(k)} - \delta^{(j)}). \tag{A.9}$$

Consider the ordering of the terms, $\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}), j = i+1, \dots, I\}$. Let $i < j < k$. Then,

$$\begin{aligned} p^{(j)} - p^{(k)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(k)}(\delta^{(i)} - \delta^{(k)}) &\geq \theta^{(k)}(\delta^{(k)} - \delta^{(j)}) - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(k)}(\delta^{(i)} - \delta^{(k)}) \\ &= (\theta^{(k)} - \theta^{(j)})(\delta^{(i)} - \delta^{(j)}) \geq 0, \end{aligned}$$

where the first inequality is due to the first inequality in (A.9), and the last inequality is due to $\theta^{(j)} \leq \theta^{(k)}$ and $\delta^{(i)} \geq \delta^{(j)}$. Therefore, $j = i+1$ maximizes the lower bound in (A.8), i.e., $\max_{j>i}\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)})\} = p^{(i+1)} - \theta^{(i+1)}(\delta^{(i)} - \delta^{(i+1)})$.

Consider the ordering of the terms $\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}), j = i+1, \dots, I\}$. Let $i < j < k$. Then,

$$\begin{aligned} p^{(j)} - p^{(k)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(i)}(\delta^{(i)} - \delta^{(k)}) &\leq \theta^{(j)}(\delta^{(k)} - \delta^{(j)}) - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(i)}(\delta^{(i)} - \delta^{(k)}) \\ &= (\theta^{(j)} - \theta^{(i)})(\delta^{(k)} - \delta^{(j)}) \leq 0, \end{aligned}$$

where the first inequality is due to the second inequality in (A.9), and the last inequality follows from $\theta^{(i)} \leq \theta^{(j)}$ and $\delta^{(j)} \geq \delta^{(k)}$. Therefore, $j = i+1$ minimizes the upper bound in (A.8), i.e., $\min_{j>i}\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)})\} = p^{(i+1)} - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)})$.

Therefore, the feasible interval for p_i in (A.8) becomes:

$$p^{(i)} \in \left[p^{(i+1)} - \theta^{(i+1)}(\delta^{(i)} - \delta^{(i+1)}),\ p^{(i+1)} - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)})\right], \tag{A.10}$$

which is clearly nonempty, because $\delta^{(i)} \geq \delta^{(i+1)}$ and $\theta^{(i)} \leq \theta^{(i+1)}$.

To see that the pricing scheme is individually rational (i.e., reservation utility is met), note that

$$(u_0 - \theta^{(i)}\delta^{(i)} - p^{(i)} - \underline{u}) - (u_0 - \theta^{(i+1)}\delta^{(i+1)} - p^{(i+1)} - \underline{u}) \geq \theta^{(i)}(\delta^{(i+1)} - \delta^{(i)}) - p^{(i)} + p^{(i+1)} \geq 0, \tag{A.11}$$

where the first inequality is due to $\theta^{(i+1)} > \theta^{(i)}$ and the second inequality follows from (A.10). That is, the net utility of a customer in class i is higher than that of a customer in class $i+1$. Given that the pricing scheme is individually rational for a customer in class I by construction, then it is also individually rational for all classes. Moreover, this result also holds if the reservation utility of customers increase in their delay-sensitivity, i.e., if $\underline{u}^{(1)} \leq \underline{u}^{(2)} \leq \cdots \leq \underline{u}^{(I)}$.[1] In this case, the inequalities in (A.11) continue to hold because

$$\begin{aligned}(u_0 - \theta^{(i)}\delta^{(i)} - p^{(i)} - \underline{u}^{(i)}) &- (u_0 - \theta^{(i+1)}\delta^{(i+1)} - p^{(i+1)} - \underline{u}^{(i+1)}) \\ &\geq \theta^{(i)}(\delta^{(i+1)} - \delta^{(i)}) - p^{(i)} + p^{(i+1)} + \underline{u}^{(i+1)} - \underline{u}^{(i)} \geq 0,\end{aligned}$$

where $\underline{u}^{(i+1)} - \underline{u}^{(i)} \geq 0$. Accordingly, the pricing scheme is individually rational even if $\underline{u}^{(1)} \leq \underline{u}^{(2)} \leq \cdots \leq \underline{u}^{(I)}$.

(ii) Using the result from part (i), substituting the first-best solution $\{\tau^{(i)*} : i = 1, \ldots, I\}$ from (15) into (12), there exists prices that satisfy incentive compatibility and individual rationality constraints. This implies that the optimal completion times to (12) is the first-best solution. ∎

Proof of Proposition 3: (i) Suppose $\tau^{(j)} < \tau^{(k)}$ for some $j < k$. Incentive compatibility requires

$$p^{(j)} + \theta^{(j)}\delta(\tau^{(j)}) \leq p^{(k)} + \theta^{(j)}\delta(\tau^{(k)}), \tag{A.12}$$

$$p^{(j)} + \theta^{(k)}\delta(\tau^{(j)}) \geq p^{(k)} + \theta^{(k)}\delta(\tau^{(k)}). \tag{A.13}$$

Subtracting (A.13) from (A.12) and simplifying, we have $(\theta^{(j)} - \theta^{(k)})(\delta(\tau^{(j)}) - \delta(\tau^{(k)})) \leq 0$, which contradicts $\theta^{(j)} < \theta^{(k)}$ and $\tau^{(j)} < \tau^{(k)}$ because $\delta(\tau)$ strictly increases in τ. Therefore, any feasible solution must have declining completion times. To prove the existence of an optimal solution, note that the objective function (13) is continuous in $(p^{(i)}, \tau^{(i)})$ because of the continuity of $C(\tau_1, \ldots, \tau_N)$ from Lemma 1. Moreover, charging prices and completion times are bounded by $0 \leq p^{(i)} \leq u_0$ and $\underline{w} \leq \tau^{(i)} \leq T$. Maximizing the continuous function in (13) over this compact set ensures the existence of an optimal solution (with sufficiently low prices that ensure that the individual rationality constraints are satisfied). Finally, because each feasible solution must have declining completion times, the optimal completion times must also be declining.

(ii) Note that the given prices in the proposition correspond to the upper bounds of the prices

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[1] We index customer class I such that it has the highest delay-sensitivity $\theta^{(I)}$ and the highest reservation utility $\underline{u}^{(I)}$, but this reservation utility can still be met by the utility firm. Formally, $u_0 - \theta^{(I)}\delta(\underline{w}) > \underline{u}^{(I)}$, where $\underline{w}$ is the minimum charging completion time. This ensures that it is feasible for the utility firm to serve all customer classes i with delay-sensitivity $\theta^{(i)} \leq \theta^{(I)}$ and reservation utility $\underline{u}^{(i)} \leq \underline{u}^{(I)}$.

in Proposition 2. Therefore, these prices continue to satisfy individual rationality constraints (4) and incentive compatibility constraints (2). Moreover, given that they are the upper bounds, they maximize the utility firm's profit. ∎

Proof of Proposition 4: The private utility firm's objective function in (13) is given as

$$\mathbb{E}_{\{\theta_1,\ldots,\theta_N\}}\left[\sum_{n=1}^{N} p_n - C(\tau_1,\ldots,\tau_N)\right] = N\mathbb{E}\left[p_n\right] - \mathbb{E}\left[C(\tau_1,\ldots,\tau_N)\right]. \tag{A.14}$$

Consider the expected revenue $N\mathbb{E}\left[p_n\right]$. Based on the iterative procedure given in (19), $p^{(I)} = u_0 - \theta^{(I)}\delta(\tau^{(I)}) - \underline{u}$, and letting $\delta^{(i)} \doteq \delta(\tau^{(i)})$, we can write

$$\begin{aligned} p^{(i)} &= u_0 - \theta^{(I)}\delta^{(I)} - \theta^{(I-1)}(\delta^{(I-1)} - \delta^{(I)}) - \cdots - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)}) - \underline{u} \\ &= u_0 - (\theta^{(I)} - \theta^{(I-1)})\delta^{(I)} - \cdots - (\theta^{(i+1)} - \theta^{(i)})\delta^{(i+1)} - \theta^{(i)}\delta^{(i)} - \underline{u} \\ &= u_0 - \theta^{(i)}\delta^{(i)} - \underline{u} - \sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}, \end{aligned}$$

for $i = 1, \ldots, I-1$. Therefore, the expected revenue is

$$\begin{aligned} N\mathbb{E}\left[p_n\right] = N\sum_{i=1}^{I}\beta^{(i)}p^{(i)} &= N\sum_{i=1}^{I}\beta^{(i)}\left(u_0 - \theta^{(i)}\delta^{(i)} - \underline{u}\right) - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} \\ &= N(u_0 - \underline{u}) - N\sum_{i=1}^{I}\beta^{(i)}\theta^{(i)}\delta^{(i)} - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} \\ &= N(u_0 - \underline{u}) - \mathbb{E}\left[\sum_{n=1}^{N}\theta_n\delta_n\right] - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}, \end{aligned}$$

where $\delta_n \doteq \delta(\tau_n)$. By substituting the above expression of the expected revenue into (A.14), the private utility firm's objective in (13) can be rewritten as

$$\max_{\{\tau^{(i)}\in[\underline{w},T],\, i=1,\ldots,I\}} \quad N(u_0 - \underline{u}) - \mathbb{E}\left[\sum_{n=1}^{N}\theta_n\delta_n\right] - \mathbb{E}\left[C(\tau_1,\ldots,\tau_I)\right] - N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}.$$

This is equivalent to the private utility firm's objective in Proposition 4 because $N(u_0 - \underline{u})$ is constant in $\tau^{(i)}$'s. Finally, note that for $\beta^{(i)} = \frac{1}{I}$, the information rent term $N\sum_{i=1}^{I-1}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} = \frac{N}{I}\sum_{i=1}^{I-1}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} = \frac{N}{I}\sum_{i=2}^{I}(i-1)\left(\theta^{(i)} - \theta^{(i-1)}\right)\delta^{(i)}$. ∎

Proof of Proposition 5: Thresholds $z_{n,j}$'s can be computed via the following problem:

$$\min_{\{z_{n,j},\, n=1,\ldots,N,\, j=1,\ldots,2N-1\}} \int_0^T c\big(q(t) \mid d(t)\big)\mathrm{d}t \tag{A.15}$$

$$\text{s.t. } (23),\ (24),\ (26), \quad z_{n,j} = 0 \quad \forall j = 1,\ldots,2N-1, \forall n \notin \mathcal{I}(t_j) \text{ and} \tag{A.16}$$

$$a_n(t) = \min\left\{\left(z_{n,j} - d(t) - \sum_{k<n}a_k(t)\right)^{+}, \overline{a}\right\}, \forall t \in [t_j, t_j+1], \forall j = 1,\ldots,2N-1, \forall n \in \mathcal{I}(t_j). \tag{A.17}$$

Note that the decision variables of the original objective function (22), i.e., $\{a_n(t),\ n = 1,\ldots,N\}$, are replaced with the thresholds $z_{n,j}$'s in (A.15). In this formulation, (A.17) implicitly defines the charging schedule as a function of the thresholds.

Similar to the simultaneous arrivals case, the charging problem (22)-(26) can be expressed in the standard form (A.1)-(A.3), with the additional constraint that vehicle n cannot be charged if it is not at the station at time t, i.e., $a_n(t) = 0, \forall t \notin [s_n, \tau_n]$.

We prove the optimality of the charging policy for non-simultaneous arrivals (given $z_{n,j}$'s) by induction. First consider the time interval $[t_{2N-1}, t_{2N}]$ in which there is only one electric vehicle at the station. Let its index be k. Consistent with the standard form (A.1)-(A.3), denote the level of electricity charged by time t_{2N-1} to vehicle k by $x_k(t_{2N-1})$. Given that vehicle k is the only vehicle in the station in this time interval, we can use Pontryagin's minimum principle to identify the optimal charging schedule as in the proof of Proposition 1: There exists a threshold $z_{k,2N-1}$, where $\int_{t_{2N-1}}^{t_{2N}} \min\left\{(z_{k,2N-1} - d(t))^+, \overline{a}\right\} \mathrm{d}t = 1 - x_k(t_{2N-1})$ such that it is optimal to set $a_k^*(t) = \min\left\{\left(z_{k,2N-1} - d(t)\right)^+, \overline{a}\right\}$ for $t \in [t_{2N-1}, t_{2N}]$. It is straightforward to verify that the charging policy for non-simultaneous arrivals essentially leads to the same charging decision. Therefore, the policy is optimal for $t \in [t_{2N-1}, t_{2N}]$.

Suppose the policy is optimal for intervals $[t_{j+1}, t_{j+2}], \ldots, [t_{2N-2}, t_{2N-1}]$ for some $j \in \{1, \ldots, 2N-3\}$. We next prove the policy is also optimal in $[t_j, t_{j+1}]$. For this time interval, let $a_n^*(t)$ be an optimal solution for $n \in \mathcal{I}(t_j)$. The charging policy for non-simultaneous arrivals (given $z_{n,j}$'s) produces a charging schedule for vehicles $k \in \mathcal{I}(t_j)$ such that each vehicle is charged $\int_{t_j}^{t_{j+1}} a_k^*(t)\mathrm{d}t$ units of electricity by time t_{j+1}. As discussed in the proof of Proposition 1, this schedule is optimal for $t \in [t_j, t_{j+1}]$ because it is equivalent to charging one electric vehicle with $\sum\limits_{k\in\mathcal{I}(t)} \int_{t_j}^{t_{j+1}} a_k^*(t)\mathrm{d}t$ units of electricity by time t_{j+1}. Given that the policy for non-simultaneous arrivals is also optimal for $[t_j, t_{j+1}]$, by induction, it is optimal for all intervals.

Finally, in the above analysis, without loss of generality, we assume that when a customer arrives at the charging station there is at least another customer whose vehicle is being charged. Otherwise, the utility firm's problem can be decoupled into multiple problems, which can be solved separately. ∎

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