Online Appendix for Manuscript

Competition between Hospitals under Bundled Payments and Fee-for-Service: An Equilibrium Analysis of Insurer's Choice

Appendix

Proof of Proposition 1. We first prove this result for the FFS/BP game. Recall the definition of $\pi_1^{FFS/BP}$ from Equation (3), i.e.,

$$\pi_1^{FFS/BP}(q_1,q_2)=\Bigg(\Big(\gamma_1^{FFS}-\alpha_{11}q_1\Big)+\Big(1-a_1\Big)\Big(\gamma_2^{FFS}-\alpha_{12}q_1\Big)\Bigg)\frac{1}{2}\big(1+\tau(q_1-q_2)\big)\,.$$

It is straightforward to verify that $\pi_1^{FFS/BP}$ is concave in q_1. By solving $\frac{\partial\pi_1^{FFS/BP}}{\partial q_1}=0$, one can obtain the best response of Hospital 1 as

$$q_1^*(q_2)=\frac{1}{2}\big(\frac{\tau q_2-1}{\tau}+\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}\big). \tag{A1}$$

Also, recall the definition of $\pi_2^{FFS/BP}$ from Equation (4),

$$\pi_2(q_2,q_1)^{FFS/BP}=\Bigg(\gamma^{BP}-\alpha_{21}q_2-\Big(1-a_2\Big)\alpha_{22}q_2\Bigg)\frac{1}{2}\big(1+\tau(q_2-q_1)\big)\,.$$

By solving $\frac{\partial\pi_2^{FFS/BP}}{\partial q_2}=0$, one can obtain the best response of Hospital 2 is

$$q_2^*(q_1)=\frac{1}{2}\big(\frac{\tau q_1-1}{\tau}+\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}\big). \tag{A2}$$

Clearly, we can get that $\frac{\partial q_1^*(q_2)}{\partial q_2}=\frac{\partial q_2^*(q_1)}{\partial q_1}=\frac{1}{2}$, i.e., both best response functions are increasing with the competitor's quality. Followed by similar procedures and straightforward algebra, the same results hold for the FFS/FFS game and the BP/BP game.

Proof of Proposition 2. For the FFS/BP game, by the best response functions obtained in Equations (A1) and (A2) from the proof of Proposition 1, one can solve for the system of linear equations:

$$\begin{cases} q_1^*=\frac{1}{2}\big(\frac{\tau q_2^*-1}{\tau}+\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}\big),\\ q_2^*=\frac{1}{2}\big(\frac{\tau q_1^*-1}{\tau}+\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}\big). \end{cases}$$

The solution is the equilibrium:

$$q_1^*=\frac{2}{3}\cdot\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}+\frac{1}{3}\cdot\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}-\frac{1}{\tau}\,, \tag{A3}$$

and

$$q_2^*=\frac{2}{3}\cdot\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}+\frac{1}{3}\cdot\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}-\frac{1}{\tau}\,. \tag{A4}$$

Similarly, for the FFS/FFS game, from Equations (3) and (5), the equilibrium is:

$$\begin{aligned} q_1^{FFS}=&\frac{2}{3}\cdot\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}+\frac{1}{3}\cdot\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_{21}+(1-a_2)\alpha_{22}}-\frac{1}{\tau}\,,\\ q_2^{FFS}=&\frac{2}{3}\cdot\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_{21}+(1-a_2)\alpha_{22}}+\frac{1}{3}\cdot\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}-\frac{1}{\tau}\,. \end{aligned}$$

And for the BP/BP game, from Equations (4) and (6), the equilibrium is:

$$\begin{aligned} q_1^{BP}=&\frac{2}{3}\cdot\frac{\gamma^{BP}}{\alpha_{11}+(1-a_1)\alpha_{12}}+\frac{1}{3}\cdot\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}-\frac{1}{\tau}\,,\\ q_2^{BP}=&\frac{2}{3}\cdot\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}+\frac{1}{3}\cdot\frac{\gamma^{BP}}{\alpha_{11}+(1-a_1)\alpha_{12}}-\frac{1}{\tau}\,. \end{aligned}$$

Proof of Proposition 3. For the FFS/BP game, from the expressions of q_1^* and q_2^* in Equations (A3) and (A4), one can get that $\frac{\partial q_1^*}{\partial \gamma_1^{FFS}} = \frac{2}{3} \cdot \frac{1}{\alpha_{11}+(1-a_1)\alpha_{12}} > 0\,$, $\frac{\partial q_1^*}{\partial \gamma_2^{FFS}} = \frac{2}{3} \cdot \frac{1-a_1}{\alpha_{11}+(1-a_1)\alpha_{12}} > 0\,$, and $\frac{\partial q_2^*}{\partial \gamma^{BP}} = \frac{2}{3} \cdot \frac{1}{\alpha_{21}+(1-a_2)\alpha_{22}} > 0\,$. Clearly, q_1^* is increasing in γ_1^{FFS} and γ_2^{FFS}, and q_2^* is increasing in γ^{BP}. Followed by similar procedures and straightforward algebra, the same results hold for the FFS/FFS game and the BP/BP game.

Proof of Proposition 4. By the results from Proposition 2, clearly, all equilibrium quality solutions are increasing in τ.

Expressions of Ω, Φ, and Ψ in Equations (11), (12) and (13).

We have

$$\begin{aligned}\Omega =&\tau^2\Big(\left(\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}\right)(\alpha_{21}+(1-a_2)\alpha_{22}) - \gamma^{BP}(\alpha_{11}+(1-a_1)\alpha_{12})\Big)\\ &+9\tau(\alpha_{11}+(1-a_1)\alpha_{12})(\alpha_{21}+(1-a_2)\alpha_{22})\left[\left(\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}\right)(\alpha_{21}+(1-a_2)\alpha_{22})\right.\\ &\left.+\gamma^{BP}(\alpha_{11}+(1-a_1)\alpha_{12})\right] - 18(\alpha_{11}+(1-a_1)\alpha_{12})^2(\alpha_{21}+(1-a_2)\alpha_{22})^2\,,\end{aligned}$$

and

$$\begin{aligned}\Phi =&\tau^2\Big(\gamma_1^{FFS}\left[\alpha_{11}-\alpha_{21}+(1-a_1)\alpha_{12}-(1-a_2)\alpha_{22}\right]+\\ &\gamma_2^{FFS}\left[(1-a_2)\alpha_{11}-(1-a_1)\alpha_{21}+(1-a_1)(1-a_2)(\alpha_{12}-\alpha_{22})\right]\Big)^2\\ &+9\tau(\alpha_{11}+(1-a_1)\alpha_{12})(\alpha_{21}+(1-a_2)\alpha_{22})\Big(\gamma_1^{FFS}\left[\alpha_{11}+\alpha_{21}+(1-a_1)\alpha_{12}+(1-a_2)\alpha_{22}\right]\\ &+\gamma_2^{FFS}\left[(1-a_2)\alpha_{11}+(1-a_1)\alpha_{21}+(1-a_1)(1-a_2)(\alpha_{12}+\alpha_{22})\right]\Big)\\ &-18(\alpha_{11}+(1-a_1)\alpha_{12})^2(\alpha_{21}+(1-a_2)\alpha_{22})^2\,,\end{aligned}$$

and lastly,

$$\begin{aligned}\Psi =&\tau^2(\gamma^{BP})^2\Big((\alpha_{11}-\alpha_{21}-(1-a_2)\alpha_{22})^2+\alpha_{12}(1-a_1)\left[2\alpha_{11}-2\alpha_{21}+(1-a_1)\alpha_{12}-2(1-a_2)\alpha_{22}\right]\Big)^2\\ &+9\tau\gamma^{BP}(\alpha_{11}+(1-a_1)\alpha_{12})(\alpha_{21}+(1-a_2)\alpha_{22})\left[\alpha_{11}+\alpha_{21}+(1-a_1)\alpha_{12}+(1-a_2)\alpha_{22}\right]\\ &-18(\alpha_{11}+(1-a_1)\alpha_{12})^2(\alpha_{21}+(1-a_2)\alpha_{22})^2\,.\end{aligned}$$

Proof of Lemma 1. The results follow from straightforward algebra by comparing the respective values from Equations (12), (13) and (11). Furthermore, the factor that defines the sign of $Q^*_{BP/BP}-Q^*_{FFS/FFS}$ is

$$\begin{aligned}&-(a_1-1)^2\left((a_2-1)\gamma_{12}+\gamma_2-\gamma_{11}\right)\left((9a_2-9)\alpha_{22}+\tau(a_2-1)\gamma_{12}+(-\gamma_2-\gamma_{11})\tau-9\alpha_{21}\right){\alpha_{12}}^2\\ &+2(a_1-1)\Bigg(-\frac{9}{2}(a_2-1)^2\left((a_1-1)\gamma_{12}+\gamma_2-\gamma_{11}\right){\alpha_{22}}^2+(a_2-1)\Big(\tau(a_2-1)(a_1-1){\gamma_{12}}^2+\big(-\gamma_{11}(a_1+a_2-2)\tau\\ &+9a_1\alpha_{21}+9a_2\alpha_{11}-9\alpha_{11}-9\alpha_{21}\big)\gamma_{12}-(\gamma_2-\gamma_{11})\left((\gamma_2+\gamma_{11})\tau-9\alpha_{11}-9\alpha_{21}\right)\Big)\alpha_{22}\\ &-\tau(a_2-1)(a_1\alpha_{21}-a_2\alpha_{11}+\alpha_{11}-\alpha_{21}){\gamma_{12}}^2+\Bigg(\left(a_1\alpha_{21}+(-2\alpha_{11}+\alpha_{21})a_2+2\alpha_{11}-2\alpha_{21}\right)\gamma_{11}\tau\\ &-\frac{9}{2}\alpha_{21}\left(a_1\alpha_{21}+2a_2\alpha_{11}-2\alpha_{11}-\alpha_{21}\right)\Bigg)\gamma_{12}-(\gamma_2-\gamma_{11})\left((\alpha_{11}-\alpha_{21})(\gamma_2+\gamma_{11})\tau+9\alpha_{21}(\alpha_{11}+\alpha_{21}/2)\right)\Bigg)\alpha_{12}\\ &-(a_2-1)^2\left((a_1-1)\gamma_{12}+\gamma_2-\gamma_{11}\right)\left(\tau(a_1-1)\gamma_{12}+(-\gamma_2-\gamma_{11})\tau-9\alpha_{11}\right){\alpha_{22}}^2\\ &+2(a_2-1)\Bigg(\tau(a_1-1)(a_1\alpha_{21}-a_2\alpha_{11}+\alpha_{11}-\alpha_{21}){\gamma_{12}}^2+\Bigg(\left((\alpha_{11}-2\alpha_{21})a_1+a_2\alpha_{11}-2\alpha_{11}+2\alpha_{21}\right)\gamma_{11}\tau\\ &-9\alpha_{11}\left(a_1\alpha_{21}+\frac{1}{2}a_2\alpha_{11}-\alpha_{11}/2-\alpha_{21}\right)\Bigg)\gamma_{12}+(\gamma_2-\gamma_{11})\left((\alpha_{11}-\alpha_{21})(\gamma_2+\gamma_{11})\tau-\frac{9}{2}\alpha_{11}(\alpha_{11}+2\alpha_{21})\right)\Bigg)\alpha_{22}\\ &-\tau(a_1\alpha_{21}-a_2\alpha_{11}+\alpha_{11}-\alpha_{21})^2{\gamma_{12}}^2+\big(-2\gamma_{11}(\alpha_{11}-\alpha_{21})(a_1\alpha_{21}-a_2\alpha_{11}+\alpha_{11}-\alpha_{21})\tau+\\ &9\alpha_{11}\alpha_{21}(a_1\alpha_{21}+a_2\alpha_{11}-\alpha_{11}-\alpha_{21})\big)\gamma_{12}+(\gamma_2-\gamma_{11})\left((\alpha_{11}-\alpha_{21})^2(\gamma_2+\gamma_{11})\tau+9\alpha_{11}\alpha_{21}(\alpha_{11}+\alpha_{21})\right)\,.\end{aligned}$$

Proof of Observation 1. Under the condition of $\gamma^{BP} = \gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS} = \gamma_1^{FFS} + (1-a_2)\gamma_2^{FFS}$ and using Equations (8)-(10), we have

$$q_1^* = q_1^{FFS} = q_1^{BP} = \frac{2}{3} \cdot \frac{\gamma^{BP}}{\alpha_{11} + (1-a_1)\alpha_{12}} + \frac{1}{3} \cdot \frac{\gamma^{BP}}{\alpha_{21} + (1-a_2)\alpha_{22}} - \frac{1}{\tau}\,,$$
$$q_2^* = q_2^{FFS} = q_2^{BP} = \frac{2}{3} \cdot \frac{\gamma^{BP}}{\alpha_{21} + (1-a_2)\alpha_{22}} + \frac{1}{3} \cdot \frac{\gamma^{BP}}{\alpha_{11} + (1-a_1)\alpha_{12}} - \frac{1}{\tau}$$

for all three subgames.

Proof of Proposition 5. Recall the results from Equations (11), (12) and (13), by algebra we have $Q^*_{FFS/BP} \geq Q^*_{BP/BP}$ if and only if

$$\left[\tau\big(\gamma^{BP} - (\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS})\big) - 9\alpha\right]\big(\gamma^{BP} - (\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS})\big) \geq 0\,, \tag{A5}$$

and $Q^*_{FFS/BP} \geq Q^*_{FFS/FFS}$ if and only if

$$\left[\tau\big(\gamma^{BP} - (\gamma_1^{FFS} + (1+a_2-2a_1)\gamma_2^{FFS})\big) + 9\alpha\right]\big(\gamma^{BP} - (\gamma_1^{FFS} + (1-a_2)\gamma_2^{FFS})\big) \geq 0\,. \tag{A6}$$

Next, we prove that the first factor in both conditions have fixed signs: recall that in the FFS/BP game, we require $\tau \leq 3|\frac{\gamma^{BP}-(\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS})}{\alpha}|^{-1}$ to ensure that the respective demand of each hospital exists in equilibrium. This condition $\tau \leq 3|\frac{\gamma^{BP}-(\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS})}{\alpha}|^{-1}$ implies that $\tau \leq 9|\frac{\gamma^{BP}-(\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS})}{\alpha}|^{-1}$, which further implies that $|\tau(\gamma^{BP} - (\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS}))| \leq 9\alpha$, and thus the first factor in Equation (A5) is non-positive. Similarly, in the FFS/FFS game, we require $\tau \leq 3|\frac{(a_2-a_1)\gamma_2^{FFS}}{\alpha}|^{-1}$ to ensure that the respective equilibrium demands exist. Combining this condition with the previous one, by triangle inequality we have $|\tau\big(\gamma^{BP} - (\gamma_1^{FFS} + (1+a_2-2a_1)\gamma_2^{FFS})\big)| \leq |\tau(\gamma^{BP} - (\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS}))| + |\tau((a_2-a_1)\gamma_2^{FFS})| \leq 3\alpha + 3\alpha \leq 9\alpha$, and thus the first factor in Equation (A6) is non-negative. Therefore, we can rewrite the first two conditions as $Q^*_{FFS/BP} \geq Q^*_{BP/BP}$ if and only if $\gamma^{BP} - (\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS}) \leq 0$, and $Q^*_{FFS/BP} \geq Q^*_{FFS/FFS}$ if and only if $\gamma^{BP} - (\gamma_1^{FFS} + (1-a_2)\gamma_2^{FFS}) \geq 0$. Hence, the insurer will pick FFS/BP if $\gamma_1^{FFS} + (1-a_2)\gamma_2^{FFS} \leq \gamma^{BP} \leq \gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS}$. Next, by algebra we have $Q^*_{BP/BP} \geq Q^*_{FFS/FFS}$ if and only if

$$9\alpha\big(2\gamma^{BP} - (2\gamma_1^{FFS} + (2-a_1-a_2)\gamma_2^{FFS})\big) - \tau(a_1-a_2)^2(\gamma_2^{FFS})^2 \geq 0\,, \tag{A7}$$

and the other results follow by algebra.

Proof of Proposition 6. The results follow from straightforward algebra by substituting $a_1 = a_2 = a$ into the results of Lemma 1.

Proof of Proposition 7. We first prove the result of $Q^*_{FFS/FFS}$ with respect to γ_1^{FFS} and γ_2^{FFS}. For notational convenience, let $\alpha_1 := \alpha_{11} + (1-a_1)\alpha_{12}$ and $\alpha_2 := \alpha_{21} + (1-a_2)\alpha_{22}$. By algebra, the sign of $\frac{\partial Q^*_{FFS/FFS}}{\partial \gamma_1^{FFS}}$ is decided by a quadratic function in τ, i.e.,

$$f(\tau) = 2(\alpha_1 - \alpha_2)\left(\frac{\gamma_1^{FFS} + (1-a_2)\gamma_2^{FFS}}{\alpha_2} - \frac{\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)\tau^2 + 9(\alpha_1 + \alpha_2)\tau\,.$$

Recall that we require

$$0 < \tau \leq 3\left|\frac{\gamma_1^{FFS} + (1-a_1)\gamma_2^{FFS}}{\alpha_1} - \frac{\gamma_1^{FFS} + (1-a_2)\gamma_2^{FFS}}{\alpha_2}\right|^{-1} \tag{A8}$$

to ensure non-negative equilibrium demand at each hospital in the FFS/FFS game. In what follows, we prove $\frac{\partial Q^*_{FFS/FFS}}{\partial \gamma_1^{FFS}} > 0$ in three cases.

(1) If $(\alpha_1-\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)>0$, $f(\tau)$ is convex and the bigger root of $f(\tau)=0$ is $\tau=0$. Therefore, we have $f(\tau)>0$ on the respective interval defined by Condition (A8), and thus $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_1^{FFS}}>0$.

(2) If $(\alpha_1-\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)<0$, $f(\tau)$ is concave and the roots of $f(\tau)=0$ are $\tau_1=0$ and $\tau_2=-9(\alpha_1+\alpha_2)/\left(2(\alpha_1-\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)\right)>0$. Note that because of $|\frac{9(\alpha_1+\alpha_2)}{2(\alpha_1-\alpha_2)}|>3$, we have $\tau_2\geq 3|\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}-\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}|^{-1}$. Therefore, we have $f(\tau)>0$ on the respective interval defined by Condition (A8), and thus $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_1^{FFS}}>0$.

(3) If $(\alpha_1-\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)=0$, then $f(\tau)=9(\alpha_1+\alpha_2)\tau>0$ on the respective interval defined by Condition (A8), and thus $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_1^{FFS}}>0$.

Next, we prove that $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_2^{FFS}}>0$. The sign of $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_2^{FFS}}$ is decided by a quadratic function in τ, i.e.,

$$\begin{aligned}g(\tau)=&\,2((1-a_2)\alpha_1-(1-a_1)\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)\tau^2\\&+9((1-a_2)\alpha_1+(1-a_1)\alpha_2)\tau\,.\end{aligned}$$

We prove the result in three cases below.

(1) If $((1-a_2)\alpha_1-(1-a_1)\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)>0$, $g(\tau)$ is convex and the bigger root of $g(\tau)=0$ is $\tau=0$. Therefore, we have $g(\tau)>0$ on the respective interval defined by Condition (A8), and thus $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_2^{FFS}}>0$.

(2) If $((1-a_2)\alpha_1-(1-a_1)\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)<0$, $g(\tau)$ is concave and the roots of $g(\tau)=0$ are $\tau_1=0$ and $\tau_2=-9((1-a_2)\alpha_1+(1-a_1)\alpha_2)/\left(2((1-a_2)\alpha_1-(1-a_1)\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)\right)>0$. Note that because of $|\frac{9((1-a_2)\alpha_1+(1-a_1)\alpha_2)}{2((1-a_2)\alpha_1-(1-a_1)\alpha_2)}|>3$, we have $\tau_2\geq 3|\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}-\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}|^{-1}$. Therefore, we have $g(\tau)>0$ on the respective interval defined by Condition (A8), and thus $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_2^{FFS}}>0$.

(3) If $((1-a_2)\alpha_1-(1-a_1)\alpha_2)\left(\frac{\gamma_1^{FFS}+(1-a_2)\gamma_2^{FFS}}{\alpha_2}-\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_1}\right)=0$, then $g(\tau)=9((1-a_2)\alpha_1+(1-a_1)\alpha_2)\tau>0$ on the respective interval defined by Condition (A8), and thus $\frac{\partial Q^*_{FFS/FFS}}{\partial\gamma_2^{FFS}}>0$.

Next, we prove the result of $Q^*_{BP/BP}$ with respect to γ^{BP}. Recall that we require

$$0<\tau\leq 3\gamma^{BP}\left|\frac{1}{\alpha_{11}+(1-a_1)\alpha_{12}}-\frac{1}{\alpha_{21}+(1-a_2)\alpha_{22}}\right|^{-1}\tag{A9}$$

to ensure non-negative equilibrium demand at each hospital in the BP/BP game. By algebra, the sign of $\frac{\partial Q^*_{BP/BP}}{\partial\gamma^{BP}}$ is decided by a quadratic and convex function in τ, with the bigger root as $\tau=0$. Thus, we have $\frac{\partial Q^*_{BP/BP}}{\partial\gamma^{BP}}>0$ on the respective interval defined by Condition (A9).

For the result of $Q^*_{FFS/BP}$ with respect to γ_1^{FFS}, γ_2^{FFS} and γ^{BP}, the proof can be done using the same argument based on the condition of $0<\tau\leq 3|\frac{\gamma_1^{FFS}+(1-a_1)\gamma_2^{FFS}}{\alpha_{11}+(1-a_1)\alpha_{12}}-\frac{\gamma^{BP}}{\alpha_{21}+(1-a_2)\alpha_{22}}|^{-1}$.

Proof of Theorem 1. First, we consider the scenario, where intermediate quality levels are the equilibrium outcome. By Theorem 7 of Cachon and Netessine (2004), for each of the three games, it suffices to show that the strategy space of this game ($[\underline{q},\overline{q}]\times[\underline{q},\overline{q}]$) is convex, and both payoff functions are quasi-concave in q_1 and q_2 respectively, and $|H|$ is positive at critical points of payoff functions, where H is the Hessian matrix of the game. Given that both $\underline{q}$ and $\overline{q}$ are finite, the strategy space is convex. We look at concavity requirement next. Note that we assume demand function is linear as in Equation (1), so $\frac{\partial D_1}{\partial q_1} = \frac{\partial D_2}{\partial q_2} = \frac{\tau}{2}$. We first prove our result for the FFS/BP game.

From Equation (14):

$$\begin{aligned}\frac{\partial \pi_1^{FFS/BP}}{\partial q_1} =& \Big(- c'_{11}(q_1) - p'_1(q_1)\Big(\gamma_2^{FFS} - c_{12}(q_1)\Big) - \Big(1 - p_1(q_1)\Big)c'_{12}(q_1)\Big) D_1(q_1, q_2) \\ &+ \Big(\Big(\gamma_1^{FFS} - c_{11}(q_1)\Big) + \Big(1 - p_1(q_1)\Big)\Big(\gamma_2^{FFS} - c_{12}(q_1)\Big)\Big)\frac{\tau}{2}\,,\end{aligned} \tag{A10}$$

$$\begin{aligned}\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2} =& \Big(- c''_{11}(q_1) - p''_1(q_1)\Big(\gamma_2^{FFS} - c_{12}(q_1)\Big) + 2p'_1(q_1)c'_{12}(q_1) - \Big(1 - p_1(q_1)\Big)c''_{12}(q_1)\Big) D_1(q_1, q_2) \\ &- \tau\Big(c'_{11}(q_1) + p'_1(q_1)\Big(\gamma_2^{FFS} - c_{12}(q_1)\Big) + \Big(1 - p_1(q_1)\Big)c'_{12}(q_1)\Big)\,.\end{aligned} \tag{A11}$$

Straightforward algebra shows that when

$$\big(- p''_1\gamma_2^{FFS} + 2p'_1c'_{12}\big)D_1 + p'_1c_{12}\tau \le \big(c''_{11} - p''_1c_{12} + (1 - p_1)c''_{12}\big)D_1 + \big(c'_{11} + p'_1\gamma_2^{FFS} + (1 - p_1)c'_{12}\big)\tau\,, \tag{A12}$$

we have $\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2} \le 0$ for all q_1, q_2 in $[\underline{q}, \overline{q}]$. Condition (A12), thus, guarantees concavity of $\pi_1^{FFS/BP}$. To simplify our exposition, we have dropped the arguments of the functions. Note that second derivatives of costs appear on the right hand side (RHS) of the inequality. Thus the interpretation of the above condition is that $\pi_1^{FFS/BP}$ is concave when the cost functions are sufficiently convex. Notice that if the critical point of $\pi_1^{FFS/BP}$ lies inside $[\underline{q}, \overline{q}]$, then q_1^* is intermediate and is the solution to $\frac{\partial \pi_1^{FFS/BP}}{\partial q_1} = 0$. If not, q_1^* will be one of the boundary values since $\pi_1^{FFS/BP}$ would be solely increasing or decreasing on $[\underline{q}, \overline{q}]$. We look at concavity of $\pi_2^{FFS/BP}$ next.

From Equation (15):

$$\begin{aligned}\frac{\partial \pi_2^{FFS/BP}}{\partial q_2} =& \Big(- c'_{21}(q_2) + p'_2(q_2)c_{22}(q_2) - \Big(1 - p_2(q_2)\Big)c'_{22}(q_2)\Big) D_2(q_1, q_2) \\ &+ \Big(\gamma^{BP} - c_{21}(q_2) - \Big(1 - p_2(q_2)\Big)c_{22}(q_2)\Big)\frac{\tau}{2}\,,\end{aligned} \tag{A13}$$

$$\begin{aligned}\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2} =& \Big(- c''_{21}(q_2) + p''_2(q_2)c_{22}(q_2) + 2p'_2(q_2)c'_{22}(q_2) - \Big(1 - p_2(q_2)\Big)c''_{22}(q_2)\Big) D_2(q_1, q_2) \\ &\tau\Big(- c'_{21}(q_2) + p'_2(q_2)c_{22}(q_2) - \Big(1 - p_2(q_2)\Big)c'_{22}(q_2)\Big)\,.\end{aligned} \tag{A14}$$

Similarly, straightforward algebra shows when

$$2p'_2c'_{22}D_2 + p'_2c_{22}\tau \le \big(c''_{21} - p''_2c_{22} + (1 - p_2)c''_{22}\big)D_2 + \big(c'_{21} + (1 - p_2)c'_{22}\big)\tau\,, \tag{A15}$$

we have $\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2} \le 0$ for all q_1, q_2 in $[\underline{q}, \overline{q}]$. Condition (A15), thus, guarantees concavity of $\pi_2^{FFS/BP}$. Again the interpretation of the above condition is that $\pi_2^{FFS/BP}$ is concave when the cost functions are sufficiently convex. Lastly, we study the determinant of Hessian matrix, $|H| = \frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2} \cdot \frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2} - \frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial q_2} \cdot \frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_1 \partial q_2}$. It can be shown that $|H| > 0$ if

$$\begin{aligned}&\big(c''_{11} + p''_1(\gamma_2^{FFS} - c_{12}) - 2p'_1c'_{12} + (1 - p_1)c''_{12}\big)D_1 \cdot \big(c''_{21} - p''_2c_{22} - 2p'_2c'_{22} + (1 - p_2)c''_{22}\big)D_2 \\ &+ \big(c''_{11} + p''_1(\gamma_2^{FFS} - c_{12}) - 2p'_1c'_{12} + (1 - p_1)c''_{12}\big)D_1 \cdot \tau\big(c'_{21} - p'_2c_{22} + (1 - p_2)c'_{22}\big) \\ &+ \big(c''_{21} - p''_2c_{22} - 2p'_2c'_{22} + (1 - p_2)c''_{22}\big)D_2 \cdot \tau\big(c'_{11} + p'_1(\gamma_2^{FFS} - c_{12}) + (1 - p_1)c'_{12}\big) \\ >& \frac{3}{4}\tau^2\big(c'_{11} + p'_1(\gamma_2^{FFS} - c_{12}) + (1 - p_1)c'_{12}\big)\big(c'_{21} - p'_2c_{22} + (1 - p_2)c'_{22}\big)\,,\end{aligned} \tag{A16}$$

for all q_1, q_2 in $[\underline{q}, \overline{q}]$. Still, because all second derivatives of cost functions appear on the left hand side of the inequality, this condition can be interpreted as that cost functions are sufficiently convex.

For notational convenience, we define

$$g_1(q_1) = c'_{11}(q_1) + p'_1(q_1)(\gamma_2^{FFS} - c_{12}(q_1)) + (1 - p_1(q_1))c'_{12}(q_1)\,,$$
$$g_2(q_2) = c'_{21}(q_2) - p'_2(q_2)c_{22}(q_2) + (1 - p_2(q_2))c'_{22}(q_2)\,.$$

It is straightforward to verify that the sufficient conditions we derived above, i.e., Equations (A12), (A15), and (A16), are equivalent to Equations (16), (17) and (18), respectively. Under the conditions described in equations (16), (17) and (18), by Theorem 7 of Cachon and Netessine (2004), there exists a unique Nash equilibrium (q_1^*, q_2^*) for the FFS/BP game. For the FFS/FFS and BP/BP game, similarly for notational convenience we define

$$g_{12}(q_1) = c'_{11}(q_1) - p'_1(q_1)c_{12}(q_1) + (1 - p_1(q_1))c'_{12}(q_1)\,,$$
$$g_{21}(q_2) = c'_{21}(q_2) + p'_2(q_2)(\gamma_2^{FFS} - c_{22}(q_2)) + (1 - p_2(q_2))c'_{22}(q_2)\,.$$

Based on the exactly same procedure for the FFS/BP game, by algebra, we can write the concavity conditions for the FFS/FFS game as $g'_1(q_1)D_1(q_1, q_2) + 2g_1(q_1)\frac{\partial D_1}{\partial q_1} \geq 0$ and $g'_{21}(q_2)D_2(q_1, q_2) + 2g_{21}(q_2)\frac{\partial D_2}{\partial q_2} \geq 0$, which are equivalent to Conditions (16) and (19), respectively. Furthermore, we can write the concavity conditions for the BP/BP game as $g'_2(q_2)D_2(q_1, q_2) + 2g_2(q_2)\frac{\partial D_2}{\partial q_2} \geq 0$ and $g'_{12}(q_1)D_1(q_1, q_2) + 2g_{12}(q_1)\frac{\partial D_1}{\partial q_1} \geq 0$, which are equivalent to Conditions (17) and (20), respectively. In addition, by algebra we find that both FFS/FFS and BP/BP games share the same Hessian determinant condition as in Equation (A16), which is equivalent to Equation (18). Thus, there exists a unique Nash equilibrium for the FFS/FFS game and for the BP/BP game.

Numerical Illustration of Theorem 1. Tables 1, 2 and 3 provide illustrations for all the nine possible equilibrium outcomes in the FFS/BP game, the FFS/FFS game, and the BP/BP game, respectively. In Table 1, for $\gamma_1^{FFS} = 30$, $\gamma^{BP} = 50$, and $\beta_{11} = \beta_{12} = 2$, both hospitals choosing intermediate qualities is the unique Nash equilibrium. This is depicted in the center cell (in bold font) of Table 1. When $\gamma^{BP} = 100$ while other parameters remain at baseline values, Hospital 2 choosing the highest quality while Hospital 1 choosing the intermediate quality becomes the unique Nash equilibrium. The remainder of the cells in Table 1, and the following Tables 2 and 3 can be interpreted similarly (as referenced to the center cell).

Table 1 Illustration of Equilibrium Outcomes under the FFS/BP game

		Hospital 1 (FFS)		
		$q_1^* = \underline{q}$	q_1^* is intermediate	$q_1^* = \overline{q}$
Hospital 2 (BP)	$q_2^* = \underline{q}$	$\gamma_1^{FFS} = 10$, $\gamma^{BP} = 15$	$\gamma^{BP} = 10$	$\gamma_1^{FFS} = 130$, $\gamma^{BP} = 3$
	q_2^* is intermediate	$\gamma_1^{FFS} = 5$	**$\gamma_1^{FFS} = 30$, $\gamma^{BP} = 50$, $\beta_{11} = \beta_{12} = 2$**	$\gamma_1^{FFS} = 90$
	$q_2^* = \overline{q}$	$\gamma_1^{FFS} = 5$, $\gamma^{BP} = 115$, $\beta_{11} = \beta_{12} = 7$	$\gamma^{BP} = 100$	$\gamma_1^{FFS} = 70$, $\gamma^{BP} = 100$

Table 2 Illustration of Equilibrium Outcomes under the FFS/FFS game

		Hospital 1 (FFS)		
		$q_1^{FFS} = \underline{q}$	q_1^{FFS} is intermediate	$q_1^{FFS} = \overline{q}$
Hospital 2 (FFS)	$q_2^{FFS} = \underline{q}$	$\beta_{11} = \beta_{12} = 5,$ $\beta_{21} = \beta_{22} = 5$	$\beta_{21} = \beta_{22} = 7$	$\gamma_1^{FFS} = \gamma_2^{FFS} = 130,$ $\beta_{21} = \beta_{22} = 25$
	q_2^{FFS} is intermediate	$\beta_{11} = \beta_{12} = 7$	$\boldsymbol{\gamma_1^{FFS} = \gamma_2^{FFS} = 50,}$ $\boldsymbol{\beta_{11} = \beta_{12} = 2,}$ $\boldsymbol{\beta_{21} = \beta_{22} = 2}$	$\gamma_1^{FFS} = \gamma_2^{FFS} = 100,$ $\beta_{21} = \beta_{22} = 2.5$
	$q_2^{FFS} = \overline{q}$	$\gamma_1^{FFS} = \gamma_2^{FFS} = 130,$ $\beta_{11} = \beta_{12} = 25,$	$\gamma_1^{FFS} = \gamma_2^{FFS} = 100,$ $\beta_{11} = \beta_{12} = 2.5$	$\gamma_1^{FFS} = \gamma_2^{FFS} = 90$

Table 3 Illustration of Equilibrium Outcomes under the BP/BP game

		Hospital 1 (BP)		
		$q_1^{BP} = \underline{q}$	q_1^{BP} is intermediate	$q_1^{BP} = \overline{q}$
Hospital 2 (BP)	$q_2^{BP} = \underline{q}$	$\beta_{11} = \beta_{12} = 7,$ $\beta_{21} = \beta_{22} = 7$	$\beta_{21} = \beta_{22} = 10$	$\gamma^{BP} = 115,$ $\beta_{21} = \beta_{22} = 25$
	q_2^{BP} is intermediate	$\beta_{11} = \beta_{12} = 10$	$\boldsymbol{\gamma^{BP} = 50,}$ $\boldsymbol{\beta_{11} = \beta_{12} = 2,}$ $\boldsymbol{\beta_{21} = \beta_{22} = 2}$	$\gamma^{BP} = 120,$ $\beta_{21} = \beta_{22} = 10$
	$q_2^{BP} = \overline{q}$	$\gamma^{BP} = 115,$ $\beta_{11} = \beta_{12} = 25,$	$\gamma^{BP} = 120,$ $\beta_{11} = \beta_{12} = 10$	$\gamma^{BP} = 90$

Next, we present our study on the distribution of equilibrium outcomes in each of the three games. For the FFS/BP game, we vary the payment parameters γ_1^{FFS}, γ^{BP}, and the parameters $\beta_{i1}(i = 1, 2)$ in the cost functions as follows to generate problem instances. We let each of β_{11} and β_{21} vary between $[2, 4]$ in increments of 0.2. Similarly, we let the payment parameter γ_1^{FFS} vary between $[62, 262]$ in steps of 20, and let the parameter γ^{BP} vary between $[65, 265]$ in steps of 20. We select the ranges of γ_1^{FFS} and γ^{BP} by setting the respective lower bounds to $\frac{1}{2} \times c_{11}(\overline{q})$ and $\frac{1}{2} \times max_{q_2 \in [\underline{q}, \overline{q}]}\{c_{21} + (1 - p_2)c_{22}\}$, i.e., half of the highest costs, and the respective upper bounds to $2 \times c_{11}(\overline{q})$ and $2 \times max_{q_2 \in [\underline{q}, \overline{q}]}\{c_{21} + (1 - p_2)c_{22}\}$, i.e., twice of the highest costs, using the baseline values of other parameters. Our choices result in 14,641 problem instances. We obtain the equilibrium outcomes by solving the system of best response functions of each instance using MATLAB. Specifically, we apply MATLAB commands such as vpasolve and diff. Table 4 summarizes the distribution of 14,641 equilibrium outcomes under nine scenarios.

Table 4 Distribution of Equilibrium Outcomes under the FFS/BP game

		Hospital 1 (FFS)		
		$q_1^* = \underline{q}$	q_1^* is intermediate	$q_1^* = \overline{q}$
Hospital 2 (BP)	$q_2^* = \underline{q}$	0%	0%	0%
	q_2^* is intermediate	0%	70.59%	10.75%
	$q_2^* = \overline{q}$	0%	11.40%	7.26%

Each cell in Table 4 represents the percentage of equilibrium outcomes (out of the 14,641 instances) that fall under the scenario described by that cell. Table 4 shows that both hospitals choosing an intermediate quality is the Nash equilibrium for 70.59% of the problem instances (10,335 out of 14,641), making it the most likely outcome. For the FFS/FFS game and the BP/BP game, we vary the payment and the cost parameters in the same fashion and summarize our results in Tables 5 and 6, respectively.

Table 5 **Distribution of Equilibrium Outcomes under the FFS/FFS game**

		Hospital 1 (FFS)		
		$q_1^{FFS}=\underline{q}$	q_1^{FFS} is intermediate	$q_1^{FFS}=\overline{q}$
Hospital 2 (FFS)	$q_2^{FFS}=\underline{q}$	0%	0%	0%
	q_2^{FFS} is intermediate	0%	72.13%	9.69%
	$q_2^{FFS}=\overline{q}$	0%	9.69%	8.49%

Table 6 **Distribution of Equilibrium Outcomes under the BP/BP game**

		Hospital 1 (BP)		
		$q_1^{BP}=\underline{q}$	q_1^{BP} is intermediate	$q_1^{BP}=\overline{q}$
Hospital 2 (BP)	$q_2^{BP}=\underline{q}$	0%	0%	0%
	q_2^{BP} is intermediate	0%	70.70%	10.37%
	$q_2^{BP}=\overline{q}$	0%	10.37%	8.56%

To test the robustness of our findings, we run several additional numerical tests and find that our insights continue to hold. For instance, we have experimented with additional parameters by letting the payment parameters (γ_1^{FFS} and γ^{BP}) vary from the lowest cost to twice the highest cost, correspondingly. In addition, we have tried alternative functional forms such as cost functions $c_{ij}(q_i)=\alpha_{ij}e^{\beta_{ij}q_i}$, non-linear demand functions $D_i(q_i,q_j)=\frac{q_i}{q_i+q_j}$, as well to find similar qualitative insights. These numerical experiments indicate that the insights we derived are rather robust.

Proof of Proposition 8.

(a) We first prove the result for the FFS/BP game. For notational convenience, we *drop* the superscript FFS/BP here. From equations (14) and (15), the profit function π_i for Hospital i can be rewritten as $\pi_i(q_i,q_j)=P_i(q_i)D_i(q_i,q_j)$ for $i=1,2$, where $P_1(q_1)=\Big(\gamma_1^{FFS}-c_{11}(q_1)\Big)+\Big(1-p_1(q_1)\Big)\Big(\gamma_2^{FFS}-c_{12}(q_1)\Big)$, and $P_2(q_2)=\gamma^{BP}-c_{21}(q_2)-\Big(1-p_2(q_2)\Big)c_{22}(q_2)$. By implicit differentiation, we have

$$\frac{\partial q_i^*}{\partial q_j}=-\frac{\frac{\partial^2\pi_i}{\partial q_i\partial q_j}}{\frac{\partial^2\pi_i}{\partial q_i^2}}\Bigg|_{(q_i^*,q_j^*)}.$$

Since $\frac{\partial^2\pi_i}{\partial q_i^2}<0$ at equilibrium, $sgn(\frac{\partial q_i^*}{\partial q_j})=sgn(\frac{\partial^2\pi_i}{\partial q_i\partial q_j})$. Now $\frac{\partial\pi_i}{\partial q_i}=P_i'D_i+P_i\frac{\partial D_i}{\partial q_i}=0$ at q_i^* implies that $P_i'\leq 0$ at q_i^* since $D_i\geq 0$, $\frac{\partial D_i}{\partial q_i}\geq 0$, and $P_i\geq 0$ at q_i^*. The last inequality holds because hospitals will always choose a non-negative margin at equilibrium, given the participation constraints of all payment parameters. Thus, $\frac{\partial^2\pi_i}{\partial q_i\partial q_j}=P_i'\frac{\partial D_i}{\partial q_j}+P_i\frac{\partial^2 D_i}{\partial q_i\partial q_j}\geq 0$ at (q_i^*,q_j^*) since $P_i'\leq 0$, $\frac{\partial D_i}{\partial q_j}<0$, $P_i\geq 0$ and $\frac{\partial^2 D_i}{\partial q_i\partial q_j}\geq 0$ at equilibrium. Hence $\frac{\partial q_i^*}{\partial q_j}\geq 0$ for $i=1,2$. For the FFS/FFS game and the BP/BP game, following the same logic above, by writing the objective function as a product of profit margin and demand function, the same result can be proved.

(b) We first prove the result for the FFS/BP game. When q_1^* is at the boundary value ($\underline{q}$ or $\overline{q}$), $\frac{\partial q_1^*}{\partial \gamma_1^{FFS}} = 0$ for a small change in γ_1^{FFS}. Thus it is sufficient to consider the case where q_1^* is intermediate. If q_2^* is also intermediate, for any arbitrary parameter ρ, by the Implicit Function Theorem (IFT):

$$\begin{aligned} \frac{\partial q_1^*}{\partial \rho} &= -\frac{1}{|H|}\left(\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2}\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \rho} - \frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial q_2}\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2 \partial \rho}\right), \\ \frac{\partial q_2^*}{\partial \rho} &= -\frac{1}{|H|}\left(\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2}\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2 \partial \rho} - \frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_1 \partial q_2}\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \rho}\right), \end{aligned} \tag{A17}$$

where H is the Hessian matrix at (q_1^*, q_2^*). Since γ_1^{FFS} does not appear in the definition of $\pi_2^{FFS/BP}$, we can get

$$\frac{\partial q_1^*}{\partial \gamma_1^{FFS}} = -\frac{1}{|H|}\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2}\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \gamma_1^{FFS}}.$$

Because $|H| > 0$, $\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2} < 0$ and $\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \gamma_1^{FFS}} = \frac{\partial D_1}{\partial q_1} > 0$ at (q_1^*, q_2^*), we have $\frac{\partial q_1^*}{\partial \gamma_1^{FFS}} > 0$. If q_2^* is at the boundary value, i.e., $q_2^* = \underline{q}$ or $\overline{q}$, from $\frac{\partial \pi_1^{FFS/BP}}{\partial q_1} = 0$ we can get

$$\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2}\frac{\partial q_1^*}{\partial \gamma_1^{FFS}} + \frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \gamma_1^{FFS}} = 0.$$

Since $\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2} < 0$ and $\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \gamma_1^{FFS}} > 0$, we have $\frac{\partial q_1^*}{\partial \gamma_1^{FFS}} > 0$. Similarly, for Hospital 2, note that $\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2 \partial \gamma^{BP}} = \frac{\partial D_2}{\partial q_2} > 0$. When q_2^* is at the boundary value, $\frac{\partial q_2^*}{\partial \gamma^{BP}} = 0$ for a small change in γ^{BP}. When q_2^* is intermediate: if q_1^* is also intermediate, by IFT

$$\frac{\partial q_2^*}{\partial \gamma^{BP}} = -\frac{1}{|H|}\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1^2}\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2 \partial \gamma^{BP}} > 0.$$

If q_1^* is one of the boundary values, since $\frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2^2}\frac{\partial q_2^*}{\partial \gamma^{BP}} + \frac{\partial^2 \pi_2^{FFS/BP}}{\partial q_2 \partial \gamma^{BP}} = 0$, we get $\frac{\partial q_2^*}{\partial \gamma^{BP}} > 0$. By repeating the same procedures and case discussions for the FFS/FFS game and the BP/BP game, we can get the same results on sensitivity of equilibrium quality to payment parameters. Hence the proof for Part (a). From Equation (14), $\frac{\partial^2 \pi_1^{FFS/BP}}{\partial q_1 \partial \gamma_2^{FFS}} = (1 - p_1)D_1\left(\frac{\frac{\partial D_1}{\partial q_1}}{D_1} - \frac{p_1'}{1-p_1}\right)$. Following the logic similar to above, we get $\frac{\partial q_1^*}{\partial \gamma_2^{FFS}} > 0$ if and only if $SE_1(D_1) > SE_1(1 - p_1)$ at (q_1^*, q_2^*). Hence the proof for Part (b). By the exactly same argument, for the FFS/FFS game, we can get the same results on sensitivity of equilibrium quality to payment parameter γ_2^{FFS}.

(c) We first prove the result for the FFS/BP game. For notational convenience, we *drop* the superscript FFS/BP here. If $q_j^* = \overline{q}$ and q_i^* is intermediate, by implicit differentiation,

$$\frac{\partial q_i^*}{\partial \tau} = -\left.\frac{\frac{\partial^2 \pi_i}{\partial q_i \partial \tau}}{\frac{\partial^2 \pi_i}{\partial q_i^2}}\right|_{(q_i^*, \overline{q})}.$$

Following the reasoning similar to the proof of the previous part, we have $\frac{\partial^2 \pi_i}{\partial q_i \partial \tau} = P_i' \frac{\partial D_i}{\partial \tau} + P_i \frac{\partial^2 D_i}{\partial q_i \partial \tau} \geq 0$ at $(q_i^*, \overline{q})$, since $\frac{\partial D_i}{\partial \tau} \leq 0$ (because $q_j^* = \overline{q}$, higher τ implies lower D_i), $P_i' \leq 0$, $P_i \geq 0$ and $\frac{\partial^2 D_i}{\partial q_i \partial \tau} \geq 0$ at equilibrium. Therefore, if $\frac{\partial^2 D_i}{\partial q_i \partial \tau} \geq 0$, then $\frac{\partial q_i^*}{\partial \tau} > 0$. For the FFS/FFS game and the BP/BP game, following the same logic above, by writing the objective function as a product of profit margin and demand function, the same conclusion holds.

Proof of Proposition 9.

(a) For the FFS/BP game and the FFS/FFS game, without loss of generality, let Hospital 1 be the hospital under FFS, we have that $\frac{\partial \pi_1}{\partial q_1}$ (for notational convenience, we *drop* the superscripts FFS/BP and FFS/FFS here) can be rewritten as

$$\frac{\partial \pi_1}{\partial q_1} = \frac{\partial D_1}{\partial q_1}\Big(\gamma_1^{FFS} - h(q_1, q_2)\Big)\,, \tag{A18}$$

where,

$$h := c_{11} + \frac{1}{\frac{\partial D_1}{\partial q_1}}\bigg(\big(c_{11}' + p_1'(\gamma_2^{FFS} - c_{12}) + (1-p_1)c_{12}'\big)D_1 - (1-p_1)(\gamma_2^{FFS} - c_{12})\frac{\partial D_1}{\partial q_1}\bigg)\,. \tag{A19}$$

By definition h is continuously differentiable. Therefore, by Weierstrass extreme value theorem, h must attain a maximum value on $[\underline{q}, \overline{q}] \times [\underline{q}, \overline{q}]$. Let

$$\hat{\gamma}_1^{FFS} = \max_{(q_1,q_2)\in[\underline{q},\overline{q}]\times[\underline{q},\overline{q}]} h(q_1, q_2)\,.$$

Thus $\gamma_1^{FFS} \geq \hat{\gamma}_1^{FFS}$ implies $\frac{\partial \pi_1}{\partial q_1} \geq 0$ for all $q_1, q_2 \in [\underline{q}, \overline{q}]$, so $q_1^* = \overline{q}$.

(b) For the FFS/BP game and the BP/BP game, without loss of generality, let Hospital 2 be the hospital under BP. Rewrite $\frac{\partial \pi_2}{\partial q_2}$ (for notational convenience, we *drop* the superscripts FFS/BP and BP/BP here) as

$$\frac{\partial \pi_2}{\partial q_2} = \Big(\gamma^{BP} - f(q_1, q_2)\Big)\frac{\partial D_2}{\partial q_2}\,,$$

where

$$f := c_{21} + (1-p_2)c_{22} + \frac{D_2}{\frac{\partial D_2}{\partial q_2}}c_{21}' + (1-p_2)c_{22}\frac{D_2}{\frac{\partial D_2}{\partial q_2}}\bigg(-\frac{p_2'}{1-p_2} + \frac{c_{22}'}{c_{22}}\bigg)\,. \tag{A20}$$

By Weierstrass extreme value theorem, f must attain a maximum value on $[\underline{q}, \overline{q}] \times [\underline{q}, \overline{q}]$. Let

$$\hat{\gamma}^{BP} = \max_{(q_1,q_2)\in[\underline{q},\overline{q}]\times[\underline{q},\overline{q}]} f(q_1, q_2)\,.$$

Thus $\gamma^{BP} \geq \hat{\gamma}^{BP}$ implies $\frac{\partial \pi_2}{\partial q_2} \geq 0$, so $q_2^* = \overline{q}$.

Proof of Proposition 10.

(a) Again for notational convenience, we *drop* the superscript FFS/BP here and in part (b). From the proof of Proposition 9, simplifying Equation (A19) we get:

$$h = c_{11} + \frac{1}{\frac{\partial D_1}{\partial q_1}}\Bigg(c_{11}'D_1 + (1-p_1)c_{12}D_1\bigg(\Big(\frac{\gamma_2^{FFS}}{c_{12}} - 1\Big)\Big(\frac{p_1'}{1-p_1} - \frac{\frac{\partial D_1}{\partial q_1}}{D_1}\Big) + \frac{c_{12}'}{c_{12}}\bigg)\Bigg)\,.$$

Furthermore, h also must attain a minimum value on $[\underline{q}, \overline{q}] \times [\underline{q}, \overline{q}]$. Define

$$\check{\gamma}_1^{FFS} = \min_{(q_1,q_2)\in[\underline{q},\overline{q}]\times[\underline{q},\overline{q}]} h(q_1, q_2)\,.$$

Thus, if $\gamma_1^{FFS} \leq \check{\gamma}_1^{FFS}$, we get $\frac{\partial \pi_1}{\partial q_1} \leq 0$ for all $q_1, q_2 \in [\underline{q}, \overline{q}]$, so $q_1^* = \underline{q}$. If $\gamma_2^{FFS} \geq c_{12}$ and $SE_1(1-p_1) \geq SE_1(D_1)$ for all $q_1, q_2 \in [\underline{q}, \overline{q}]$, then $h > c_{11}$ on $[\underline{q}, \overline{q}] \times [\underline{q}, \overline{q}]$, and thus $\check{\gamma}_1^{FFS} > \underline{\gamma}_1^{FFS}$.

(b) Define $\check{\gamma}^{BP} = \min_{(q_1,q_2)\in[\underline{q},\overline{q}]\times[\underline{q},\overline{q}]} f(q_1, q_2)$, where f is given by Equation (A20). If $\gamma^{BP} \leq \check{\gamma}^{BP}$, then $q_2^* = \underline{q}$ since $\frac{\partial \pi_2}{\partial q_2} \leq 0$. And if $SE_2(c_{22}) \geq SE_2(1-p_2)$ for all $q_2 \in [\underline{q}, \overline{q}]$, we can get $\check{\gamma}^{BP} > \underline{\gamma}^{BP}$.

Proof of Proposition 11. From the proof of Proposition 9, recall that $\hat{\gamma}^{BP} = \max_{(q_1,q_2)\in[\underline{q},\overline{q}]\times[\underline{q},\overline{q}]} f(q_1,q_2)$, where f is given by Equation (A20). Rewrite f as

$$f = c_{21} + (1-p_2)c_{22} + \frac{1}{SE_2(D_2)}\left(c'_{21} + (1-p_2)c_{22}\left(-\frac{p'_2}{1-p_2} + \frac{c'_{22}}{c_{22}}\right)\right).$$

The term $c'_{21} + (1-p_2)c_{22}\big(-\frac{p'_2}{1-p_2} + \frac{c'_{22}}{c_{22}}\big)$ on the right hand side is independent of the demand function D_2, and is positive if $SE_2(c_{22}) \geq SE_2(1-p_2)$ for all $q_2 \in [\underline{q},\overline{q}]$. Therefore, $SE_2(D_2) \to 0^+$ implies $f \to \infty$, and hence, $\hat{\gamma}^{BP} \to \infty$.

Numerical Study of System Benefit under the FFS/FFS and the BP/BP game.

Table 7 **Equilibrium System Benefit under Different Costs and Readmission Probabilities (FFS/FFS)**

	Low payments ($0.5\times\bar{c}$)	Medium payments ($1\times\bar{c}$)	High payments ($2\times\bar{c}$)
High readmission prob. × Low costs	9,656	10,025	**10,739**
Low readmission prob. × Low costs	10,803	11,153	**11,612**
High readmission prob. × High costs	**10,034**	9,922	9,480
Low readmission prob. × High costs	**11,068**	10,898	10,395

Table 8 **Equilibrium System Benefit under Different Costs and Readmission Probabilities (BP/BP)**

	Low payments ($0.5\times\bar{c}$)	Medium payments ($1\times\bar{c}$)	High payments ($2\times\bar{c}$)
High readmission prob. × Low costs	9,749	10,287	**10,954**
Low readmission prob. × Low costs	10,907	11,314	**11,664**
High readmission prob. × High costs	**10,167**	10,071	9,632
Low readmission prob. × High costs	**11,131**	10,955	10,442

References

Cachon, G. P., Netessine, S. 2004. Game theory in supply chain analysis. In Handbook of Quantitative Supply Chain Analysis (13-65). Springer US.