

Appendix A: Supplementary Results

In this appendix we provide several supplementary results for the main paper.

A.1. The Impact of Quick Response

We first show that compared to the benchmark model, quick response can lead to either more or less production of finished goods in expectation.

PROPOSITION 10. *Suppose $Y \sim \text{Uniform}[0, 1]$, fewer finished goods are produced in expectation in the quick response model than in the benchmark model (i.e., $q^Q + \mathbb{E}q_\delta^Q < q^B$) if and only if $p_f > \frac{2(c+\delta)^2}{c+2\delta}$ and $0 \leq c_m < \left(\frac{p_f - 2(c+\delta)}{p_f} + \frac{\delta}{c+\delta}\right)(p_f - c - \delta)$.*

We then establish the result that the amount of demand fulfilled in expectation is larger with quick response.

PROPOSITION 11. *Compared to the benchmark model, more units of demand are fulfilled in expectation in the quick response model, i.e., $\mathbb{E} \min\{D_f(Y), q^Q + q_\delta^Q\} \geq \mathbb{E} \min\{D_f(Y), q^B\}$.*

Next we numerically calculate the conditions under which quick response leads to higher carbon emissions over the fashion supply chain, using the impact coefficients in §3.2.3 and assuming that quick response is used if available (i.e., $c_m \leq \frac{(p_f - c - \delta)c}{c + \delta}$). Specifically, by the proof of Proposition 4(ii), $EI^Q - EI^B > 0$ holds if $\frac{c}{p_f} > \bar{c}^Q(I) = \sqrt{\left(\frac{i_{fabric} + i_l + i_{FG}}{i_{use} - i_l}\right)^2 + \frac{i_{FG} + i_l - i_m}{i_{use} - i_l} - \frac{i_{fabric} + i_l + i_{FG}}{i_{use} - i_l}} \approx 0.0478$, or equivalently, $\frac{p_f}{c} \lesssim 20.9208$. That is, a sufficient condition for quick response to increase carbon emissions in the supply chain (compared to the benchmark model) is that the relevant product's revenue per unit is less than 21 times its production cost. If we assume fabric cost is three times the production cost (i.e., $c_m = 3c$), which is reasonable based on industry estimates¹⁵, then this condition means that the gross profit margin of the product without quick response should be less than $1 - \frac{4c}{21c} \approx 81\%$.

A.2. The Impact of Upcycling

In this section we first demonstrate that in the upcycling model with no demand encroachment, the expected amount of finished goods produced may increase *or* decrease with the introduction of upcycling opportunities (as stated in Table 2). Figure 9 illustrates this result numerically. In this figure, the shaded area represents cases where the expected amount of finished goods produced is lower in the upcycling model than in the quick response model. This expected amount is either higher or the same in the unshaded area.

We then turn to the upcycling model in the presence of the demand encroachment effect. We first detail the firm's optimal upcycling decision in stage 2 .

LEMMA 4. *In the upcycling model with demand encroachment, for any given decisions (x, q) in stage 1 and any realized market size y , there exist thresholds W_1 and W_2 such that the focal firm's optimal decision in stage 2 are given by*

$$S^{U\dagger}(x, q, y) = \begin{cases} 0 & \text{if } w \leq W_1, \\ \frac{((1-p_f)y - q)^+}{\min\{\alpha, (1-p_f)/(1-p_s/\alpha)\}} & \text{if } W_1 < w < W_2, \\ \min\left\{x - q, x - \left(1 - \frac{p_f - p_s}{1 - \alpha}\right)^+ y, \frac{(x - (1-p_f)y)^+}{1 - \min\{\alpha, (1-p_f)/(1-p_s/\alpha)\}}\right\} & \text{if } w \geq W_2. \end{cases} \quad (5)$$

and $q_\delta^{U\dagger}(x, q) = \min\{(D_f^{U\dagger}(S^{U\dagger}(x, q, y), y) - q)^+, x - q\}$.

¹⁵ <https://tegentl.com/clothes-line-cost/>.

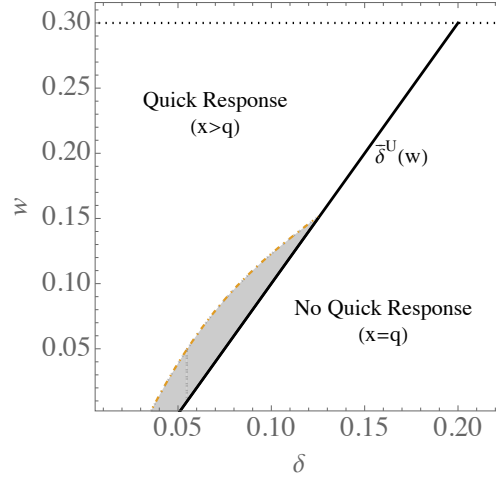


Figure 9 The firm's optimal quick response decision under upcycling opportunities (without demand encroachment effect). In this figure, $Y \sim Uniform[0, 1]$, $c = 0.1$, $c_m = 0.3$, $p_f = 0.6$. In the shaded region, we have $q^U + \mathbb{E}q_\delta^U < q^Q + \mathbb{E}q_\delta^Q$. In the un-shaded regions, we have $q^U + \mathbb{E}q_\delta^U > q^Q + \mathbb{E}q_\delta^Q$ for $\delta < \bar{\delta}^U(w)$ and $q^U + \mathbb{E}q_\delta^U = q^Q + \mathbb{E}q_\delta^Q$ for $\delta \geq \bar{\delta}^U(w)$.

The key insight from Lemma 4 is that in the presence of demand encroachment, the focal firm may no longer upcycle all of its deadstock fabric as is the case without demand encroachment. Instead, it may be optimal for the focal firm to upcycle none, partial or all of the deadstock fabric depending on the price of the deadstock fabric w and the realized market size y (as the W_1 and W_2 threshold depend on y , see the proof of Lemma 4 in Appendix D and the Electronic Companion for details). What drives the focal firm's choice is the tradeoff between the salvage value of deadstock fabric (i.e., w) and the sales revenue loss due to demand encroachment (which depends on the market size y). When w is small (i.e., $w \leq W_1$), the demand encroachment effect dominates and it is optimal not to engage in upcycling. When w is large (i.e., $w \geq W_2$), the salvage value effect dominates, thus the focal firm should upcycle as much as possible. When w is intermediate (i.e., $W_1 < w < W_2$), it may be optimal for the focal firm to upcycle only a portion of the deadstock fabric to balance between the marginal loss from demand encroachment and the unit salvage value from deadstock fabric. Specifically, in this case (which only happens when the realized market size is sufficiently large), we can calculate that $D_f^{U\dagger}(S^{U\dagger}(x, q, y), y) = q$ and $q_\delta^{U\dagger}(x, q, y) = 0$. This indicates that it is optimal for the focal firm to upcycle just enough fabric such that its demand equals the finished goods inventory already produced in stage 1. That is, the focal firm allows a certain level of demand encroachment but not so much that it is left with finished goods inventory.

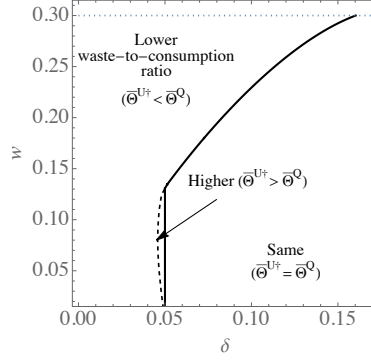
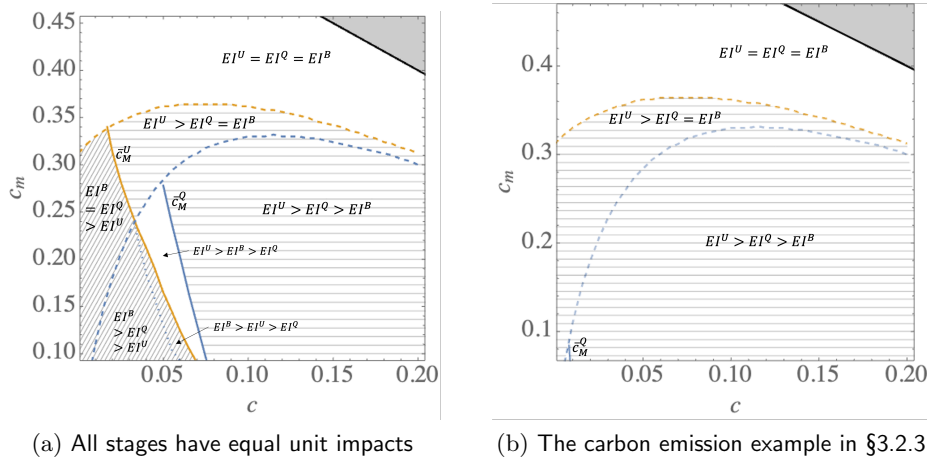
Based on Proposition 9 and extensive numerical experiments, we summarize in Table 3 the impact of the firm's optimal decisions $(x^{U\dagger}, q^{U\dagger})$ in the upcycling model with the demand encroachment effect. As in Table 2, the comparison is with respect to the quick response model.

Finally, Figure 10 illustrates that the introduction of upcycling opportunities may in fact lead to a higher waste-to-consumption ratio, and Figure 11 illustrates the relative environmental impacts of the three models when the encroachment effect exists.

Table 3 Life Cycle Impact of Upcycling (with Demand Encroachment Effect).

Fabric acquisition (x)	Finished goods production			Deadstock generation			Total demand fulfilled
	Stage 1 (q)	Stage 2 ($\mathbb{E}q_\delta$)	Total ($q + \mathbb{E}q_\delta$)	Fabric (Θ_m)	Finished goods (Θ_l)	Total ($\Theta_m + \Theta_l$)	
+	-	+	+/-	+/-	-	+/-	+

Notes: The symbol “+” (“-”) indicates that the amount increases (decreases) with the introduction of upcycling.

**Figure 10 The waste-to-consumption ratio comparison with and without upcycling opportunities.**

(a) All stages have equal unit impacts

(b) The carbon emission example in §3.2.3

Figure 11 Comparison of $EI^{U\dagger}$, EI^Q and EI^B when the encroachment effect exists. In this figure,

$$Y \sim \text{Uniform}[0, 1], p_f = 0.6, \alpha = 0.3, w = 0.1, \delta = 0.04.$$

Appendix B: Policy Simulations

B.1. Subsidizing Quick Response or/and Upcycling

We conduct two sets of numerical simulations to (i) gauge the effectiveness of the different subsidy policies, and (ii) explore the fabric acquisition implications of such subsidies.

Effectiveness of different subsidy policies: We first compare the deadstock reduction outcomes from three types of government subsidy policies: (1) quick response subsidy only, (2) upcycling subsidy only, and (3) joint quick response and upcycling subsidies. We compare the deadstock reduction under these three policies given a fixed budget. Specifically, to determine the optimal subsidies under the third policy, we solve

$$\min_{s_w, s_\delta} \Theta_m^{U\dagger} + \Theta_l^{U\dagger},$$

under the constraints

$$s_\delta \mathbb{E} q_\delta^{U^\dagger}(x^{U^\dagger}, q^{U^\dagger}, Y) + s_w \mathbb{E} S^{U^\dagger}(x^{U^\dagger}, q^{U^\dagger}, Y) \leq \Omega,$$

$$s_w \in [0, c_m - w], \quad s_\delta \in [0, \delta].$$

The first constraint is to ensure that the policy cost does not exceed the given budget Ω . The second and third constraints limit the subsidy amounts. Similarly, to determine the optimal subsidy on w or δ only, we solve the constrained optimization problem above while setting s_δ or s_w to zero, respectively.

We illustrate two examples of the results in Figure 12, with and without the demand encroachment effect. First consider the case without demand encroachment. Observe from Figure 12(a) that in most regions, subsidizing δ is more effective than subsidizing w , so that the optimal subsidy policy consists of δ subsidies only. This is because in absence of the demand encroachment effect, the firm always upcycles all leftover fabric, so the objective becomes one of minimizing leftover finished goods. Between the two subsidies, subsidizing δ may achieve this objective more efficiently because it more directly targets the production of finished goods. To see this, note that if $\delta = 0$, deadstock is eliminated (i.e., $\Theta_t^{U^\dagger} = 0$). In contrast, if $\delta > 0$, then regardless of how large w is, there will be deadstock (i.e., $\Theta_t^{U^\dagger} > 0$). Nevertheless, the subsidies can only reduce leftover finished goods when the firm engages in quick response. When the firm is currently not engaging in quick response, then in order to nudge the firm to use quick response, it may be optimal to subsidize both δ and w (due to the complementarity between the two strategies, as discussed in §4).

Next consider the case with the demand encroachment effect. Comparing Figure 12(b) with Figure 12(a) shows that the presence of the demand encroachment effect makes subsidizing δ less effective: the region where subsidizing both δ and w is optimal becomes larger, and there is also a region where it is optimal to subsidize only w . This is because the firm, concerned about the demand encroachment effect, may adopt a selective upcycling strategy. As a result, decreasing δ could lead to higher deadstock as the firm acquires more fabric that it does not end up upcycling. In contrast, we confirm numerically that increasing w typically always reduces the total deadstock. Moreover, because the threshold for engaging in quick response is smaller with than without the demand encroachment effect (i.e., $\bar{\delta}^{U^\dagger}(w) \leq \bar{\delta}^U(w)$), it becomes more likely that no subsidy package under the budget constraint can induce the firm to engage in quick response (or upcycling). This explains the region in Figure 12(b) labeled “No subsidy”.

We conduct the same analyses over a range of numerical values for the key parameters, and confirm that the insights from Figure 12 are robust.

Impact of subsidies on fabric acquisition: Finally, we explore whether there exists subsidy policies that reduce both deadstock and fabric acquisition. To do so, we impose an additional constraint to the numerical studies introduced above: that the amount of fabric acquired (i.e., x^{U^\dagger}) cannot increase. We find that the only feasible subsidies are $s_\delta = 0$ and $s_w = 0$. This indicates that any subsidy policy that is effective in reducing total deadstock would inevitably increase fabric acquisition.

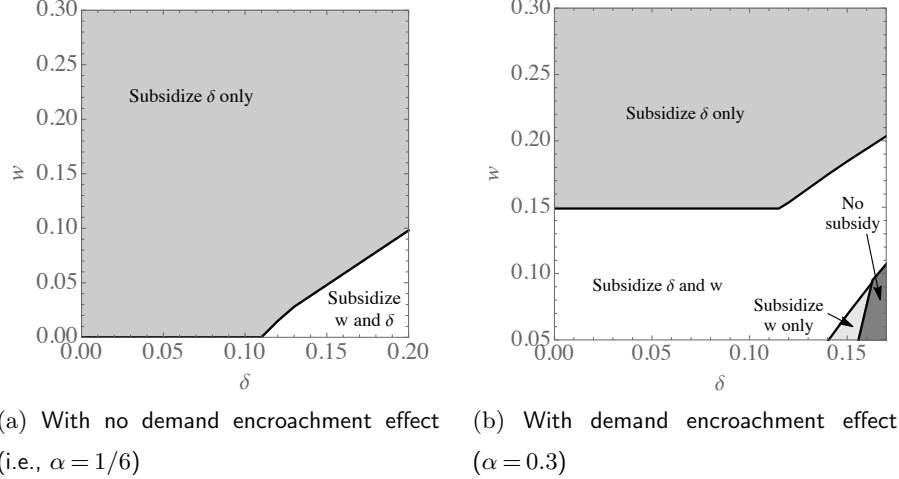


Figure 12 The most effective subsidy policy to reduce deadstock under budget constraint. In this figure,

$Y \sim \text{Uniform}[0, 1]$, $c = 0.1$, $c_m = 0.3$, $p_f = 0.6$, $p_s = 0.1$, $\Omega = 0.03$.

B.2. Banning Deadstock Destruction

We consider three scenarios in this analysis (i.e., quick response, upcycling without demand encroachment, and upcycling with demand encroachment). The results presented in the paper are based on the last scenario which is the most comprehensive in terms of the impact of the ban. The other two scenarios lead to similar insights but are less nuanced. Below we present details of the analysis for each scenario and discuss their differences. Throughout the analysis, we assume $r_m \geq r_l$.

We first study the case when there are no upcycling opportunities, which we denote by the superscript “QB”. Specifically, in stage 1, the firm chooses x and q to maximize its profit function under the ban, which is formulated as

$$\mathbb{E}\Pi^{QB}(x, q) = \mathbb{E}\Pi^Q(x, q) - r_m \mathbb{E}(x - q - q_\delta^{QB}(x, q, Y))^+ - r_l \mathbb{E}(q - D_f(Y))^+,$$

where $\mathbb{E}\Pi^Q(x, q)$ is as given in equation (1), and it is easy to see that $q_\delta^{QB}(x, q, y) = q_\delta^Q(x, q, y) = \min\{x - q, (D_f(y) - q)^+\}$. Denote the firm’s optimal decisions as (x^{QB}, q^{QB}) .

LEMMA 5. *In the quick response model with a ban on deadstock destruction, there exists a threshold $\bar{\delta}^{QB} = \frac{(p_f - c - c_m)(c + r_l - r_m)}{c + c_m + r_l}$ such that*

- (i) if $\delta < \bar{\delta}^{QB}$, then the focal firm engages in quick response (i.e., $x^{QB} > q^{QB}$), and its optimal decisions in stage 1 are given by $F\left(\frac{x^{QB}}{1 - p_f}\right) = \frac{p_f - c - \delta - c_m}{p_f - c - \delta + r_m}$ and $F\left(\frac{q^{QB}}{1 - p_f}\right) = \frac{\delta}{c + \delta - r_m + r_l}$.
- (ii) if $\delta \geq \bar{\delta}^{QB}$, then the firm does not engage in quick response (i.e., $x^{QB} = q^{QB}$), and its optimal decisions in stage 1 are given by $F\left(\frac{x^{QB}}{1 - p_f}\right) = \frac{p_f - c - c_m}{p_f + r_l}$ and $q^{QB} = x^{QB}$.

Comparing (x^{QB}, q^{QB}) with (x^Q, q^Q) yields the following result.

PROPOSITION 12. *Compared to the quick response model with no ban:*

- (i) When $r_m > 0$, or $r_m = 0$ and $\delta > \bar{\delta}^Q$, the ban induces the firm to acquire less fabric and generate less deadstock (i.e., $x^{QB} < x^Q$ and $\Theta^{QB} < \Theta^Q$);
- (ii) When $r_m = 0$ and $\delta \leq \bar{\delta}^Q$, the ban does not affect the firm’s fabric decision or its deadstock generation (i.e., $x^{QB} = x^Q$ and $\Theta^{QB} = \Theta^Q$).

Proposition 12 indicates that as long as $r_m > 0$ (i.e., the firm incurs a penalty for leftover fabric), the ban can always reduce both fabric acquisition and deadstock generation. Interestingly, it also shows that even when $r_m = 0$, the ban can be effective as long as δ is sufficiently large. In other words, the ban can reduce fabric acquisition (and overall deadstock) even if it only penalizes leftover finished goods. This is because the ban induces the firm to produce fewer products in stage 1. When δ is relatively small and $r_m = 0$, the firm simply shifts the inventory it holds from finished goods to on-hand fabric, and does not change its fabric acquisition decision (as shown in Proposition 12(ii)). However, when δ is relatively large, this strategy becomes expensive, and hence the firm reduces the amount of inventory it holds overall (as shown in Proposition 12(i)).

Next, we analyze the model where upcycling opportunities exist, starting with the case without demand encroachment. The firm's profit function in stage 2 becomes $\Pi^{UB} = \Pi^U - r_m(x - q - S - q_\delta)^+ - r_l(q - D_f(y))^+$. It is easy to see that, as in the case with no ban, the firm produces to satisfy its demand, i.e., $q_\delta^{UB}(x, q, y) = q_\delta^Q(x, q, y) = \min\{x - q, (D_f(y) - q)^+\}$, and upcycles the remaining fabric, i.e., $S^{UB}(x, q, y) = \min\{x - q, (x - D_f(y))^+\}$. Then, in stage 1, it chooses (x, q) to maximize its expected profit function, which is formulated as

$$\mathbb{E}\Pi^{UB}(x, q) = \mathbb{E}\Pi^U(x, q) - r_l\mathbb{E}(q - D_f(Y))^+.$$

Note that because the firm upcycles all of its leftover fabric, it does not incur any penalty related to deadstock fabric under the ban. Thus, the value of r_m does not influence the firm's decisions. The firm's decisions are described in the following lemma.

LEMMA 6. For any $w > 0$, there exists a threshold $\bar{\delta}^{UB} = \frac{(p_f - c - c_m)(c + r_l + w)}{c + c_m + r_l}$ such that

(i) if $\delta < \bar{\delta}^{UB}$, then the firm engages in quick response and upcycling (i.e., $x^{UB} > q^{UB}$) and its optimal decisions are given by $F\left(\frac{x^{UB}}{1 - p_f}\right) = \frac{p_f - c - \delta - c_m}{p_f - c - \delta - w}$ and $F\left(\frac{q^{UB}}{1 - p_f}\right) = \frac{\delta}{c + \delta + w + r_l}$;

(ii) if $\delta \geq \bar{\delta}^{UB}$, then the firm does not engage in quick response nor upcycling (i.e., $x^{UB} = q^{UB}$) and its optimal decisions are given by $F\left(\frac{x^{UB}}{1 - p_f}\right) = \frac{p_f - c - c_m}{p_f + r_l}$ and $q^{UB} = x^{UB}$.

Comparing the fabric acquisition and deadstock outcomes with those in absence of the ban, we can summarize the impact of the ban as follows.

PROPOSITION 13. Suppose there is no demand encroachment effect. Compared to the upcycling model with no ban:

(i) when $\delta > \bar{\delta}^U(w)$, the ban induces the firm to acquire less fabric and generate less deadstock (i.e., $x^{UB} < x^U$ and $\Theta_l^{UB} < \Theta_l^U$);

(ii) when $\delta \leq \bar{\delta}^U(w)$, the ban does not affect the firm's fabric decision but still leads to less deadstock generation (i.e., $x^{UB} = x^U$ and $\Theta_l^{UB} < \Theta_l^U$).

Figure 13(a) illustrates the thresholds $\bar{\delta}^U(w)$ and $\bar{\delta}^{UB}(w)$, while Figure 13(b) shows how the firm's stage 1 decisions change in response to the ban. The intuition for Proposition 13 is similar to the quick response case with $r_m = 0$. That is, when δ is small, the firm responds to the ban (which penalizes leftover finished goods) by changing the form of inventory it holds from finished goods to fabric, and does not change the amount of fabric it acquires. When δ is relatively large, however, on-hand fabric is less valuable because quick response

becomes more expensive, and the firm reduces the amount of fabric it acquires. This explains the effect of the ban on fabric acquisition. The ban always reduces total deadstock because it reduces the amount of finished goods the firm produces in stage 1, which in turn reduces the expected amount of leftover finished goods.

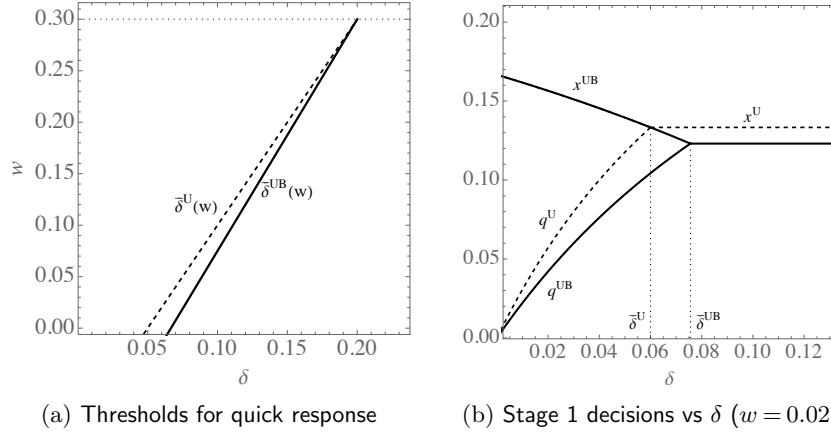


Figure 13 Effect of ban on firm's optimal decisions. We let $r_l = 0.05$, and the other parameters are the same as those used in Figure 5.

Similarly, we can set up the firm's maximization problem for the upcycling model with the demand encroachment effect. We use numerical simulation to solve for the focal firm's optimal decisions in this case, and the key insights are summarized in §5.2.

We next briefly discuss how the incorporation of the demand encroachment effect impacts the effectiveness of a ban. Specifically, comparing Figure 13 (and Proposition 13) with the results highlighted in §5.2 yields two insights. First, the presence of demand encroachment could *weaken* the effectiveness of a ban by inducing more fabric acquisition with than without the ban (see Region II in Figure 8). This relates to the two ways the firm can react to a ban, as discussed in §5.2: (1) the firm may reduce its inventory holdings (in terms of both fabric acquired and finished goods produced in stage 1); (2) the firm could become more likely to engage in upcycling. Without demand encroachment concerns, the firm upcycles as much leftover fabric as possible even without the ban, and hence the first strategy dominates. In contrast, when there is demand encroachment, the second strategy may dominate, leading to more fabric acquired. Second, the presence of demand encroachment could also *strengthen* the effectiveness of a ban. Specifically, when both w and δ are small, the ban could reduce fabric acquired when there is demand encroachment (i.e., lower-left corner of Region III in Figure 8), even though it has no effect on fabric acquired without demand encroachment (see Figure 13). To further highlight the ban's effectiveness, Figure 14 illustrates the regions (in light gray) where a ban can induce less fabric acquisition but also more demand fulfilled. As discussed in §5.2, this shows that even though the ban leads to less fabric acquired, more of that fabric may be used rather than wasted.

Appendix C: Endogenous Pricing

In this section we extend the base model to allow the focal firm's pricing decision (p_f) to be endogenous.¹⁶ Similar to most previous literature on price-dependent newsvendors, we assume that the focal firm chooses the

¹⁶ We still keep p_s fixed because upcycling firms in practice often source from multiple brands and price based on styles that can be made with different materials.

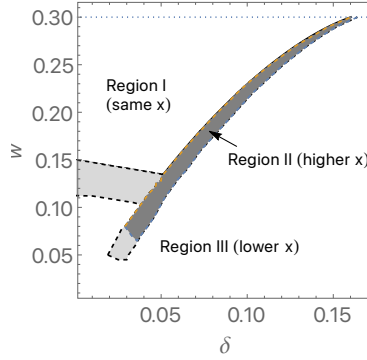


Figure 14 Effect of ban on fabric acquisition and demand fulfilled. The light gray area represents the region where the firm acquires less fabric and fulfilled more demand (in expectation) after the ban.

price before demand uncertainty is resolved. This model does not allow for tractable solutions, so we conduct numerical studies to show that our main insights are robust. For ease of reference, we add a superscript “ E ” to all notations related to this extension.

C.1. The Impact of Quick Response

The firm’s production decision in stage 2 remains unchanged (i.e., $q_\delta^{QE}(x, q, y) = q_\delta^Q(x, q, y)$). In stage 1, the firm chooses x , q and p_f jointly to maximize its expected profit in stage 1, which is given by $\mathbb{E}\Pi^Q(x, q)$ in the proof of Lemma 1. Define $\tilde{x} \doteq \frac{x}{1-p_f}$, $\tilde{q} \doteq \frac{q}{1-p_f}$. The optimization problem becomes one of choosing \tilde{x} , \tilde{q} and p_f to maximize

$$\mathbb{E}\hat{\Pi}^{QE}(\tilde{x}, \tilde{q}, p_f) \doteq (1-p_f) \left(p_f \int_a^{\tilde{x}} y dF(y) + p_f \tilde{x} \bar{F}(\tilde{x}) - (c_m \tilde{x} + c \tilde{q}) - (c + \delta) \left(\int_{\tilde{q}}^{\tilde{x}} (y - \tilde{q}) dF(y) + (\tilde{x} - \tilde{q}) \bar{F}(\tilde{x}) \right) \right),$$

subject to the constraint $\tilde{q} \leq \tilde{x}$.

As in the proof of Lemma 1, we can write the Lagrangian of the problem as $L = \mathbb{E}\hat{\Pi}^{QE}(\tilde{x}, \tilde{q}, p_f) + \lambda(\tilde{x} - \tilde{q})$, where $\lambda \geq 0$ is the Lagrangian multiplier. The KKT conditions are:

$$\begin{aligned} \frac{\partial L}{\partial \tilde{q}} &= \delta - (c + \delta)F(\tilde{q}) - \lambda \leq 0, \\ \frac{\partial L}{\partial \tilde{x}} &= (p_f - c - \delta)\bar{F}(\tilde{x}) - c_m + \lambda \leq 0, \\ \frac{\partial L}{\partial p_f} &= c_m \tilde{x} + c \tilde{q} + (c + \delta) \left(\int_{\tilde{q}}^{\tilde{x}} (y - \tilde{q}) dF(y) + (\tilde{x} - \tilde{q}) \bar{F}(\tilde{x}) \right) + (1 - 2p_f) \left(\int_a^{\tilde{x}} y dF(y) + \tilde{x} \bar{F}(\tilde{x}) \right) \leq 0, \\ \frac{\partial L}{\partial \lambda} &= \tilde{x} - \tilde{q} \geq 0, \\ \tilde{x} \frac{\partial L}{\partial \tilde{x}} &= 0, \quad \tilde{q} \frac{\partial L}{\partial \tilde{q}} = 0, \quad p_f \frac{\partial L}{\partial p_f} = 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0, \quad \tilde{q} \geq 0, \quad \tilde{x} \geq 0, \quad \lambda \geq 0, \quad p_f \geq 0. \end{aligned}$$

Solving the KKT conditions yields two possible solutions. In the first solution, the constraint does not bind, and the solution is obtained by solving $F(\tilde{x}) = 1 - \frac{c_m}{p_f - c - \delta}$, $F(\tilde{q}) = \frac{\delta}{c + \delta}$, and $\frac{\partial L}{\partial p_f} = 0$. Note that the condition $\tilde{x} - \tilde{q} > 0$ requires that $p_f > \frac{(c + \delta)(c + c_m)}{c}$.

In the second solution, the constraint binds (i.e., $\tilde{x} = \tilde{q}$) and the solution is obtained by solving $F(\tilde{x}) = 1 - \frac{c + c_m}{p_f}$, $\tilde{q} = \tilde{x}$, and

$$\frac{\partial L}{\partial p_f} = (c_m + c)\tilde{x} + (1 - 2p_f) \left(\int_a^{\tilde{x}} y dF(y) + \tilde{x} \bar{F}(\tilde{x}) \right) = 0.$$

Under the condition $\lambda \geq 0$, this solution is only feasible if $p_f \leq \frac{(c+\delta)(c+c_m)}{c}$.

Finally, note that each of these two solutions is a local maximizer if at the solution, the Hessian matrix is negative definite. It is easy to check that this simplifies to the condition $\frac{\partial^2 L}{\partial p_f^2} \frac{\partial^2 L}{\partial \tilde{x}^2} - \left(\frac{\partial^2 L}{\partial \tilde{x} \partial p_f}\right)^2 > 0$.

Based on the conditions described above, we numerically solve the optimization problem and compare the quick response solution with the benchmark case (which we denote by the superscript “BE”). We confirm that in this extension, access to quick responses opportunities leads the firm to order more fabric and generate more overall deadstock. Figure 15 illustrates an example of the firm’s decisions and deadstock generation outcomes as functions of δ , with and without quick response. Note that compared to the case with exogenous price (represented by the dot-dashed lines), the fabric acquired and total deadstock are slightly more sensitive to changes in δ (i.e., the slopes are steeper) when prices are endogenous and δ is small. In contrast, the waste-to-consumption ratio is less sensitive to changes in δ when prices are endogenous and δ is small.

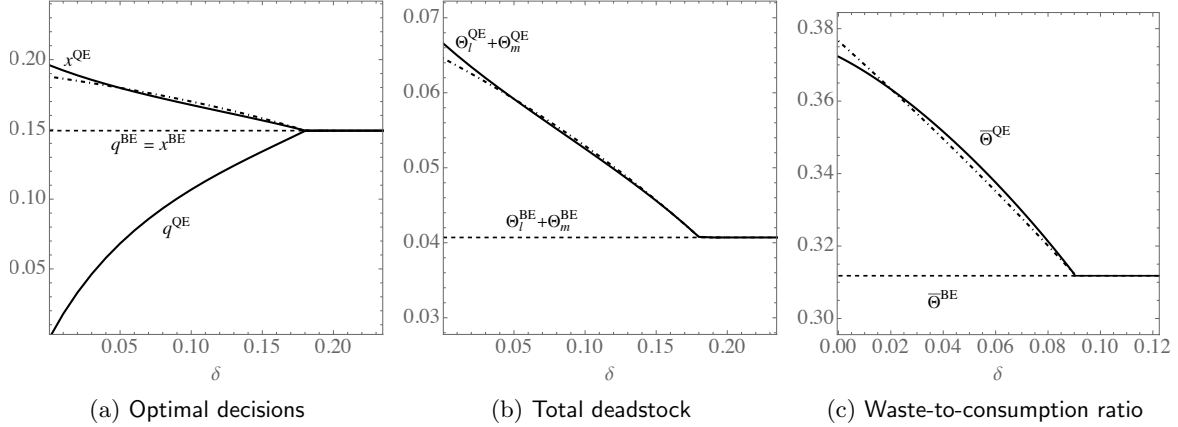


Figure 15 The firm’s decisions and deadstock outcome with and without quick response when price is endogenous. In this figure, $c = 0.1$, $c_m = 0.3$. The dot-dashed lines in (a), (b) and (c) represent x^Q , Θ^Q and $\bar{\Theta}^Q$, respectively, under $p_f = 0.762$ (the optimal price in the benchmark model with no quick response).

C.2. The Impact of Upcycling

Next we explore the impact of upcycling when pricing is endogenous, starting with the case without demand encroachment (i.e., the model in §4.1).

Again, the firm’s decisions in stage 2 (i.e., $q_\delta^U(x, q, y)$ and $S^U(x, q, y)$) remain unchanged. In stage 1, the firm chooses x , q and p_f jointly to maximize its expected profit, which is given by $\mathbb{E}\Pi^U(x, q)$ in equation (4). Specifically, again using $\tilde{x} \doteq \frac{x}{1-p_f}$, $\tilde{q} \doteq \frac{q}{1-p_f}$, the optimization problem becomes one of choosing \tilde{x} , \tilde{q} and p_f to maximize

$$\mathbb{E}\hat{\Pi}^{UE}(\tilde{x}, \tilde{q}, p_f) \doteq (1-p_f) \left(p_f \int_a^{\tilde{x}} y dF(y) + p_f \tilde{x} \bar{F}(\tilde{x}) - (c_m \tilde{x} + c \tilde{q}) - (c+\delta) \left(\int_{\tilde{q}}^{\tilde{x}} (y-\tilde{q}) dF(y) + (\tilde{x}-\tilde{q}) \bar{F}(\tilde{x}) \right) + w(\tilde{x}-\tilde{q})F(\tilde{q}) + w \int_{\tilde{q}}^{\tilde{x}} (\tilde{x}-y) dF(y) \right),$$

subject to the constraint $\tilde{q} \leq \tilde{x}$. Solving the KKT conditions (similar to those for the quick response model) yields two possible solutions. In the first solution, the constraint does not bind, and the solution is obtained by solving $F(\tilde{x}) = \frac{p_f - c - \delta - c_m}{p_f - c - \delta - w}$, $F(\tilde{q}) = \frac{\delta}{c + \delta + w}$, and $\frac{\partial}{\partial p_f} \mathbb{E} \hat{\Pi}^{UE}(\tilde{x}, \tilde{q}, p_f) = 0$.

In the second solution, the constraint binds (i.e., $\tilde{x} = \tilde{q}$) and the solution is the same as the second solution in the quick response model (in Section C.1).

Figure 16 illustrates an example of the parameter conditions under which the firm engages in upcycling and quick response (i.e., $\delta < \bar{\delta}^U(w)$). We confirm that in the numerical cases we tested, introducing the upcycling option always leads to (weakly) more fabric acquired and (weakly) less total deadstock. Therefore, the key tradeoff between deadstock reduction and reducing fabric production/acquisition continues to hold. We can also derive similar policy results to those in the main model. In particular, we confirm that subsidizing w and/or δ can be effective in reducing deadstock, but that this benefit comes at the expense of increasing the fabric needed from the upstream supply chain.

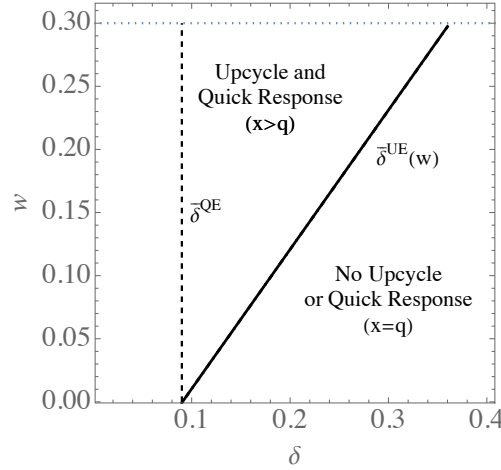


Figure 16 The firm's optimal quick response and upcycling decisions with endogenous price. In this figure, $Y \sim Uniform[0, 1]$, $c = 0.1$, $c_m = 0.3$, $p_f = 0.6$. Note that $\bar{\delta}^{UE}(w) > \bar{\delta}^U(w)$ in this example, but the difference is very small (i.e., $\bar{\delta}^{UE}(w) - \bar{\delta}^U(w) < 0.001$ for all $w \in (0, c_m)$).

Finally, we consider the upcycling model with the demand encroachment effect. Using a similar approach to that described above, we numerically solve for the firm's optimal decisions, and confirm the key insights from §4.3. Specifically, we find that, just as shown in Figure 5, the firm may choose full, selective, or no upcycling (see Figure 17). Moreover, similar to the results illustrated in Figure 6, we can show that upcycling could lead to more total deadstock, due to an increase in leftover fabric (see Figure 18).

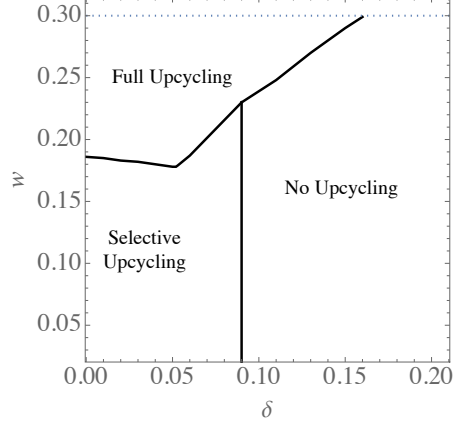


Figure 17 Upcycling decisions under demand encroachment with endogenous price. The parameters are the same as those used in Figure 5.

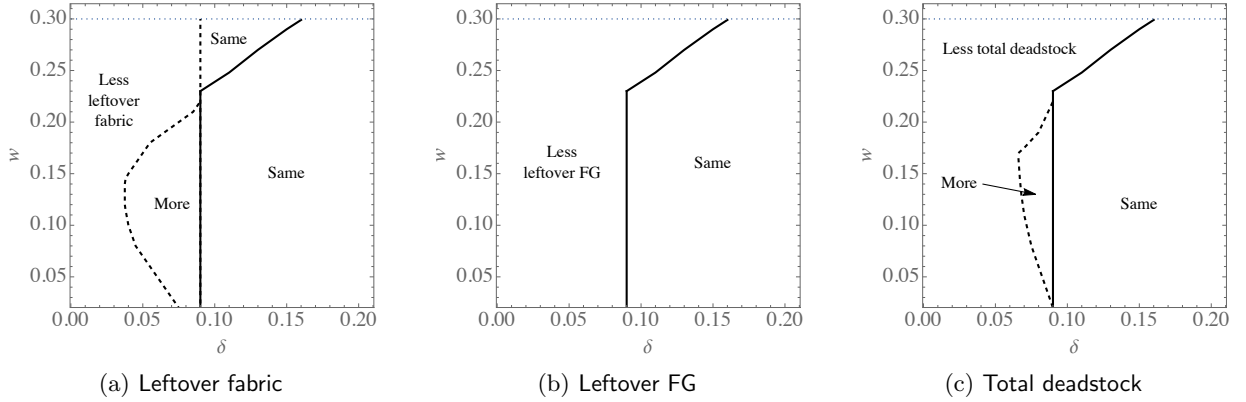


Figure 18 Comparison of deadstock outcomes under upcycling with endogenous price. The parameters are the same as those used in Figure 5.

Appendix D: Proofs

Proof of Lemma 1. If $\delta \geq p_f - c$, then $q_\delta^Q(x, q, y) = 0$ and the firm's problem reverts to benchmark case, with the optimal solution $(x^Q, q^Q) = (x^B, q^B)$. If $\delta < p_f - c$, then $q_\delta^Q(x, q, y) = \min\{(D_f(y) - q)^+, x - q\}$ which implies $\min\{D_f(y), q + q_\delta^Q(x, q, y)\} = \min\{D_f(y), x\}$. So the firm's expected profit in stage 1 is

$$\begin{aligned} \mathbb{E}\Pi^Q(x, q) &= p_f \mathbb{E} \min\{D_f(Y), x\} - c_m x - cq - (c + \delta) \mathbb{E} \min\{(D_f(Y) - q)^+, x - q\} \\ &= p_f \int_a^{\frac{x}{1-p_f}} (1-p_f)y dF(y) + p_f x \bar{F}\left(\frac{x}{1-p_f}\right) - c_m x - cq - (c + \delta) \left(\int_{\frac{q}{1-p_f}}^{\frac{x}{1-p_f}} ((1-p_f)y - q) dF(y) + (x - q) \bar{F}\left(\frac{x}{1-p_f}\right) \right). \end{aligned}$$

The firm solves $\max_{x, q} \mathbb{E}\Pi^Q(x, q)$ subject to the constraints $q \leq x$, $q \geq 0$, and $x \geq 0$. The Lagrangian of this constrained optimization problem is $L = \mathbb{E}\Pi^Q(x, q) + \lambda(x - q)$, where $\lambda \geq 0$ is the Lagrangian multiplier. The critical points satisfy the Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{aligned} \frac{\partial L}{\partial q} &= -c + (c + \delta) \bar{F}\left(\frac{q}{1-p_f}\right) - \lambda \leq 0, \\ \frac{\partial L}{\partial x} &= (p_f - c - \delta) \bar{F}\left(\frac{x}{1-p_f}\right) - c_m + \lambda \leq 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \lambda} &= x - q \geq 0, \\ x \frac{\partial L}{\partial x} &= 0, \quad q \frac{\partial L}{\partial q} = 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0, \quad q \geq 0, \quad x \geq 0, \quad \lambda \geq 0. \end{aligned}$$

Solving these conditions yields two solutions, depending on the cost of quick response δ :

Solution 1: When $\delta < \bar{\delta}^Q = \frac{c \cdot p_f}{c_m + c} - c$, the constraint $q \leq x$ does not bind, and the optimal decisions are given by

$$F\left(\frac{x^Q}{1-p_f}\right) = 1 - \frac{c_m}{p_f - c - \delta}, \quad F\left(\frac{q^Q}{1-p_f}\right) = 1 - \frac{c}{c + \delta}.$$

Solution 2: When $\bar{\delta}^Q \leq \delta < p_f - c$, the constraint $q \leq x$ binds, and the optimal decisions are given by

$$F\left(\frac{x^Q}{1-p_f}\right) = 1 - \frac{c_m + c}{p_f}, \quad q^Q = x^Q.$$

Note that Solution 2 is the same as (x^B, q^B) , and the region always exists since $\bar{\delta}^Q = \frac{c \cdot p_f}{c_m + c} - c < p_f - c$. Combining these two solutions with the solution under $\delta \geq p_f - c$ concludes the proof. \square

Proof of Proposition 1. By inspecting the optimal decisions (x^Q, q^Q) in Lemma 1, we observe that as δ increases from zero, x^Q first decreases when $\delta \in [0, \bar{\delta}^Q)$ and then equals x^B (which is constant in δ) when $\delta \in [\bar{\delta}^Q, +\infty)$. Similarly, q^Q first increases when $\delta \in [0, \bar{\delta}^Q)$ and then equals q^B (which is constant in δ) when $\delta \in [\bar{\delta}^Q, +\infty)$. It follows that $x^Q \geq x^B$ and $q^Q \leq q^B$. \square

Proof of Proposition 2. We first prove the sensitivity results in (iv) by considering two possible cases. In the first case, $\delta \geq \bar{\delta}^Q$, Θ_m^Q and Θ_l^Q do not change in δ (because q^Q and x^Q are constant in δ). In the second case, $\delta < \bar{\delta}^Q$, we differentiate $\Theta_l^Q = \int_a^{q^Q/(1-p_f)} (q^Q - (1-p_f)y) dF(y)$ with respect to δ to obtain $\frac{\partial \Theta_l^Q}{\partial \delta} = \int_a^{q^Q/(1-p_f)} \frac{dq^Q}{d\delta} dF(y) = \frac{dq^Q}{d\delta} \cdot F\left(\frac{q^Q}{1-p_f}\right)$, which is positive because $\frac{dq^Q}{d\delta} > 0$ (see Proposition 1). Similarly, we can differentiate $\Theta_m^Q = (x^Q - q^Q)F\left(\frac{q^Q}{1-p_f}\right) + \int_{q^Q/(1-p_f)}^{x^Q/(1-p_f)} (x^Q - (1-p_f)y) dF(y)$ to obtain $\frac{\partial \Theta_m^Q}{\partial \delta} = \frac{dx^Q}{d\delta} F\left(\frac{x^Q}{1-p_f}\right) - \frac{dq^Q}{d\delta} F\left(\frac{q^Q}{1-p_f}\right)$, which is negative because $\frac{dq^Q}{d\delta} < 0$. Put together, we have $\frac{\partial \Theta_l^Q}{\partial \delta} + \frac{\partial \Theta_m^Q}{\partial \delta} = \frac{dx^Q}{d\delta} F\left(\frac{x^Q}{1-p_f}\right) < 0$. Since Θ_m^Q and Θ_l^Q are continuous at $\delta = \bar{\delta}^Q$, we conclude that as δ increases from 0 to $+\infty$, Θ_m^Q decreases, Θ_l^Q increases, and $\Theta_m^Q + \Theta_l^Q$ decreases.

The results in (i)-(iii) follow from (iv) and observing that when $\delta \geq \bar{\delta}^Q$, the firm does not use the quick response option, and hence $\Theta_l^Q = \Theta_l^B$ and $\Theta_m^Q = \Theta_m^B$. \square

Proof of Proposition 3. Denote $\Theta^Q \doteq \Theta_m^Q + \Theta_l^Q$ and $\Theta^B \doteq \Theta_m^B + \Theta_l^B$. We need to prove $\bar{\Theta}^Q = \frac{\Theta^Q}{\mathbb{E}\min\{D_f, q^Q + q_\delta^Q\}} \geq \frac{\Theta^B}{\mathbb{E}\min\{D_f, q^B\}} = \bar{\Theta}^B$. Equality holds when $\delta \geq \bar{\delta}^Q$. For $\delta < \bar{\delta}^Q$, note that $x^Q = \Theta^Q + \mathbb{E}\min\{D_f, q^Q + q_\delta^Q\}$, i.e., total inventory equals the sum of deadstock and demand fulfilled in expectation. So $\bar{\Theta}^Q = \frac{\Theta^Q}{x^Q - \Theta^Q} = \frac{1}{x^Q/\Theta^Q - 1}$. It suffices to prove $\bar{\Theta}^Q$ decreases in $\delta \in [0, \bar{\delta}^Q)$, or equivalently, $\frac{x^Q}{\Theta^Q}$ increases in $\delta \in [0, \bar{\delta}^Q)$. To do so, observe that $\frac{d}{d\delta} \frac{x^Q}{\Theta^Q} = \frac{1}{\Theta^Q} \frac{dx^Q}{d\delta} - \frac{x^Q}{(\Theta^Q)^2} \frac{d\Theta^Q}{d\delta} = \frac{1}{\Theta^Q} \frac{dx^Q}{d\delta} - \frac{x^Q}{(\Theta^Q)^2} \frac{d\Theta^Q}{d\delta} F\left(\frac{x^Q}{1-p_f}\right) = \frac{1}{\Theta^Q} \frac{dx^Q}{d\delta} \left(1 - \frac{x^Q}{\Theta^Q} F\left(\frac{x^Q}{1-p_f}\right)\right) > 0$, where the last inequality holds because $\frac{dx^Q}{d\delta} < 0$ and $\Theta^Q < x^Q F\left(\frac{x^Q}{1-p_f}\right)$. The latter holds because $\Theta^Q = \int_a^{q^Q/(1-p_f)} (q^Q - (1-p_f)y) dF(y) + (x^Q - q^Q)F\left(\frac{q^Q}{1-p_f}\right) + \int_{q^Q/(1-p_f)}^{x^Q/(1-p_f)} (x^Q - (1-p_f)y) dF(y) = x^Q F\left(\frac{x^Q}{1-p_f}\right) - \int_a^{x^Q/(1-p_f)} (1-p_f)y dF(y) < x^Q F\left(\frac{x^Q}{1-p_f}\right)$. This concludes the proof. \square

Proof of Proposition 4 For $Y \sim \text{Uniform}[0, 1]$ and $c_m \leq \frac{(p_f - c - \delta)c}{c + \delta}$, we can substitute (x^Q, q^Q) and (x^B, q^B) into the environmental impact functions to obtain

$$\begin{aligned} EI^Q - EI^B = & - \frac{c_m^2 \left((p_f^2 - (p_f - c - \delta)^2) (i_{FGQ} + i_{use} - i_l) + (p_f - c - \delta)^2 i_{FGQ} + p_f^2 (i_l - i_m) \right) (1 - p_f)}{2p_f^2 (p_f - c - \delta)^2} \\ & + \frac{c_m \left((p_f - c - \delta) \left((p_f - c) i_l + c i_{use} + p_f i_{FG} \right) - p_f (c + \delta) i_{fabric} - p_f^2 i_m \right) (1 - p_f)}{p_f^2 (p_f - c - \delta)} \\ & + \frac{c \left(2p_f (c + \delta)^2 (i_{fabric} + i_{FG} + i_l) - 2p_f^2 (c + \delta) i_{FG} + c p_f^2 i_{FGQ} - p_f^2 (c + 2\delta) (i_l - i_m) + c (c + \delta)^2 (i_{use} - i_l) \right)}{2p_f^2 (c + \delta)^2 / (1 - p_f)}, \end{aligned} \quad (6)$$

which is concave in c_m given the assumptions $i_{FGQ} + i_{use} - i_l \geq 0$ and $i_l - i_m \geq 0$. The two roots of (6) are

$$c_m = \frac{(p_f - c - \delta)c}{c + \delta} \text{ and}$$

$$c_m^{root} \doteq \frac{(p_f - c - \delta) \left(2p_f^2 (c + \delta) i_{FG} - c p_f^2 i_{FGQ} - 2p_f (c + \delta)^2 (i_{fabric} + i_l + i_{FG}) + p_f^2 (c + 2\delta) (i_l - i_m) + c (c + \delta)^2 (i_l - i_{use}) \right)}{(c + \delta) \left((p_f^2 - (p_f - c - \delta)^2) (i_{FGQ} + i_{use} - i_l) + i_{FGQ} (p_f - c - \delta)^2 + p_f^2 (i_l - i_m) \right)}.$$

Recall that $EI^Q - EI^B = 0$ for $c_m \geq \frac{(p_f - c - \delta)c}{c + \delta}$. Hence, $EI^Q - EI^B \geq 0$ if and only if $c_m \geq \bar{c}_m^Q(p_f, c, \delta, I) \doteq \min\{c_m^{root}, \frac{(p_f - c - \delta)c}{c + \delta}\}$. This proves (i).

For (ii), note that a sufficient condition for (i) to hold is $c_m^{root} \leq 0$, which holds when

$$\begin{aligned} & 2\left(\frac{p_f}{c + \delta}\right) i_{FG} - \frac{c}{p_f} \left(\frac{p_f}{c + \delta}\right)^2 i_{FGQ} - 2(i_{fabric} + i_l + i_{FG}) + \frac{p_f (c + 2\delta)}{(c + \delta)^2} (i_l - i_m) + \frac{c}{p_f} (i_l - i_{use}) \\ & = 2\left(\frac{p_f}{c + \delta}\right) i_{FG} - \frac{c}{p_f} \left(\left(\frac{p_f}{c + \delta}\right)^2 - 1 \right) i_{FGQ} - 2(i_{fabric} + i_l + i_{FG}) + \frac{p_f (c + 2\delta)}{(c + \delta)^2} (i_l - i_m) - \frac{c}{p_f} (i_{FGQ} + i_{use} - i_l) \\ & \leq 2\left(\frac{p_f}{c}\right) i_{FG} - 2(i_{fabric} + i_l + i_{FG}) + \frac{p_f}{c} (i_l - i_m) - \frac{c}{p_f} (i_{FGQ} + i_{use} - i_l) \leq 0, \end{aligned}$$

or equivalently, $2i_{FG} + i_l - i_m - 2(i_{fabric} + i_l + i_{FG}) \frac{c}{p_f} - \frac{c^2}{p_f^2} (i_{FGQ} + i_{use} - i_l) \leq 0$. Since the left-hand-side of the inequality is concave in $\frac{c}{p_f}$, a sufficient condition for the inequality to hold is

$$\frac{c}{p_f} \geq \frac{-i_{fabric} - i_{FG} - i_l + \sqrt{(i_{fabric} + i_{FG} + i_l)^2 + (i_{FGQ} - i_l + i_{use})(2i_{FG} + i_l - i_m)}}{i_{FGQ} + i_{use} - i_l} \doteq \bar{c}_1(I).$$

Note that when $i_{FG} = i_{FGQ}$ and $i_{use} > i_l$, we can further tighten the bound by inspecting

$$\begin{aligned} & 2\left(\frac{p_f}{c + \delta}\right) i_{FG} - \frac{c}{p_f} \left(\frac{p_f}{c + \delta}\right)^2 i_{FG} - 2(i_{fabric} + i_l + i_{FG}) + \frac{p_f (c + 2\delta)}{(c + \delta)^2} (i_l - i_m) + \frac{c}{p_f} (i_l - i_{use}) \\ & = \frac{p_f}{c} \left(1 - \frac{\delta^2}{(c + \delta)^2} \right) (i_{FG} + i_l - i_m) - 2(i_{fabric} + i_l + i_{FG}) - \frac{c}{p_f} (i_{use} - i_l) \leq 0 \end{aligned}$$

to obtain the sufficient condition $\frac{c}{p_f} \geq \sqrt{\left(\frac{i_{fabric} + i_l + i_{FG}}{i_{use} - i_l}\right)^2 + \frac{i_{FG} + i_l - i_m}{i_{use} - i_l}} - \frac{i_{fabric} + i_l + i_{FG}}{i_{use} - i_l} \doteq \bar{c}_2(I)$. Letting $\bar{c}^Q(I) = \bar{c}_2(I)$ for $i_{FG} = i_{FGQ}$ and $i_{use} > i_l$ and $\bar{c}^Q(I) = \bar{c}_1(I)$ otherwise completes the proof. \square

Proof of Lemma 2 For $\delta < p_f - c - w$, we have $q_\delta^U(x, q, y) = \min\{(D_f(y) - q)^+, x - q\}$ which implies $\min\{D_f(y), q + q_\delta^U(x, q, y)\} = \min\{D_f(y), x\}$. The firm chooses x and q to maximize its expected profit:

$$\begin{aligned} \mathbb{E}\Pi^U(x, q) & = p_f \mathbb{E} \min\{D_f(Y), x\} - c_m x - cq - (c + \delta) \mathbb{E} q_\delta^U(x, q, Y) + w \mathbb{E}(x - q - q_\delta^U(x, q, Y))^+ \\ & = p_f \left(\int_a^{x/(1-p_f)} (1-p_f) y dF(y) + x \bar{F}\left(\frac{x}{(1-p_f)}\right) \right) - c_m x - cq - (c + \delta) \left((x - q) \bar{F}\left(\frac{x}{(1-p_f)}\right) + \int_{q/(1-p_f)}^{x/(1-p_f)} ((1-p_f)y - q) dF(y) \right) \\ & \quad + w \left((x - q) F\left(\frac{q}{(1-p_f)}\right) + \int_{q/(1-p_f)}^{x/(1-p_f)} (x - (1-p_f)y) dF(y) \right), \end{aligned}$$

subject to the constraints $q \leq x$, $x \geq 0$ and $q \geq 0$.

Similar to the proof of Lemma 1, there are two solutions, depending on the cost of quick response δ :

Solution 1: When $\delta < \bar{\delta}^U = \frac{(p_f - c - c_m)(c + w)}{c + c_m}$, the constraint $q \leq x$ does not bind, and the optimal decisions are given by

$$F\left(\frac{x^U}{1 - p_f}\right) = \frac{p_f - c_m - c - \delta}{p_f - c - \delta - w}, \quad F\left(\frac{q^U}{1 - p_f}\right) = \frac{\delta}{c + \delta + w}.$$

Solution 2: When $\bar{\delta}^U \leq \delta < p_f - c - w$, the constraint $q \leq x$ binds, and the optimal decisions are given by

$$F\left(\frac{x^U}{1 - p_f}\right) = 1 - \frac{c_m + c}{p_f}, \quad q^U = x^U.$$

Note that the region for Solution 2 always exists because $\bar{\delta}^U = \frac{p_f(c+w)}{c+c_m} - c - w < p_f - c - w$. Furthermore, it is easy to see that the threshold $\bar{\delta}^U = \frac{(p_f - c - c_m)(c + w)}{c + c_m}$ equals $\bar{\delta}^Q$ at $w = 0$ and increases in w . This concludes the proof. \square

Proof of Proposition 5 The result follows directly from inspecting the expressions of the optimal solutions (x^U, q^U) in Lemma 2, and comparing them with those in Lemma 1. \square

Proof of Proposition 6 It suffices to compare the expected amounts of leftover finished goods. If $\delta \geq \bar{\delta}^U$ then the firm does not engage in upcycling, and so $\Theta_i^U = \Theta_i^Q$. If $\delta < \bar{\delta}^U$, then we compare $\Theta_i^U = \mathbb{E}(q^U - D_f(Y))^+$ with $\Theta_i^Q = \mathbb{E}(q^Q - D_f(Y))^+$. Since $q^U \geq q^Q$ (by Proposition 5), we have $\Theta_i^U \geq \Theta_i^Q$. Moreover, since q^U decreases in w and increases in δ , Θ_i^U also decreases in w and increases in δ . \square

Proof of Proposition 7 Since $\Theta_m^U + \Theta_i^U \leq \Theta_m^Q + \Theta_i^Q$ (Proposition 6), it suffices to show that more units of demand are fulfilled in expectation under quick response, that is, $\mathbb{E} \min\{D_f(Y), x^U\} \geq \mathbb{E} \min\{D_f(Y), x^Q\}$. This inequality holds because $x^U \geq x^Q$ (Proposition 5). \square

Proof of Proposition 8 For $Y \sim \text{Uniform}[0, 1]$ and $c_m \leq \frac{p_f(c+w)}{c+\delta+w} - c$, we can substitute (x^U, q^U) and (x^Q, q^Q) into the environmental impact functions to obtain

$$\begin{aligned} EI^U - EI^B &= \frac{c_m^2}{2} \left(\frac{i_{use} - i_l}{p_f^2} - \frac{i_{FGQ} + i_{use}}{(p_f - c - \delta - w)^2} \right) (1 - p_f) \\ &\quad + c_m \left(\frac{wi_{FGQ} - i_{fabric}}{(c + \delta - p_f + w)^2} + \frac{p_f^2 i_{fabric} + p_f^2 wi_{use}}{p_f^2 (c + \delta - p_f + w)^2} + \frac{i_{fabric}}{c + \delta - p_f + w} + \frac{(p_f - c)i_l + ci_{use}}{p_f^2} + \frac{i_{fabric} + i_{FG}}{p_f} \right) (1 - p_f) \\ &\quad + \left(\frac{c^2(i_{use} - i_l)}{2p_f^2} + \frac{c(i_l + i_{fabric} + i_{FG})}{p_f} + i_l \left(\frac{\delta^2}{2(c + \delta + w)^2} - \frac{1}{2} \right) - \frac{w^2 i_{use}}{2(c + \delta - p_f + w)^2} - \frac{wi_{fabric}}{c + \delta - p_f + w} - \frac{(c + w)i_{FG}}{c + \delta + w} \right. \\ &\quad \left. + i_{FGQ} \left(\frac{(c + w)^2}{2(c + \delta + w)^2} - \frac{w^2}{2(c + \delta - p_f + w)^2} \right) \right) (1 - p_f), \end{aligned}$$

which is concave in c_m . Since $EI^U - EI^B = 0$ for $c_m \geq \frac{p_f(c+w)}{c+\delta+w} - c$ and we assume $c_m \geq w$, a sufficient condition for $EI^U - EI^B \geq 0$ to hold for all parameter settings is $(EI^U - EI^B)|_{c_m=w} \geq 0$. That is,

$$\begin{aligned} &\frac{1}{2} \left(\frac{2i_{fabric}(c+w)}{p_f} + i_{FG} \left(-\frac{2(p_f - c - w)}{p_f} + \frac{2\delta}{c + \delta + w} \right) + \frac{i_{FGQ}(c+w)^2}{(c + \delta + w)^2} + i_l \left(\frac{\delta^2}{(c + \delta + w)^2} - \frac{(c - p_f + w)^2}{p_f^2} \right) + \frac{i_{use}(c+w)^2}{p_f^2} \right) \\ &\geq \frac{1}{2} \left(\frac{2ci_{fabric}}{p_f} - \frac{2i_{FG}(p_f - c)}{p_f} + \frac{i_{FGQ}(c+w)^2}{(c + \delta + w)^2} - \frac{i_l(p_f - c)^2}{p_f^2} + \frac{c^2 i_{use}}{p_f^2} \right) \\ &\geq \frac{1}{2} \left(\frac{2ci_{fabric}}{p_f} - \frac{2i_{FG}(p_f - c)}{p_f} - \left(1 - \frac{2c}{p_f}\right) i_l + \frac{c^2(i_{FGQ} + i_{use} - i_l)}{p_f^2} \right) \geq 0. \end{aligned}$$

which is equivalent to $\frac{c}{p_f} \geq \bar{c}^U(I) \doteq \frac{\sqrt{(i_{fabric} + i_{FG} + i_l)^2 + (2i_{FG} + i_l)(i_{FGQ} - i_l + i_{use})} - i_{fabric} - i_{FG} - i_l}{i_{FGQ} - i_l + i_{use}}$. \square

Proof of Lemma 3. See the Electronic Companion of the paper. \square

Proof of Proposition 9. See the Electronic Companion of the paper. \square

Proof of Proposition 10. It suffices to only consider the case of $\delta < \bar{\delta}^Q$, or equivalently, $c_m < \frac{c(p_f - c - \delta)}{c + \delta}$.

When $Y \sim Uniform[0, 1]$, the difference in the expected amount of production with and without quick response is

$$q^Q + \mathbb{E}q_\delta^Q - q^B = x^Q - \frac{(x^Q)^2 - (q^Q)^2}{2(1 - p_f)} - q^B = (1 - p_f) \left(\frac{p_f - c - \delta - c_m}{p_f - c - \delta} - \frac{(p_f - c - \delta - c_m)^2}{2(p_f - c - \delta)^2} + \frac{\delta^2}{2(c + \delta)^2} - \frac{p_f - c - c_m}{p_f} \right),$$

which is a concave function of c_m and equals zero at the upper bound $c_m = \frac{c(p_f - c - \delta)}{c + \delta}$. At the lower bound $c_m = 0$, the expression above is negative if and only if $p_f > \frac{2(c + \delta)^2}{c + 2\delta}$. The result directly follows. \square

Proof of Proposition 11. For $\delta \geq \bar{\delta}^Q$, we have $\mathbb{E} \min\{D_f, q^Q + q_\delta^Q\} = \mathbb{E} \min\{D_f, q^Q\} = \mathbb{E} \min\{D_f, q^B\}$. For $\delta < \bar{\delta}^Q$, we have $\mathbb{E} \min\{D_f, q^Q + q_\delta^Q\} = \mathbb{E} \min\{D_f, x^Q\} > \mathbb{E} \min\{D_f, q^B\}$ because $x^Q > q^B$ by Proposition 2. \square

Proof of Lemma 4. The firm's optimal decisions in stage 2 are analyzed and summarized in the Electronic Companion (Section EC.1). See Figure EC.1 for illustrations of the two thresholds W_1 and W_2 . \square

Proof of Lemma 5. The proof is similar to that for Lemma 1 and hence omitted. \square

Proof of Proposition 12. The result follows from comparing the fabric acquisition decisions x^{QB} and x^Q , and observing that total deadstock depends only on the amount of fabric acquired, i.e., $\Theta^i = x^i F\left(\frac{x^i}{1 - p_f}\right) - \int_a^{x^i/(1 - p_f)} (1 - p_f)y dF(y)$ for $i \in \{QB, Q\}$ (see the proof of Proposition 2). \square

Proof of Lemma 6. The proof is similar to that for Lemma 2 and hence omitted. \square

Proof of Proposition 13. The result follows directly from comparing the optimal decisions (x^{UB}, q^{UB}) with (x^U, q^U) , and observing that $\Theta^i = \Theta_i^i = \mathbb{E}(q^i - D_f)^+$ for $i \in \{UB, U\}$. \square

Appendix E: Truncated Normal Demand Distribution

In this appendix we discuss the robustness of the results in §4.3 and §5 to alternative demand distributions. In particular, we numerically replicate the main results using a truncated normal distribution for market size Y , with mean $\mu = 0.5$, standard deviation $\sigma = 0.3$, and support $[0, 1]$. Note that when $\sigma \rightarrow +\infty$, the truncated normal distribution approximates a uniform distribution (which is what we assume in §4.3 and the numerical simulations of §5). Thus, by comparing the results in this appendix with those in the main paper, we can also assess the effect of lower demand uncertainty (i.e., a smaller σ) on the firm's decisions and environmental outcomes. To ease the comparison, we use the same model parameter values as those used to generate the figures in §4.3 and §5.

First, Figure 19 illustrates a numerical example of the firm's upcycling and quick response decisions in the presence of the encroachment effect. The three regions are similar to those shown in Figure 5(b), and the same insights apply as those discussed after Lemma 3. We also numerically confirm that compared to the model with quick response only, the focal firm acquires more fabric but produces fewer units in stage 1 in the upcycling model with demand encroachment (similar to Proposition 9).

Interestingly, the regions of no upcycling (with no quick response) and selective upcycling (with quick response) are both larger in Figure 19 than in Figure 5(b), and the region of full upcycling (i.e., with quick response) is smaller. These differences suggest that as demand uncertainty reduces, the firm becomes less inclined to upcycle or to use quick response. Intuitively, as demand uncertainty reduces, the value of each potential customer for the firm becomes higher because the expected cost of demand-supply mismatch is now lower; as a result, the firm becomes less willing to cede any demand to the upcycling firm through upcycling. Given the complementary effect between upcycling and quick response, the firm also becomes less willing to engage in quick response.

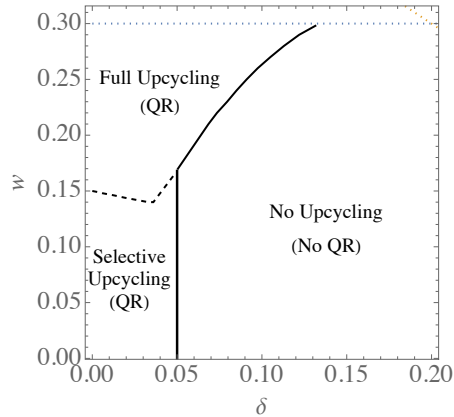


Figure 19 Upcycling decisions under demand encroachment. In this example, Y follows a truncated normal distribution on $[0, 1]$ with $\mu = 0.5$, $\sigma = 0.3$; $p_f = 0.6$, $c = 0.1$, $c_m = 0.3$, $\alpha = 0.3$, $p_s = 0.1$.

Next we examine the impact of upcycling on deadstock outcomes, which is illustrated in Figure 20. The results are similar to those depicted in Figure 6. We observe that the region where introducing upcycling leads to more leftover fabric (i.e., $\Theta_m^{U^\dagger} > \Theta_m^Q$) is larger in Figure 20 than in Figure 6. This is consistent with

the previous observation that as demand uncertainty reduces, the focal firm—due to demand encroachment concerns—becomes less inclined to fully upcycle and more likely to upcycle only a portion of its deadstock fabric. As a result, the effectiveness of upcycling in reducing deadstock is undermined.

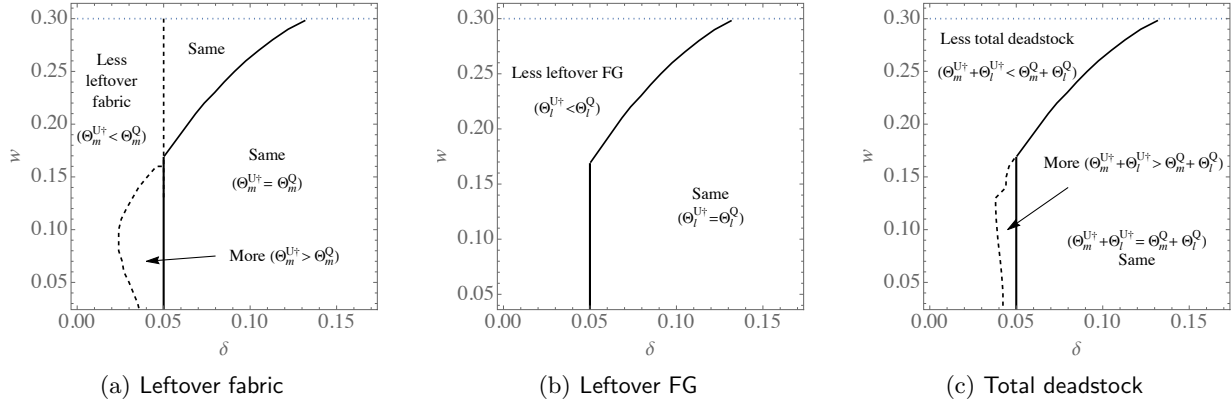
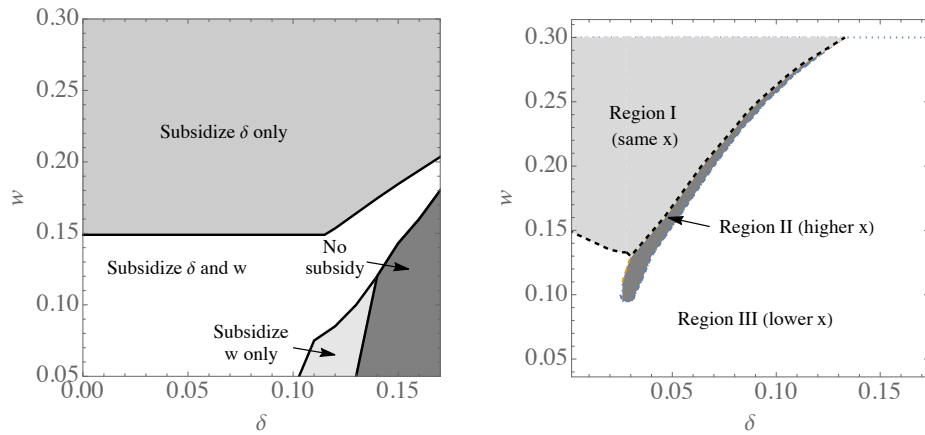


Figure 20 Impact of upcycling on deadstock outcomes (truncated normal distribution).

Finally, Figures 21(a) and 21(b) illustrate the policy implications, which are similar to those shown in Figures 12(b) and Figure 8. In particular, Figure 21(a) illustrates the optimal subsidy design for deadstock reduction under a given budget, where the parameter values are the same as those used in Figure 12(b). Comparison of those two figures shows that the area where subsidies do not work is larger in Figure 21(a) than in Figure 12(b). This is again because when demand uncertainty is lower, the firm becomes less inclined to upcycle due to demand encroachment concerns, and hence it would take a larger budget for subsidies to achieve an effect. Last but not least, Figure 21(b) reveals that, in comparison with Figure 8, the region where a deadstock ban results in lower x is larger. Thus, our results show that when demand is less uncertain, policies that focus on inducing firms to upcycle become less effective, while direct deadstock bans may become more effective in the sense that it is more likely to achieve reductions in both deadstock and fabric input.



(a) Most effect subsidy policy to reduce deadstock under budget constraint (b) Effect of deadstock ban on focal firm's fabric acquired

Figure 21 Policy implications (truncated normal distribution).