Appendices

Online Appendix for: "Cost learning and strategic inventory: who should carry inventory in a vertical supply chain?"

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EC.1. Proof of results in Section 4

Proof of Theorem 1. The equilibrium results are obtained using backward induction. The retailer's second period profit function is given by $\Pi_{2r} = p_2(Q_2 + I_r) - w_2Q_2$ where $p_2 = 1 - Q_2 - I_r$. Since Π_{2r} is concave in Q_2, from first order conditions, we obtain $Q_2(w_2) = \frac{1-2I_r-w_2}{2}$. The manufacturer's second period profit function is given by $\Pi_{2m} = w_2Q_2 - c_2(Q_2 - I_m)$. Substituting $Q_2 = \frac{1-2I_r-w_2}{2}$ in Π_{2m}, we obtain $\Pi_{2m} = w_2(\frac{1-2I_r-w_2}{2}) - c_2(\frac{1-2I_r-w_2}{2} - I_m)$, which is concave in w_2. From the first order condition, we obtain $w_2 = \frac{1+c_2-2I_r}{2}$. Substituting Q_2 and w_2, $\Pi_{2r}(c_2) = \frac{1-2c_2+12I_r+c_2^2+4c_2I_r-12I_r^2}{16}$. As c_2 is the realized value of the random variable $C_2 = c_1 - \Lambda(Q_1 + I_m)$, we take the expectation over C_2 to yield $\mathbb{E}[\Pi_{2r}] = \frac{1}{16}[1 + c_1^2 + 4c_1I_r - 12I_r^2 + 2\mu(I_m + Q_1) - 4I_r(\mu Q_1 + \mu I_m - 3) - 2c_1(1 + \mu(I_m + Q_1)) + (\mu^2 + \sigma^2)(I_m + Q_1)^2]$.

The retailer's total expected profit is given by $\mathbb{E}[\Pi_r] = p_1q_1 - w_1(q_1 + I_r) - h_rI_r + \mathbb{E}[\Pi_{2r}]$. Substituting $\mathbb{E}[\Pi_{2r}]$ in $\mathbb{E}[\Pi_r]$, we obtain $\mathbb{E}[\Pi_r] = \frac{1}{16}[1 + c_1^2 + 4c_1I_r - 12I_r^2 + 2\mu(I_m + Q_1) - 4I_r(\mu Q_1 + \mu I_m - 3) - 2c_1(1 + \mu(I_m + Q_1)) + (\mu^2 + \sigma^2)(I_m + Q_1)^2] - w_1(q_1 + I_r) + q_1(1 - q_1) - h_rI_r$. $\mathbb{E}[\Pi_r]$ is concave in q_1 and I_r since the determinant of the Hessian matrix is 3 and the second derivatives of $\mathbb{E}[\Pi_r]$ with respect to q_1 and I_r are strictly negative making the Hessian negative definite. The retailer maximizes $\mathbb{E}[\Pi_r]$ subject to $I_r \geq 0$. Thus, we have the Lagrangian $L_r(q_1, I_r) = \Pi_r + \lambda_r(I_r)$. Solving this Lagrangian, we obtain $I_r > 0$ when $w_1 < \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$ and $I_r = 0$ when $w_1 > \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$. We elaborate these two cases below.

Case 1: $I_r > 0$ when $w_1 < \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$. Note that the retailer orders $Q_1 = q_1 + I_r$ units in the first period. The optimal values from the first order conditions are obtained as $q_1 = \frac{1-w_1}{2}$ and $I_r = \frac{6+2c_1-8h_r-\mu-2I_m\mu-8w_1-\mu w_1}{2(6+\mu)}$. Substituting q_1 and I_r, we have

$$\Pi_{2m}(c_2) = \frac{1}{8(\mu+6)^2}[4c_1^2 + 4c_1(c_2(\mu+6) - 8h_r - 2I_m\mu + \mu w_1 - 2\mu - 8w_1) + c_2^2(\mu+6)^2$$
$$+ 2c_2(\mu+6)(-8h_r + 2I_m(\mu+12) + \mu w_1 - 2\mu - 8w_1) + (8h_r + \mu(2I_m - w_1 + 2) + 8w_1)^2]$$

Note that c_2 is the realized value of the random variable $C_2 = c_1 - \Lambda(q_1 + I_r + I_m)$. The manufacturer's total expected profit is given by $\mathbb{E}[\Pi_m] = w_1(q_1 + I_r) - c_1(q_1 + I_r + I_m) - h_m I_m + \mathbb{E}[\Pi_{2m}(C_2)]$. The expected profit of the manufacturer in the second period is obtained as:

$$\mathbb{E}[\Pi_{2m}(C_2)] = \frac{1}{8(\mu+6)^2}[c_1^2(\sigma^2+64) + 2c_1(4h_r(8\mu - \sigma^2 - 16) + I_m(-40\mu + 6\sigma^2 + 144) + 64\mu w_1 - 64\mu - 7\sigma^2 w_1)$$
$$+ 2c_1(6\sigma^2 - 64w_1) + 16h_r^2(\mu^2 - 4\mu + \sigma^2 + 4) - 8h_r(4\mu^2(I_m - 2w_1 + 2) - 8\mu(5I_m - 3w_1 + 2)$$
$$+ \sigma^2(6I_m - 7w_1 + 6) - 16w_1) + 16I_m^2\mu^2 - 288I_m^2\mu + 36I_m^2\sigma^2 - 72I_m\mu^2 w_1 + 80I_m\mu^2 + 464I_m\mu w_1$$
$$- 288I_m\mu - 84I_m\sigma^2 w_1 + 72I_m\sigma^2 + 64\mu^2 w_1^2 - 128\mu^2 w_1 + 64\mu^2 - 128\mu w_1^2 + 128\mu w_1 + 49\sigma^2 w_1^2$$
$$- 84\sigma^2 w_1 + 36\sigma^2 + 64w_1^2]$$

Substituting $\mathbb{E}[\Pi_{2m}(C_2)]$ in $\mathbb{E}[\Pi_m]$ yields

$$\mathbb{E}[\Pi_m] = \frac{1}{8(\mu+6)^2}[c_1^2(-8\mu + \sigma^2 + 16) + 2c_1(4h_r(12\mu - \sigma^2 + 8) - 8\mu(8I_m - 12w_1 + 11) + 6I_m\sigma^2 - 7\sigma^2 w_1 + 6\sigma^2 + 128w_1)$$
$$+ 16h_r^2\left(\mu^2 - 4\mu + \sigma^2 + 4\right) - 8h_r\left(4\mu^2(I_m - 2w_1 + 2) - 4\mu(10I_m - 7w_1 + 4) + \sigma^2(6I_m - 7w_1 + 6) + 8w_1\right) - 8h_m I_m\mu^2$$
$$+ 16I_m^2\mu^2 - 96h_m I_m\mu - 288h_m I_m - 288I_m^2\mu + 36I_m^2\sigma^2 - 80I_m\mu^2 w_1 + 80I_m\mu^2 + 416I_m\mu w_1 - 288I_m\mu - 84I_m\sigma^2 w_1$$
$$+ 72I_m\sigma^2 + 64\mu^2 w_1^2 - 128\mu^2 w_1 + 64\mu^2 - 184\mu w_1^2 + 176\mu w_1 + 49\sigma^2 w_1^2 - 84\sigma^2 w_1 + 36\sigma^2 - 272w_1^2 + 288(w_1 - c_1)]$$

$\mathbb{E}[\Pi_m]$ is concave in w_1 and I_m, since the determinant of the Hessian matrix is obtained as $\frac{(6+\mu)^2(72\mu - 4\mu^2 - 9\sigma^2)(272 + 184\mu - 64\mu^2 - 49\sigma^2) - (104\mu - 20\mu^2 - 21\sigma^2)^2}{4(6+\mu)^4}$, which is strictly positive in our model and the second derivatives of $\mathbb{E}[\Pi_m]$ with respect to w_1 and I_m are strictly negative, making the Hessian negative definite. Since the manufacturer maximizes $\mathbb{E}[\Pi_m]$ subject to $I_m \geq 0$, we solve the Lagrangian $L_s(w_1, I_m) = \mathbb{E}[\Pi_m] + \lambda_{m1} I_m$ that yields two solutions based on whether the constraint $I_m \geq 0$ is binding or not.

Subcase 1.1: The constraint is binding, that is, $I_m = 0$ when:

(a) $h_r \leq h_{r1} = \frac{(1-c_1)(32\mu - 13\sigma^2)}{128\mu - 32\mu^2 - 18\sigma^2}$ and $h_m > 0$.

(b) $h_r > h_{r1}$ and $h_m > h_{m1}(h_r) = \frac{h_r(256\mu - 64\mu^2 - 36\sigma^2) - (1-c_1)(64\mu - 26\sigma^2)}{272 + 184\mu - 64\mu^2 - 49\sigma^2}$.

For sub-cases 1.1 (a) and (b), since the constraint $I_m = 0$, we get $w_1 = \frac{144 + 88\mu - 64\mu^2 - 42\sigma^2 + c_1(128 + 96\mu - 7\sigma^2) - 4h_r(8 + 28\mu - 8\mu^2 - 7\sigma^2)}{272 + 184\mu - 64\mu^2 - 49\sigma^2}$. For the sub-case 1.1. (a), the optimal values of w_1 and I_m satisfies the condition $w_1 < \frac{3 + c_1 - 4h_r - \mu Q_1 + I_m}{4}$ for $I_r > 0$. For the sub-case 1.1. (b), $w_1 < \frac{3 + c_1 - 4h_r - \mu(Q_1 + I_m)}{4}$ is satisfied when $h_r < \widehat{h}_r = \frac{(1-c_1)(80 + 32\mu + 7\sigma^2)}{320 + 48\mu - 32\mu^2 - 28\sigma^2}$. Note that $0 < h_{r1} < \widehat{h}_r$. Hence, in sub-case 1.1 (a) where $0 < h_r \leq h_{r1}$, we obtain $I_m = 0$ and $I_r > 0$. In sub-case 1.1 (b) where $h_r > h_{r1}$ and $h_m > h_{m1}(h_r)$, we obtain $I_m = 0$ and $I_r > 0$ when $h_{r1} < h_r < \widehat{h}_r$ and $h_m > h_{m1}(h_r)$.

Sub-case 1.2: $I_m > 0$ when $h_r > h_{r1}$ and $h_m < h_{m1}(h_r)$. Solving the first order conditions with respect to I_m and w simultaneously, we get $I_m = \frac{4h_r(64\mu-9\sigma^2-16\mu^2)-(1-c_1)(64\mu-26\sigma^2)-h_m(272+184\mu-64\mu^2-49\sigma^2)}{4(136\mu-36\mu^2-17\sigma^2)}$ and $w_1 = \frac{144\mu-18\sigma^2-72\mu^2+c_1(128\mu-16\sigma^2)-4h_r(8\mu-4\mu^2-\sigma^2)-h_m(104\mu-20\mu^2-21\sigma^2)}{2\left(136\mu-36\mu^2-17\sigma^2\right)}$. The above optimal values of w_1 and I_m satisfies the condition $w_1 < \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$ for $I_r > 0$ when h_r and h_m satisfies (i) $h_{r1} < h_r < h_{r2} = \frac{(1-c_1)(8\mu-\sigma^2)}{32\mu-8\mu^2-4\sigma^2}$ and $h_m < h_{m1}(h_r)$ (ii) $h_{r2} < h_r < \widehat{h}_r$ and $h_{m1}(h_r) > h_m > h_{m2}(h_r) = \frac{h_r(160\mu-40\mu^2-20\sigma^2)-(1-c_1)(40\mu-5\sigma^2)}{92\mu-22\mu^2-14\sigma^2}$. Hence, in sub-case 1.2, $I_r > 0$ and $I_m > 0$, that is, both members carry inventory when either of the conditions mentioned in (i) and (ii) are satisfied.

Case 2: $I_r = 0$ when $w_1 > \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$. Solving the first order conditions with respect to q_1, we obtain $q_1 = \frac{1-w_1}{2}$. Substituting q_1 and $I_r = 0$, we obtain $\Pi_{2m}(c_2) = \frac{(1-c_2)^2+8c_2I_m}{8}$ where c_2 is the realized value of the random variable $C_2 = c_1 - \Lambda(q_1 + I_r + I_m)$. The manufacturer's total expected profit is given by $\mathbb{E}[\Pi_m] = w_1q_1 - c_1(q_1 + I_r + I_m) - h_m(I_m) + \mathbb{E}[\Pi_{2m}(C_2)]$. The expected profit of the manufacturer in the second period

$$\begin{aligned}\mathbb{E}[\Pi_{2m}(C_2)] = &\frac{1}{32}[4c_1^2 - 4c_1(2I_m(\mu-4) - \mu w_1 + \mu + 2) + 4I_m^2\sigma^2 + \mu^2(w_1 - 2I_m - 1)^2] \\ &+ \frac{1}{32}[4I_m\sigma^2 - 4(4I_m - 1)\mu(2I_m - w_1 + 1) - 4I_m\sigma^2 w_1 + \sigma^2 w_1^2 - 2\sigma^2 w_1 + \sigma^2 + 4].\end{aligned}$$

Substituting $\mathbb{E}[\Pi_{2m}(C_2)]$ in $\mathbb{E}[\Pi_m]$, we get

$$\begin{aligned}\mathbb{E}[\Pi_m] = &\frac{4c_1^2 - 4c_1(2I_m(\mu-4) - \mu w_1 + \mu + 2) - 16c_1(2I_m - w_1 + 1) - 32h_mI_m + 4I_m^2\sigma^2}{32} \\ &+ \frac{\mu^2(w_1 - 2I_m - 1)^2 - 4(4I_m - 1)\mu(2I_m - w_1 + 1) - 4I_m\sigma^2 w_1 4I_m\sigma^2 + \sigma^2 w_1^2}{32} \\ &+ \frac{\sigma^2 - 2\sigma^2 w_1 - 16(w_1 - 1)w_1 + 4}{32}.\end{aligned}$$

$\mathbb{E}[\Pi_m]$ is concave in w_1 and I_m since the determinant of the Hessian matrix is obtained as $\frac{\left(8\mu-2\mu^2-\sigma^2\right)}{4}$, which is strictly positive in our model and the Hessian is negative definite. Since the manufacturer maximizes $\mathbb{E}[\Pi_m]$ such that $I_m \geq 0$, we solve the Lagrangian $L_s(w_1, I_m) = \Pi_s + \lambda_{m2}I_m$, which yields two solutions based on whether the constraint $I_m \geq 0$ is binding or not.

Sub-case 2.1: $I_m = 0$ when $h_m > \widehat{h}_m = \frac{\sigma^2(1-c_1)}{16-\mu^2-\sigma^2}$. Solving the first order conditions, we obtain $w_1 = \frac{2c_1(\mu+4)-\mu^2-2\mu-\sigma^2+8}{16-\mu^2-\sigma^2}$. The optimal w_1 satisfies the condition $w_1 > \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$ when

$h_m > \widehat{h}_m$ and $h_r > h_{r3} = \frac{(1-c_1)(16+4\mu+\sigma^2)}{64-4\mu^2-4\sigma^2}$. Note that $h_{r3} < \widehat{h}_r$. Therefore, $I_m = 0$ and $I_r = 0$ when $h_m > \widehat{h}_m$ and $h_r > \widehat{h}_r$.

Sub-case 2.2: $I_m > 0$ when $h_r > \widehat{h}_r$ and $h_m < \widehat{h}_m$. Solving the first order conditions, we obtain $w_1 = \frac{(1+c_1)(8\mu-\sigma^2)-h_m(4\mu-\mu^2-\sigma^2)-4\mu^2}{2\left(8\mu-2\mu^2-\sigma^2\right)}$ and $I_m = \frac{(1-c_1)\sigma^2-h_m\left(16-\mu^2-\sigma^2\right)}{4\left(8\mu-2\mu^2-\sigma^2\right)}$.The optimal w_1 satisfies the condition $w_1 > \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$ when (i) $h_{r2} < h_r < \widehat{h}_r$ and $h_m < h_{m2}$ (ii)$h_r \geq \widehat{h}_r$ and $h_m < \widehat{h}_m$. Thus, $I_m > 0$ and $I_r = 0$ when either of the conditions in (i) and (ii) are satisfied. All the sub-cases and the corresponding equilibrium decisions are summarized in Table 2. From the equilibrium decisions, the manufacturer's inventory I_m and the retailer inventory I_r in the four regions are given by

(a) *Region RS*: $I_m = 0$ and $I_r = \frac{(1-c_1)\Theta_2-4h_r\Theta_3}{2\left(272+184\mu-64\mu^2-49\sigma^2\right)} > 0$ where $\Theta_2 \equiv 32\mu + 7\sigma^2 + 80$; $\Theta_3 \equiv 80 + 12\mu - 8\mu^2 - 7\sigma^2$ hold for all values of h_r satisfying conditions for Region RS and for all values of μ, σ, and c_1 satisfying (1).

(b) *Region RMS*: $I_m = \frac{4h_r(64\mu-9\sigma^2-16\mu^2)-(1-c_1)(64\mu-26\sigma^2)-h_m\Theta_8}{4(136\mu-36\mu^2-17\sigma^2)} > 0$ and $I_r = \frac{(1-c_1)(40\mu-5\sigma^2)-20h_r\Theta_9+h_m(23-22\mu^2-14\sigma^2)}{2(136\mu-36\mu^2-17\sigma^2)} > 0$ where $\Theta_8 \equiv 272 + 184\mu - 64\mu^2 - 49\sigma^2$; $\Theta_9 \equiv 8\mu - 2\mu^2 - \sigma^2$ hold for all values of h_r satisfying conditions for Region RMS and for all values of μ, σ, and c_1 satisfying (1).

(c) *Region MS*: $I_m = \frac{(1-c_1)\sigma^2-h_m\left(16-\mu^2-\sigma^2\right)}{4\left(8\mu-2\mu^2-\sigma^2\right)} > 0$ and $I_r = 0$ hold for all values of h_r and h_m satisfying conditions for Region MS and for all values of μ, σ, and c_1 satisfying (1).

(d) *Region NS*: $I_m = 0$ and $I_r = 0$ hold for all values of h_r and h_m satisfying conditions for Region NS and for all values of μ, σ, and c_1 satisfying (1). □

Proof of Lemma 1. The interim step of backward induction (to find the subgame perfect Nash equilibrium), gives the reaction function of the retailer's strategic inventory, $I_r = \frac{6+2c_1-8h_r-\mu-2I_m\mu-8w_1-\mu w_1}{2(6+\mu)}$ (Case 1 in the Proof of Theorem 1). Therefore, we obtain (a) $\frac{\partial I_r}{\partial I_m} = \frac{-\mu}{6+\mu}$ and (b) $\frac{\partial I_r}{\partial w_1} = \frac{-(8+\mu)}{2(6+\mu)}$. □

Proof of Lemma 2. From the interim step of backward induction while solving for the equilibrium solution provided in Theorem 1, we obtain $I_m = 0$ when $h_m > \widehat{h}_m$ (see Sub-case 2.1 in the Proof of Theorem 1). From Case 2 of the proof of Theorem 1, note that when $h_r > \widehat{h}_r$, we obtain $w_1 > \frac{3+c_1-4h_r-\mu(Q_1+I_m)}{4}$, which is the Lagrangian condition for $I_r = 0$. □

Proof of Lemma 3. (a) From the interim step of backward induction while solving for the equilibrium solution provided in Theorem 1, we obtain $I_m > 0$ when $h_m < \widehat{h}_m$ and $h_r \geq \widehat{h}_r$ (from condition (ii) of Sub-case 2.2 in the Proof of Theorem 1) or when $h_m < h_{m1}$ and $h_r < \widehat{h}_r$ (from conditions (i) and (ii) in Sub-case 1.2). (b) The sufficient conditions for $I_r > 0$ similarly follow from combining the conditions obtained in Sub-case 1.1 and 1.2 in the Proof of Theorem 1. $\square$

Proof of Lemma 4. Lemma 2 provides the expressions of $\widehat{h}_r$ and $\widehat{h}_m$ and Lemma 3 along with Table 1 provide the expressions of h_{m1} and h_{m2} obtained in the Proof of Equilibrium decisions.

(a) $\frac{\partial \widehat{h}_m}{\partial c_1} = \frac{-\sigma^2}{16-\mu^2-\sigma^2} < 0$ and $\frac{\partial \widehat{h}_r}{\partial c_1} = \frac{-(80+32\mu+7\sigma^2)}{320+48\mu-32\mu^2-28\sigma^2} < 0$ for all values of μ and σ satisfying (1), which is valid throughout our model.

(b) $\frac{\partial h_{m1}}{\partial c_1} = \frac{64\mu-26\sigma^2}{272-64\mu^2+184\mu-49\sigma^2} > 0$, $\frac{\partial h_{m2}}{\partial c_1} = \frac{5\left(8\mu-\sigma^2\right)}{92\mu-22\mu^2-14\sigma^2} > 0$ for all values of μ and σ satisfying (1), which is valid throughout our model.

(c) $\frac{\partial h_{m1}}{\partial h_r} = \frac{4(64\mu-16\mu^2-9\sigma^2)}{272+184\mu-64\mu^2-49\sigma^2} > 0$ and $\frac{\partial h_{m2}}{\partial h_r} = \frac{10(8\mu-2\mu^2-\sigma^2)}{46\mu-11\mu^2-7\sigma^2} > 0$ for all values of μ and σ satisfying (1), which is valid throughout our model.

(d) $h_{m1} = \frac{h_r\left(256\mu-64\mu^2-36\sigma^2\right)-(1-c_1)(64\mu-26\sigma^2)}{272+184\mu-64\mu^2-49\sigma^2}$ and $h_{m2} = \frac{h_r\left(160\mu-40\mu^2-20\sigma^2\right)-(1-c_1)(40\mu-5\sigma^2)}{92\mu-22\mu^2-14\sigma^2}$. Substituting $h_r = \widehat{h}_r$, we get $h_{m1} = h_{m2} = \frac{5\sigma^2(1-c_1)}{80+12\mu-8\mu^2-7\sigma^2} < \widehat{h}_m = \frac{\sigma^2(1-c_1)}{16-\mu^2-\sigma^2}$ for all values of μ, σ, and c_1 satisfying (1). $\square$

Proof of Proposition 1. $\widehat{h}_r|_{\mu>0} = \frac{(1-c_1)(80+32\mu+7\sigma^2)}{320+48\mu-32\mu^2-28\sigma^2} = \left(\frac{1-c_1}{4}\right)\left(\frac{80+32\mu+7\sigma^2}{80+12\mu-8\mu^2-7\sigma^2}\right) > \widehat{h}_r|_{\mu=0} = \frac{1-c_1}{4}$ holds for all values of μ, σ, and c_1 satisfying (1). $\square$

Proof of Proposition 2. From Table 2, we obtain w_1 for the four regions.

(a) (i) *Region RS:* $w_1 = \frac{144+88\mu-64\mu^2-42\sigma^2+c_1(128+96\mu-7\sigma^2)-4h_r\Theta_1}{272+184\mu-64\mu^2-49\sigma^2}$ where $\Theta_1 \equiv 8 + 28\mu - 8\mu^2 - 7\sigma^2$. Taking the first derivative with respect to μ, we obtain $\frac{\partial w_1}{\partial \mu} = \frac{-(8(1-c_1)(320+2048\mu+768\mu^2-427\sigma^2-112\mu\sigma^2)-h_r(3072-1664\mu+160\mu^2-42\sigma^2-56\mu\sigma^2))}{(272+184\mu-64\mu^2-49\sigma^2)^2} < 0$ holds for all the values of h_r and h_m satisfying the conditions for Region RS and values of μ, σ, and c_1 satisfying (1). *Region RMS:* $w_1 = \frac{144\mu-18\sigma^2-72\mu^2+c_1(128\mu-16\sigma^2)-4h_r\Theta_7-h_m\Theta_8}{2\left(136\mu-36\mu^2-17\sigma^2\right)}$ where $\Theta_7 = 8\mu - 4\mu^2 - \sigma^2$ and $\Theta_8 = 104\mu - 20\mu^2 - 21\sigma^2$. Taking the first derivative with respect to μ, we obtain $\frac{\partial w_1}{\partial \mu} = \frac{-((1-c_1)(72\mu^2-18\mu\sigma^2)-16\mu^2(h_r-h_m)+17h_m\sigma^2+\mu\sigma^2(4h_r-13h_m))}{\left(136\mu-36\mu^2-17\sigma^2\right)^2} < 0$ holds for all values of h_r and h_m satisfying the conditions for Region RMS and values of μ, σ, and c_1 satisfying (1). *Region MS:* $w_1 = \frac{(1+c_1)(8\mu-\sigma^2)-h_m(4\mu-\mu^2-\sigma^2)-4\mu^2}{2\left(8\mu-2\mu^2-\sigma^2\right)}$. Taking the first derivative with respect to μ, we obtain $\frac{\partial w_1}{\partial \mu} = \frac{-\left(2\mu(1-c_1)(4\mu-\sigma^2)+h_m\sigma^2(2-\mu)\right)}{\left(8\mu-2\mu^2-\sigma^2\right)^2} < 0$ holds for all values of h_r and h_m satisfying the conditions for Region MS and values of μ, σ, and c_1 satisfying (1). *Region NS:* $w_1 = \frac{2c_1(\mu+4)-\mu^2-2\mu-\sigma^2+8}{16-\mu^2-\sigma^2}$. Taking the first derivative with respect to μ, we obtain $\frac{\partial w_1}{\partial \mu} = \frac{-2(1-c_1)(16+8\mu+\mu^2-\sigma^2)}{(16-\mu^2-\sigma^2)^2} < 0$ holds for all values of h_r and h_m satisfying the conditions for Region NS and values of μ, σ, and c_1 satisfying (1).

(ii) For Region RS, $\frac{\partial w_2}{\partial \mu} = \frac{4\left(c_1\left(256\mu^2+16\mu(7\sigma^2+80)-357\sigma^2-752\right)+h_r\left(-352\mu^2-56\mu\sigma^2+2944\mu+350\sigma^2-5728\right)\right)}{\left(-64\mu^2+184\mu-49\sigma^2+272\right)^2}$ $+\frac{4(752+357\sigma^2-256\mu^2-112\mu\sigma^2-1280\mu)}{\left(-64\mu^2+184\mu-49\sigma^2+272\right)^2}$. For values of h_r and h_m in Region RS and for μ, σ, and c_1 satisfying (1), we obtain $\frac{\partial w_2}{\partial \mu} < 0$ only when $\sigma^2 < \hat{\sigma^2} = \frac{32h_r(179-92\mu+11\mu^2)-16(1-c_1)(47-80\mu+16\mu^2)}{7(51+h_r(50-8\mu)-16\mu-c_1(51-16\mu)))}$ and $\frac{\partial w_2}{\partial \mu} > 0$, otherwise. For Region RMS, $\frac{\partial w_2}{\partial \mu} = \frac{10\left(8\mu^2(9c_1+2h_r-2h_m-9)+\mu\sigma^2(-18c_1-4h_r+13h_m+18)-17h_m\sigma^2\right)}{\left(36\mu^2-136\mu+17\sigma^2\right)^2}$. For values of h_r and h_m in Region RMS and for μ, σ, and c_1 satisfying (1), we obtain $\frac{\partial w_2}{\partial \mu} < 0$ only when $\sigma^2 < \hat{\sigma^2}$ and $\frac{\partial w_2}{\partial \mu} > 0$, otherwise. For Region MS and Region NS, since $w_2 = \frac{1+c_2}{2}$, $\frac{\partial w_2}{\partial \sigma^2} = 0$.

(iii) From Theorem 1, $I_r = 0$ in Region MS and Region NS; $I_r > 0$ in Region RS and Region RMS. *Region RS:* $I_r = \frac{(1-c_1)\Theta_2-4h_r\Theta_3}{2\left(272+184\mu-64\mu^2-49\sigma^2\right)}$ where $\Theta_2 \equiv 32\mu + 7\sigma^2 + 80$ and $\Theta_3 \equiv 80 + 12\mu - 8\mu^2 - 7\sigma^2$. Taking the derivative with respect to μ, we obtain $\frac{\partial I_r}{\partial \mu} = \frac{-4\left((1-c_1)(752-1280\mu-256\mu^2+357\sigma^2-112\mu\sigma^2)-h_r(5728+352\mu^2-350\sigma^2-8\mu(368-7\sigma^2)\right)}{(272+184\mu-64\mu^2-49\sigma^2)^2}$. Note that $\frac{\partial I_r}{\partial \mu}$ is linear in h_r. Solving $\frac{\partial I_r}{\partial \mu} = 0$, we obtain $h_r = \frac{(1-c_1)(752-1280\mu-256\mu^2+357\sigma^2-112\mu\sigma^2)}{5728-2944\mu+352\mu^2-350\sigma^2+56\mu\sigma^2}$. Since $\frac{(1-c_1)(752-1280\mu-256\mu^2+357\sigma^2-112\mu\sigma^2)}{5728-2944\mu+352\mu^2-350\sigma^2+56\mu\sigma^2} < \widehat{h}_r$, it lies strictly within Region RS. Due to the above stated linearity, $\frac{\partial I_r}{\partial \mu} \leq 0$ holds when $h_r \leq \frac{(1-c_1)(752-1280\mu-256\mu^2+357\sigma^2-112\mu\sigma^2)}{5728-2944\mu+352\mu^2-350\sigma^2+56\mu\sigma^2}$. Rearranging this inequality, we obtain $\sigma^2 \geq \hat{\sigma^2} = \frac{32h_r(179-92\mu+11\mu^2)-16(1-c_1)(47-80\mu+16\mu^2)}{7(51+h_r(50-8\mu)-16\mu-c_1(51-16\mu)))}$. $\frac{\partial I_r}{\partial \mu} > 0$ holds when $h_r > \frac{(1-c_1)(752-1280\mu-256\mu^2+357\sigma^2-112\mu\sigma^2)}{5728-2944\mu+352\mu^2-350\sigma^2+56\mu\sigma^2}$. Rearranging $h_r > \frac{(1-c_1)(752-1280\mu-256\mu^2+357\sigma^2-112\mu\sigma^2)}{5728-2944\mu+352\mu^2-350\sigma^2+56\mu\sigma^2}$, we obtain $\sigma^2 < \hat{\sigma^2}$. *Region RMS:* $I_r = \frac{(1-c_1)(40\mu-5\sigma^2)-20h_r(8\mu-2\mu^2-\sigma^2)+h_m(23-22\mu^2-14\sigma^2)}{2(136\mu-36\mu^2-17\sigma^2)}$. Taking the first derivative with respect to μ, we obtain $\frac{\partial I_r}{\partial \mu} = \frac{10\left((4\mu-\sigma^2)(18\mu(1-c_1)-4h_r\mu)+h_m(16\mu^2+17\sigma^2-13\mu\sigma^2)\right)}{(136\mu-36\mu^2-17\sigma^2)^2} > 0$ holds for all values of h_m and h_r satisfying the conditions for Region RMS and for all values of μ, σ, and c_1 satisfying (1).

(b) (i) *Region RS:* $w_1 = \frac{144+88\mu-64\mu^2-42\sigma^2+c_1(128+96\mu-7\sigma^2)-4h_r\Theta_1}{272+184\mu-64\mu^2-49\sigma^2}$ where $\Theta_1 \equiv 8 + 28\mu - 8\mu^2 - 7\sigma^2$. Taking the first derivative with respect to σ^2, we obtain $\frac{\partial w_1}{\partial \sigma^2} =$

$\frac{56(\mu+6)(c_1(8\mu+13)+h_r(18-4\mu)-8\mu-13)}{\left(-64\mu^2+184\mu-49\sigma^2+272\right)^2}<0$ holds for all the values of h_r and h_m satisfying the conditions for Region RS and values of μ, σ, and c_1 satisfying (1). *Region RMS:* $w_1=\frac{144\mu-18\sigma^2-72\mu^2+c_1(128\mu-16\sigma^2)-4h_r\Theta_7-h_m\Theta_8}{2\left(136\mu-36\mu^2-17\sigma^2\right)}$ where $\Theta_7=8\mu-4\mu^2-\sigma^2$ and $\Theta_8=104\mu-20\mu^2-21\sigma^2$. Taking the first derivative with respect to σ^2, we obtain $\frac{\partial w_1}{\partial\sigma^2}=\frac{16\mu(2\mu(9c_1+2h_r-9)+h_m(34-13\mu))}{\left(36\mu^2-136\mu+17\sigma^2\right)^2}<0$ holds for all values of h_r and h_m satisfying the conditions for Region RMS and values of μ, σ, and c_1 satisfying (1). *Region MS:* $w_1=\frac{(1+c_1)(8\mu-\sigma^2)-h_m(4\mu-\mu^2-\sigma^2)-4\mu^2}{2\left(8\mu-2\mu^2-\sigma^2\right)}$. Taking the first derivative with respect to σ^2, we obtain $\frac{\partial w_1}{\partial\sigma^2}=\frac{\mu(-2(1-c_1)\mu+h_m(4-\mu))}{2\left(2\mu^2-8\mu+\sigma^2\right)^2}<0$ holds for all values of h_r and h_m satisfying the conditions for Region MS and values of μ, σ, and c_1 satisfying (1). *Region NS:* $w_1=\frac{2c_1(\mu+4)-\mu^2-2\mu-\sigma^2+8}{16-\mu^2-\sigma^2}$. Taking the first derivative with respect to σ^2, we obtain $\frac{\partial w_1}{\partial\sigma^2}=\frac{-2(1-c_1)(\mu+4)}{\left(\mu^2+\sigma^2-16\right)^2}<0$ holds for all values of h_r and h_m satisfying the conditions for Region NS and values of μ, σ, and c_1 satisfying (1).

(ii) For Region RS and Region RMS, $\frac{\partial w_2}{\partial\sigma^2}$ is obtained as $\frac{28(8-\mu)(c_1(8\mu+13)-2h_r(2\mu-9)-8\mu-13)}{\left(272-64\mu^2+184\mu-49\sigma^2\right)^2}$ and $\frac{5\mu(2\mu(9c_1+2h_r-9)+h_m(34-13\mu))}{\left(36\mu^2-136\mu+17\sigma^2\right)^2}$, respectively. For both the regions, taking the corresponding range of values of h_r and h_m, $\frac{\partial w_2}{\partial\sigma^2}<0$ always hold for all values of μ, σ, and c_1 satisfying (1). For Region MS and Region NS, since $w_2=\frac{1+c_2}{2}$, $\frac{\partial w_2}{\partial\sigma^2}=0$.

(iii) From Theorem 1, $I_r=0$ in Region MS and Region NS; $I_r>0$ in Region RS and Region RMS. *Region RS:* $I_r=\frac{(1-c_1)\Theta_2-4h_r\Theta_3}{2\left(272+184\mu-64\mu^2-49\sigma^2\right)}$ where $\Theta_2\equiv 32\mu+7\sigma^2+80$ and $\Theta_3\equiv 80+12\mu-8\mu^2-7\sigma^2$. Taking the first derivative with respect to σ^2, we obtain $\frac{\partial I_r}{\partial\sigma^2}=\frac{28(8-\mu)((1-c_1)(8\mu+13)-h_r(18-4\mu))}{\left(-64\mu^2+184\mu-49\sigma^2+272\right)^2}>0$ for all values of h_r in Region RS and all values of μ, σ, and c_1 satisfying (1). *Region RMS:* $I_r=\frac{(1-c_1)(40\mu-5\sigma^2)-20h_r(8\mu-2\mu^2-\sigma^2)+h_m(23-22\mu^2-14\sigma^2)}{2(136\mu-36\mu^2-17\sigma^2)}$. Taking the first derivative with respect to σ, we obtain $\frac{\partial I_r}{\partial\sigma^2}=\frac{5\mu(2\mu(9(1-c_1)-2h_r)-h_m(34-13\mu))}{\left(36\mu^2-136\mu+17\sigma^2\right)^2}>0$ holds for all values of h_m and h_r satisfying the conditions for Region RMS and for all values of μ, σ, and c_1 satisfying (1). □

Proof of Lemma 5. From Theorem 1, $I_m=0$ in Regions RS and NS; $I_m>0$ only in Regions RMS and MS. Hence, in the rest of the proof, we focus on Region RMS and MS only. The proof is structured as follows. First, we prove that Region RMS and Region MS cease to exist when $\sigma=0$, which implies $I_m=0$ when $\sigma=0$. Next, we prove that when $\sigma>0$, $\frac{\partial I_m}{\partial\sigma^2}>0$ in Regions

RMS and MS. From Lemma 2, we obtain $\widehat{h}_m = \frac{\sigma^2(1-c_1)}{16-\mu^2-\sigma^2}$. From this expression, we obtain $\widehat{h}_m = 0$ when $\sigma = 0$. Region RMS: The holding cost range for satisfying the conditions is $h_{m2} \le h_m < h_{m1}$. In Region RMS, we find that $\widehat{h}_m > Max\{h_{m2}, h_{m1}\}$ always holds for all values of μ, σ, and c_1 satisfying (1). Hence, when $\sigma = 0$, Region RMS ceases to exist as the required conditions $h_{m2} \le h_m < h_{m1}$ are violated for positive values of h_m. Region MS: From Table 1, the condition for this region to exist is $h_m < h_{m2}$ when $h_r < \widehat{h}_r$ and $h_m < \widehat{h}_m$ when $h_r \ge \widehat{h}_r$. Since $h_{m2} < \widehat{h}_m$ holds for all $h_r < \widehat{h}_r$ and $\widehat{h}_m = 0$ when $\sigma = 0$, the conditions for Region MS is violated when $\sigma = 0$. Hence, we prove that Region RMS and Region MS cease to exist when $\sigma = 0$. In Region RMS, $I_m = \frac{4h_r(64\mu-9\sigma^2-16\mu^2)-(1-c_1)(64\mu-26\sigma^2)-h_m\Theta_8}{4(136\mu-36\mu^2-17\sigma^2)} > 0$ where $\Theta_8 \equiv 272 + 184\mu - 64\mu^2 - 49\sigma^2$. Taking the first derivative with respect to σ^2, we obtain $\frac{\partial I_m}{\partial \sigma^2} = \frac{2\mu(9-9c_1-2h_r)(34-13\mu)-h_m(34-13\mu)^2}{\left(36\mu^2-136\mu+17\sigma^2\right)^2} > 0$ for all values of μ, σ, and c_1 satisfying (1). In Region MS, we obtain, $\frac{\partial I_m}{\partial \sigma^2} = \frac{(4-\mu)(2(1-c_1)\mu-h_m(4-\mu))}{4\left(2\mu^2-8\mu+\sigma^2\right)^2}$, which is positive for all values of μ, σ, and c_1 satisfying (1). □

Proof of Lemma 6. The first derivatives of the manufacturer's profit with respect to the learning rate and variance are as follows.

(i) *Region RS:*

$$\frac{\partial \Pi_m}{\partial \mu} = \frac{64\left((13+8\mu)(1-c_1) - h_r(18-4\mu)\right)\left((1-c_1)(5+8\mu-2\sigma^2) + h_r(14-4\mu-\sigma^2)\right)}{(272+184\mu-64\mu^2-49\sigma^2)^2} > 0$$
$$\frac{\partial \Pi_m}{\partial \sigma^2} = \frac{8((13+8\mu)(1-c_1) - 18h_r + 4h_r\mu)^2}{(272+184\mu-64\mu^2-49\sigma^2)^2} > 0$$

hold for all values of h_m and h_r satisfying the conditions for Region RS and for all values of μ, σ, and c_1 satisfying (1).

(ii) *Region RMS:*

$$\frac{\partial \Pi_m}{\partial \mu} = \frac{(2\mu(9-9c_1-2h_r) - h_m(34-13\mu))(2(9-9c_1-2h_r)(4\mu-\sigma^2) + h_m(136-20\mu-13\sigma^2))}{(136\mu-36\mu^2-17\sigma^2)^2} > 0$$
$$\frac{\partial \Pi_m}{\partial \sigma^2} = \frac{(h_m(34-13\mu) - 2\mu(9-9c_1-2h_r))^2}{2(136\mu-36\mu^2-17\sigma^2)^2} > 0$$

hold for all values of h_m and h_r satisfying the conditions for Region RMS and for all values of μ, σ, and c_1 satisfying (1).

(iii) *Region MS:*

$$\frac{\partial \Pi_m}{\partial \mu} = \frac{(2\mu(1-c_1) - h_m(4-\mu))(2(1-c_1)(4\mu-\sigma^2) + h_m(16-4\mu-\sigma^2))}{4(8\mu-2\mu^2-\sigma^2)^2} > 0$$
$$\frac{\partial \Pi_m}{\partial \sigma^2} = \frac{(2\mu(1-c_1) - 4h_m(4-\mu))^2}{8(8\mu-2\mu^2-\sigma^2)^2} > 0$$

hold for all values of h_m and h_r satisfying the conditions for Region MS and for all values of μ, σ, and c_1 satisfying (1).

(iv) *Region NS:*

$$\frac{\partial \Pi_m}{\partial \mu} = \frac{(1-c_1)^2(4+\mu)(16+4\mu-\sigma^2)}{4(16-\mu^2-\sigma^2)^2} > 0$$
$$\frac{\partial \Pi_m}{\partial \sigma^2} = \frac{(1-c_1)^2(4+\mu)^2}{8(16-\mu^2-\sigma^2)^2} > 0$$

hold for all values of h_m and h_r satisfying the conditions for Region NS and for all values of μ, σ, and c_1 satisfying (1).

The first derivatives of the retailer profit with respect to the learning rate and variance are as follows.

For Region RS, $\frac{\partial \Pi_r}{\partial \mu} = \frac{4c_1(h_r(4096\mu^3(2\sigma^2-3)-4416\mu^2(15\sigma^2+16)+8\mu(588\sigma^2-16745\sigma^2-58480)+23(49\sigma^4+11808\sigma^2+25856)))}{(64\mu^2-184\mu+49\sigma^2-272)^3}+$
$\frac{-8c_1^2(4096\mu^3(\sigma^2+30)-768\mu^2(37\sigma^2-528)+16\mu(147\sigma^2-5546\sigma^2+28192)-13(49\sigma^4+6712\sigma^2-9088))}{(64\mu^2-184\mu+49\sigma^2-272)^3}+$
$\frac{-8h_r^2(128\mu^3(8\sigma^2+79)-48\mu^2(197\sigma^2+2480)+4\mu(147\sigma^4+7978\sigma^2+120928)+9(49\sigma^4-2304\sigma^2-70912))}{(64\mu^2-184\mu+49\sigma^2-272)^3}+$
$\frac{-4h_r(4096\mu^3(2\sigma^2-3)-4416\mu^2(15\sigma^2+16)+8\mu(588\sigma^4-16745\sigma^2-58480)+23(49\sigma^4+11808\sigma^2+25856))}{(64\mu^2-184\mu+49\sigma^2-272)^3}+$
$\frac{-32768\mu^3(\sigma^2+30)-1536\mu^2(37\sigma^2-528)+32\mu(147\sigma^4-5546\sigma^2+28192)-26(49\sigma^4+6712\sigma^2-9088)}{(64\mu^2-184\mu+49\sigma^2-272)^3}+$
$\frac{+16c_1(4096\mu^3(\sigma^2+30)-768\mu^2(37\sigma^2-528)+16\mu(147\sigma^4-5546\sigma^2+28192)-13(49\sigma^4+6712\sigma^2-9088))}{(64\mu^2-184\mu+49\sigma^2-272)^3}$ and $\frac{\partial \Pi_r}{\partial \sigma^2}$ is obtained as
$\frac{4(-c_1(8\mu+13)+2h_r(2\mu-9)+8\mu+13)(c_1(-512\mu^3+4672\mu^2-294\mu\sigma^2+13752\mu+637\sigma^2+12272)+h_r(256\mu^3-1160\mu^2+147\mu\sigma^2-2368\mu+882\sigma^2+16992))}{(64\mu^2-184\mu+49\sigma^2-272)^3}+$
$\frac{(4(-c_1(8\mu+13)+2h_r(2\mu-9)+8\mu+13))(512\mu^3-4672\mu^2+294\mu\sigma^2-13752\mu-637\sigma^2-12272)}{(64\mu^2-184\mu+49\sigma^2-272)^3}.$

For Region RMS, $\frac{\partial \Pi_r}{\partial \mu} = \frac{-4\mu^3(36c_1^2(81\sigma^2+2480)-4c_1(-2h_r(279\sigma^2+6728)+1287h_m\sigma^2+13456h_m+1458\sigma^2+44640)+32h_r^2(11\sigma^2+236))}{2(36\mu^2-136\mu+17\sigma^2)^3}+$

$\frac{-4\mu^3(-4h_r(455h_m\sigma^2+3776h_m+558\sigma^2+13456)+2197h_m^2\sigma^2+7552h_m^2+5148h_m\sigma^2+53824h_m+2916\sigma^2+89280)}{2(36\mu^2-136\mu+17\sigma^2)^3}+$

$\frac{+24\mu^2\sigma^2(5580c_1^2+4c_1(841h_r-2227h_m-2790)+472h_r^2-2h_r(1309h_m+1682)+3523h_m^2+8908h_m+5580)}{2(36\mu^2-136\mu+17\sigma^2)^3}+$

$\frac{-\mu\sigma^2(4\sigma^2(1413c_1^2+2c_1(535h_r-1413)+168h_r^2-1070h_r+1413)-4h_m(c_1(2483\sigma^2+41888)+871h_r\sigma^2+12648h_r-2483\sigma^2-41888)+5h_m^2(845\sigma^2+35088))}{2(36\mu^2-136\mu+17\sigma^2)^3}+$

$\frac{+1664\mu^4(h_r-h_m)(9c_1+2h_r-2h_m-9)+68h_m\sigma^2(\sigma^2(-2c_1-25h_r+2)+h_m(26\sigma^2+2312))}{2(36\mu^2-136\mu+17\sigma^2)^3}$ and $\frac{\partial \Pi_r}{\partial \sigma^2}$ is obtained as
$\frac{(h_m(13\mu-34)-2\mu(9c_1+2h_r-9))(h_m(676\mu^3-4880\mu^2+\mu(325\sigma^2+4624)+578\sigma^2)-2\mu(4\mu^2(81c_1+44h_r-81)-8\mu(463c_1+152h_r-463)+\sigma^2(157c_1+84h_r-157)))}{4(36\mu^2-136\mu+17\sigma^2)^3}.$

For Region MS, $\frac{\partial \Pi_r}{\partial \mu} = \frac{-2\mu^3(4c_1^2(\sigma^2+32)-4c_1((h_m+2)\sigma^2+64)+(h_m+2)^2\sigma^2+128)}{8(2\mu^2-8\mu+\sigma^2)^3}+$
$\frac{-\mu\sigma^2(-4(c_1-1)h_m(\sigma^2+32)+4(c_1-1)^2\sigma^2+h_m^2(\sigma^2+96))+24\mu^2\sigma^2(-2c_1+h_m+2)^2+128h_m^2\sigma^2}{8(2\mu^2-8\mu+\sigma^2)^3}$ and $\frac{\partial \Pi_r}{\partial \sigma^2}$ is obtained as
$\frac{(h_m(\mu-4)-2(c_1-1)\mu)(h_m(2\mu^3-16\mu^2+\mu(\sigma^2+32)+4\sigma^2)-2(c_1-1)\mu(2\mu^2-24\mu+\sigma^2))}{16(2\mu^2-8\mu+\sigma^2)^3}.$

For Region NS, $\frac{\partial \Pi_r}{\partial \mu} = \frac{(1-c_1)^2(\mu^3(\sigma^2+32)+384\mu^2+\mu(\sigma^4-48\sigma^2+1536)+128(16-\sigma^2))}{8(16-\mu^2-\sigma^2)^3}$ and $\frac{\partial \Pi_r}{\partial \sigma^2}$ is obtained as
$\frac{(1-c_1)^2(\mu+4)(192+4\sigma^2-\mu^3+4\mu^2+80\mu-\mu\sigma^2)}{16(16-\mu^2-\sigma^2)^3}.$

From the above expressions, we obtain that $\frac{\partial \Pi_r}{\partial \mu} > 0$ and $\frac{\partial \Pi_r}{\partial \sigma^2} > 0$ hold for all values of μ, σ, and c_1 satisfying (1) and all ranges of h_r and h_m applicable to the four regions. $\square$

EC.2. Continuous Learning

The manufacturer's and the retailer's equilibrium decisions under continuous learning are provided in Table EC.1.

Table EC.1 Equilibrium solutions: Continuous Learning

	$h_r < \hat{h_r}$	$h_r \geq \hat{h_r}$
Period 1		
w_1	$\frac{h_r(4-\mu)^2\psi_1+\psi_2+2c_1\psi_3}{2(4-\mu)^2\psi_4}$	$\frac{(1+c_1)(256-224\mu+64\mu^2-4\mu^3+16\sigma^2-4\mu\sigma^2-2\mu^2\sigma^2)-\psi_{17}}{(4-\mu)^2\left(2\mu^2-\mu(24-\sigma^2)-2\sigma^2+32\right)}$
Q_1	$\frac{h_r(4-\mu)^2\psi_5+\psi_6-2c_1\psi_7}{2\psi_4}$	$\frac{(16+\sigma^2-2\mu)(1-c1)-\mu\sigma^2}{4\mu^2-2\mu(24-\sigma^2)-4\sigma^2+64}$
I_r	$\frac{\psi_8-2c_1\psi_9-h_r(4-\mu)^2\psi_{10}}{4\psi_4}$	0
I_m	0	0
Period 2		
w_2	$\frac{h_r(4-\mu)^2\psi_{11}-\psi_{12}+2c_1\psi_{13}}{2(4-\lambda)\psi_4}$	$\frac{(1+c_1)(16\lambda+4\mu^2-2\lambda\mu+2\mu\sigma^2-48\mu+\lambda\sigma^2-4\sigma^2+64)-64\lambda+28\mu\lambda-2\mu^2\lambda}{(2\mu^2-\mu(24-\sigma^2)-2\sigma^2+32)(4-\lambda)}$
Q_2	$\frac{h_r(4-\mu)^2\psi_{14}-2c_1\psi_{15}+\psi_{16}}{2(4-\lambda)\psi_4}$	$\frac{(1+c_1)(16\lambda+4\mu^2-2\lambda\mu+2\mu\sigma^2-48\mu+\lambda\sigma^2-4\sigma^2+64)-\mu\sigma^2\lambda}{2(2\mu^2-\mu(24-\sigma^2)-2\sigma^2+32)(4-\lambda)}$

$$\hat{h_r} = \frac{(1-c_1)(2560-3200\mu+1632\mu^2-408\mu^3+48\mu^4-2\mu^5+576\sigma^2-376\mu\sigma^2+76\mu^2\sigma^2-4\mu^3\sigma^2+28\sigma^4-2\mu\sigma^4)}{2\mu^6-62\mu^5+\mu^4\sigma^2+716\mu^4-32\mu^3\sigma^2-4064\mu^3-\mu^2\sigma^4+240\mu^2\sigma^2+12096\mu^2+8\mu\sigma^4-640\mu\sigma^2-17920\mu-16\sigma^4+512\sigma^2+10240}$$
$$+\frac{640\sigma^2-800\mu\sigma^2+316\mu^2\sigma^2-46\mu^3\sigma^2+2\mu^4\sigma^2-24\mu\sigma^4+\mu^2\sigma^4+32\sigma^4-\sigma^6}{2\mu^6-62\mu^5+\mu^4\sigma^2+716\mu^4-32\mu^3\sigma^2-4064\mu^3-\mu^2\sigma^4+240\mu^2\sigma^2+12096\mu^2+8\mu\sigma^4-640\mu\sigma^2-17920\mu-16\sigma^4+512\sigma^2+10240}.$$

ψ_i , $i \in \{1, 2, ..17\}$, are functions of μ and σ^2. The explicit expressions are omitted for brevity and are available upon request.

Proof of Theorem 2 The equilibrium results are obtained using backward induction. The retailer's second period profit function is given by $\Pi_{2r} = p_2(Q_2 + I_r) - w_2Q_2$ where $p_2 = 1 - Q_2 - I_r$. Since Π_{2r} is concave in Q_2 $\left(\frac{\partial^2\Pi_{2r}}{\partial Q_2^2} = -2 < 0\right)$, from first order conditions, we obtain $Q_2(w_2) = \frac{1-2I_r-w_2}{2}$. The manufacturer's second period profit function is given by $\Pi_{2m} = w_2Q_2 - \overline{c_2}(Q_2 - I_m)$. Substituting $Q_2 = \frac{1-2I_r-w_2}{2}$ and $\overline{c_2} = \frac{c_2+c_3}{2}$ where $c_3 = c_2 - \lambda(Q_2 - I_m)$ in Π_{2m}, we obtain $\Pi_{2m} = \frac{1}{8}((2I_m + 2I_r + w_2 - 1)(4c_2 + \lambda(2I_m + 2I_r + w_2 - 1)) - 4w_2(2I_r + w_2 - 1))$, which is concave in w_2 $\left(\frac{\partial^2\Pi_{2m}}{\partial w_2^2} = \frac{-(4-\lambda)}{4} < 0\right)$. Hence, from the first order condition, we obtain $w_2 = \frac{2c_2+2I_m\lambda-2I_r(2-\lambda)-\lambda+2}{4-\lambda}$. Substituting Q_2 and w_2, we obtain $\Pi_{2r}(c_2) = \frac{c_2^2+2c_2(I_m\lambda+2I_r-1)+I_r\left(4(I_m-2)\lambda+\lambda^2+12\right)+(I_m\lambda-1)^2-I_r^2\left(\lambda^2-8\lambda+12\right)}{(\lambda-4)^2}$. The retailer sets the first period decisions q_1 and I_r, maximizing his total expected profit given by $\mathbb{E}[\Pi_r] = p_1q_1 - w_1(q_1 + I_r) - h_rI_r + \mathbb{E}[\Pi_{2r}]$, where the expected second period profit is obtained as $\mathbb{E}[\Pi_{2r}] =$

$\frac{c_1^2-2c_1(I_r(\mu-2)+\mu q_1+1)+4I_r^2(\mu-3)+I_r\left(2q_1\left(\mu^2+\sigma^2\right)+\mu^2-4\mu q_1-6\mu+\sigma^2+12\right)+(\mu q_1+1)^2}{(4-\mu)^2}$. Substituting $\mathbb{E}[\Pi_{2r}]$, the total expected profit for the retailer is obtained as $\mathbb{E}[\Pi_r] = (1-q_1)q_1 - w_1(I_r+q_1) - h_r I_r + \frac{c_1^2-2c_1(I_r(\mu-2)+\mu q_1+1)+4I_r^2(\mu-3)+I_r\left(2q_1\left(\mu^2+\sigma^2\right)+\mu^2-4\mu q_1-6\mu+\sigma^2+12\right)+(\mu q_1+1)^2}{(\mu-4)^2}$. $\mathbb{E}[\Pi_r]$ is concave in q_1 and I_r since the determinant of the Hessian matrix is positive and the second derivatives of $\mathbb{E}[\Pi_r]$ with respect to q_1 and I_r are strictly negative making the Hessian negative definite. The retailer maximizes $\mathbb{E}[\Pi_r]$ subject to $I_r \geq 0$. Solving the Lagrangian, we obtain $I_r > 0$ when $w_1 < \frac{4c_1-2c_1\mu-h_r\mu^2+8h_r\mu-16h_r+\mu^2-6\mu+\sigma^2+12}{\mu^2-8\mu+16}$ and $I_r = 0$ otherwise. We elaborate these two cases below.

Case 1: $I_r > 0$ when $w_1 < \frac{4c_1-2c_1\mu-h_r\mu^2+8h_r\mu-16h_r+\mu^2-6\mu+\sigma^2+12}{\mu^2-8\mu+16}$. The optimal values of q_1 and I_r are obtained from the first order conditions as $q_1 = \frac{2\mu^3(c_1-5h_r-7w_1+6)+\mu^2(-16c_1+h_r(\sigma^2+32)+\sigma^2w_1-2\sigma^2+76w_1-60)+2\mu(c_1(\sigma^2+16)-4h_r(\sigma^2+4)-4\sigma^2w_1+4\sigma^2-96w_1+80)}{2\left(\mu^4-4\mu^3+2\mu^2(\sigma^2-14)-4\mu(\sigma^2-40)+\sigma^4-192\right)} + \frac{\mu^4(h_r+w_1-1)-4\sigma^2(c_1-4h_r-4w_1+3)-\sigma^4+192(w_1-1)}{2\left(\mu^4-4\mu^3+2\mu^2(\sigma^2-14)-4\mu(\sigma^2-40)+\sigma^4-192\right)}$, and $I_r = \frac{2c_1\left(\mu^3-10\mu^2+\mu(\sigma^2+32)-32\right)-8h_r(\mu-4)^2(\mu-2)}{2\left(\mu^4-4\mu^3+2\mu^2(\sigma^2-14)-4\mu(\sigma^2-40)+\sigma^4-192\right)} + \frac{\mu^4w_1-\mu^4-18\mu^3w_1+16\mu^3+\mu^2\sigma^2w_1-\mu^2\sigma^2+112\mu^2w_1-92\mu^2-8\mu\sigma^2w_1+14\mu\sigma^2-288\mu w_1+224\mu+16\sigma^2w_1-32\sigma^2+256w_1-192}{2\left(\mu^4-4\mu^3+2\mu^2(\sigma^2-14)-4\mu(\sigma^2-40)+\sigma^4-192\right)}$. Substituting the optimal q_1 and I_r in the manufacturer's total expected profit, we obtain the manufacturer's total expected profit to be decreasing in I_m, that is, $\frac{\partial\mathbb{E}[\Pi_m]}{\partial I_m} = -h_m$. Hence, the optimal $I_m = 0$. Since the expected manufacturer profit is concave in w_1, from the first order conditions, we obtain the optimal w_1 provided in Table EC.1. Note that the optimal w_1 satisfies $w_1 < \frac{4c_1-2c_1\mu-h_r\mu^2+8h_r\mu-16h_r+\mu^2-6\mu+\sigma^2+12}{\mu^2-8\mu+16}$ only when $h_r < \hat{h_r}$ (see Table EC.1 notes for $\hat{h_r}$). Otherwise, the equilibrium solutions correspond to those under case 2 mentioned below.

Case 2: $I_r = 0$ when $w_1 \geq \frac{4c_1-2c_1\mu-h_r\mu^2+8h_r\mu-16h_r+\mu^2-6\mu+\sigma^2+12}{\mu^2-8\mu+16}$. Since retailer's total expected profit is concave in q_1, $\left(\frac{\partial^2\mathbb{E}[\Pi_r]}{\partial q_1^2} = -\frac{16(2-\mu)}{(4-\mu)^2} < 0\right)$, we obtain optimal q_1 from the first order condition as $q_1 = \frac{2\mu(4w_1-c_1-3)+(\mu^2+16)(1-w_1)}{16(2-\mu)}$. Substituting this q_1 in the manufacturer's total expected profit, we obtain the manufacturer's total expected profit to be decreasing in I_m, that is, $\frac{\partial\mathbb{E}[\Pi_m]}{\partial I_m} = -h_m$. Hence, optimal $I_m = 0$. Since the expected manufacturer profit is concave in w_1, from the first order conditions, we obtain the optimal $w_1 = \frac{2c_1\left(\mu^2(32-\sigma^2)-2\mu^3-2\mu(\sigma^2+56)+8(\sigma^2+16)\right)+2\mu^4-\mu^3(36-\sigma^2)+16\mu^2(12-\sigma^2)-52\mu(8-\sigma^2)-48\sigma^2+256}{(4-\mu)^2\left(2\mu^2+\mu(\sigma^2-24)-2\sigma^2+32\right)}$. Substituting the optimal w_1 in the best response functions, we obtain the equilibrium decisions provided in Table EC.1. From this table, we obtain the retailer's inventory $I_r > 0$ when $h < \hat{h}$ and $I_r = 0$ when $h \geq \hat{h}$. Also, from the equilibrium solutions provided in Table EC.1, we obtain $I_m = 0$. $\square$

Proof of Lemma 7 From Table EC.1, we obtain the retailer's strategic inventory for continuous learning. Taking the first derivative of I_r with respect to σ^2, we obtain $\frac{\partial I_r}{\partial \sigma^2} = \frac{\Psi_1 + 4h_r(4-\mu)^2\Psi_2 + 4c_1\Psi_3}{4\left(11\mu^5 + 2\mu^4(\sigma^2-100) + \mu^3(1396-6\sigma^2) + 4\mu^2\left(\sigma^4 - 18\sigma^2 - 1164\right) + \mu\left(-17\sigma^4 + 304\sigma^2 + 7360\right) + 2\left(\sigma^6 + 16\sigma^4 - 128\sigma^2 - 2176\right)\right)^2}$, where Ψ_i , $i \in \{1,2,3\}$, are functions of μ and σ^2. The explicit expressions are omitted for brevity and are available upon request. The numerator of $\frac{\partial I_r}{\partial \sigma^2}$ is positive for all values of μ, σ^2, and c_1 satisfying (1) and the denominator is a square function which takes only positive values for any μ and σ in the Real domain. Hence, we obtain $\frac{\partial I_r}{\partial \sigma^2} > 0$. Using a similar algebraic approach, we obtain $\frac{\partial I_r}{\partial \mu} > 0$. □

EC.3. Retail Duopoly

Under a retail duopoly setting, the sequence of decisions is as follows. At the beginning of period 1, the manufacturer announces the wholesale price w_1 and commits to carry I_m units of inventory from period 1 over to period 2. As in Section 3, we assume complete information, wherein inventory held by one supply chain agent is completely visible to the other. After observing the first period wholesale price and the manufacturer's inventory, each retailer j, $j \in \{1,2\}$, orders $Q_{1j} = q_{1j} + I_{rj}$ units, sells q_{1j} units at a retail price $p_1 = 1 - \Sigma_j q_{1j}$ and carries I_{rj} units as inventory over to period 2 incurring a holding cost of h_r per unit. At the beginning of the second period, the reduced unit production cost c_2 (due to cost learning) is observed. Given c_2, the manufacturer sets the wholesale price w_2. After observing w_2, retailer j, $j \in \{1,2\}$, orders Q_{2j} units and sells $Q_{2j} + I_{rj}$ units at a price $p_2 = 1 - \Sigma_j(Q_{2j} + I_{rj})$.

The manufacturer produces $Q_{11} + Q_{12} + I_m$ units in the first period at a production cost c_1. She experiences a stochastic cost learning, as a result of which the second period unit production cost is given by the random variable $C_2 = c_1 - \Lambda(Q_{11} + Q_{12} + I_m)$. In the second period, she produces $Q_{21} + Q_{22} - I_m$ units at a unit production cost c_2. The rest of the framework is similar to the retailer monopoly case discussed in Section 4. Retailer j's second period profit is given by $\Pi_{2rj} = p_2(Q_{2j} + I_{rj}) - w_2 Q_{2j}$, $j \in \{1,2\}$. The manufacturer's second period profit is $\Pi_{2m} = w_2(Q_{21} + Q_{22}) - c_2(Q_{21} + Q_{22} - I_m)$. Retailer j's total expected profit can be expressed as $\mathbb{E}[\Pi_{rj}] = p_1 q_{1j} - w_1(q_{1j} + I_{rj}) - h_r I_{rj} + \mathbb{E}[\Pi^*_{2rj}]$, $j \in \{1,2\}$. The manufacturer's total expected profit is $\mathbb{E}[\Pi_m] = (w_1 - c_1)(q_{11} + q_{12} + I_{r1} + I_{r2}) - (c_1 + h_m)I_m + \mathbb{E}[\Pi^*_{2m}]$.

Proof of Lemma 8 The proof proceeds similarly to that of the proof provided for retailer monopoly (Appendix EC.1). We use superscripts M and D to denote the monopoly and duopoly settings, respectively. Similar to the equilibrium decisions under retailer monopoly setting, we obtain four regions based on the values of h_r and h_m each with a unique Subgame Perfect Nash Equilibrium solution. The regions are defined in Table EC.2.

Table EC.2 Manufacturer's and Retailer's inventory decisions for various regions in the h_r-h_m plane for Retailer Duopoly

Region	Definition	Inventory decisions
RS	$h_r < \widehat{h}_r^D$ & $h_m \geq h_{m1}^D$	$I_m = 0, I_{rj} > 0$
RMS	$h_{m2}^D \leq h_m < h_{m1}^D$	$I_m > 0, I_{rj} > 0$
MS	$MS1 \cup MS2$	$I_m > 0, I_{rj} = 0$
NS	$h_r \geq \widehat{h}_r^D$ & $h_m \geq \widehat{h}_m^D$	$I_m = 0, I_{rj} = 0$

$MS1 \equiv (h_r < \widehat{h}_r^D \;\&\; h_m < h_{m2}^D)$; $MS2 \equiv (h_r \geq \widehat{h}_r^D \;\&\; h_m < \widehat{h}_m^D)$

$\widehat{h}_r^D \equiv \dfrac{(1-c_1)(126+75\mu+100\sigma^2)}{1512+396\mu-300\mu^2-240\sigma^2}$; $h_{m1}^D \equiv \dfrac{h_r\,(1800\mu-600\mu^2-312\sigma^2)-150(1-c_1)(\mu-\sigma^2)}{1116+1500\mu-625\mu^2-400\sigma^2}$;

$h_{m2}^D \equiv \dfrac{h_r\,(1008\mu-336\mu^2-168\sigma^2)-(1-c_1)(84\mu-14\sigma^2)}{678\mu-225\mu^2-120\sigma^2}$; $\widehat{h}_m^D \equiv \dfrac{\sigma^2(1-c_1)}{9-\mu^2-\sigma^2}$;

From Appendix EC.3, for retail duopoly, we obtain the second period profit functions of the two competing retailers and the manufacturer as

$$\Pi_{2rj} = p_2(Q_{2j} + I_{rj}) - w_2 Q_{2j}\,, \quad j \in \{1,2\},$$
$$\Pi_{2m} = w_2(Q_{21} + Q_{22}) - c_2(Q_{21} + Q_{22} - I_m).$$

Since Π_{2rj} is concave in Q_{2j}, from first order conditions, we obtain the optimal second period order quantities as $Q_{2j} = \frac{1-w_2-3I_{rj}}{3}$. Substituting optimal Q_{2j} in Π_{2m}, we obtain Π_{2m} to be concave in w_2. Solving the first order condition, we obtain optimal second period wholesale price as $w_{2j}(I_{rj}) = \frac{1}{2} - \frac{3}{4}I_{rj} - \frac{3}{4}I_{rk}$, $j \in \{1,2\}$, $k = 3-j$ which proves that the second period wholesale price decreases for both retailers not only with their own strategic inventory level, but also with their competitor's strategic inventory level to the same degree. Hence, both retailers benefit (free-ride) from their competitor's strategic inventory holding. □

EC.3.1. Downstream competition with Price Discrimination

Here, we explore the effect of wholesale price discrimination by the manufacturer between the competing retailers based on their respective inventory levels. Let w_{tj} denote the wholesale price offered to retailer j in period t, $t \in \{1,2\}$ and $j \in \{1,2\}$. The rest of the model is the same as in Section 6. The proof of equilibrium solutions in retailer duopoly with wholesale

Table EC.3 Equilibrium decisions under retailer duopoly and uniform wholesale pricing scheme

$j \in \{1,2\}$	*Region RS*	*Region RMS*
w_1	$\frac{1134+1275\mu-1250\mu^2-600\sigma^2+c_1(1098+1725\mu-200\sigma^2)-12h_r\Phi_1}{2232-1250\mu^2+3000\mu-800\sigma^2}$	$\frac{756\mu-126\sigma^2-500\mu^2+c_1(732\mu-122\sigma^2)-12h_r\Phi_5-h_m\Phi_6}{4\left(372\mu-125\mu^2-62\sigma^2\right)}$
Q_{1j}	$\frac{(1-c_1)(625\mu+450)+3h_r(100\mu-312)}{2232-1250\mu^2+3000\mu-800\sigma^2}$	$\frac{(1-c_1)(900\mu-150\sigma^2)-24h_r(78\mu-13\sigma^2-25\mu^2)+h_m\Phi_7}{12\left(372\mu-125\mu^2-62\sigma^2\right)}$
I_m	0	$\frac{24h_r(75\mu-13\sigma^2-25\mu^2)-(1-c_1)(150\mu-150\sigma^2)-h_m\Phi_8}{6\left(372\mu-125\mu^2-62\sigma^2\right)}$
I_{rj}	$\frac{(1-c_1)\Phi_2-12h_r\Phi_3}{3\left(1116-625\mu^2+1500\mu-400\sigma^2\right)}$	$\frac{(1-c_1)(84\mu-14\sigma^2)-168h_r\Phi_9+h_m(678\mu-120\sigma^2-225\mu^2)}{6\left(372\mu-125\mu^2-62\sigma^2\right)}$
w_2	$\frac{c_1\Phi_2+12h_r\Phi_3+5\Phi_4}{2232-1250\mu^2+3000\mu-800\sigma^2}+\frac{c_2}{2}$	$\frac{660\mu-250\mu^2-110\sigma^2+c_1(84\mu-14\sigma^2)+168h_r\Phi_9-h_m\Phi_{10}}{4\left(372\mu-125\mu^2-62\sigma^2\right)}+\frac{c_2}{2}$
Q_{2j}	$\frac{c_1\Phi_2+12h_r\Phi_3+5\Phi_4}{6696-3750\mu^2+9000\mu-2400\sigma^2}-\frac{c_2}{6}$	$\frac{660\mu-250\mu^2-110\sigma^2+c_1(84\mu-14\sigma^2)+168h_r\Phi_9-h_m\Phi_{10}}{12\left(372\mu-125\mu^2-62\sigma^2\right)}-\frac{c_2}{6}$

	Region MS	*Region NS*
w_1	$\frac{(1+c_1)(6\mu-\sigma^2)-h_m(3\mu-\sigma^2-\mu^2)-4\mu^2}{2\left(6\mu-2\mu^2-\sigma^2\right)}$	$\frac{3c_1(\mu+3)-2\mu^2-3\mu-2\sigma^2+9}{2\left(9-\mu^2-\sigma^2\right)}$
Q_{1j}	$\frac{(1-c_1)(6\mu-\sigma^2)+h_m(3\mu-\sigma^2-\mu^2)}{6\left(6\mu-2\mu^2-\sigma^2\right)}$	$\frac{(1-c_1)(\mu+3)}{2\left(9-\mu^2-\sigma^2\right)}$
I_m	$\frac{(1-c_1)\sigma^2-h_m\left(9-\mu^2-\sigma^2\right)}{3\left(6\mu-2\mu^2-\sigma^2\right)}$	0
I_{rj}	0	0
w_2	$\frac{1+c_2}{2}$	$\frac{1+c_2}{2}$
Q_{2j}	$\frac{1-c_2}{6}$	$\frac{1-c_2}{6}$

$\Phi_1 \equiv 18+141\mu-50\mu^2-40\sigma^2$; $\Phi_2 \equiv 75\mu+100\sigma^2+126$; $\Phi_3 \equiv 126+33\mu-25\mu^2-20\sigma^2$; $\Phi_4 \equiv 198-125\mu^2+285\mu-100\sigma^2$; $\Phi_5 \equiv 12\mu-2\sigma^2-6\mu^2$; $\Phi_6 \equiv 594-160\sigma^2-175\mu^2$; $\Phi_7 \equiv 1950\mu-625\mu^2-400\sigma^2$; $\Phi_8 \equiv 1116+1500\mu-625\mu^2-400\sigma^2$; $\Phi_9 \equiv 6\mu-2\mu^2-\sigma^2$; $\Phi_{10} \equiv 678\mu-225\mu^2-120\sigma^2$.

price discrimination proceeds similar to that of the proof provided for retailer monopoly (Appendix EC.1). Similar to the equilibrium decisions under the retailer monopoly setting, we obtain four regions based on the values of h_r and h_m each with a unique subgame perfect Nash equilibrium solution. The regions are defined in Table EC.4 and the equilibrium solutions are provided in Table EC.5. Although the equilibrium wholesale prices remain identical between the two retailers, they are different compared to the case of no possible price discrimination (refer to Table EC.5 and Table EC.3).

EC.3.2. Bertrand Competition

In Section 6, we considered a Cournot competition between symmetric retailers. In this section, we consider a Bertrand competition model, where two retailers compete to sell two differentiated

Table EC.4 Manufacturer's and Retailer's inventory decisions for various regions in the h_r-h_m plane for Retailer Duopoly with wholesale price discrimination

Region	Definition	Inventory decisions
RS	$h_r < \widehat{h}_r^D$ & $h_m \geq h_{m1}^D$	$I_m = 0,\ I_{rj} > 0$
RMS	$h_{m2}^D \leq h_m < h_{m1}^D$	$I_m > 0,\ I_{rj} > 0$
MS	$MS1 \cup MS2$	$I_m > 0,\ I_{rj} = 0$
NS	$h_r \geq \widehat{h}_r^D$ & $h_m \geq \widehat{h}_m^D$	$I_m = 0,\ I_{rj} = 0$

$$MS1 \equiv (h_r < \widehat{h}_r^D \;\&\; h_m < h_{m2}^D);\ MS2 \equiv (h_r \geq \widehat{h}_r^D \;\&\; h_m < \widehat{h}_m^D)$$
$$\widehat{h}_r^D \equiv \frac{(1-c_1)\,(63+36\mu+20\sigma^2)}{378+90\mu-72\mu^2-60\sigma^2}, h_{m1}^D \equiv \frac{h_r\,(432\mu-144\mu^2-78\sigma^2)-(1-c_1)\,(72\mu-44\sigma^2)}{279+336\mu-144\mu^2-100\sigma^2},$$
$$h_{m2}^D \equiv \frac{h_r\,(252\mu-84\mu^2-42\sigma^2)-(1-c_1)\,(42\mu-7\sigma^2)}{159\mu-52\mu^2-30\sigma^2}, \widehat{h}_m^D \equiv \frac{\sigma^2(1-c_1)}{9-\mu^2-\sigma^2}.$$

products. We assume that the manufacturer charges the same wholesale price for both retailers. For ease of exposition, we consider only downstream inventory. Following Desai et al. (2010), we take the demand functions in both the periods to be $q_{i1} = \frac{1}{2}(1-p_{i1}+\theta(p_{i2}-p_{i1}))$ and $q_{i2} = \frac{1}{2}(1-p_{i2}+\theta(p_{i1}-p_{i2}))$, where $0 \leq \theta \leq 1$ represents the degree of substitution between the products. These demand functions are obtained from the quadratic utility function developed by Shubik and Levitan (1980). The sequence of decisions is as follows. At the beginning of period 1, the manufacturer announces the wholesale price w_1. After observing the first period wholesale price, each retailer $j,\ j \in \{1,2\}$, sets price p_{1j} such that q_{1j} units are sold in period 1 and I_j units are carried over to period 2 incurring a holding cost of h_r per unit. At the beginning of the second period, the reduced unit production cost c_2 (due to cost learning) is observed. Given c_2, the manufacturer sets the wholesale price w_2. After observing w_2, retailer $j,\ j \in \{1,2\}$, sets p_{2j} such that $q_{2j}+I_{rj}$ units are sold in period 2.

The manufacturer produces $q_{11}+I_1+q_{12}+I_2$ units in the first period at a production cost c_1. She experiences a stochastic cost learning, as a result of which the second period unit production cost is given by the random variable $C_2 = c_1 - \Lambda(q_{11}+I_1+q_{12}+I_2)$. The rest of the framework is similar to the retailer monopoly case discussed in Section 4. Similar to the proof of equilibrium solutions in Section 6, we solve the game backward to arrive at the equilibrium solutions. Each retailer's second period optimal responses are given by $p_{2j} = \frac{1+w_2+w_2\theta}{2+\theta},\ j \in \{1,2\}$. The manufacturer's optimal response in the second period is obtained as $w_2 = \frac{1+c_2(1+\theta)}{2(1+\theta)} - I_1\frac{(2+\theta)}{2(1+\theta)} - I_2\frac{(2+\theta)}{2(1+\theta)}$. The free-riding effect observed in Section 6, wherein one retailer benefits from the other

Table EC.5 Equilibrium decisions under retailer duopoly and possible wholesale price discrimination

$j \in \{1,2\}$	*Region RS*	*Region RMS*
w_{1j}	$\frac{4(36+39\mu-36\mu^2-20\sigma^2)-5c_1(27+36\mu-4\sigma^2)-3h_r\Phi_1}{279+336\mu-144\mu^2-100\sigma^2}$	$\frac{96\mu-16\sigma^2-64\mu^2+c_1(90\mu-15\sigma^2)-3h_r\Phi_5-h_m\Phi_6}{186\mu-64\mu^2-31\sigma^2}$
Q_{1j}	$\frac{(1-c_1)(72\mu+66)+h_r(36\mu-117)}{279+336\mu-144\mu^2-100\sigma^2}$	$\frac{(1-c_1)(132\mu-22\sigma^2)-3h_r(78\mu-13\sigma^2-24\mu^2)+h_m\Phi_7}{3\left(186\mu-64\mu^2-31\sigma^2\right)}$
I_m	0	$\frac{6h_r(72\mu-13\sigma^2-24\mu^2)-(1-c_1)(72\mu-44\sigma^2)-h_m\Phi_8}{3\left(186\mu-64\mu^2-31\sigma^2\right)}$
I_{rj}	$\frac{(1-c_1)\Phi_2-h_r\Phi_3}{3(279+336\mu-144\mu^2-100\sigma^2)}$	$\frac{(1-c_1)(42\mu-7\sigma^2)-48h_r\Phi_9+h_m(159\mu-30\sigma^2-52\mu^2)}{3\left(186\mu-64\mu^2-31\sigma^2\right)}$
w_{2j}	$\frac{c_1\Phi_2+h_r\Phi_3+6\Phi_4}{2(279+336\mu-144\mu^2-100\sigma^2)}+\frac{c_2}{2}$	$\frac{144\mu-64\mu^2-24\sigma^2+c_1(42\mu-7\sigma^2)+42h_r\Phi_9-h_m\Phi_{10}}{2\left(186\mu-64\mu^2-31\sigma^2\right)}+\frac{c_2}{2}$
Q_{2j}	$\frac{c_1\Phi_2+h_r\Phi_3+6\Phi_4}{6(279+336\mu-144\mu^2-100\sigma^2)}-\frac{c_2}{6}$	$\frac{144\mu-64\mu^2-24\sigma^2+c_1(42\mu-7\sigma^2)+42h_r\Phi_9-h_m\Phi_{10}}{6\left(186\mu-64\mu^2-31\sigma^2\right)}-\frac{c_2}{6}$

	Region MS	*Region NS*
w_{1j}	$\frac{(1+c_1)(6\mu-\sigma^2)-h_m(3\mu-\sigma^2-\mu^2)-4\mu^2}{2\left(6\mu-2\mu^2-\sigma^2\right)}$	$\frac{3c_1(\mu+3)-2\mu^2-3\mu-2\sigma^2+9}{2\left(9-\mu^2-\sigma^2\right)}$
Q_{1j}	$\frac{(1-c_1)(6\mu-\sigma^2)+h_m(3\mu-\sigma^2-\mu^2)}{6\left(6\mu-2\mu^2-\sigma^2\right)}$	$\frac{(1-c_1)(\mu+3)}{2\left(9-\mu^2-\sigma^2\right)}$
I_m	$\frac{(1-c_1)\sigma^2-h_m\left(9-\mu^2-\sigma^2\right)}{3\left(6\mu-2\mu^2-\sigma^2\right)}$	0
I_{rj}	0	0
w_{2j}	$\frac{1+c_2}{2}$	$\frac{1+c_2}{2}$
Q_{2j}	$\frac{1-c_2}{6}$	$\frac{1-c_2}{6}$

$\Phi_1 \equiv 9+66\mu-24\mu^2-20\sigma^2$; $\Phi_2 \equiv 63+36\mu+20\sigma^2$; $\Phi_3 \equiv 378+90\mu-72\mu^2-60\sigma^2$; $\Phi_4 \equiv 36+50\mu-24\mu^2-20\sigma^2$; $\Phi_5 \equiv 6\mu-\sigma^2-4\mu^2$; $\Phi_6 \equiv 75\mu-20\sigma^2-20\mu^2$; $\Phi_7 \equiv 234\mu-72\mu^2-50\sigma^2$; $\Phi_8 \equiv 279+336\mu-144\mu^2-100\sigma^2$; $\Phi_9 \equiv 6\mu-2\mu^2-\sigma^2$; $\Phi_{10} \equiv 159\mu-52\mu^2-30\sigma^2$.

retailer's inventory holding, is prevalent in the Bertrand competition model as well. Note that $w_2(I_1, I_2)$ is decreasing in I_1 and I_2. When $\theta = 0$, that is, when the products are independent, the optimal response for the second period wholesale price is obtained as $w_2 = \frac{1+c_2}{2} - I_1 - I_2$. Whereas, when $\theta = 1$, that is, when the products are perfect substitutes $w_2 = \frac{1+2c_2}{4} - \frac{3}{4}I_1 - \frac{3}{4}I_2$. Hence, the marginal impact of the retailers' strategic inventory on the second period wholesale price, w_2, is more profound when the product substitutability is low.