Online Appendix

Is Full Price the Full Story When Consumers Have Time and Budget Constraints?

A. Proofs

A.1. Proof of Lemma 1

First assume that the agent is a full-price decision-maker. The linearity of the iso-utility curves follows immediately from Definition 1. Next suppose that the iso-utility curves are linear. That implies that any two points between which the agent is indifferent are linearly related, i.e., $r+\eta\delta = \hat{r}+\eta\hat{\delta}$ and the full-price requirement holds.

A.2. Proof of Proposition 1

We start by determining the optimal spending on overhead c_O^j and the optimal work time t_w^j, where $j \in \{NB, B\}$ denotes a non-buyer's and a buyer's decision respectively. Time and spending on leisure are then found from $t_l^{NB} = T - t_w^{NB} - O(c_O^{NB})$, $c_l^{NB} = \theta t_w^{NB} - c_O^{NB}$ for those who do not buy the discrete service and $t_l^B = T - t_w^B - O(c_O^B) - \delta$, $c_l^B = \theta t_w^B - c_O^B - r$, for those who do buy the discrete service. Define $\chi(t_l, c_l) = \frac{\partial U(t_l,c_l)}{\partial t_l} / \frac{\partial U(t_l,c_l)}{\partial c_l}$. We write down Lemma 4 from the first-order conditions of (P_C) to solve for consumers' optimal actions.

Lemma 4. *Under the linear compensation structure defined in (1),*

1. *The optimal actions for a consumer who does purchase the discrete service are found from* $\chi(T - t_w^B - O(c_O^B) - \delta, \theta t_w^B - c_O^B - r) = \theta$, *and* $O'(c_O^B) = -1/\theta$, *and*
2. *The optimal actions for a consumer who does not purchase the discrete service are found from* $\chi(T - t_w^{NB} - O(c_O^{NB}), \theta t_w^{NB} - c_O^{NB}) = \theta$, *and* $O'(c_O^{NB}) = -1/\theta$.

Proof of Lemma 4. To prove part 1, we first rewrite (P_C) of a consumer who does purchase the discrete service as $U(T - t_w - O(c_O) - \delta y, \theta t_w - c_O - ry) + Vy$. The optimal work time, and overhead expenditure when a linearly paid consumers buy satisfy the first-order conditions,

$$\frac{\partial U(t_l,c_l)}{\partial t_l}(-1) + \frac{\partial U(t_l,c_l)}{\partial c_l}\theta = 0, \text{ and } \frac{\partial U(t_l,c_l)}{\partial t_l}(-O'(c_O)) + \frac{\partial U(t_l,c_l)}{\partial c_l}(-1) = 0.$$

Note that the former equality indicates $\chi(t_l, c_l)|_{(t_l^B, c_l^B)} = \theta$. The latter indicates, $\chi(t_l, c_l)|_{(t_l^B, c_l^B)} = -1/O'(c_O^B)$. Therefore, further, $O'(c_O^B) = -1/\theta$. Set $y = 0$, the argument above applies to the consumers who do not buy, which proves part 2. This completes the proof of Lemma 4.

Note that, $\chi(t_l^B, c_l^B)$ is the slope of the iso-utility curve, which can also be interpreted as the marginal rate of substitution of time for money (DeSerpa 1971). Thus, consumers paid under linear compensation structures have iso-utility curves with constant slopes, and that slope is equal to their wages. By Lemma 1, these consumers, i.e., the gig workers, are full-price decision-makers and furthermore, $\eta = \theta$.

A.3. Proof of Proposition 2

First we state the full conditions for part 2 as follows.

2. The indifference price for gig workers is higher than for shift workers, i.e., $r_L(\theta;\delta) \geq r_S(\theta;\delta)$, $\forall\theta$, under certain conditions.
   (i) $r_L(\theta_0';\delta) > r_S(\theta_0';\delta)$;
   (ii) $t^j_{w,L}(\theta)$ increases in θ, for $j \in \{B, NB\}$;
   (iii) $S\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq t^{NB}_{w,L}(\theta)\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$ when $\theta > \max\{\theta_0', \theta_0\}$, $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}} + (1 - \frac{t^B_{w,L}(\theta)}{t^{NB}_{w,L}(\theta)}) \cdot \mathbb{I}\{t^B_{w,L}(\theta) < t^{NB}_{w,L}(\theta)\}$ when $\theta \in (\min\{\theta_0', \theta_0\}, \max\{\theta_0', \theta_0\})$, and $S\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \leq t^{NB}_{w,L}(\theta)\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$ when $\theta < \min\{\theta_0', \theta_0\}$.

Next, we prove the proposition.

Note, by definition of indifference prices, $r_{\tau_c}(\theta;\delta)$ makes consumers indifferent to buying or not buying the service, that is,

$$U(T - t^B_{w,\tau_c} - O(c^B_{O,\tau_c}) - \delta, \theta t^B_{w,\tau_c} - c^B_{O,\tau_c} - r_{\tau_c}(\theta;\delta)) + V = U(T - t^{NB}_{w,\tau_c} - O(c^{NB}_{O,\tau_c}), \theta t^{NB}_{w,\tau_c} - c^{NB}_{O,\tau_c}). \tag{7}$$

Note that, $t^B_{w,S} = t^{NB}_{w,S} = S$. To simplify the notations, we denote the left-hand side of the above equation to be $u^B_{\theta,\tau_c}(r_{\tau_c}(\theta;\delta),\delta) + V$ and the right-hand side to be u^{NB}_{θ,τ_c}. Note that, utility on both sides are at optimality. We simplify the notation $r_{\tau_c}(\theta;\delta)$ as $r_{\tau_c}(\theta)$ hereafter in the proof.

Consider an indifferent gig worker, and an indifferent shift worker. There exists a θ_0 at which the gig worker is willing to work exactly S hours when they do not buy the service; that is, $t^{NB}_{w,L}(\theta_0) = S$. By construction, we have $u^B_{\theta_0,L}(r_L(\theta_0),\delta) + V = u^{NB}_{\theta_0,L} = u^{NB}_{\theta_0,S} = u^B_{\theta_0,S}(r_S(\theta_0),\delta) + V$. Note that the price charged to gig workers and shift workers can be different at the optimal buying utility. We also know that given the price and time commitment menu, $(r_S(\theta_0),\delta)$, the optimal utility of the shift worker, is lower than the optimal utility of the gig worker; that is, $u^B_{\theta_0,L}(r_S(\theta_0),\delta) \geq u^B_{\theta_0,S}(r_S(\theta_0),\delta)$, since gig workers have a larger feasible region of (t_w, c_O). Furthermore, Therefore, if we can show $u^B_{\theta_0,L}(r_L,\delta)$ decreases as r_L increases, then $r_L(\theta_0) \geq r_S(\theta_0)$ directly follows.

Given θ_0, fix δ, suppose (t^*_w, c^*_O) optimizes a consumer's utility at the price r_2. Consider a price $r_2 > r_1$, then $U^B_{\theta_0',L}(t_l(t^*_w,c^*_O), c_l(t^*_w,c^*_O); r_1,\delta) \geq U^B_{\theta_0',L}(t_l(t^*_w,c^*_O), c_l(t^*_w,c^*_O); r_2,\delta) = u^B_{\theta_0',L}(r_2,\delta)$. Note that the optimal solution $(t^{*'}_w, c^{*'}_O)$ at price r_1 will lead to an optimal utility at price r_1, which is at least as high as $U^B_{\theta_0,L}(t_l(t^*_w,c^*_O), c_l(t^*_w,c^*_O); r_1,\delta)$. Therefore, $u^B_{\theta_0,L}(r_1,\delta) \geq u^B_{\theta_0,L}(r_2,\delta)$. We therefore conclude that, $u^B_{\theta_0,L}(r,\delta)$ decreases in r. Therefore, $r_L(\theta_0) \geq r_S(\theta_0)$. We have thus proved the stated inequality at the point θ_0. When $r_L(\theta_0) > r_S(\theta_0)$, $r_L(\theta_0) \geq r_S(\theta_0)$ in the neighbourhood of θ_0 directly follows from the continuity of $r_{\tau_c}(\theta)$. This completes the proof of part 1.

Furthermore, there exists a θ_0' at which the gig worker is willing to work exactly S hours when they buy the service; that is $t^{NB}_{w,L}(\theta_0') = S$. Note we have $r_L(\theta_0';\delta) > r_S(\theta_0';\delta)$ from condition (i).

It is then easy to see that if we can further show that $\frac{\partial r_L(\theta)}{\partial\theta} > \frac{\partial r_L(\theta)}{\partial\theta}$ on $\theta > \min(\theta_0', \theta_0)$, while $\frac{\partial r_L(\theta)}{\partial\theta} < \frac{\partial r_L(\theta)}{\partial\theta}$ on $\theta < \min(\theta_0', \theta_0)$, then the statement $r_L(\theta) \geq r_S(\theta)$ is proved for all θ.

Denote $u^B_{\theta,\tau_c,c}(r_{\tau_c}(\theta), \delta)$ as the partial derivative of the buying utility with respect to leisure consumption c_l evaluated at optimality for the compensation structure τ_c for type θ consumers. Denote $u^{NB}_{\theta,\tau_c,c}$ as the partial derivative of not-buying utility with respect to leisure consumption c_l evaluated at optimality. $t^B_{w,L}(\theta, r_L(\theta))$ denotes the optimal work time of a linearly paid type θ consumer when she buys the service at price $r_L(\theta)$, while, $t^{NB}_{w,L}(\theta)$ denotes the optimal work time of a linearly paid type θ consumer when she does not buy the service.

Given δ, take derivative w.r.t. θ on both sides of Equation (7), and rearrange the terms by noticing, $\forall\theta$, $\chi^j_{\tau_c} = -1/O'(c^j_{O,\tau_c})$ and $\chi^j_L = \theta$, where $j \in \{B, NB\}$, from Lemma 4 and Lemma 2. For any θ, we have

$$\frac{\partial r_L(\theta)}{\partial\theta} = \frac{t^B_{w,L}(\theta, r_L(\theta))u^B_{\theta,L,c}(r_L(\theta),\delta) - t^{NB}_{w,L}(\theta)u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}(r_L(\theta),\delta)}, and$$
$$\frac{\partial r_S(\theta)}{\partial\theta} = \frac{S(u^B_{\theta,S,c}(r_S(\theta),\delta) - u^{NB}_{,\theta,S,c})}{u^B_{\theta,S,c}(r_S(\theta),\delta)}.$$

Next, we compare the magnitude of $\frac{\partial r_L(\theta)}{\partial\theta}$, $\frac{\partial r_S(\theta)}{\partial\theta}$ in different ranges of θ to complete the proof.

Case 1: $\theta > \max(\theta_0', \theta_0)$, when shift workers are *underemployed*, whether buy or not buy. If we can prove $\frac{\partial r_L(\theta)}{\partial\theta} > \frac{\partial r_S(\theta)}{\partial\theta}$ in this case, we then have proved $r_L(\theta) > r_S(\theta)$ on $\theta > \max(\theta_0', \theta_0)$.

Recall that, by construction, $t^{NB}_{w,L}(\theta_0) = S$. Therefore, it follows from condition (ii) that, whenever $\theta > \theta_0$, we have $t^{NB}_{w,L}(\theta) \geq S$. We omit the parameters in the brackets for ease of notation.

1. $t^{NB}_{w,L}(\theta) > t^B_{w,L}(\theta, r_L(\theta)) > S$.

$$\frac{\partial r_L(\theta)}{\partial\theta} - \frac{\partial r_S(\theta)}{\partial\theta} \geq S - t^{NB}_{w,L}\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}} - S + S\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq 0.$$

   The last inequality follows from $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq \frac{t^{NB}_{w,L}}{S}\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$.

2. $t^B_{w,L}(\theta, r_L(\theta)) > t^{NB}_{w,L}(\theta) > S$.

$$\frac{\partial r_L(\theta)}{\partial\theta} - \frac{\partial r_S(\theta)}{\partial\theta} \geq S(1 - \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}) - S(1 - \frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}}) \geq 0.$$

   The last inequality follows from $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$

Case 2: $\theta \in \big(\min(\theta_0', \theta_0), \max(\theta_0', \theta_0)\big)$. Again, we want to prove $\frac{\partial r_L(\theta)}{\partial\theta} > \frac{\partial r_S(\theta)}{\partial\theta}$ in this interval.

Recall θ_0' as the type at which $t^B_{w,L}(\theta_0') = S$. By construction, $t^B_{w,L}(\theta, r_L(\theta)) > S > t^{NB}_{w,L}(\theta)$ when $\theta_0' < \theta_0$ or $t^{NB}_{w,L}(\theta, r_L(\theta)) > S > t^B_{w,L}(\theta)$ when $\theta_0 < \theta_0'$.

1. $t^B_{w,L}(\theta, r_L(\theta)) > S > t^{NB}_{w,L}(\theta)$ when $\theta_0' < \theta_0$.

$$\frac{\partial r_L(\theta)}{\partial\theta} - \frac{\partial r_S(\theta)}{\partial\theta} \geq S(1 - \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}) - S(1 - \frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}}) \geq 0.$$

   The last inequality follows from $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$.

2. $t^{NB}_{w,L}(\theta, r_L(\theta)) > S > t^B_{w,L}(\theta)$ when $\theta_0 < \theta'_0$.

$$\frac{\partial r_L(\theta)}{\partial\theta} - \frac{\partial r_S(\theta)}{\partial\theta} \geq t^B_{w,L} - t^{NB}_{w,L} - t^{NB}_{w,L}\left(\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}} - \frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}}\right) \geq 0.$$

The last inequality follows from $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \geq \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}} + (1 - \frac{t^B_{w,L}}{t^{NB}_{w,L}})$.

Case 3: $0 < \theta < \min(\theta'_0, \theta_0)$, when shift workers are *overemployed*, whether buy or not buy. If we can prove $\frac{\partial r_L(\theta)}{\partial\theta} < \frac{\partial r_S(\theta)}{\partial\theta}$ in this case, we then have proved $r_L(\theta) > r_S(\theta)$ on $0 < \theta < \min(\theta'_0, \theta_0)$.

1. $S > t^{NB}_{w,L}(\theta) > t^B_{w,L}(\theta, r_L(\theta))$.

$$\frac{\partial r_L(\theta)}{\partial\theta} - \frac{\partial r_S(\theta)}{\partial\theta} \leq S\left(1 - \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}\right) - S\left(1 - \frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}}\right) \leq 0.$$

The last inequality follows from $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \leq \frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$.

2. $S > t^B_{w,L}(\theta, r_L(\theta)) > t^{NB}_{w,L}(\theta)$.

$$\frac{\partial r_L(\theta)}{\partial\theta} - \frac{\partial r_S(\theta)}{\partial\theta} \leq S - t^{NB}_{w,L}\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}} - S + S\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \leq 0.$$

The last inequality follows from $\frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}} \leq \frac{t^{NB}_{w,L}}{S}\frac{u^{NB}_{\theta,L,c}}{u^B_{\theta,L,c}}$.

That is, $r_L(\theta)$ increases and approaches $r_L(\theta'_0)$ at a smaller rate than $r_S(\theta)$ approaches $r_S(\theta'_0)$. Therefore, $r_L(\theta) \geq r_S(\theta)$ when $0 < \theta < \theta'_0$. We have thus shown $r_L(\theta) \geq r_S(\theta)$ for all θ. This completes the proof.

A.4. Proof of Corollary 1

Note first that, $\frac{\partial r_S(\theta)}{\partial\theta} = \frac{S(u^B_{\theta,S,c} - u^{NB}_{\theta,S,c})}{u^B_{\theta,S,c}} = S(1 - \frac{u^{NB}_{\theta,S,c}}{u^B_{\theta,S,c}}) > 0$. For gig workers, by the proof shown in A.3, when $\theta > \min(\theta'_0, \theta_0)$, i.e., under Case 1, Case 2, given that the conditions in Proposition 2 hold, $\frac{\partial r_L(\theta)}{\partial\theta} > \frac{\partial r_S(\theta)}{\partial\theta} > 0$. This completes the proof.

A.5. Uniqueness of solution to the maximization problem solved by the seller managing a single-server, Markovian, hidden queue

Note that $\bar{\delta}(\beta\lambda) = \frac{s}{1-\beta\lambda s}$. There is an indifference price r that induces this expected time commitment for type θ consumers, i.e., $r = r(\bar{\delta}(\beta\lambda); \theta)$. Suppress θ since homogeneous consumer type is considered, we have $\bar{\delta}(\beta\lambda) = r^{-1}_{\tau_c}(r)$. That is, $\beta_{\tau_c}(r) := \frac{r^{-1}_{\tau_c}(r) - s}{\lambda s r^{-1}_{\tau_c}(r)}$.

Omit the subscript τ_c on β_{τ_c} and write down the FOC of the maximization problem, we have

$$\beta(r) + r\beta'(r) = 0.$$

Furthermore, $\beta'(r) = \frac{(r^{-1}_{\tau_c}(r))'(r^{-1}_{\tau_c}(r)) - (r^{-1}_{\tau_c}(r) - s)(r^{-1}_{\tau_c}(r))'}{\lambda s \left(r^{-1}_{\tau_c}(r)\right)^2} = \frac{r^{-1}_{\tau_c}(r)'}{\lambda\left(r^{-1}_{\tau_c}(r)\right)^2}$.

Lemma 5. *$r_L(\delta;\theta)$ and $r_S(\delta;\theta)$ strictly decrease in δ. Furthermore, $\partial r_L(\delta;\theta)/\partial\delta = -\theta$ and $\partial r_S(\delta;\theta)/\partial\delta = 1/O'(c^B_{O,S})$.*

We prove Lemma 5 in A.6.

Let $r_{\tau_c}^{-1}(r) = \delta$. Lemma 5 shows that, given θ, $r_{\tau_c}(\delta)$ is an invertible function. Therefore, $(r_{\tau_c}^{-1}(r))' = \frac{1}{\partial r_{\tau_c}(\delta)/\partial\delta}$. Note further, from Equation (8), we know that, $\frac{\partial r_L(\delta;\theta)}{\partial\delta} = -\chi_L^B = -\theta$, and $\frac{\partial r_S(\delta;\theta)}{\partial\delta} = -\chi_S^B = 1/O'(c_{O,S}^B)$.

We next show that the revenue function have a unique maximizer. For gig workers, optimal r solves $\frac{\beta(r)}{\beta'(r)} = -r \Rightarrow \frac{r_L^{-1}(r)(r_L^{-1}(r)-s)}{r} = s\theta$. It is easy to see that there is a unique r_L^* since the right hand side is a constant, while the left hand side is strictly decreasing following from Lemma 5. For shift workers, under the condition of $\frac{\partial^2 r_S(\delta)}{\partial\delta^2} > -1/\delta$, we can show that $\beta_S(r)$ is log-concave (i.e., $\frac{\beta'_S(r)}{\beta_S(r)}$ decreases in r). Hence, left hand side of the FOC increases in r while right hand side decreases in r, subsequently a unique r_S^*. The sufficient condition can be seen as follows. Denote $\delta_{\tau_c}(r) = r_{\tau_c}^{-1}(r)$, and omit τ_c to simplify notations.

$$\begin{aligned}
\log(\beta(r)) &= \log(\delta(r) - s) - \log(\lambda s) - \log(\delta(r)) \\
(\log(\beta(r)))'' &= \left(\frac{\delta'(r)}{\delta(r)-s} - \frac{\delta'(r)}{\delta(r)}\right)' = \left(\frac{s\delta'(r)}{\delta(r)(\delta(r)-s)}\right)' \quad \text{suppress } (r) \text{ hereafter} \\
&= \frac{s(\delta-s)(\delta''\delta - (\delta')^2) - s(\delta')^2\delta}{(\delta(\delta-s))^2}
\end{aligned}$$

Note that, $\delta - s > 0$, otherwise there is no consumer joining the system. Therefore, $\delta''\delta - (\delta')^2 < 0$ is sufficient to ensure the log-concavity of $\beta(r)$ in r. That is, $\delta''\delta - (\delta')^2 = \frac{-\delta r''(\delta)-1}{\left(r'(\delta)\right)^2} < 0 \Rightarrow r''(\delta) > -1/\delta$. This completes the proof. Back to page(s) 14,38.

A.6. Proof of Lemma 5 and Lemma 6

Lemma 6. *Given consumer type θ, suppose that $\partial c_{O,S}^B(\delta)/\partial\delta > 0$; we have $r_L(\delta;\theta) \geq r_S(\delta;\theta)$.*

Taking derivative w.r.t. δ on both sides of (7), we have

$$\begin{aligned}
\frac{\partial r_L(\delta;\theta)}{\partial\delta} &= -\chi_L^B \cdot ({t_{w,L}^B}' + O'(c_{O,L}^B){c_{O,L}^B}' + 1) + \theta {t_{w,L}^B}' - {c_{O,L}^B}' \Rightarrow \frac{\partial r_L(\delta;\theta)}{\partial\delta} = -\chi_L^B = -\theta, \text{ and} \\
\frac{\partial r_S(\delta;\theta)}{\partial\delta} &= -\chi_S^B \cdot (O'(c_{O,S}^B){c_{O,S}^B}' + 1) - {c_{O,S}^B}' \Rightarrow \frac{\partial r_S(\delta;\theta)}{\partial\delta} = -\chi_S^B = 1/O'(c_{O,S}^B).
\end{aligned} \tag{8}$$

Note that $O'(c_{O,S}^B) < 0$, which follows from the assumption that overhead decreases in overhead spending. We have thus proved Lemma 5. That is, indifference prices strictly monotonically decrease in δ for both compensation structures.

Let $f(\delta;\theta) = r_L(\delta;\theta) - r_S(\delta;\theta)$. If we can show that $f(\delta;\theta)$ is convex and has a minimum greater or equal to 0 for any θ, Lemma 6 then follows. For simplicity of notation, we omit θ in the bracket hereafter.

First, $f'(\delta) = 0$ when $\theta = -1/O'(c_{O,S}^B(\delta))$. Denote $\tilde{\delta}$ as the service time that solves this FOC. Suppose all conditions in Proposition 2 are satisfied at this θ; by Proposition 2, we know $r_L(\tilde{\delta}) \geq r_S(\tilde{\delta})$, i.e., $f(\tilde{\delta}) \geq 0$. We now have only left to prove the convexity, that is, $f''(\delta) > 0$.

$$f''(\delta) = (O'(c_{O,S}^B(\delta)))^{-2} \cdot O''(c_{O,S}^B(\delta)) \cdot {c_{O,S}^B}'(\delta)$$

We have assumed the overhead expenditure function to be convex in c_O; that is, $O''(\cdot) > 0$. We apply the condition ${c_{O,S}^B}'(\delta) > 0$. This completes the proof. Back to page(s) 15,35,37.

A.7. Uniqueness of solution to the maximization problem solved by the seller managing a single-server, Markovian, visible queue

We omit the subscript τ_c. Denote $g(n) = \frac{1-\rho^n}{1-\rho^{n+1}}$, $g(n) > 0$. We can write the second-order derivative of $R(n)$ as follows:

$$R''(n) = \lambda(r''(sn)sg(n) + 2r'(sn)sg'(n) + r(sn)g''(n)).$$

First, following from the analysis on $f(\delta)$ in A.6, $r''_L(sn) = 0$, $r''_S(sn) = -f''(sn)/s^2 < 0$. Therefore, the first term in $R''(n)$ is smaller than or equal to zero. From Lemma 5, we know that $r'(sn) < 0$. Further, notice that $g'(n) = \frac{-\rho^n \log\rho(1-\rho)}{(1-\rho^{n+1})^2} > 0$. Lastly, we obtain

$$g''(n) = \frac{((\rho^n(\log\rho)^2+\rho^{n-1})(-1+\rho)+\rho^n\log\rho)\cdot(1-\rho^{n+1})^2-\rho^n\log\rho\cdot(-1+\rho)\cdot 2(1-\rho^{n+1})\cdot(-\rho^{n+1}\log\rho)}{(1-\rho^{n+1})^4} < 0.$$

That is, the objective function is strictly concave, which ensures the unique optimum. This completes the proof. Back to page(s) 16.

A.8. Proof of Proposition 5 and Proposition 3

$$\max_{n_{\tau_c}} R_{\tau_c} = \max_{n_{\tau_c}} r_{\tau_c}(sn_{\tau_c};\theta)\cdot\lambda\frac{1-\rho^{n_{\tau_c}}}{1-\rho^{n_{\tau_c}+1}}, \forall n_{\tau_c} \in \mathbb{Z}^+,$$

First, we fix θ and apply FOC. We have the following. To simplify the notations, we omit the compensation structure notation τ_c in the following expression:

$$r(sn;\theta)\cdot\lambda\frac{-\rho^n\log\rho(1-\rho^{n+1})-(1-\rho^n)(-\rho^{n+1}\log\rho)}{(1-\rho^{n+1})^2} + r'(sn;\theta)\cdot s\cdot\lambda\frac{1-\rho^n}{1-\rho^{n+1}} = 0;$$
$$\frac{r(sn;\theta)}{r'(sn;\theta)} = \frac{-s(1-\rho^n)(1-\rho^{n+1})}{\rho^n(1-\rho)^2}.$$

First-order Taylor expansion approximation, $\log\rho \approx (\rho-1)$, is used to get the second equation above. The optimal threshold $n^*_{\tau_c}$ satisfies the above expression. We omit the integer constraint on n_{τ_c} for now. Note that the right-hand side of the above expression is independent of consumer type θ and compensation structure type τ_c. Denote the right-hand side as $f(n)$. Furthermore, $f'(n) = \frac{-s(\rho^{2n+1}-1)}{\rho^n(\rho-1)} < 0$ indicates that $f(n)$ decreases in n for all $\rho < 1$, i.e., a stable queue.

Next we want to compare $\frac{r_L(sn;\theta)}{r'_L(sn;\theta)}$ and $\frac{r_S(sn;\theta)}{r'_S(sn;\theta)}$, varying θ. Recall from Equations (8) that we can simplify the two ratios as follows. We omit the superscript B, and obtain

$$\frac{r_L(sn;\theta)}{r'_L(sn;\theta)} = \frac{r_L(sn;\theta)}{-\theta\cdot s} = \frac{r_L(sn;\theta)O'(c_{O,L}(sn;\theta))}{s}, \text{ and}$$
$$\frac{r_S(sn;\theta)}{r'_S(sn;\theta)} = \frac{r_S(sn;\theta)O'(c_{O,S}(sn;\theta))}{s}.$$

Consider the curves $r_L(sn;\theta)O'(c_{O,L}(sn;\theta))$ and $r_S(sn;\theta)O'(c_{O,S}(sn;\theta))$. We suppress θ in the bracket of price and expenditure expressions, $r_{\tau_c}(sn;\theta)$ and $c_{O,\tau_c}(sn;\theta)$ in the rest of the proof.

$$(r_{\tau_c}(sn;\theta)O'(c_{O,\tau_c}(sn;\theta)))' = r'_{\tau_c}(sn)sO'(c_{O,\tau_c}(sn)) + r_{\tau_c}(sn)O''(c_{O,\tau_c}(sn))c'_{O,\tau_c}(sn)s$$

Given θ, Lemma 5 shows $r'_{\tau_c}(\delta) < 0$; by the monotone decreasing assumption $O'(c_{O,\tau_c}(sn))) < 0$, therefore the first term is positive. The second term is also positive following from the convexity of function $O(\cdot)$ and also $\partial c^B_{O,S}(\delta)/\partial\delta > 0$ stated as an assumption in Lemma 6. Hence, function $\frac{r_{\tau_c}(sn)}{r'_{\tau_c}(sn)}$ increases in n. This also shows that the optimal solutions $n^*_{\tau_c}$ are unique, recall that the right hand side of the FOC decreases in n.

As stated in the condition, when θ is small, $\theta \le \theta_e$, $c^B_{O,S}(sn) > c_{O,L}(sn)$. Intuitively, this condition suggests that consumers with a shift pay compensation structure who are indifferent to joining the queue are overemployed. They are time constrained but have cash from the their longer-than-they're-willing-to-work shift. They therefore want to spend more on buying their overhead time back, i.e., $c^B_{O,S}(sn) > c_{O,L}(sn)$. Given the convexity of the overhead function, $O(c_O)$, we then have $0 > O'(c^B_{O,S}(sn)) > O'(c_{O,L}(sn))$. Recall also from Lemma 6 that, we have $r_L(sn) \ge r_S(sn)$. That is,

$$r_L(sn)O'(c_{O,L}(sn)) \le r_S(sn)O'(c_{O,L}(sn)) \le r_S(sn)O'(c_{O,S}(sn)).$$

Consequently, r_L/r'_L crosses function $f(n)$ at a larger n than r_S/r'_S; that is $n^*_L \ge n^*_S$, when $\theta \le \theta_e$.

Next, consider when θ is large, i.e., $\theta > \theta_e$. As stated in the condition, $c^B_{O,S}(sn) < c_{O,L}(sn)$. Intuitively, this condition suggests that consumers with shift pay compensation structure who are indifferent to joining the queue are underemployed. The shift pay consumers are earnings constrained compared to the gig workers. They will choose to invest less in overhead expenditure; that is, $c^B_{O,S}(sn) < c_{O,L}(sn)$. Again, due to convexity of $O(c_O)$, we then have $O'(c^B_{O,S}(sn)) < O'(c_{O,L}(sn)) < 0$. We therefore have,

$$r_L(sn)O'(c_{O,L}(sn)) \ge r_S(sn)O'(c^B_{O,S}(sn)).$$

Consequently, r_L/r'_L now crosses function $f(n)$ at a smaller n than r_S/r'_S; that is $n^*_S \ge n^*_L$, when $\theta > \theta_e$. This completes the proof of Proposition 5.

We now modify the proof above to prove Proposition 3. Note first we have proved in A.5 that $r \to \beta(r)$ is a bijection. Therefore, we can rewrite the maximization problem (4) as solving for an optimal β, that is,

$$\max_\beta R_{\tau_c} = \max_\beta r_{\tau_c}(\frac{s}{1-\beta\lambda s})\lambda\beta.$$

Let $\delta = \frac{s}{1-\beta\lambda s}$. We further transform the revenue function to $r(\delta)\lambda(\frac{1}{\lambda s} - \frac{1}{\lambda\delta}) = r(\delta)(\frac{1}{s} - \frac{1}{\delta})$. If we can show $\delta^*_L > \delta^*_S$ ($\delta^*_L < \delta^*_S$), then it easily follows that $\beta^*_L > \beta^*_S$ ($\beta^*_L < \beta^*_S$).

We again start by writing down the FOC and omit the subscript τ_c,

$$r'(\delta)(\frac{1}{s}-\frac{1}{\delta})+r(\delta)(\frac{1}{\delta^2})=0; \quad \frac{r(\delta)}{r'(\delta)}=\frac{s\delta-\delta^2}{s}$$

Denote the right hand side as $g(\delta)$. We have $g'(\delta)=\frac{s-2\delta}{s}$. Note that, δ is the total expected sojourn time in the system, while s is the average time commitment of each consumer in the system. It is natural to assume $s<\delta$, otherwise, there is never any consumers joining the system. Therefore, $s<2\delta$. Hence, $g'(\delta)<0$. Again, we arrive at the conclusion that the right hand side of the FOC decreases in the decision variable, in this case, δ. The next step is to compare $\frac{r_L(\delta)}{r'_L(\delta)}$ and $\frac{r_S(\delta)}{r'_S(\delta)}$. Similarly,

$$\frac{r_L(\delta)}{r'_L(\delta)}=\frac{r_L(\delta)}{-\theta}=r_L(\delta)O'(c_{O,L}(\delta)), \text{ and } \frac{r_S(\delta)}{r'_S(\delta)}=r_S(\delta)O'(c_{O,S}(\delta)).$$

Next, we again apply the conditions given in the statement to compare $r_L(\delta)O'(c_{O,L}(\delta))$ and $r_S(\delta)O'(c_{O,S}(\delta))$. Rest of the proof follows the same arguments as above in the proof of Proposition 5. This completes the proof of Proposition 3.

A.9. Proof of Proposition 6 and Proposition 4

Consider the revenue function when the seller serves a group of shift workers. Suppose the optimal decision of the service provider is to admit n_s^* consumers. Given θ, n_s^* is a feasible solution to the problem of serving a community of gig workers, $\max_{n_L} R_L(\theta)$. We thus have the following:

$$r_L(sn_L^*;\theta)\cdot\lambda\frac{1-\rho^{n_L^*}}{1-\rho^{n_L^*+1}}\geq r_L(sn_S^*;\theta)\cdot\lambda\frac{1-\rho^{n_S^*}}{1-\rho^{n_S^*+1}}\geq r_S(sn_S^*;\theta)\cdot\lambda\frac{1-\rho^{n_S^*}}{1-\rho^{n_S^*+1}}, \quad \forall\theta.$$

The first inequality follows from the optimality of n_L^*. The second inequality follows from Lemma 6, $r_L(sn_S^*;\theta)\geq r_S(sn_S^*;\theta)$. We therefore conclude that $R_L^*\geq R_S^*$. This completes the proof of Proposition 6.

We follow the same arguments to prove Proposition 4. That is,

$$r_L(\delta_L^*;\theta)\cdot(\frac{1}{s}-\frac{1}{\delta_L^*})\geq r_L(\delta_S^*;\theta)\cdot(\frac{1}{s}-\frac{1}{\delta_S^*})\geq r_S(\delta_S^*;\theta)\cdot(\frac{1}{s}-\frac{1}{\delta_S^*}), \quad \forall\theta.$$

This completes the proof of Proposition 4.

A.10. Threshold policy of admitting heterogeneous consumers to a service with fixed time commitment

The following proof applies to both compensation structures. We therefore omit the subscript τ_c. Following from Corollary 1, we know that, $r(\tilde{\theta};\delta)$ is the unique indifference price for the type $\tilde{\theta}$ consumer. We want to show that consumer buys the service at $r(\tilde{\theta};\delta)$ when $\theta\geq\tilde{\theta}$ and otherwise, does not buy.

We start with $\theta \geq \tilde{\theta}$. We have shown in Corollary 1 that the indifference price increases in consumer types; i.e., $r(\theta;\delta) \geq r(\tilde{\theta};\delta)$. Suppose the consumer buys at price $r(\theta;\delta)$; it's easy to see that they definitely buy at a lower price as follows. Denote $(\widetilde{t_w}, \widetilde{c_O})$ as the solutions of the type θ indifferent consumers at optimality when they buy the service. We have

$$U(T - \widetilde{t_w} - O(\widetilde{c_O}) - \delta, \theta\widetilde{t_w} - \widetilde{c_O} - r(\tilde{\theta};\delta)) + V \geq u_\theta^B(r(\theta;\delta), \delta) + V = u_\theta^{NB}.$$

That is, consumers of type $\theta \geq \tilde{\theta}$ are better off buying the service at the price $r(\tilde{\theta};\delta)$.

Next, we want to show by contradiction that, type $\theta < \tilde{\theta}$ would not purchase at price $r(\tilde{\theta};\delta)$, the contrapositive of the "only if" part. Note that $r(\theta;\delta) < r(\tilde{\theta};\delta)$. Suppose for type θ consumers there exists a pair $(\widetilde{t_w}, \widetilde{c_O})$ that makes them want to buy the service priced at $r(\tilde{\theta};\delta)$. Then

$$u_\theta^B(r(\theta), \delta) + V > U(T - \widetilde{t_w} - O(\widetilde{c_O}) - \delta, \theta\widetilde{t_w} - \widetilde{c_O} - r(\tilde{\theta};\delta)) + V \geq u_\theta^{NB}.$$

The above inequality indicates that $u_\theta^B(r(\theta), \delta) + V > u_\theta^{NB}$, which violates the definition of an indifference price. This is a contradiction. That is, there is no feasible allocation of time and money that would make a type $\theta < \tilde{\theta}$ better off buying the service. This completes the proof. Back to page(s) 17.

A.11. Proof of Lemma 3

We start with the proof of Part (i). We omit the subscript τ_c for ease of notation in the following proof. Since δ is fixed in the analysis, we also omit δ in $r(\theta;\delta)$. Therefore, the service provider's revenue function is given by $R(\theta) := r(\theta)\bar{F}(\theta)$. Taking derivative w.r.t. θ, we have $R'(\theta) = r'(\theta)\overline{F}(\theta) - r(\theta)f(\theta)$. From FOC, θ^* solves $\frac{r'(\theta)}{r(\theta)} = \frac{f(\theta)}{\bar{F}(\theta)}$. Given the log-concavity of $r(\theta)$, we have $r'(\theta)/r(\theta)$ decreases in θ. Additionally, $\frac{f(\theta)}{\bar{F}(\theta)}$ increases in θ since F has an increasing failure rate. Therefore, FOC has a unique solution. That is, revenue maximization problem (6) has a unique optimum θ^*.

Now we move on to Part (ii). Let $R(\theta, S) := r_S(\theta;\delta)\overline{F}(\theta)$. If we can show that $R(\theta, S)$ satisfies single crossing, then $\theta^*_{S'} \geq \theta^*_S$ if $S' \geq S$ by the Milgrom-Shannon theorem. The statement is then proved.

By definition, $R(\theta, S)$ satisfies single crossing if and only if $\forall\ \theta' > \theta$ and $S' > S$, the statement $R(\theta', S) \geq R(\theta, S) \Rightarrow R(\theta', S') > R(\theta, S')$, holds. Rewrite $R(\theta', S) \geq R(\theta, S)$ to get $\overline{F}(\theta') \geq \overline{F}(\theta)r(\theta, S)/r(\theta', S)$ and substitute the inequality to $R(\theta', S') - R(\theta, S')$. We obtain

$$R(\theta', S') - R(\theta, S') \geq \overline{F}(\theta)\big(\frac{r(\theta, S)r(\theta', S')}{r(\theta', S)} - r(\theta, S')\big).$$

We need to show that $\frac{r(\theta,S)r(\theta',S')}{r(\theta',S)} - r(\theta, S') > 0$; that is, $\frac{r(\theta,S)}{r(\theta',S)} > \frac{r(\theta,S')}{r(\theta',S')}$. That is,

$$\frac{r(\theta, S)}{r(\theta', S)} \quad \text{strictly decreases in } S.$$

Equivalently, we are left to show, $\partial \frac{r(\theta,S)}{r(\theta',S)}/\partial S < 0$. That is,

$$\partial \frac{r(\theta,S)}{r(\theta',S)}/\partial S = \frac{\partial r(\theta,S)/\partial S \cdot r(\theta',S) - r(\theta,S)\cdot \partial r(\theta',S)/\partial S}{(r(\theta',S))^2}.$$

To simplify the notations, we denote $r'_\theta := \frac{\partial r(\theta,S)}{\partial S}$ and $r'_{\theta'} := \frac{\partial r(\theta',S)}{\partial S}$.

Note first that it follows from the proof of Corollary 1 in A.4 that $r(\theta',S) > r(\theta,S) > 0$. Additionally, from the condition (ii)(1), we have $\partial r(\theta,S)/\partial S < \partial r(\theta',S)/\partial S$. Therefore, as S increases, we might encounter the following three cases: (i) $r'_{\theta'} > r'_\theta > 0$, (ii) $r'_{\theta'} > 0 > r'_\theta$, (iii) $0 > r'_{\theta'} > r'_\theta$. It is easy to show $\partial \frac{r(\theta,S)}{r(\theta',S)}/\partial S < 0$ for the last two cases, while for the first case, when both types are underemployed, it follows directly from condition (ii)(2). This completes the proof.

Remark 4. The intuition of why condition (ii)(1) can hold is as follows. For any S, when both type θ and θ' consumers are underemployed, given the increasing work time assumption we made in Proposition 2, the type θ' consumer would be richer in time; further, they earn more extra cash with the increase of shift length. Therefore, a higher type would have a larger increase in willingness to pay for the discrete service. If the higher-type consumer is underemployed but the lower type is overemployed at shift length S, the increase of shift length will benefit the higher type but make the shortage of time even worse for the lower type. Type θ consumer's indifference price might reduce, but the type θ' consumer might be willing to pay a higher price. Lastly, if both types are overemployed, an increase of shift length pushes both types farther away from optimal work time under linear pay structure, but the time shortage is worse for the lower type. The lower type would incur a larger decrease in the indifference price.

A.12. Proof of Proposition 8

Consider the shift-pay consumers' problem $\max_{\theta_s} r_S(\theta_S;\delta)\overline{F}(\theta_S)$. Denote the optimal solution to this problem to be θ^*_S. Following directly from Corollary 1, we know, given S, when conditions of Proposition 2 are satisfied, $r_L(\theta;\delta) \geq r_S(\theta;\delta)$ $\forall\theta$. Therefore, given S, the function value $r_L(\theta^*_S;\delta)\overline{F}(\theta^*_S)$ is always larger than the function value of problem $r_S(\theta^*_S;\delta)\overline{F}(\theta^*_S)$. Denote the optimal solution to the problem involving the linearly paid consumer as θ^*_L. We therefore have $R^*_L = r_L(\theta^*_L;\delta)\overline{F}(\theta^*_L) \geq r_L(\theta^*_S;\delta)\overline{F}(\theta^*_S) \geq r_S(\theta^*_S;\delta)\overline{F}(\theta^*_S) = R^*_S$.

B. Mixed compensation structures analysis

In this section, we denote the proportion of gig workers among the arrival consumers to be $\alpha \in (0,1)$, therefore $1-\alpha$ are shift workers. In all numerical studies conducted in this section, we assume that there are 20% gig workers among the arrival consumers, i.e., $\alpha = 0.2$. The results for the numerical studies in this section are compiled in Figure 5.

B.1. Fixed time commitment seller, heterogeneous consumer types

To start with, we look at a seller offering service with a fixed time commitment to a heterogeneous group of consumers. Given a fixed time commitment δ, for any price r, there is a unique type $\theta_{\tau_c}(r)$ that makes consumers indifferent in purchasing the service. The consumers whose types are $\theta \geq \theta_{\tau_c}(r)$ will also purchase the service. We assume the consumer type distributions, denoted as F, to be the same for gig and shift workers. Therefore, the revenue function can be written as

$$\max_r R_{mix} = \max_r r\big(\alpha\overline{F}(\theta_L(r)) + (1-\alpha)\overline{F}(\theta_S(r))\big).$$

Note that, using Proposition 8, $R^*_{mix} \in (R^*_S, R^*_L)$ easily follows for any S. We conduct a numerical analysis to investigate the congestion level the seller observes. We adopt the same notation as in the main text, $p^* = \alpha\overline{F}(\theta_L(r^*)) + (1-\alpha)\overline{F}(\theta_S(r^*))$.

Figure 5 (right) shows that our key result in congestion level is robust to a mixed population setting—when there are some underemployed shift workers in the population mix, a larger proportion of consumers are admitted to the service (i.e., resulting in a more-congested system) than it is in the case when there are only gig workers, and vice versa. Furthermore, it is easy to see that $r^*_{mix} \neq r^*_L$ in general since $\theta_S(r)$ and $\theta_L(r)$ are different in general. In fact, in this numerical analysis, we see $r^*_{mix} \in (\min\{r^*_S, r^*_L\}, \max\{r^*_S, r^*_L\})$. Therefore, a seller who fails to see that there are shift workers in the population and prices their service as if all consumers are gig workers will experience revenue loss.

B.2. Single-server Markovian visible queue, homogeneous consumer type

We next check the case when the seller manages a single-server, Markovian, and visible queue serving a group of homogeneous consumers with type θ. The consumers make a purchase decision upon arrival to the queue, i.e., after they see the queue.

Suppose the overall arrivals to the system follow a Poisson distribution with rate λ. The expected number of consumers joining the queue in unit time is then $\lambda - \lambda_b$, where λ_b is the balk rate. Given a price r, a gig worker and a shift worker will be indifferent in joining the service when they see $n_L(r;\theta)$ and $n_S(r;\theta)$ consumers in the queue respectively. Denote the queue length consumers see upon arrival to be n. We can specify the balk rate as $\lambda_b = (1-\alpha)\lambda\mathbb{P}(n = n_S(r;\theta)) + \alpha\lambda\mathbb{P}(n = n_L(r;\theta))$. The revenue function can then be written as

$$\max_r R_{mix} = \max_r r(\lambda - \lambda_b).$$

Next, we calculate the two probabilities in the balk rate expression. Let b_{τ_c} be the balk probability, i.e., $b_{mix}(r;\theta) = (1-\alpha)\mathbb{P}(n = n_S(r;\theta)) + \alpha\mathbb{P}(n = n_L(r;\theta))$; $b_{\tau_c}(r_{\tau_c};\theta) = \mathbb{P}(n = n_{\tau_c}(r_{\tau_c};\theta))$, $\tau_c \in \{L, S\}$. Denote the optimal balk rate as $b^*_{\tau_c} = b_{\tau_c}(r^*_{\tau_c};\theta)$, $\tau_c \in \{L, S, \text{mix}\}$. Let $n_2 = \max\{n_L(r;\theta), n_S(r;\theta)\}$, and $n_1 = \min\{n_L(r;\theta), n_S(r;\theta)\}$.

When $i=0,1,\ldots,n_1$, arriving consumers with neither compensation structure will balk. The probability of seeing a queue length of i should be the same as in a standard visible MM1 queue with an arrival rate λ. Therefore, $\pi_i=\pi_0\rho^i$. When $i>n_2$, all arriving consumers balk. Therefore, $\pi_i=0$. When $i=n_1+1,\ldots,n_2$, arriving consumers with the pay structure that has a smaller n_{τ_c} will balk, the arrival rate of the system is the arrival rate of the remaining compensation type, denoted as λ_2, and $\rho_2=\lambda_2 s$. Therefore, $\pi_i=\pi_0\rho^{n_1}\rho_2^{i-n_1}$.

Therefore, $\pi_0=\left(\sum_{i=0}^{n_1}\rho^i+\sum_{i=n_1+1}^{n_2}\rho^{n_1}\rho_2^{i-n_1}\right)^{-1}=\left(\frac{1-\rho^{n_1+1}}{1-\rho}+\frac{\rho^{n_1}\rho_2(1-\rho_2^{n_2-n_1})}{1-\rho_2}\right)^{-1}$. Substitute π_0 into $\mathbb{P}(n=n_1)=\pi_0\rho^{n_1}$; $\mathbb{P}(n=n_2)=\pi_0\rho^{n_1}\rho_2^{n_2-n_1}$. Note that, when $n_2=n_1$, i.e., there is only one type effectively. This expression reduces to $\pi_0=(\sum_{i=0}^{n_1}\rho^i)^{-1}$. Consequently, the balk rate and the revenue function also reduce to the expression for one consumer compensation structure case.

The key insight in congestion level is again preserved (see Figure 5, center). From left to right on the horizontal axis, consumer type θ changes from low to high, the shift workers in the population therefore change from overemployed to underemployed. The seller then sees a less-congested system, i.e., $1-b^*_{mix}<1-b^*_L$ [more-congested system, i.e., $1-b^*_{mix}>1-b^*_L$] compared to serving a gig-worker-only population when there are overemployed [underemployed] shift workers in their consumer population. Note that, in this numerical analysis, the optimal admission thresholds of serving the mixed, or the gig-worker-only population crosses at a consumer type that is smaller than θ_e. Also, $r^*_{mix}\neq r^*_L$ since $n_S(r;\theta)\neq n_L(r;\theta)$ in general.

B.3. Single-server Markovian hidden queue, homogeneous consumer type

Lastly, we discuss the case when the seller manages a hidden queue serving a group of homogeneous consumers (workers) with type θ. Suppose all workers have exponentially distributed service time with mean s. All shift workers have a common shift length of S time units.

For $\beta_S,\beta_L\in[0,1]$ define $\lambda(\beta_S,\beta_L)=\lambda((1-\alpha)\beta_S+\alpha\beta_L)$. $\lambda(\beta_S,\beta_L)$ is the average arrival rate when shift [gig] workers randomize between purchasing the discrete service with probability $\beta_S[\beta_L]$ and not purchasing with probability $(1-\beta_S)[(1-\beta_L)]$. The expected sojourn time in the system is then $\overline{\delta}(\beta_S,\beta_L)=s/(1-\lambda(\beta_S,\beta_L)s)$. Note that both types of consumers have the same expected sojourn times. In this analysis, we suppose the conditions in Proposition 2 hold, hence we have $r_L(\theta;\overline{\delta}(\beta_S,\beta_L))\geq r_S(\theta;\overline{\delta}(\beta_S,\beta_L))$, that is, given the same expected sojourn time, at type θ, the indifference price of gig workers is higher than the indifference price of shift workers.

Given any price r, suppose $r\leq r_S(\theta;\overline{\delta}(1,1))$, all consumers of type θ join the service. As r increases to $r\in(r_S(\theta;\overline{\delta}(1,1)),r_S(\theta;\overline{\delta}(0,1)))$, then $\beta_S\in(0,1)$ shift workers and all gig workers join the service, i.e., $(\beta_S,\beta_L)=(\beta_S,1)$. Note that, in this price interval, only shift workers would randomize their purchase decisions. This can be seen from the following argument. Suppose $\exists\beta_S,\beta_L\in(0,1)$, such that β_S shift workers, and β_L gig workers join the service at menu $r=r(\theta;\overline{\delta}(\beta_S,\beta_L)),\delta=$

$\overline{\delta}(\beta_S, \beta_L)$. Then $r = r_S(\theta;\delta) \leq r_L(\theta;\delta)$. That is, at δ, all gig workers will be at least indifferent to joining the service, i.e., $\beta_L = 1$. Hence $\beta_L \notin (0,1)$. Further increase r to $r \in [r_S(\theta;\overline{\delta}(0,1)), r_L(\theta;\overline{\delta}(0,1))]$. In this price interval, the lowest possible price is too high for any shift workers to join, and the highest possible price is lower than what induces all $\alpha\lambda$ gig workers to join, therefore $(\beta_S, \beta_L) = (0,1)$. Further increase r to $r > r_L(\theta;\overline{\delta}(0,1))$, then $(\beta_S, \beta_L) = (0, \beta_L), \beta_L \in (0,1)$. β_L gig workers purchase the service at $r = r_L(\theta;\overline{\delta}(0,\beta_L))$.

Given θ, at any price r, we can then invert the indifference price function $r = r_{\tau_c}(\theta;\overline{\delta}(\beta_S, \beta_L))$ to get $\overline{\delta}(\beta_S(r), \beta_L(r))$. And therefore, $\lambda(\beta_S(r), \beta_L(r)) = 1/s - 1/\overline{\delta}(\beta_S(r), \beta_L(r))$. We write the maximization problem for serving consumers with mixed compensation structures as follows.

$$\max_r R_{mix} = \max_r r\lambda\big(\beta_S(r), \beta_L(r)\big).$$

Let $\beta_{mix} = \lambda((1-\alpha)\beta_S + \alpha\beta_L)/\lambda$. From Figure 5 (left), the seller sees $\beta^*_{mix} > \beta^*_L$, i.e., a more-congested system, if there are underemployed shift workers in the population than in the case when there are only gig workers. And the seller sees $\beta^*_{mix} < \beta^*_L$, i.e., a less-congested system, if there are overemployed shift workers in the population. In general $r^*_{mix} \neq r^*_L$.

C. Multiple discrete services

In this section, we want to show that our main insights are robust to the case when there are multiple discrete services. While we focus on two services in the following analysis, the arguments can be easily extended to more than two discrete services.

Specifically, we show the following two main findings in Section 3 to continue to hold when there are multiple discrete services: 1) the gig workers are full-price decision-makers when they consider buying the second discrete service, while shift workers are not; 2) given both the gig worker and the shift worker purchased the first service, the indifference price of the second service for gig workers and shift workers are different, i.e., $r_L \neq r_S$ still holds for the second discrete service.

We start by writing down the utility-maximizing problem of consumers when there are two distinct discrete services. Denote the monetary price of discrete service i as r_i, time required as δ_i, and utility gain upon completing the service as V_i.

$$\begin{aligned}
(P_{C,1}) \qquad \max_{t_l, c_l, t_w, c_O, y_1, y_2} \quad & U(t_l, c_l) + V_1 y_1 + V_2 y_2 \\
\text{s.t.} \quad & t_w + O(c_O) + t_l + \delta_1 y_1 + \delta_2 y_2 \leq T \\
& c_O + c_l + r_1 y_1 + r_2 y_2 \leq \Pi_\theta(t_w) \\
& y_1 \in \{0,1\}, y_2 \in \{0,1\}, t_l, c_l, t_w, c_O \geq 0
\end{aligned}$$

Write down the FOC of $(P_{C,1})$, we have $\chi(t_l^j, c_l^j) = \theta$ for the gig workers, where $j \in \{NB, B_1, B_2\}$ denotes the optimal actions of not buying, buying Service 1 only, and buying both services, respectively. For shift workers, we have $\chi(T - S - O(c_O^{B_2}) - \delta_1 - \delta_2, \theta S - c_O^{B_2} - r_1 - r_2) = -\frac{1}{O'(c_O^{B_2})}$, $\chi(T-S-O(c_O^{B_1})-\delta_1, \theta S - c_O^{B_1} - r_1) = -\frac{1}{O'(c_O^{B_1})}$, $\chi(T-S-O(c_O^{NB}), \theta S - c_O^{NB}) = -\frac{1}{O'(c_O^{NB})}$. Following the same argument as in A.2 in the manuscript, we again conclude, that gig workers are full-price decision-makers when they consider the second discrete service, while the shift workers are not.

Next, we want to show part 2), in which we assume the consumers make purchase decisions for the two services sequentially and we investigate their purchase decision when they have decided to purchase the first service. We construct the following utility-maximizing problem.

$$
\begin{aligned}
(P_{C,2}) \qquad \max_{t_l, c_l, t_w, c_O, y_2} \quad & U(t_l, c_l) + V_2 y_2 \\
\text{s.t.} \quad & t_w + O(c_O) + t_l + \delta_2 y_2 \leq T - \delta_1 \\
& c_O + c_l + r_2 y_2 \leq \Pi_\theta(t_w) - r_1 \\
& y_2 \in \{0, 1\}, t_l, c_l, t_w, c_O \geq 0
\end{aligned}
$$

From the construction of $(P_{C,2})$, it is not surprising that the inequality between the two indifference prices, r_L and r_S, preserves for the second discrete service. Given both gig and shift workers have decided to purchase Service 1, $(P_{C,2})$ is essentially reducing the total time available by δ_1 and reducing the total budget available by r_1 from our base model (P_C).