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Online Appendix

O.1. Extensions

O.1.1. Impact of Changes in Price Sensitivity During Economic Distress

In §3, we made two key assumptions to derive the direct impact of brand switching on low-income consumers: (i) price sensitivity of low-income consumers is the same as that of high-income consumers ($\theta = 1$); (ii) price sensitivity of consumers does not change during economic distress. We test the robustness of our results with regards to these assumptions. Increase in price sensitivity during economic distress for low-income consumers is significantly more relative to high-income consumers (Ni Mhurchu et al. 2013). To model this phenomenon, we assume that price sensitivity for low-income consumers changes from θ to $\nu\theta$ during economic distress for some $\nu \geq 1$. The case we analyze in §3 assumes that $\theta = \nu = 1$. Note that when $\theta > 1$ and $\nu = 1$, we are in the setting where low-income consumers have higher price sensitivity then high-income consumers without economic distress but it does not increase further during economic distress. In contrast, when $\theta > 1$ and $\nu > 1$, we are in the setting where low-income consumers have higher price sensitivity then high-income consumers during no economic distress, and it increases further during times of economic distress.[18] We start by again characterizing the demand for the two products in this more general case.

LEMMA O.1. *Define $v_R^u \equiv v_R(1 - \lambda_H^2/4)$. Total demand for product N, d_N, and for product R, d_R, is*

$$d_N(p_N, p_R, \lambda_H) = \Big(1 - \max\{\frac{p_N}{v_N}, \frac{p_N - p_R}{v_N - v_R^u}\}\Big)^+ + \Big(1 - \max\{\frac{\nu\theta p_N}{v_N}, \frac{\nu\theta(p_N - p_R)}{v_N - v_R}\}\Big)^+$$

and

$$d_R(p_N, p_R, \lambda_H) = \Big(\min\{1, \frac{p_N - p_R}{v_N - v_R^u}\} - \frac{p_R}{v_R^u}\Big)^+ + \Big(\min\{1, \frac{\nu\theta(p_N - p_R)}{v_N - v_R}\} - \frac{\nu\theta p_R}{v_R}\Big)^+$$

Using this demand function, the following result characterizes the equilibrium market prices for the retailer and the national-brand manufacturer.

THEOREM O.1. *Define $\mu \equiv (1 - \lambda_H^2/4)$. If $s \geq 1$,*

(i) The equilibrium wholesale price of H is $w_N^* = \dfrac{(1 - v_R)(1 - \mu v_R)}{(1 - v_R - \nu\theta(1 - \mu v_R))}$

(ii) The equilibrium retail price of the two products charged by the retailer is,

$$p_N^* = \frac{w_N^*}{2} + \frac{1 - v_R + \nu\theta\mu(1 - \mu v_R)}{(1 + \nu\theta\mu)(1 - v_R - \nu\theta(1 - \mu v_R))}, p_R^* = \frac{\mu v_R}{1 + \nu\theta\mu}$$

Further, $d_n + d_R = 1$ for all $\lambda_H \in [0, 1]$.

[18] We assume that $\theta\nu < (8 - 8v_R)/(4 - 4v_R + \lambda_H^2 v_R)$ and $\nu < (-4 + 4\lambda_H^2 - 12\theta + 3\lambda_H^2\theta)/(-12\theta + 3\lambda_H^2\theta)$. These conditions are sufficient to ensure that at least some low-income consumers buy national-brand product in the market during economic distress.

Our next result shows the impact of economic distress on the prices of private-label and national-brand products. Let $p_k^*(\lambda_H, \nu, \theta)$ be the price of product $k \in \{N, R\}$. Without economic distress, $\lambda_H = 1$ and $\nu = 1$. With economic distress, perception of private-label product among high-income consumers improves, thus $\lambda_H < 1$. Similarly, during economic distress, low-income consumers may become more price sensitive and $\nu \geq 1$. In what follows we show the key insights from our analysis with regards to market prices and revenue in the market.

Proposition O.1. *If $s \geq 1$, (i) If $\nu < 4/3$, $\theta < \dfrac{1}{3(\nu-1)}$, and $\lambda_H < 2\sqrt{\dfrac{1+3\theta-3\nu\theta}{4+3\theta-3\nu\theta}}$, $p_R^*(\lambda_H, \nu, \theta) > p_R^*(1, 1, \theta)$. (ii) $w_N^*(\lambda_H, \nu, \theta) < w_N^*(1, 1, \theta)$. Further, the manufacturer's revenue always decreases for any $\nu > 1$ and $\theta \geq 1$, while the retailer's revenue may increase only if λ_H is smaller than a threshold.*

In contrast to results in §3, we find that prices of private-label product increase only when the decrease in quality threshold is large and the increase in price sensitivity is small. This result can be explained by the following dynamics. On the one hand, as low-income consumers' price sensitivity increases, there are fewer low-income consumers willing to buy private-label products at the same price. This incentivizes the retailer to reduce prices of the private-label product. On the other hand, as brand switching from high-income consumers increases, there are more high-income consumers who are willing to buy private-label products during economic distress. This incentivizes the retailer to increase the prices of private-label products. When the price sensitivity effect dominates (i.e. $\nu\theta$ is large), price of private-label product decreases. However, when the brand-switching effect dominates (i.e. λ_H is small), price of private-label product increases. Finally, similar to §3, we find that while the wholesale price of the national-brand product always decreases under economic distress, the retail price of the national-brand product may increase or decrease under economic distress in the market. Despite the potential decrease in prices of the private-label product under economic distress, our next result confirms that the market access and consumer surplus of low-income consumers always decreases under economic distress.

Proposition O.2. *If $s \geq 1$, (i) The demand served of low-income consumers always decreases, and that of high-income consumers always increases under economic distress. (ii) The consumer surplus of low-income consumers always decreases, and that of high-income consumers always increases under economic distress.*

Since the results for cash-subsidy policies follow in a straightforward manner from the analysis presented in §3, we do not repeat them here and instead focus on Price Control policies. A key insight from our analysis is that price control policies can in fact hurt low-income consumers by reducing access in the market. Our next result confirms that the negative effect on access from PC continues to persist in this setting.

Proposition O.3. *If $s=1$, (i) The demand served of low-income consumers always decreases, and that of high-income consumers always increases under PC. (ii) The consumer surplus of low-income and high-income consumers increases under PC.*

Results in Proposition O.3 confirm that Price Control policy decreases access for low-income consumers and increases access for high-income consumers. Similarly, in line with the results in Proposition 7, we find that Price Control policy also increases inequity in the market with increased surplus for those who can purchase either product, but a decrease in the total number of low-income consumers who are able to buy either product in the market.

O.1.2. Markets with More L Consumers

In §3, we assumed that there are an equal proportion of high-income and low-income consumers in the market. Since many countries have significantly more low-income consumers than high-income consumers, we relax this assumption in this extension. In particular, we assume that $\alpha_L \in [1/2, 1]$ fraction are low-income consumers and $1-\alpha_L$ are high-income consumers in the market. Our first result characterizes the equilibrium retail prices and the wholesale price in the market.

Theorem O.2. *Define $\mu \equiv (1-\lambda_H^2/4)$. If $s \geq 1$,*

(i) The equilibrium wholesale price of H is $w_N^* = \dfrac{(1-v_R)(1-\mu v_R)}{2(1-v_R-\alpha_L(1-\mu v_R))}$

(ii) The equilibrium retail price of the two products is,

$$p_N^* = \frac{w_N^*}{2} + \frac{1-v_R-\alpha_L(1-\mu)(1-v_R-\mu v_R)}{2(1-\alpha_L(1-\mu))(1-v_R-\alpha_L(1-\mu)v_R)}, p_R^* = \frac{\mu v_R}{2-2\alpha_L(1-\mu)}$$

Proposition O.4. *(i) As λ_H decreases, p_R^* increases and w_N^* decreases.*

(ii) As λ_H decreases, revenue of the retailer increases and of the manufacturer decreases.

In line with Proposition 1, we again find that the price of private-label product increases during times of economic distress due to the phenomenon of brand switching. Importantly, for any value of λ_H, as the fraction of low-income consumers in the market, α_L, increases, we find that the price of the private-label product also increases. In contrast to the increase in the price of the private-label product, the price of the national-brand product may again increase or decrease during economic distress. Finally, since the wholesale price decreases during economic distress, we find that the revenue of the manufacturer also decreases. The following result confirms that the negative effects of brand switching continue to persist for low-income consumers who experience a decrease in consumer surplus and access during economic distress.

Proposition O.5. *If $s \geq 1$*

(i) As λ_H decreases, both the consumer surplus and the demand served of low-income consumers at equilibrium always decreases.

(ii) As λ_H decreases, both the consumer surplus and the demand served of high-income consumers at equilibrium always increases.

Similar to the previous section, we focus our analysis on Price Control policy and confirm that the key results continue to hold in this setting. In particular, the probabilistic allocation of private-label product again hurts low-income consumers whose market access in fact decreases under Price Control policy. Further, inequity in the market for low-income consumers also increases as surplus of consumers who are able to purchase either of the two products increases while the total number of consumers who are unable to purchase either of the two products also increases.

Proposition O.6. *If $s = 1$, (i) The demand served of low-income consumers always decreases, and that of high-income consumers always increases under PC. (ii) The consumer surplus of low-income and high-income consumers increases under PC.*

O.1.3. When a Fraction of Consumers Update their Quality Threshold

In §3, we assumed that all high-income consumers update their quality threshold (λ_H), and that consumers' change in price sensitivity is uniform across the two products. In order to further check the robustness of our results to these assumptions, we relax both of these assumption.

First, we analyze a setting where only consumers who have consumed the product without economic distress change their perceived value of the product in the following manner. Recall from §3 that all high-income consumers with $b \geq \frac{p_R^{\phi *}}{v_R^u}$ consume the product without economic distress. In this extension, we assume that only high-income consumers with utility, $b \geq \frac{p_R^{\phi *}}{v_R^u}$ update their quality threshold. Since $\lambda_H = 1$ without any economic distress, from Theorem 1 we have that $p_R^{\phi *}$ without economic distress is $\frac{3v_R}{7}$. Thus, if $b < \frac{3v_R}{7\mu v_R} = 4/7$, high-income consumers do not update their quality threshold and their $\lambda_H = 1$ even during economic distress. Finally, if $b > \frac{4}{7}$, high-income consumers behave as before and update their quality threshold λ_H.

Second, in order to model drop in willingness for the national brand product (which may affect low-income consumers even more), we model an additional price sensitivity parameter ($\theta \in [1, 2]$) that allows for low-income consumers to exhibit higher price sensitivity towards national brand products during economic distress.[19] More formally, if $v_N b - \theta p_N > 0$ and $v_N b - \theta p_N > v_R b - p_R$, low-income consumers buy national brand product. Otherwise, if $v_N b - \theta p_N < v_R b - p_R$ and $v_R b - p_R > 0$, low-income consumers buy private-label product. For analytical tractability, we will not consider the shelf space constraint in this setting. We again start by characterizing the demand function for the two products in this setting.

[19] Our results continue to hold if we model increased price sensitivity for both high-income (θ_H) and low-income consumers (θ_L) as long as $\theta_L > \theta_H$. This assumption is reasonable since increase in price sensitivity should be more prominent among low-income consumers vis-a-vis high income consumers during periods of economic distress.

Lemma O.2. *Let $\mu_0 = 3/4$ and $v_R^{u_o} = v_R\mu_0$. The total demand for the two products is as follows:*

$$\begin{aligned} d_N(p_N,p_R,\lambda_H) &= \Big(\frac{1}{1+\mu_0} - \max\{\frac{p_N}{v_N}, \frac{p_N-p_R}{v_N-v_R^{u0}}\}\Big)^+ + \Big(1-\max\{\frac{p_N}{v_N}, \frac{p_N-p_R}{v_N-v_R^{u}}, \frac{1}{1+\mu_0}\}\Big)^+ \\ &\quad + \Big(1-\max\{\frac{\theta p_N}{v_N}, \frac{\theta p_N - p_R}{v_N - v_R}\}\Big)^+ \\ d_R(p_N,p_R,\lambda_H) &= \Big(\min\{\frac{1}{1+\mu_0}, \frac{p_N-p_R}{v_N-v_R^{u0}}\} - \frac{p_R}{v_R^{u0}}\Big)^+ + \Big(\min\{1, \frac{p_N-p_R}{v_N-v_R^{u}}\} - \max\{\frac{1}{1+\mu_0}, \frac{p_R}{v_R^{u}}\}\Big)^+ \\ &\quad + \Big(\min\{1, \frac{\theta p_N - p_R}{v_N-v_R}\} - \frac{p_R}{v_R}\Big)^+ \end{aligned}$$

Using the demand function characterized in the above lemma, we next characterize the equilibrium outcomes for the retailer and the national brand manufacturer.

Theorem O.3. *Let $\psi \equiv (28(1+\theta) - (59+\theta(22+28\mu+3\theta))v_R + (28+\mu(3+\theta(22+3\theta)))v_R^2)$.*

(i) The equilibrium wholesale price is

$$w_N^* = \frac{(1-\mu v_R)(-14+(11+3\theta)v_R)}{14(-1+v_R+\theta(-1+\mu v_R))}$$

(ii) The equilibrium retail price of the two products is,

$$\begin{aligned} p_R^* &= \frac{3v_R(70+42\theta - (129+3\theta(12+\theta)+14\mu(1+3\theta))v_R + (56+\mu(17+3\theta(12+\theta)))v_R^2)}{14\psi} \\ p_N^* &= \frac{-4(-1+v_R)\,(7+v_R(-1-3v_R+\mu(-7+4v_R)))}{\psi} \\ &\quad + \frac{(14(1+\theta) - 2(17+(4+7\mu)\theta)v_R + (17-3\theta+\mu(3+11\theta))v_R^2)\,w_N^*}{\psi} \end{aligned}$$

Proposition O.7. *(i) As λ_H decreases, p_R^* increases while w_N^* and p_N^* decrease.*
(ii) As λ_H decreases, revenue of both the retailer and the manufacturer decrease.

The above result aligns with the findings in Section §3, demonstrating that even in this setting, an increase in economic distress (indicated by a decrease in λ_H) leads to higher prices for private-label products. However, the impact on p_N^* is no longer non-monotonic; instead, it consistently decreases as λ_H declines in this scenario. This outcome is driven by several dynamics. As low-income consumers are more price-sensitive towards national brand products, the retailer faces increased pressure to lower retail prices for these products. While previously, under the conditions in Section §3, the markup over the wholesale price rose with economic distress, it now declines. Additionally, as before, the wholesale price continues to drop as λ_H decreases. Consequently, both the markup and the wholesale price of national brand products decline with increasing economic distress, leading to lower retail prices for national brands as λ_H decreases.

Further, while the retail prices of national brand products are falling, the prices of private-label products are rising with increasing economic distress. Therefore, it is not immediately obvious whether the retailer's overall revenue will increase or decrease. However, Proposition O.7 confirms that in contrast to §3, retailer's revenue in this scenario declines as λ_H decreases. This result is driven by the fact that the aggregate demand across the two consumer classes also decrease as economic distress intensifies. As a result, the retailer not only suffers from reduced prices of national brand products but also from the decline in overall demand, leading to a loss of revenue in the market.

PROPOSITION O.8. *For any $\theta \geq 1$, market access for both low-income and high-income consumers decreases as λ_H decreases. As λ_H decreases, surplus of high income consumers always increases. Further, if θ is sufficiently large, surplus of low income consumers always decreases as λ_H decreases.*

Similar to the analysis in §3, we observe that access for low-income consumers decreases as λ_H increases in this setting. However, the surplus for low-income consumers only decreases when θ is sufficiently large. When θ is small, the surplus of low-income consumers actually increases as λ_H decreases. This occurs because a significant portion of consumers continue to purchase the national brand at equilibrium and benefit from the resulting decrease in its prices. Conversely, when θ is large, the heightened price sensitivity toward the national brand means that fewer consumers benefit from its reduced prices. The resulting increase in the prices of private-label products outweighs these benefits, leading to a net decrease in the aggregate surplus for low-income consumers.

Finally, as in §3, we find that the surplus for high-income consumers increases with rising economic distress. However, unlike in §3, access for high-income consumers decreases under these conditions. This is because a portion of high-income consumers do not adjust their quality thresholds, and the significant increase in the prices of private-label products deters them from purchasing these products, leading to reduced access. Nonetheless, those consumers who continue to purchase at equilibrium benefit from the lower prices of national brand products, resulting in an overall increase in aggregate surplus.

O.1.4. Price Control Policy with Regulated Wholesale Price

In §3, we assume that while the retailer is required to charge the same retail price under PC, the manufacturer can optimize the wholesale price. In this section, we test the robustness of our results to this assumption and assume $s = 1$ for simplicity.

The objective function of the retailer in this case is,

$$\pi_{Re}^{PC}(i_N, i_R) = \max_{i_N, i_R} \left\{ (p_N^{\phi*} - w_N^{\phi*}) \min\{d_N^{pot}, i_N\} + p_R^{\phi*} \min\{d_R^{pot}, i_R\} | \min\{d_N^{pot}, i_N\} + \min\{d_R^{pot}, i_R\} \leq 1 \right\}$$

The next theorem characterizes the equilibrium solution to the Stackelberg game.

THEOREM O.4. *Equilibrium shelf space of national-brand product is,* $i_N^* = d_N^{pot}$ *and that of private-label brand product is,* $i_R^* = 1 - d_N^{pot}$*, where* d_N^{pot} *is the solution to the following equation:*

$$d_N^{pot} = d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) + \Big(1 - \frac{1 - d_N^{pot}}{d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)}\Big)\Big(\frac{p_N^{\phi*} - p_R^{\phi*}}{1 - v_R} - p_N^{\phi*} + \frac{p_N^{\phi*} - p_R^{\phi*}}{1 - v_R^u} - p_N^{\phi*}\Big)$$

The above result shows that the additional control on the manufacturer in the supply chain interestingly has no effect on the retailer's optimal decision. Since the per unit revenue from the national-brand product is higher than that of the private-label product, it is still optimal for the retailer to prioritize the shelf space for the national-brand product. As such, our results related to the consumer surplus of the two consumer classes continue to hold in an identical manner in this setting.

O.2. Results from the Parallel Trends Tests

The key identifying assumption for a DID approach is that the difference in private-label share between the treatment and control groups would have remained constant over time in the absence of the treatment; i.e., the private-label shares are parallel between the two income groups prior to the treatment. We verify that the parallel trends assumption is satisfied by estimating the following model following the literature (Jensen 2007):

$$\log(\bar{S}_{hmy}) = \gamma_0 T + \gamma_1 \alpha_h \times T + \gamma_2 \alpha_h + \gamma_3 p_{hmy} + X_{hy} + \epsilon_{hmy}. \tag{O.1}$$

The dependent variable is the logarithm of the percentage of the total expenditure spent on private-label for household h in month m of year y. The variable T denotes the number of months since the start of the data and controls for linear time trend. The dummy variable α_h is 1 if household h is in the high-income group and 0 otherwise. p_{hmy} controls for the weighted price of private-label products bought by the household h in month m and year y. X_{hy} includes a set of control variables at the household-year level that were discussed previously and include the following: race of the household, marital status, type of residence and household composition and County Code in which the household lies. Finally, we cluster standard errors at the County Code level to account for potential correlations across observations from households in the same County Code. A non-significant γ_1 indicates that we cannot reject the null hypothesis that private-label share in the high-income and low-income households follow the same linear pre-trend.

Similarly, the parallel trends assumption to evaluate our second insight regarding prices of private-label brands would require that prices of private-label and national-brands would have remained parallel without treatment. In order to verify this assumption, we estimate the following equation:

$$\log(\bar{P}_{bmy}) = \gamma_0 T + \gamma_1 \alpha_b \times T + \gamma_2 \alpha_b + \psi_d + \epsilon_{bmy}. \tag{O.2}$$

The dependent variable is the logarithm of the average monthly price of brand b for month m in year y. The variable T denotes the number of months since the start of the data and controls for linear time trend. The dummy variable α_b is 1 if if the observation is from private-label brands and 0 otherwise. ψ_d controls for department-level fixed effects. Finally, we cluster standard errors at the the department level to account for potential correlations across observations from the same department. A non-significant γ_1 indicates that we cannot reject the null hypothesis that prices of national and private-label brands follow the same linear pre-trend.

Table O.1 presents the results from estimating Equation O.1 and Equation O.2. γ_1 is not significant for either of the specifications and this confirms that a common parallel trend between the two groups prior to the implementation cannot be rejected.

Table O.1 Parallel Trends Assumption

Variable	Equation (O.1)	Equation (O.2)
γ_0	-0.062	0.004**
	(1.03)	(0.001)
γ_1	0.004	0.001
	(0.001)	(0.001)
γ_2	-0.152***	-0.200**
	(0.007)	(0.045)
Fixed effects	Y	Y
Observations	186,636	3,279,831

Notes. "–" means the variable is not present in the model. Standard errors (in parentheses) are clustered at the zip code level. ***: $p < 0.01$; **: $p < 0.05$; *: $p < 0.1$.

The results in the above table confirm that since γ_1 is not significant, we cannot reject the null hypothesis that the two groups followed the same time trend pre-tretament.

O.3. Robustness Checks

In order to further strengthen our empirical insights, we consider the following robustness tests. First, since consumers who switch from high-income groups to low-income groups (and vice-versa) may bias our results, we exclude them from our data and re-estimate Eq. (6) (Column 2). Second, we classify consumers with income above USD $50,000 as high-income consumers and re-estimate Eq. (6) to ensure that our results are not driven by our choice of the threshold (Column 3). Third, since our panel data has more post treatment data that may bias results, we re-estimate Equation (6) (Column 4) and Equation (7) (Column 5) by only using data from January, 2007 to December, 2008. Table O.2 presents the results from these robustness checks and confirms a similar positive and statistically significant value of γ_2 as in the main DID model.

Table O.2 Robustness Checks for Estimated Impact of Economic Recession on private-label Consumption

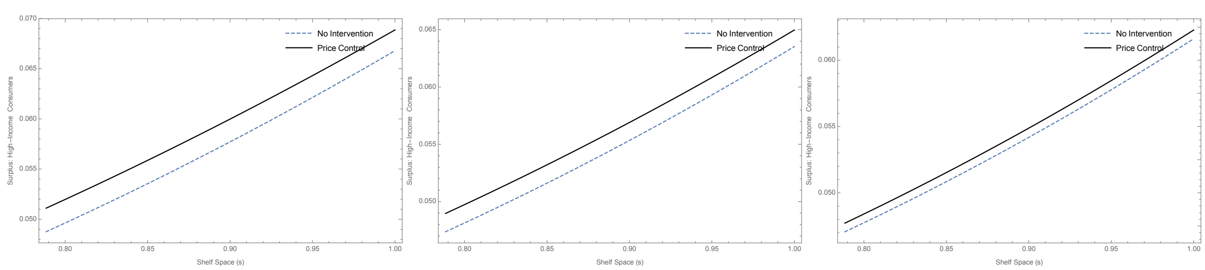
Variable	No Switch	Different Threshold	2007-2008 Data	2007-2008 Data (Price)
γ_0	-0.167***	-0.150***	-0.148***	-0.198***
	(0.005)	(0.005)	(0.004)	(0.028)
γ_1	0.017***	0.019***	0.010**	0.012**
	(0.004)	(0.004)	(0.004)	(0.003)
γ_2	0.083***	0.078***	0.071***	0.053***
	(0.002)	(0.003)	(0.003)	(0.008)
γ_3	1.610***	1.612***	1.652***	–
	(0.007)	(0.007)	(0.007)	
Fixed effects	Y	Y	Y	Y
Observations	670,049	751,249	498,263	7,851,590

Notes. "–" means the variable is not present in the model. Standard errors (in parentheses) are clustered at the zip code level. ***: $p < 0.01$; **: $p < 0.05$; *: $p < 0.1$.

O.4. Additional Numerical Simulations

We analyze the impact of Price Control (PC) on the consumer surplus of low-income and high-income consumers in scenarios where shelf space is less than 1. Our simulations confirm that the key insight from Proposition 7 remains valid: PC increases surplus compared to the no intervention setting for both consumer types. Specifically, we use three different values of λ_H to simulate and plot the surplus values for various shelf space levels for both high-income (Figure O.1) and low-income (Figure O.2) consumers. Furthermore, the increase in surplus is more pronounced when economic distress is higher, highlighting the effectiveness of PC during such periods.

Figure O.1 Surplus of High-Income Consumers under PC

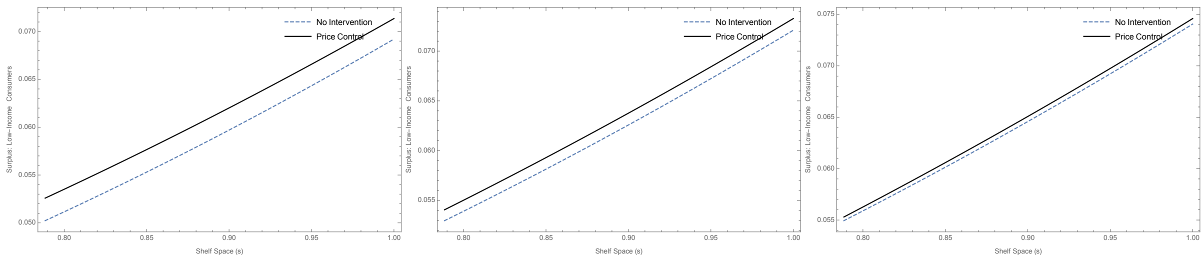


(a) High Distress (b) Intermediate Distress (c) Low Distress

Note. We use the following parameter values to generate the plots: $v_R = 0.4$ for all three plots and $\mu = 0.9$ (left), 0.8 (middle) and 0.7 (right). On the y-axis, we plot the consumer surplus of high-income consumers under both PC and no-intervention. On the x-axis, we vary values of shelf space, s from 0.75 to 1. Note that economic distress is accompanied with increase in switching intensity (or a decrease in λ_H).

Finally, in Figure O.3, we confirm that the key insights with regards to the comparison of different policies as discussed in §5 continue to hold. For instance, we again find that for high-income consumers, Price Control increases surplus while Cash Subsidy decreases surplus vis-a-vis no intervention. Similarly, for low-income consumers, we find that when μ is lower than threshold

Figure O.2 Surplus of Low-Income Consumers under PC

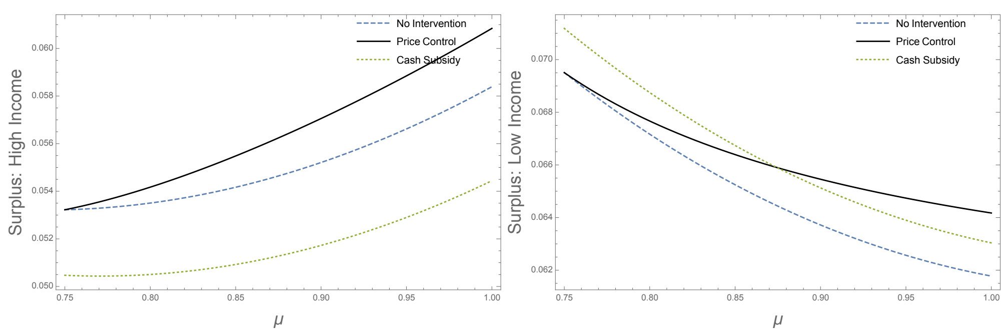


(a) High Distress (b) Intermediate Distress (c) Low Distress

Note. We use the following parameter values to generate the plots: $v_R = 0.4$ for all three plots and $\mu = 0.9$ (left), 0.8 (middle) and 0.7 (right). On the y-axis, we plot the consumer surplus of high-income consumers under both PC and no-intervention. On the x-axis, we vary values of shelf space, s from 0.75 to 1. Note that economic distress is accompanied with increase in switching intensity (or a decrease in λ_H).

(economic distress is not severe), Cash Subsidy is optimal, while when μ is high (economic distress is severe), Price Control is optimal.

Figure O.3 Surplus Comparison of High-Income and Low-Income Consumers Under Different Policies



(a) High Income Consumers (b) Low Income Consumers

Note. We use the following parameter values to generate the plots: $v_R = 0.4$, $c = 0.02$ and $s = 0.9$. On the y-axis, we plot the consumer surplus of high-income (left) and low-income (right) consumers under all three policies.

O.5. Proofs of Key Results

We include proofs of key results in this section due to space constraints. All other proofs can be obtained from the authors upon request.

Proof of Theorem 1 First consider the case when $s \geq 1$. In order to characterize the equilibrium solution for this bi-level optimization problem, we start by solving for the retailer's optimal p_N^* and p_R^* as a function of manufacturer's national-brand product price, w_N. The objective of the retailer is $(p_N - w_N)d_N(p_N, p_R, \lambda_H) + p_R d_R(p_N, p_R, \lambda_H)$ where $d_N(p_N, p_R, \lambda_H)$ and $d_R(p_N, p_R, \lambda_H)$ are characterized in Lemma 1. Note that if $p_R \geq p_N v_R^u$, then demand of H consumers for R would be 0 due to the high price. Thus we solve for the optimal solution under the two cases separately.

First, consider the retailer's problem under the constraint $p_R \leq p_N v_R^u$. In this case it is easy to check that the revenue function is concave in p_R and p_N. Thus, the optimal solution is,

$$p_N^* = \frac{w_N^*}{2} + \frac{1 - v_R + \mu(1 - \mu v_R)}{(1+\mu)(2 - v_R - \mu v_R)}, p_R^* = \frac{\mu v_R}{1+\mu}$$

Finally, the optimal solution satisfies the constraint $p_R^* \leq p_N^* v_R^u$ if and only if $w_N > w_t$ where w_t is a threshold, $w_t = \dfrac{-8\lambda_H^2 + 8\lambda_H^2 v_R - 2\lambda_H^4 v_R}{(-64 + 8\lambda_H^2 + 64 v_R - 16\lambda_H^2 v_R + \lambda_H^4 v_R}$. Thus, to summarize, if $w_N > w_t$, p_N^* and p_R^* are optimal. Otherwise, the constraint, $p_R \leq p_N v_R^u$, is strict and resolving the optimization problem with this constraint gives, $p_N = \dfrac{2(8(1+w_N) + v_R(-8 + (-8 + \lambda_H^2) w_N)}{(32 + (-32 + \lambda_H^4) v_R)}$.

Next, consider the retailer's optimization problem with the constraint $p_R \geq p_N v_R^u$. In this case, high-income consumers do not buy any private-label products. Demand for N is, $d_N = (1 - p_N) + 1 - (p_N - p_R)/(1 - v_R)$, and R is, $d_R = (p_N - p_R)/(1 - v_R) - p_R/v_R$. Solving the retailer's optimization problem, we get, $p_N^* = (1 + w_N)/2, p_R^* = v_R/2$. This optimal solution satisfies the constraint $p_R^* \geq p_N^* v_R^u$ if and only if $w_N \leq \dfrac{\lambda_H^2}{4 - \lambda_H^2}$. Otherwise, the constraint is strict and resolving the optimization problem with this constraint gives, $p_N = \dfrac{2(8(1+w_N) + v_R(-8 + (-8 + \lambda_H^2) w_N)}{(32 + (-32 + \lambda_H^4) v_R)}$. Finally, it is easy to check that in both the cases, the shelf space constraint is satisfied if $s \geq 1$.

Combining the results in these two regions, we get that there exists another threshold w_T, such that when $w_N < w_T$, $p_N^* = (1 + w_N)/2, p_R^* = v_R/2$ is optimal, while if $w_N > w_T$, $p_N^* = \dfrac{w_N^*}{2} + \dfrac{8(1 - v_R) + \lambda_H^2(v_R + v_R^u - 1)}{(8 - \lambda_H^2)(2 - v_R - v_R^u)}, p_R^* = \dfrac{v_R}{1 + v_R/v_R^u}$ is optimal.

Now that we have characterized the retailer's best response to manufacturer's national-brand price, w_N, we can optimize the wholesale's objective function taking this best response into account. In particular, if $w_N \leq w_T$, using the retailer's best-response function, revenue for the manufacturer is $w_N - ((-2 + v_R) w_N^2)/(2(-1 + v_R))$ and if $w_N \geq w_t$, revenue is $\dfrac{w_N(8(-1 + w_N) - 8 v_R(-2 + v_R + w_N) + \lambda_H^2 v_R(-2 + 2v_R + w_N))}{2(-1 + v_R)(4 + (-4 + \lambda_H^2) v_R)}$. Finally, if $v_R \leq 0.4$, it is easy to check that the revenue function is always increasing when $w_N < w_T$. And the optimal revenue when $w_N > w_T$ is $\dfrac{-((-1 + v_R)(4 + (-4 + \lambda_H^2) v_R))}{2(8 + (-8 + \lambda_H^2) v_R)}$, and $w_N^* = \dfrac{(1 - v_R)(1 - v_R^u)}{2 - (v_R + v_R^u)}$. The last step is to compare the revenue at w_T and w_N^*. We confirm that the revenue at w_N^* is indeed higher than that at w_T when $v_R \leq 0.4$.

Next, consider the case when $s < 1$. We will again start by solving for the retailer's problem. We again start with the case when $p_R \leq p_N v_R^u$. Since $s \leq 1$, the shelf space constraint is tight and $d_N + d_R = s$ at optimality. Replacing the values of d_N and d_R in terms of p_N and p_R, this constraint is equivalent to $1 - \frac{p_r}{V_r} + 1 - \frac{p_R}{v_R^u} = s$. This implies that $p_R = \frac{\mu(2-s)v_R}{1+\mu}$. Replacing this value in the objective function and solving for p_N, we get,

$$p_N^{\phi*} = \frac{w_N^{\phi*}}{2} + \frac{1 - v_R + \mu(1 - v_R(\mu - 2(1 - s) + (1+\mu)(1 - s) v_R))}{(1+\mu)(2 - v_R - \mu v_R)}$$

Note that the constraint $p_R \le p_N v_R^u$ is again satisfied when w_N is greater than a threshold, w_t. Next, consider the retailer's optimization problem with the constraint $p_R \ge p_N v_R^u$. In this case, high-income consumers do not buy any private-label products. If $1 > s \ge \frac{(3-v_R)}{(4-2v_R)}$, then the optimal is again $p_N^* = (1+w_N)/2, p_R^* = v_R/2$. Further, the shelf space constraint, $d_N(p_N^*, p_R^*) + d_R(p_N^*, p_R^*) \le s$ is indeed satisfied. If $s \le \frac{(3-v_R)}{(4-2v_R)}$, the shelf space constraint is tight and $p_N^* = \frac{(1+3v_R-2sv_R+w_N}{2+2v_R}, p_R^* = (2-s-p_N)v_R$. Combining the results from both the cases, $p_N^{\phi*} = \frac{w_N^{\phi*}}{2} + \frac{1-v_R+\mu(1-v_R(\mu-2(1-s)+(1+\mu)(1-s)v_R))}{(1+\mu)(2-v_R-\mu v_R)}$ when w_N is greater than a threshold, w_t. Otherwise if $w_N < w_t$: if $1 > s \ge \frac{(3-v_R)}{(4-2v_R)}$, then the optimal is $p_N^* = (1+w_N)/2, p_R^* = v_R/2$ and if $s < \frac{(3-v_R)}{(4-2v_R)}$, $p_N^* = \frac{(1+3v_R-2sv_R+w_N}{2+2v_R}, p_R^* = (2-s-p_N)v_R$. Solving the manufacture's problem as before using the retailer's best response function, we again find that the objective function is always increasing when $w_N < w_t$. And in the region when $w_N \ge w_t$, the optimal $w_N^* = \dfrac{(1-v_R)(1-v_R^u)}{2-(v_R+v_R^u)}$. We confirm that the revenue at w_N^* is indeed higher than that at w_T when $v_R \le 0.4$. Finally, since $s \ge \frac{1}{2} + \frac{3}{16-14v_R}$, substituting the value of w_N^*, in the definition of p_N^* we can confirm that $p_R^* < p_N^* v_R^u$. Finally, it is easy to confirm that $p_N^* - p_R^*$ is larger when $\lambda_H = 0$ than when $\lambda_H > 0$.

Proof of Proposition 1 First consider the case when $s \ge 1$. Note that $p_R^{\phi*} = \dfrac{(-4+\lambda_H^2)v_R}{(-8+\lambda_H^2)}$. Differentiating with respect to λ_H, we get $\dfrac{-8\lambda_H v_R}{(-8+\lambda_H^2)^2}$ which is always less than 0. Next, consider the case when $s < 1$. In this case, $p_R^{\phi*} = \dfrac{(-4+\lambda_H^2)v_R(2-s)}{(-8+\lambda_H^2)}$. Differentiating with respect to λ_H, we get $\dfrac{-8\lambda_H v_R(2-s)}{(-8+\lambda_H^2)^2}$ which is always less than 0. This proves the first part.

Next, consider the result for $p_N^{\phi*}$. Substituting the value of $w_N^{\phi*}$ in the definition of $p_N^{\phi*}$. If $s \ge 1$,

$$p_N^{\phi*} = \frac{-(32(-3+v_R)(-1+v_R) + \lambda_H^4(-3+v_R)v_R - 4\lambda_H^2(3+v_R(-8+3v_R)))}{2(-8+\lambda_H^2)(8+(-8+\lambda_H^2)v_R)}$$

Differentiating with respect to λ_H, we get

$$\frac{4\lambda_H v_R(-16\lambda_H^2(-3+v_R)(-1+v_R) + 64(-1+v_R)^2 + \lambda_H^4(3+(-6+v_R)v_R))}{(-8+\lambda_H^2)^2(8+(-8+\lambda_H^2)v_R)^2}$$

It is easy to check that this term is negative if v_R is larger than a threshold, and λ_H is also larger than a threshold. We can similarly show the result for the case when $s < 1$. Finally, consider $w_N^{\phi*} = \dfrac{(1-v_R)(1-v_R^u)}{2-(v_R+v_R^u)}$. Differentiating with respect to λ_H, we get $\dfrac{8\lambda_H(-1+v_R)^2 v_R}{(8+(-8+\lambda_H^2)v_R)^2}$ which is always positive.

Proof of Proposition 2 We start with the case when $s \ge 1$. Differentiating the retailer's revenue with respect to λ_H, we get

$$\frac{\lambda_H}{2}\frac{v_R(-3+\mu+v_R+\mu v_R)(5+\mu-3(1+\mu)v_R)}{4(1+\mu)^2(2-v_R-\mu v_R)^2}$$

Next in the case when $s<1$, again differentiating with respect to λ_H,

$$\frac{\lambda_H}{2}\frac{v_R(1+\mu(2+\mu)-16(2-s)s-2(1+\mu)(1+\mu+8(-2+s)s)v_R-(1+\mu)^2(1+4(-2+s)s)v_R^2)}{4(1+\mu)^2(2-v_R-\mu v_R)^2}$$

Since the second term in both the cases is negative, the retailer's revenue increases as λ_H decreases. Similarly, differentiating the manufacturer's revenue with respect to λ_H, we get $\dfrac{(4\lambda_H(-1+v_R)^2v_R)}{(8+(-8+\lambda_H^2)v_R)^2}$ which is always positive. This proves the result.

Proof of Proposition 3 Consider the case when $s\geq 1$. We start by showing the result for low-income consumers. Aggregate demand for low-income consumers across the two product classes is $(1-p_R^{\phi*}/v_R)$. Replacing the value of p_R^*, we get $4/(8-\lambda_H^2)$ which is increasing in λ_H. Next, consider the surplus for low-income consumers. Plugging in the values of optimal $p_N^{\phi*}$ and $p_R^{\phi*}$, we get,

$$\frac{-\lambda_H^8(-1+v_R)v_R^2+1024(-1+v_R)^2(1+3v_R)+8\lambda_H^6v_R(-1+v_R^2)-256\lambda_H^2(-1+v_R)(-1+5v_R^2)}{8(-8+\lambda_H^2)^2(8+(-8+\lambda_H^2)v_R)^2} \tag{O.3}$$

$$+\frac{16\lambda_H^4(1+v_R(5+v_R(-9+7v_R)))}{8(-8+\lambda_H^2)^2(8+(-8+\lambda_H^2)v_R)^2} \tag{O.4}$$

It is easy to check that both the terms are increasing in λ_H This proves the first part of the proposition. Next, consider high-income consumers. We get that the total demand served of high-income consumers is $1-4/(8-\lambda_H^2)$. It is easy to check that the demand served of high-income consumers is indeed decreasing in λ_H. We can similarly calculate the surplus of high-income consumers and confirm that the surplus for high-income consumers always increases as λ_H decreases.

Next, consider the case when $s<1$. Note that the aggregate demand for low-income consumers across the two products is $\frac{(1-\mu(1-s))}{1+\mu}$. Differentiating with respect to λ_H, we get $\frac{\lambda_H(2-s)}{2(1+\mu)^2}$ which is always positive since $s<1$. We can similarly show the result for the consumer surplus of low-income consumers in a straightforward manner. Next, note that the aggregate demand for high-income consumers across the two products is $\frac{\mu+s-1}{1+\mu}$. Differentiating with respect to λ_H, we get $\frac{-\lambda_H(2-s)}{2(1+\mu)^2}$ which is always negative. Next, note that the consumer surplus of high-income consumers is convex in μ. Thus, if $\frac{\partial CW_H^\phi}{\partial\mu}$ evaluated at $\mu=3/4$ is positive then by the convexity property, CW_H^ϕ will always be increasing with μ (equivalently, decreasing with λ_H). Evaluating $\frac{\partial CW_H^\phi}{\partial\mu}$ at $\mu=3/4$, we get $\frac{2v_R(16224+40s(-8+7v_R)^3+4s^2(-8+7v_R)^3-21v_R(1783+7v_R(-194+49v_R)))}{343(-8+7v_R)^3}$. It is easy to check that this is indeed positive when $s>-5+\frac{7}{2}\sqrt{\frac{-1376+3507v_R-2982v_R^2+847v_R^3}{(-8+7v_R)^3}}$.

Proof of Theorem 2 We will again start by first characterizing the optimal solution for the retailer as a function of w_N. Solving the retailer's problem without the shelf space constraint, we get $p_R^*=\frac{\mu(c+2v_R)}{2(1+\mu)}$ and $p_N^*=\frac{-2(1+w_N)+v_R(2+w_N)+\mu^2v_R(2+c+w_N)+\mu(-2+c(-2+v_R)+2(-1+v_R)w_N)}{2(1+\mu)(-2+v_R+\mu v_R)}$. Further, the aggregate demand across the two products is $1+\frac{c}{2v_R}$. Thus, if $s\geq 1+\frac{c}{2v_R}$, then the optimal solution is indeed feasible and otherwise, if $s<1+\frac{c}{2v_R}$, it is not and the shelf space constraint is binding. We will consider both the cases separately.

First, consider the retailer's problem with a constraint $p_R \le p_N v_R^u$ and $s \le 1 + \frac{c}{2v_R}$. Using the condition on the shelf space, we get $p_{1R}^* = \frac{\mu(c+v_R(2-s))}{(1+\mu)}$. Solving for p_{1N}^*, we get $p_{1N}^* = \frac{w_N}{2} + \frac{1-v_R+\mu(1-\mu v_R)}{(1+\mu)(2-v_R-\mu v_R)} + \frac{c\mu - v_R(s-1)\mu}{(1+\mu)}$. Finally, checking the condition on prices and rewriting in terms of w_N, we get that there exists a threshold w_t such that if $w_N > w_t$, then the optimal solution satisfy the constraint. Otherwise, the constraint $p_R = p_N(1-v_R^u)$ is strict. We can similarly consider the other case where $p_R \ge p_N v_R^u$. In this case, none of the high-income consumers buy the private-label product. Resolving the retailer's problem with updated demand functions, we get $p_{2N}^* = (2+c+2w_N)/4$ and $p_{2R}^* = (-c(-2+v_R)+2v_R)/4$. Further, if w_N is less than another threshold w_t', then $p_r^* \ge p_N^* v_R^u$ is satisfied but otherwise it is not. Comparing the objective function values, we again have that if $w_N > w_t''$, where w_t'' is another threshold, then p_{1N}^*, p_{1R}^* is optimal, and otherwise p_{2N}^*, p_{2R}^* is optimal for the retailer. Next, we look at the manufacturer's objective function as a function of w_N and use the above characterization of the retailer's best response function. If $w_N > w_t''$, the manufacturer's revenue function is $w_N + \frac{(-2+v_R+\mu v_R)w_N^2}{2(-1+v_R)(-1+\mu v_R)}$. Finally, note that the objective function is always increasing in w_N when $w_N \le w_t''$ since $c \le 3v_R/28$ and $w_N^* = \dfrac{(1-v_R)(1-\mu v_R)}{2-v_R-\mu v_R}$ is the optimal solution and indeed greater than w_t''. In this case, p_{1N}^*, p_{1R}^* is the retailer's best response to the manufacturer's actions.

We can similarly consider the case when $s > 1 + \frac{c}{2v_R}$. In this case, the shelf space constraint is not binding and if w_N is larger than a threshold, then the optimal solution indeed satisfy the constraint $p_R^* \ge p_N^* v_R^u$. Similar to the above case, we can again show that the optimal solution for the wholesaler in this range is $w_N^* = \dfrac{(1-v_R)(1-\mu v_R)}{2-v_R-\mu v_R}$ and since $w_N^* > w_t'$, the retailer's best response is indeed $p_R^* = \frac{\mu(c+2v_R)}{2(1+\mu)}$ and $p_N^* = \frac{-2(1+w_N)+v_R(2+w_N)+\mu^2 v_R(2+c+w_N)+\mu(-2+c(-2+v_R)+2(-1+v_R)w_N)}{2(1+\mu)(-2+v_R+\mu v_R)}$.

Proof of Theorem 3 We will again start by solving for the retailer's optimal solution as a function of the manufacturer' price, w_N. Further, it will always be optimal to hold maximum shelf space (upto its potential demand) for the product whose per unit revenue is higher, and leave the remaining shelf space (upto its demand) for the other product. Thus, if $p_N^{\phi*} - w_N \ge p_R^{\phi*} \implies w_N \le p_N^{\phi*} - p_R^{\phi*}$, $i_N = d_N^{pot}$ and $i_R = \max\{0, \min\{s - d_N^{pot}, d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)\}$. The result for i_R follows because $i_R \le s - i_N$ (shelf space constraint) and $i_R \le d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$ (demand constraint) and the objective function is increasing in i_R.

If $p_N^{\phi*} - w_N < p_R^{\phi*} \implies p_N^{\phi*} > w_N > p_N^{\phi*} - p_R^{\phi*}\}$, $i_R = \min\{s, d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) + d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)\}$ and $i_N = 0$. This is because all consumers who have a positive utility but are unable to buy N will switch to buying R in the latter case and thus $d_R^{pot} = d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) + d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$. Thus, it is best for the retailer to let all consumers switch to the private-label product that gives a higher per unit revenue.

Next, we consider the manufacturer's problem taking the retailer's best response into account. If $w_N \le p_N^{\phi*} - p_R^{\phi*}$, manufacturer's objective is $w_N d_H^{pot}$. Otherwise, if $p_N^{\phi*} > w_N > p_N^{\phi*} - p_R^{\phi*}$, it is 0 since the retailer does not stock any national-brand product. Thus, it is optimal for the manufacturer to charge $p_N^{\phi*} - p_R^{\phi*}$. Finally, in order to characterize d_N^{pot}, note that the potential demand for N consists of two sets of consumers. The first set of consumers are all those who prefer buying N at $p_N^{\phi*}$ and $p_R^{\phi*}$, $d_N^{\phi*}(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$. In addition, we also have consumers who prefer buying R but can afford buying N in case they are unable to find R due to inventory unavailability. Since each customer who prefers buying R can buy R with probability $1 - i_R^*/d_R^{\phi}(p_N^{\phi*}, p_R^{\phi*})$, the second term captures all those who are unable to buy R and can afford buying N. Thus, the equilibrium shelf space allocation of the national-brand product is, $i_N^* = d_N^{pot}$ where d_N^{pot} is the solution to the following equation: $d_N^{pot} = d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) + \Big(1 - \frac{i_R^*}{d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)}\Big) d_{switch}$ and and that of the private-label product is, $i_R^* = \min\{s - d_N^{pot}, d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)\}$.

If $s - d_N^{pot} \le d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$, replacing the value of $i_R^* = s - d_N^{pot}$, $d_N^{pot} = d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) + \Big(1 - \frac{s - d_N^{pot}}{d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)}\Big) d_{switch}$. Notice that this equation is linear in d_N^{pot} and solving for d_N^{pot} gives $\dfrac{d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) + d_{switch}\Big(d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) - s\Big)}{d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H) - d_{switch}}$. Finally, replacing the values of $d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$,$d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$ and d_{switch} using $p_N^{\phi*}$ and $p_R^{\phi*}$ from Theorem 1 in the equation $s - d_N^{pot} \le d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$, we get $s \le \frac{11}{7} - \frac{3}{7\mu}$. Next, note that if $s - d_N^{pot} > d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$, we similarly get that, $d_N^{pot} = d_N(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$ since $i_R^* = d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$ and replacing this value in the equation $s - d_N^{pot} > d_R(p_N^{\phi*}, p_R^{\phi*}, \lambda_H)$, we get $s > \frac{11}{7} - \frac{3}{7\mu}$.

Proof of Lemma 4 We have already shown in Proposition 5 and Proposition 7 that $D_L^{CS} < D_H^{\phi}$ and $D_H^{\phi} < D_H^{PC}$ for H consumers. Further, $D_L^{\phi} \ge D_L^{PC}$ for L consumers if $s \le \frac{11}{7} - \frac{3}{7\mu}$, and $D_L^{\phi} < D_L^{PC}$ otherwise. Combining the results, if $s \le \frac{11}{7} - \frac{3}{7\mu}$, $D_L^{CS} \ge D_L^{\phi} \ge D_L^{PC}$. If $s > \frac{11}{7} - \frac{3}{7\mu}$, $D_L^{PC} = \frac{4}{7} > \frac{1}{2} + \frac{c}{2v_R} \ge D_L^{CS} \ge D_L^{\phi}$, where the second inequality follows from the assumption that $c \le \frac{3v_R}{28}$.

The result for the consumer surplus of high-income consumers also follows directly Proposition 5 and Proposition 7. The only result for consumer surplus that remains to be shown is that of the comparison between cash subsidy and price control policies for low income consumers. Note that there are two cases for CW_L^{PC} characterized in Proposition 7 depending on whether s is less than or greater than $11/7 - 3/7\mu$. Similarly, CW_L^{CS} characterized in Proposition 5 also has three cases depending on the parameter regions for s. Since $\mu = 1 - \lambda_H^2/4$, we will show the statement in the Lemma in terms of μ instead. There are four possible cases we need to consider: Case 1: $\frac{c+2v_R}{2v_R} < s < 8/7$ and $\frac{3}{11-7s} < \mu < 1$. Case 2: $1 < s < \frac{c+2v_R}{2v_R}$ and $\frac{3}{11-7s} < \mu < 1$. Case 3: $\frac{c+2v_R}{2v_R} < s < \frac{8}{7}$ and $\frac{3}{4} < \mu < \frac{3}{11-7s}$. Case 4: $1 < s < \frac{c+2v_R}{2v_R}$ and $\frac{3}{4} < \mu < \frac{3}{11-7s}$. In each of the cases, we need to only show that if there exists a $\mu^t \in (0,1)$ s.t. $CW_L^{PC} > CW_L^{CS}$, then for all $\mu > \mu^t$, $CW_L^{PC} > CW_L^{CS}$. We will show this result by contradiction. Note that since the surplus is strictly decreasing in μ under both CS and PC, there can at most be one crossing point between the two surplus curves. By contradiction, assume that there exists a μ^t such that if $\mu < \mu^t$, $CW_L^{PC} > CW_L^{CS}$ and if $\mu >$

μ^t, $CW_L^{PC} < CW_L^{CS}$. Consider Case 1. The above condition implies that $CW_L^{PC} > CW_L^{CS}$ when evaluated at $\mu = \frac{3}{11-7s}$ and $CW_L^{PC} < CW_L^{CS}$ when evaluated at $\mu = 1$. By substituting the surplus values under PC and CS evaluated at these points, it is easy to check that the two conditions cannot simultaneously be true. We can similarly show the result for each of the cases.