E-Companion for "The Effects of Selling Formats and Upstream Competition on Product Pricing and Quality Design"

This e-companion consists of six parts: Appendix A (summary of notation), Appendix B (main proofs), Appendix C (a channel with one manufacturer and one intermediary when $K = 1$), Appendix D (Comparison Analysis with the Identical K), Appendix E (Prioritize agency fee), and Appendix F (Numerical Illustration of Result 2).

Appendix A. Summary of Notation

Table A.1: Summary of Notation

Notation	Description
Decision variables	
q	quality of a product
p	retail price of a product
w	wholesale price of a product
γ	agency fee charged by the intermediary in the agency selling format
Parameters	
c	unit production cost
Q	market demand of a product
θ	consumers' willingness-to-pay for product quality q
K	markets differ in the concentration of consumers with different levels of θ
Performance measures	
$\mathcal{U}$	utility of each consumer
$f(\theta)$	probability density function of θ
$F(\theta)$	cumulative distribution function of θ
Π^i	channel profit in retail configuration i, $i \in \{0, AS, RS\}$
CS^i	consumer surplus in retail configuration i, $i \in \{0, AS, RS\}$
Π^j_{INT}	intermediary's profit in retail configuration j, $j \in \{AS, RS\}$
Π^j_M	manufacturer's profit in retail configuration j, $j \in \{AS, RS\}$

Appendix B. Proofs of Main Results

Proof of Proposition 1

In the centralized distribution channel, the channel profit is $\Pi^0 = (p - cq^2/2)Q$, where $Q = (1 - p/q)^K$. We solve $\max \Pi^0$ subject to $p > 0$ and $q > 0$. The Hessian matrix of Π^0_M is developed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \Pi^0_M}{\partial p^2} & \frac{\partial^2 \Pi^0_M}{\partial p \partial q} \\ \frac{\partial^2 \Pi^0_M}{\partial q \partial p} & \frac{\partial^2 \Pi^0_M}{\partial p^q} \end{bmatrix}. \tag{A.1}$$

Let H_i be the leading principal minor of order i; we have $H_1 = -\frac{1}{2}cK(1+K)\left(\frac{K}{2+K}\right)^{-2+K} < 0$, and $H_2 = \frac{1}{4}c^2K(2+K)\left(\frac{K}{2+K}\right)^{2(-1+K)} > 0$. Thus, H is negative definite; this implies that Π^0_M is jointly concave in (p, q). From the first-order conditions, we have

$$\frac{\partial \Pi^0}{\partial p} = \left(1 - \frac{p}{q}\right)^K - \frac{1}{q}\left(p - \frac{1}{2}cq^2\right)K\left(1 - \frac{p}{q}\right)^{K-1} = 0, \tag{A.2}$$

$$\frac{\partial \Pi^0}{\partial q} = -cq\left(1-\frac{p}{q}\right)^K + \frac{p}{q^2}\left(p-\frac{1}{2}cq^2\right)K\left(1-\frac{p}{q}\right)^{K-1} = 0. \tag{A.3}$$

From Equation (A.2), we have $\left(1-\frac{p}{q}\right)^K = \frac{1}{q}\left(p-\frac{1}{2}cq^2\right)K\left(1-\frac{p}{q}\right)^{K-1}$. Substituting $\left(1-\frac{p}{q}\right)^K$ into Equation (A.3) yields $p = cq^2$. Then, substituting $p = cq^2$ back into Equation (A.3) can yield the equilibrium product quality $q^0 = \frac{2}{c(2+K)}$; accordingly, we have the equilibrium retail price $p^0 = c(q^0)^2 = \frac{4}{c(2+K)^2}$. Q.E.D.

Proof of Proposition 2

Under the agency selling agreement, the the manufacturer's profit is $\Pi_M^{AS} = \left((1-r)p - cq^2/2\right)(1-p/q)^K$. The second-order condition of π_M^{AS} with respect to p yields $\frac{\partial^2 \Pi_M^{AS}}{\partial p^2} = \frac{-(1-\gamma)(1+K)}{q} < 0$. Thus, π_M^{AS} is concave in p. The first-order condition of π_M^{AS} with respect to p yields $p^* = \frac{q(2-2\gamma+Kcq)}{2(1+K)(1-\gamma)}$. Then, we obtain $Q^* = \left(\frac{K(2-2\gamma-cq)}{2(1+K)(1-\gamma)}\right)^K$. Next, substituting p^* and Q^* into the intermediary's payoff function, we have $\Pi_I^{AS} = \gamma p^* Q^* = \gamma \frac{q(2-2\gamma+Kcq)}{2(1+K)(1-\gamma)}\left(\frac{K(2-2\gamma-cq)}{2(1+K)(1-\gamma)}\right)^K$. The first-order condition of π_{INT}^{AS} with respect to γ, we have γ^* which should satisfy $4(1-\gamma)^3 - 2(1-K)cq(1-\gamma)^2 + K^2c^2q^2(1-\gamma) - K(1+K)c^2q^2 = 0$. Thus, the manufacturer's problem can be reformulated as follows:

$$\begin{aligned} \max_{q>0} \Pi_M^{AS} &= \left[\frac{q(2-2\gamma+Kcq)}{2(1+K)} - \frac{1}{2}cq^2\right]\left(\frac{K(2-2\gamma-cq)}{2(1+K)(1-\gamma)}\right)^K, \\ \text{s.t. } & 4(1-\gamma)^3 - 2(1-K)cq(1-\gamma)^2 + K^2c^2q^2(1-\gamma) - K(1+K)c^2q^2 = 0. \end{aligned} \tag{A.4}$$

First, given $4(1-\gamma)^3 - 2(1-K)cq(1-\gamma)^2 + K^2c^2q^2(1-\gamma) - K(1+K)c^2q^2 = 0$ and $q > 0$, we can obtain the unique closed-form expression for q, i.e., $q = \frac{-(1-K)(1-\gamma)^2 + (1-\gamma)\sqrt{(1+K)(1-\gamma)(1+K-\gamma+3\gamma K)}}{K(1+\gamma K)c}$. Then, substituting q into Π_M^{AS} and taking the first-order condition of Π_M^{AS} with respect to γ, we can obtain $4(1+K)^2 - (1+K)(12+K)\gamma + 8(1-3K-2K^2)\gamma^2 - 4K(3K-1)\gamma^3 = 0$; solving this yields

$$\gamma^* = \begin{cases} \frac{\alpha_0 + (1+i\sqrt{3})(\alpha_1-\sqrt{27\alpha_2})^{\frac{1}{3}} + (1-i\sqrt{3})(\alpha_1+\sqrt{27\alpha_2})^{\frac{1}{3}}}{12K(3K-1)} & \text{if } K \geqslant \frac{1}{3}, \\ \frac{\alpha_0 + (1-i\sqrt{3})(\alpha_1-\sqrt{27\alpha_2})^{\frac{1}{3}} + (1+i\sqrt{3})(\alpha_1+\sqrt{27\alpha_2})^{\frac{1}{3}}}{12K(3K-1)} & \text{if } K < \frac{1}{3}, \end{cases} \tag{A.5}$$

where $\alpha_0 = -8(-1+3K+2K^2)$, $\alpha_1 = -64 + 360K - 390K^2 - 270K^3 + 78K^4 - 522K^5 - 568K^6$ and $\alpha_2 = K^2(2-5K-4K^2+3K^3)^2(-16-328K+527K^2+1242K^3+643K^4)$. Q.E.D.

Proof of Lemma 1

Given w and p, taking the first-order condition of Π_{INT}^{RS} with respect to p yields the unique solution for retail price, i.e., $p = \frac{q+Kw}{1+K}$. Note that $\frac{\mathrm{d}\Pi_{INT}^{RS}}{\mathrm{d}p} = q\left(1-\frac{p}{q}\right)^{K-1}(q+Kw-(1+K)p) > 0$ when $p < \frac{q+Kw}{1+K}$ and $\frac{\mathrm{d}\Pi_{INT}^{RS}}{\mathrm{d}p} = q\left(1-\frac{p}{q}\right)^{K-1}(q+Kw-(1+K)p) < 0$ when $p > \frac{q+Kw}{1+K}$. Thus, given $p = \frac{q+Kw}{1+K}$, the function $\Pi_{INT}^{RS}(p|w,q)$ is unimodal in p. Q.E.D.

Proof of Proposition 3

By Lemma 1, we have the intermediary's best response $p^* = \frac{q+Kw}{1+K}$. Substituting $p = p^*$ and $Q = \left(1-\frac{p^*}{q}\right)^K$ into the manufacturer's payoff function formulated in Equation (6), we obtain $\Pi_M^{RS} = (w - cq^2/2)\left(1-\frac{q+wK}{q(1+K)}\right)^K$. Taking the first-order conditions of Π_M^{RS} with respect to w and q, respectively, we have the following results:

$$\frac{\partial \Pi_M^{RS}}{\partial w} = \left(1 - \frac{q+wK}{q(1+K)}\right)^K - \frac{K}{q(K+1)}\left(w - \frac{1}{2}cq^2\right) K \left(1 - \frac{q+wK}{q(1+K)}\right)^{K-1} = 0, \tag{A.6}$$

$$\frac{\partial \Pi_M^{RS}}{\partial q} = -cq\left(1 - \frac{q+wK}{q(1+K)}\right)^K + \frac{wK}{q^2(K+1)}\left(w - \frac{1}{2}cq^2\right) K \left(1 - \frac{q+wK}{q(1+K)}\right)^{K-1} = 0. \tag{A.7}$$

From Equation (A.6), we can show $\left(1 - \frac{q+wK}{q(1+K)}\right)^K = \frac{K}{q(K+1)}\left(w - \frac{1}{2}cq^2\right) K \left(1 - \frac{q+wK}{q(1+K)}\right)^{K-1}$. Then, substituting $\left(1 - \frac{q+wK}{q(1+K)}\right)^K$ into Equation (A.7) yields $w = cq^2$. Next, substituting $w = cq^2$ back into Equation (A.6) yields the equilibrium product quality $q^{RS} = \frac{2}{(2+K)c}$; accordingly, we have the equilibrium wholesale price $w^{RS} = \frac{4}{(K+2)^2c}$ and the equilibrium retail price $p^{RS} = \frac{4+6K}{(1+K)(2+K)^2c}$. Q.E.D.

Proof of Corollary 1

By incorporating different values of K under agency selling and reselling agreements (i.e., K^{AS} and K^{RS}), we compare the equilibrium outcomes of the product quality, the retail price, and the market demand, respectively.

We first examine the quality levels in the decentralized selling channels. Specifically, by comparing q^{RS} and q^{AS}, we have following results:

$$\begin{cases} q^{AS} < q^{RS} & \text{if } (K^{AS}, K^{RS}) \in \Lambda_1, \\ q^{AS} \geqslant q^{RS} & \text{otherwise,} \end{cases} \tag{A.8}$$

where $\Lambda_1 = \left\{(K^{AS}, K^{RS}) : K^{AS}, K^{RS} \in (0, \infty), K^{RS} < \frac{2}{\Delta(K^{AS})} - 2\right\}$ and
$\Delta(K^{AS}) = \frac{-(1-K^{AS})(1-\gamma^*)^2 + (1-\gamma^*)\sqrt{(1+K^{AS})(1-\gamma^*)(1+K^{AS}-\gamma^*+3\gamma^*K^{AS})}}{K^{AS}(1+\gamma^*K^{AS})}$.

We then examine the retail prices between the agency selling and reselling channels. By comparing p^{RS} and p^{AS}, we get the following results:

$$\begin{cases} p^{AS} < p^{RS} & \text{if } (K^{AS}, K^{RS}) \in \Lambda_2, \\ p^{AS} \geqslant p^{RS} & \text{otherwise,} \end{cases} \tag{A.9}$$

where $\Lambda_2 = \left\{(K^{AS}, K^{RS}) : K^{AS}, K^{RS} \in (0, \infty), \Delta(K^{AS})\frac{2-2\gamma^*+K^{AS}\Delta(K^{AS})}{2(1+K^{AS})(1-\gamma^*)} < \frac{4+6K^{RS}}{(1+K^{RS})(2+K^{RS})^2}\right\}$.

Finally, by comparing Q^{AS} and Q^{RS}, we take the natural log with respect to their quotient. Then, we have

$$\frac{\ln Q^{AS}}{\ln Q^{RS}} = \frac{K^{AS}\ln(1 - \xi(K^{AS}))}{K^{RS}\ln(1 - \psi(K^{RS}))}, \tag{A.10}$$

where $\xi(K^{AS}) = \frac{2-2\gamma^*+K^{AS}\Delta(K^{AS})}{2(1+K^{AS})(1-\gamma^*)}$ and $\psi(K^{RS}) = \frac{2+3K^{RS}}{(1+K^{RS})(2+K^{RS})}$. Given $\ln(x)$ is strictly increasing on $x > 0$, we have $Q^{AS} < K^{RS}$ if $\frac{K^{AS}\ln(1-\xi(K^{AS}))}{K^{RS}\ln(1-\psi(K^{RS}))} < 1$; otherwise, $Q^{AS} \geqslant K^{RS}$. That is, we obtain mutually exclusive results:

$$\begin{cases} Q^{AS} < Q^{RS} & \text{if } (K^{AS}, K^{RS}) \in \Lambda_3, \\ Q^{AS} \geqslant Q^{RS} & \text{otherwise,} \end{cases} \tag{A.11}$$

where $\Lambda_3 = \left\{(K^{AS}, K^{RS}) : K^{AS}, K^{RS} \in (0, \infty), K^{AS}\ln(1 - \xi(K^{AS})) < K^{RS}\ln(1 - \psi(K^{RS}))\right\}$. Q.E.D.

Proof of Proposition 4

(i) Taking the first order condition of Π^0 with respect to K^0, we can show $\frac{d\Pi_M^0}{dK^0} = 2\left(\frac{K^0}{2+K^0}\right)^{K^0} \frac{\ln\left(\frac{K^0}{2+K^0}\right)}{c(2+K^0)^2} < 0$, due to $\ln\left(\frac{K^0}{2+K^0}\right) < 0$. Thus, Π^0 strictly decreases in K. Next, taking the first order condition of Π_M^{RS} with respect to K^{RS}, then we have $\frac{d\Pi_M^{RS}}{dK^{RS}} = 2\left(\frac{(K^{RS})^2}{2+3K^{RS}+(K^{RS})^2}\right)^{K^{RS}} \frac{1+(1+K^{RS})\ln\left(\frac{(K^{RS})^2}{2+3K^{RS}+(K^{RS})^2}\right)}{c(1+K^{RS})(2+K^{RS})^2} < 0$, due to $1+(1+K^{RS})\ln\left(\frac{(K^{RS})^2}{2+3K^{RS}+(K^{RS})^2}\right) < 0$. Thus, Π_M^{RS} is strictly decreasing in K as well.
(ii) Taking the first order condition of Π_{INT}^{RS} with respect to K^{RS}, we have the following result $\frac{d\Pi_{INT}^{RS}}{dK^{RS}} = 2\left(\frac{(K^{RS})^2}{2+3K^{RS}+(K^{RS})^2}\right)^{K^{RS}} \frac{1+K^{RS}\ln\left(\frac{(K^{RS})^2}{2+3K^{RS}+(K^{RS})^2}\right)}{c(1+K^{RS})(2+K^{RS})^2}$. Solving the equation $\frac{d\Pi_{INT}^{RS}}{dK^{RS}} = 0$ with respect to K^{RS}, which is equivalent to $1+K^{RS}\ln\left(\frac{(K^{RS})^2}{2+3K^{RS}+(K^{RS})^2}\right) = 0$, we can obtain the unique solution that is denoted by τ_I. Thus, we have $\frac{d\Pi_{INT}^{RS}}{dK^{RS}} \geqslant 0$ if $0 \leqslant K^{RS} \leqslant \tau_I$; $\frac{d\Pi_{INT}^{RS}}{dK^{RS}} < 0$ if $K^{RS} > \tau_I$. That is, Π_I^{RS} increases in K^{RS} when $K^{RS} \in [0, \tau_I]$; otherwise, Π_I^{RS} decreases in K^{RS}. Q.E.D.

Proof of Corollary 2

We first compare the entire channel profit under the reselling and centralized channels with the same K, and we then examine the effects of different K on the profits. (i) Given the same K and the equilibrium profits of the reselling and centralized distribution channels, we have $\frac{\Pi^0}{\Pi^{RS}} = \frac{1+K}{1+2K}\left(1+\frac{1}{K}\right)^K$. Since $K \in (0,\infty)$, $\lim_{K\to\infty}\left(1+\frac{1}{K}\right)^K = e$, where e is a mathematical constant approximately equal to 2.71828. It is also noted that $\frac{1+K}{1+2K} \in (0.5, 1)$. Thus, we have $\Pi^0 > \Pi^{RS}$. That is, for the entire channel profit, choosing the reselling agreement is less profitable than selecting the centralized selling agreement. For consumer of type θ, the total consumer surplus is formulated by $CS^i = \int_{\frac{p^i}{q^i}}^1 \left((\theta q^i - p^i)K(1-\theta)^{K-1}\right) d\theta$, where $i \in \{0, AS, RS\}$. Based on the equilibrium outcomes in centralized and reselling channels, we have $CS^0 = \frac{2K}{c(1+K)(2+K)^2}\left(\frac{K}{2+K}\right)^K$ and $CS^{RS} = \frac{2K}{c(1+K)(2+K)^2}\frac{K^{1+2K}((1+K)(2+K))^{-K}}{1+K}$. By comparing CS^0 and CS^{RS}, we can obtain $\frac{CS^0}{CS^{RS}} = \left(\frac{1+K}{K}\right)^{K+1} > 1$ as $\lim_{K\to\infty}\frac{CS^0}{CS^{RS}} > 1$ and $\lim_{K\to 0}\frac{CS^0}{CS^{RS}} > 1$. Thus, we can show $CS^0 > CS^{RS}$. (ii) Given different K (K^0 and K^{RS}) and the equilibrium profits, we have $\frac{\Pi^0}{\Pi^{RS}} = \frac{1+K^{RS}}{1+2K^{RS}}\left(\frac{2+K^{RS}}{2+K^0}\right)^2\left(\frac{K^0}{2+K^0}\right)^{K^0}\left(\frac{(K^{RS})^2}{2+3k^{RS}+(K^{RS})^2}\right)^{-K^{RS}}$. As $1+K^{RS} < 1+2K^{RS}$, we can show $\frac{\Pi^0}{\Pi^{RS}} < \left(\frac{2+K^{RS}}{2+K^0}\right)^2\left(\frac{K^0}{2+K^0}\right)^{K^0}\left(\frac{(K^{RS})^2}{2+3k^{RS}+(K^{RS})^2}\right)^{-K^{RS}}$. In addition, we have $\frac{\Pi^0}{\Pi^{RS}} \leqslant \left(\frac{2+K^{RS}}{2+K^0}\right)^2$ if $K^0 > \frac{2(K^{RS})^2}{2+3K^{RS}}$ and $K^0 > K^{RS}$. It is also noted that $K^{RS} > \frac{2(K^{RS})^2}{2+3K^{RS}}$. Given this condition, we further can show $\frac{2+K^{RS}}{2+K^0} < 1$ as $K^0 > K^{RS}$. Thus, we have $\Pi^0 \leqslant \Pi^{RS}$ only if $K^0 > K^{RS}$. Q.E.D.

Proof of Proposition 5

Before analyzing the equilibrium outcomes for two competing manufacturers, we first characterize the demands of two manufacturers. When $q_2 > q_1$, the market demand of two manufacturers are

$$Q_1 = \left(\frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1}\right)^+ \quad \text{and} \quad Q_2 = \left[1 - \max\left(\frac{p_2 - p_1}{q_2 - q_1}, \frac{p_2}{q_2}\right)\right]^+,$$

where $x^+ = \max\{0, x\}$. As $q_2 > q_1$ and $Q_1 = \left(\frac{p_2-p_1}{q_2-q_1} - \frac{p_1}{q_1}\right)^+$, we have $\frac{p_2-p_1}{q_2-q_1} > \frac{p_1}{q_1}$; otherwise, the market demand of the manufacturer 1 will be zero, which is not an ideal solution for the competition scenario. Suppose that $\frac{p_2-p_1}{q_2-q_1} \leqslant \frac{p_2}{q_2}$, we can obtain $p_2q_1 - p_1q_2 \leqslant 0$. However, $\frac{p_2-p_1}{q_2-q_1} > \frac{p_1}{q_1}$ implies that $p_2q_1 - p_1q_2 > 0$. This is a contradiction result; thus, we have $1 > \frac{p_2-p_1}{q_2-q_1} > \frac{p_1}{q_1}$. Consequently, the market demands of the manufacturers 1 and 2 are $Q_1 = \frac{p_2-p_1}{q_2-q_1} - \frac{p_1}{q_1}$ and $Q_2 = 1 - \frac{p_2-p_1}{q_2-q_1}$, respectively.

Similarly, in the case $q_2 < q_1$, the market demand of two manufacturers are

$$Q_1 = \left[1 - \max\left(\frac{p_2-p_1}{q_2-q_1}, \frac{p_1}{q_1}\right)\right]^+ \quad \text{and} \quad Q_2 = \left(\frac{p_2-p_1}{q_2-q_1} - \frac{p_2}{q_2}\right)^+,$$

where $x^+ = \max\{0, x\}$. Following the similar analysis, we can show $Q_1 = 1-\frac{p_2-p_1}{q_2-q_1}$ and $Q_2 = \frac{p_2-p_1}{q_2-q_1} - \frac{p_2}{q_2}$. As we can see, the market demands are symmetric in these two cases. With out of generality, we assume $q_2 > q_1$.

In addition, when $q_2 = q_1$ and $p_1 < p_2$, we have $Q_1 = 0$ and $Q_2 = \left(1 - \frac{p_2}{q_2}\right)^+$; this implies that all demand is covered by manufacturer 2; similarly, when $q_2 = q_1$ and $p_1 \geqslant p_2$, we have $Q_1 = \left(1 - \frac{p_1}{q_1}\right)^+$ and $Q_2 = 0$. Note that these results cannot meet the assumption in a competing environment because the market is only covered by one manufacturer. Thus, we do not consider this case in our analysis.

Under the reselling agreement, we first analyze the intermediary's best response. Given q_1, q_2, w_1, and w_2, the first-order condition of Π_{INT} with respect to p_1 and p_2 yields $p_1^* = \frac{q_1+w_1}{2}$ and $p_2^* = \frac{q_2+w_2}{2}$. Then, we have $Q_1^* = \frac{q_1 w_2 - q_2 w_1}{2q_1(q_2-q_1)}$ and $Q_2^* = \frac{q_2-w_2-(q_1-w_1)}{2(q_2-q_1)}$. Substituting $p_1 = p_1^*$, $p_2 = p_2^*$, $Q_1 = Q_1^*$, and $Q_2 = Q_2^*$ into Π_{M1} and Π_{M2}, we have

$$\Pi_{M1} = \left(w_1 - \frac{1}{2}cq_1^2\right)\frac{q_1 w_2 - q_2 w_1}{2q_1(q_2-q_1)}, \tag{A.12}$$

$$\Pi_{M2} = \left(w_2 - \frac{1}{2}cq_2^2\right)\frac{q_2 - w_2 - (q_1-w_1)}{2(q_2-q_1)}. \tag{A.13}$$

Next, taking the first-order condition of Π_{M1} and Π_{M2} with respect to w_1 and w_2, respectively, yields $w_1^* = \frac{q_1(2q_2-2q_1+2cq_1q_2+cq_2^2)}{2(4q_2-q_1)}$ and $w_2^* = \frac{q_2(4q_2-4q_1+cq_1^2+2cq_2^2)}{2(4q_2-q_1)}$. Then, substituting w_1^* and w_2^* into manufacturers' payoff functions yields

$$\Pi_{M1} = \frac{q_1 q_2 (q_2-q_1)(2-cq_1+cq_2)^2}{8(4q_2-q_1)^2}, \tag{A.14}$$

$$\Pi_{M2} = \frac{q_2^2(q_2-q_1)(4-cq_1-2cq_2)^2}{8(4q_2-q_1)^2}. \tag{A.15}$$

Taking the first-order condition of Π_{M1} and Π_{M2} with respect to q_1 and q_2, respectively, yields $\tilde{q}_1 = \frac{0.3987}{c}$ and $\tilde{q}_2 = \frac{0.8195}{c}$. The equilibrium wholesale prices, retail prices, and market demands can be obtained by substituting $p_1^* = \tilde{q}_1$ and $p_2^* = \tilde{q}_2$ into w_1^*, w_2^*, p_1^*, p_2^*, Q_1^*, and Q_2^*, respectively.

Under the agency selling agreement, we first analyze two manufacturers' best responses. Given agency fee γ, taking the the first-order condition of Π_{M1} and Π_{M2} with respect to p_1 and p_2, respectively, yields $p_1^* = \frac{q_1(2(1-\gamma)(q_2-q_1)+cq_2(q_2+2q_1))}{2(1-\gamma)(4q_2-q_1)}$ and $p_2^* = \frac{q_2(4(1-\gamma)(q_2-q_1)+c(2q_2^2+q_1^2))}{2(1-\gamma)(4q_2-q_1)}$. Thus, we can obtain $Q_1^* = \frac{q_2(2(1-\gamma)+c(q_2-q_1))}{2(1-\gamma)(4q_2-q_1)}$ and $Q_2^* = \frac{q_2(4(1-\gamma)-c(2q_2+q_1))}{2(1-\gamma)(4q_2-q_1)}$. Here, $Q_2 \geqslant 0$ if and only if $\gamma \leqslant \frac{4-c(2q_2+q_1)}{4}$. Next, substituting p_1^*, p_2^*, Q_1^*, and Q_2^*, into the intermediary's payoff function, and taking the first-order condition of Π_{INT} with respect to γ yields $\gamma^* = 1 - \frac{\sqrt[3]{24c\sqrt{324(cz_2)^2+3c(2z_1+cz_2)^3}+432c^2z_2}}{12} + \frac{\sqrt[3]{24c\sqrt{324(cz_2)^2+3c(2z_1+cz_2)^3}-432c^2z_2}}{12}$, where $z_1 = \frac{q_1(2q_2+q_1)^2}{(q_2-q_1)(4q_2+q_1)}$ and $z_1 = \frac{q_2(3q_1^3+q_1^2q_2+q_1q_2^2+4q_2^3)}{(q_2-q_1)(4q_2+q_1)}$. Substituting γ^*, Q_1^*, and Q_2^* into the manufacturers' payoff functions yields

$$\Pi_{M1} = \frac{q_1 q_2 (q_2-q_1)(2-2\gamma^*+cq_2)^2}{4(1-\gamma^*)(4q_2-q_1)^2}, \tag{A.16}$$

$$\Pi_{M2} = \frac{q_2^2(q_2-q_1)(4-4\gamma^*-cq_1-2cq_2)^2}{4(1-\gamma^*)(4q_2-q_1)^2}. \tag{A.17}$$

Taking the first-order condition of Π_{M1} and Π_{M2} with respect to q_1 and q_2, respectively, yields $\tilde{q}_1 = \frac{0.3930}{c}$ and $\tilde{q}_2 = \frac{0.8965}{c}$. The equilibrium retail prices, and market demands can be obtained by substituting $q_1^* = \tilde{q}_1$ and $q_2^* = \tilde{q}_2$ into p_1^*, p_2^*, Q_1^*, and Q_2^*, respectively. Q.E.D.

Proof of Corollary 3

By comparison equilibrium outcomes obtained in Proposition 6 and Proposition C.1, we can obtain the results of Corollary 3. As comparison processes are straightforward, we thus omit the proof. Q.E.D.

Proof of Corollary 4

By comparison equilibrium outcomes obtained in Proposition 6 and Proposition C.1, we can obtain the results of Corollary 4. As comparison processes are straightforward, we thus omit the proof. Q.E.D.

Proof of Proposition 6

As $q_1 < q_2$, we can show that the market demands of the manufacturers 1 and 2 are $Q_1 = \frac{p_2-p_1}{q_2-q_1} - \frac{p_1}{q_1}$ and $Q_2 = 1 - \frac{p_2-p_1}{q_2-q_1}$, respectively. We divide the proof into two cases: (1) a low-type manufacturer (denoted by manufacturer 1) accepts the agency selling format and the high-type manufacturer (denoted by manufacturer 2) adopts reselling in a hybrid channel; (2) a low-type manufacturer (manufacturer 1) accepts the reselling format and the high-type manufacturer (manufacturer 2) adopts agency selling in a hybrid channel.

Case 1: By backward induction, we first examine the pricing decisions of the intermediary and manufacturer 1. Given γ and w_2, taking the first-order conditions of Π_{INT} and Π_{M1} with respect to p_2 and p_1, respectively, yields: $p_1^* = \frac{q_1(q_1(\gamma+cq_2-1)+(1-\gamma)(q_2+w_2))}{(1-\gamma)(4q_2-(1+\gamma)q_1)}$ and $p_2^* = \frac{q_2(4(1-\gamma)(q_2+w_2)+c(1+\gamma)q_1^2-4(1-\gamma)q_1)}{2(1-\gamma)(4q_2-(1+\gamma)q_1)}$. Then, substituting p_1^* and p_2^* into Q_2 and Π_{M2}, we have

$$\Pi_{M2}^{\dagger} = \frac{(q_1^2(2\gamma + cq_2) + 4q_2(q_2 - w_2) - 2q_1((2+\gamma)q_2 - w_2))(cq_2^2 - 2w_2)}{4((1-\gamma)q_1 - 4q_2)(q_2 - q_1)}. \tag{A.18}$$

Taking the first-order condition of Π_{M2} with respect to w_2 yields $w_2^* = \frac{2q_2^2(2+cq_2)+q_1^2(2\gamma+cq_2)-q_1q_2(4+2\gamma+cq_2)}{4(2q_2-q_1)}$. Utilizing the equilibrium results derived in Proposition 6, we use $\gamma = 0.3728$ and $c = 1$; then, solving the quality decisions for the two manufacturers can yield $q_1^{\dagger} = 0.3293$ and $q_2^{\dagger} = 0.7607$. By substituting $q_1^{\dagger}$, $q_2^{\dagger}$ and $\tilde{\gamma} = 0.3728$ into p_1, p_2, w_1, Π_{M1}, and Π_{M2}, respectively, we can obtain the equilibrium prices and profits in a hybrid channel.

In this hybrid channel structure, M1 selects the agency selling format that is different from two competing manufacturers adopt the reselling format simultaneously. Next, by comparing the equilibrium profits of both manufacturers obtained in a sole format (i.e., reselling) and hybrid selling format, we find $\Pi_{M1}^{\dagger} > \tilde{\Pi}_{M1}^{RS}$ and $\Pi_{M2}^{\dagger} > \tilde{\Pi}_{M2}^{RS}$. That is, the manufacturers 1's transitioning from reselling to agency selling can benefit both manufacturers in this case. Therefore, the format of the hybrid selling channel established in the case 1 can be sustained as an equilibrium for both manufacturers. In such a scenario, we also find $q_i^{\dagger} < q_i^{RS}$ and $p_i^{\dagger} < p_i^{RS}$, where $i = \{1, 2\}$.

Case 2: Similarly, we first analyze the pricing decisions of the intermediary and manufacturer 2, i.e., p_1 and p_2. Given γ and w_1, taking the first-order conditions of Π_{INT} and Π_{M2} with respect to p_1 and p_2, respectively, yields: $p_1^* = \frac{4(1-\gamma)q_2w_1+(1+\gamma)(2-2\gamma-cq_2)q_1q_2-2(1-\gamma^2)q_1^2}{2(1-\gamma)(4q_2-(1+\gamma)q_1)}$ and $p_2^* = \frac{q_2(2(1-\gamma)q_2+cq_2^2+(1-\gamma)w_1-2(1-\gamma)q_1)}{(1-\gamma)(4q_2-(1+\gamma)q_1)}$. Then, substituting p_1^* and p_2^* into Q_1 and Π_{M1}, we have

$$\Pi_{M1}^{\dagger} = \frac{q_2(cq_1^2 - 2w_1)(2(1-\gamma)q_1^2 + 4q_2w_1 - q_1(2(1-\gamma)q_2 + cq_2^2 + 2w_1))}{4q_1((4q_2 - (1-\gamma)q_1)(q_2 - q_1))}. \tag{A.19}$$

Taking the first-order condition of Π_{M1} with respect to w_1 yields $w_1^* = \frac{q_1(q_2(2-2\gamma+cq_2)-cq_1^2-2q_1(1-\gamma-cq_2))}{4(2q_2-q_1)}$. Leveraging the equilibrium outcomes obtained in Proposition 6, we use $\gamma = 0.3728$ and $c = 1$; then, solving the quality decisions for the two manufacturers can yield $q_1^\dagger = 0.2942$ and $q_2^\dagger = 0.5360$. By substituting $q_1^\dagger$, $q_2^\dagger$ and $\tilde{\gamma} = 0.3728$ into p_1, p_2, w_1, Π_{M1}, and Π_{M2}, respectively, we can obtain the equilibrium prices and profits in a hybrid channel.

In this hybrid channel configuration, M2 chooses the agency selling format that is different from the two competing manufacturers adopt the reselling format simultaneously. By comparing the equilibrium profits of both manufacturers obtained in a sole format (i.e., reselling) and a hybrid selling format, we find $\Pi_{M1}^\dagger < \tilde{\Pi}_{M1}^{RS}$ and $\Pi_{M2}^\dagger > \tilde{\Pi}_{M2}^{RS}$. That is, only manufacturer 2 has an incentive to make such a move, transitioning from reselling to agency selling; however, this would be detrimental to manufacturer 1's profit. Therefore, the format of the hybrid selling channel established in the case 1 cannot be sustained as an equilibrium for both manufacturers. Q.E.D.

Proof of Corollary 5

By Propositions 6 and 7, we have the equilibrium outcomes obtained under the sole and hybrid selling format. By offering both high- and low-quality products, the total consumer surplus can be defined as $CS = CS_1 + CS_2 = \int_{\frac{p_1}{q_1}}^{1} (\theta q_1 - p_1)\,\mathrm{d}\theta + \int_{\frac{p_2}{q_2}}^{1} (\theta q_2 - p_2)\,\mathrm{d}\theta$. Plugging the equilibrium product qualities and prices into CS, we can show $CS^\dagger > \tilde{CS}^{AS} > \tilde{CS}^{RS}$. In addition, we find that the surplus of the high-valuation consumers is always lower than that of the low-valuation consumers in the hybrid and agency selling channels, i.e., $CS_2^\dagger < CS_1^\dagger$ and $\tilde{CS}_2^{AS} < \tilde{CS}_1^{AS}$. By contrast, in the reselling channel, the surplus of the high-valuation consumers is always higher than that of the low-valuation consumers, i.e., $\tilde{CS}_2^{RS} > \tilde{CS}_1^{RS}$. Q.E.D.

Appendix C. A Channel with One Manufacturer & One Intermediary

In this appendix, we investigate the strategic decisions of a manufacturer and an intermediary under the agency selling and reselling agreements, respectively, when $K = 1$. In such a scenario, the market demand is $Q = \Pr\{\theta q - p \geqslant 0\} = 1 - \frac{p}{q}$. We use the superscript '$\hat{}$' to denote the payoff functions and analytical outcomes in this scenario.

Under the reselling agreement, the manufacturer decides the quality of its product q and the wholesale price w. Then, the intermediary determines the retail prices p. The payoff functions of the manufacturer and the intermediary are formulated as follows:

$$\hat{\Pi}_M^{RS}(w, q|p) = \left(w - \frac{1}{2}cq^2\right) Q, \tag{B.1}$$

$$\hat{\Pi}_{INT}^{RS}(p) = (p - w)Q. \tag{B.2}$$

We solve this game model by backward induction. For any given products quality and wholesale price, we first analyze the intermediary's best response. We then investigate the equilibrium outcomes that can maximize the profits of the manufacturer and the intermediary.

Following the similar procedure, we next investigate the decisions under the agency selling agreement. In such a scenario, the intermediary announces the agency fee first. Then, the manufacturer determines its products quality and retail price, respectively. The payoff functions of the manufacturer and the intermediary are developed as follows:

$$\hat{\Pi}_M^{AS}(p, q|\gamma) = \left((1-\gamma)p - \frac{1}{2}cq^2\right) Q, \tag{B.3}$$

$$\hat{\Pi}_{INT}^{AS}(\gamma) = \gamma p Q. \tag{B.4}$$

The equilibrium outcomes under these two agreements are summarized in the following proposition.

Proposition C.1. *Under reselling and agency agreements with a sole competing manufacturers, the equilibrium outcomes are summarized in Table C.1 with respect to the product quality, wholesale price,* *retail price, agency fee, and profits.*

Table C.1: The equilibrium outcomes of a channel with a sole manufacturer and a intermediary

Agreements \ Decisions	$\hat{q}$	$\hat{p}$	$\hat{w}$	$\hat{\gamma}$	$\hat{\Pi}_M$	$\hat{\Pi}_R$	$\hat{\Pi}_{\text{Channel}}$
Reselling (RS)	$\frac{0.6667}{c}$	$\frac{0.5556}{c}$	$\frac{0.4444}{c}$	-	$\frac{0.0370}{c}$	$\frac{0.0185}{c}$	$\frac{0.0556}{c}$
Agency Selling (AS)	$\frac{0.7869}{c}$	$\frac{0.6512}{c}$	-	0.3994	$\frac{0.0141}{c}$	$\frac{0.0449}{c}$	$\frac{0.0589}{c}$

Note: Dash indicates not applicable because stakeholders make different decisions under RS and AS formats.

Proof of Proposition C.1

To derive the equilibrium outcomes for both the manufacturer and intermediary, we divide the proof into the following two cases.

(i) Under the reselling agreement, we first analyze the intermediary's best response. Given q, and w, the first-order condition of $\hat{\Pi}_{INT}^{RS}$ with respect to p yields $p^* = \frac{q+w}{2}$. Then, we have $Q^* = \frac{q-w}{2}$. Substituting $p = p^*$ and $Q = Q^*$ into $\hat{\Pi}_M^{RS}$, we have $\hat{\Pi}_M^{RS} = \left(w - \frac{1}{2}cq^2\right)\frac{q-w}{2q}$. Next, taking the first-order condition of $\hat{\Pi}_M^{RS}$ with respect to w yields $w^* = \frac{q(2+cq)}{4}$. Then, substituting w^* into the manufacturer's payoff function yields $\hat{\Pi}_M^{RS} = \frac{q(2-cq)^2}{32}$. Taking the first-order condition of $\hat{\Pi}_M^{RS}$ with respect to q yields $q^* = \frac{0.6667}{c}$. Thus, the equilibrium wholesale prices, retail prices, and market demands can be obtained by substituting q^* into w^*, p^*, and Q^*, respectively.

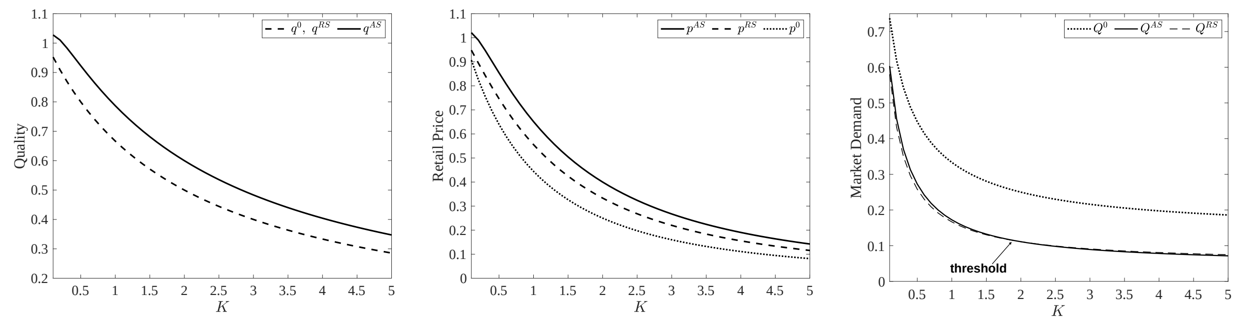
(ii) Under the agency selling agreement, we first analyze the manufacturers' best response. Given the agency fee γ, the first-order condition of $\hat{\Pi}_M^{AS}$ with respect to p yields $p^* = \frac{q(2(1-\gamma)+cq)}{4(1-\gamma)}$. Thus, we have $Q^* = \frac{q(2(1-\gamma)-cq)}{4(1-\gamma)}$. Next, substituting p^* and Q^*, into the intermediary's payoff function, and taking the first-order condition of $\hat{\Pi}_{INT}^{AS}$ with respect to γ yields $\gamma^* = 1 - \frac{1}{2}(\frac{cq}{3})^{2/3}[(\sqrt{324 + 3c^2q^2} + 18)^{1/3} - (\sqrt{324 + 3c^2q^2} - 18)^{1/3}]$. Substituting γ^* and Q^* into the manufacturer's payoff functions and taking the first-order condition of $\hat{\Pi}_M^{AS}$ with respect to q yields $q^* = \frac{0.7869}{c}$. The equilibrium retail price, and market demand can be obtained by substituting q^* into p^* and Q^*, respectively. Q.E.D.

Appendix D. Comparison Analysis with Identical K

In this appendix, we conduct comparison analysis for a special case under identical K in each selling channel. We first compare the equilibrium quality levels, pricing, and demand, respectively; then, we numerically illustrate the channel profits and consumer surplus in three selling channels.

Proposition D.1. *Given the same K, the following comparison results hold: (i) $q^{AS} > q^{RS} = q^0$, (ii) $p^{AS} > p^{RS} > p^0$, and (iii) $Q^0 > Q^{AS} \geqslant Q^{RS}$ if $K \leqslant \eta^{-1}(1)$; otherwise, $Q^0 > Q^{RS} > Q^{AS}$.*

Interestingly, Proposition D.1 shows that with the same K, the manufacturer tends to set a higher quality level in the agency selling channel than in the centralized and reselling channels (see Figure D.1(a) for illustration). In the agency selling channel, the manufacturer is induced to increase the retail price (see Figure D.1(b) for illustration) that can balance marginal profits, production cost, and agency payment and that results in a greater market share than if it selects the reselling agreement. In addition, the manufacturer prefers to target low-end consumers by offering the product at a lower retail price of lower quality for both reselling and centralized agreements. There is another interesting result. As we can see from Figure D.1(c), the market demand obtained in an agency selling channel is not larger than that in the centralized distribution and reselling channels. This result implies that offering top-notch quality cannot bring a significant advantage that enables retail market domination. That is, the pursuit of high product quality impacts the manufacturer's profit-driven objective, as it diminishes the benefits of a greater market share. These outcomes may inspire managers to rethink



(a) Illustration of q w.r.t. K (b) Illustration of p w.r.t. K (c) Illustration of Q w.r.t. K

Figure D.1: The effect of K on equilibrium outcomes (parameter values: $K \in [0.1, 5]$ and $c = 1$)

Nike's DTC sales; recently, Nike and Macy's are reuniting after over a year after Nike halted the sale of its products through the store's channel (Forbes 2023).

Result D.1. *In the case with the same K, $\Pi_C^0 > \Pi_C^{AS} > \Pi_C^{RS}$ and $CS^0 > CS^{AS} > CS^{RS}$.*

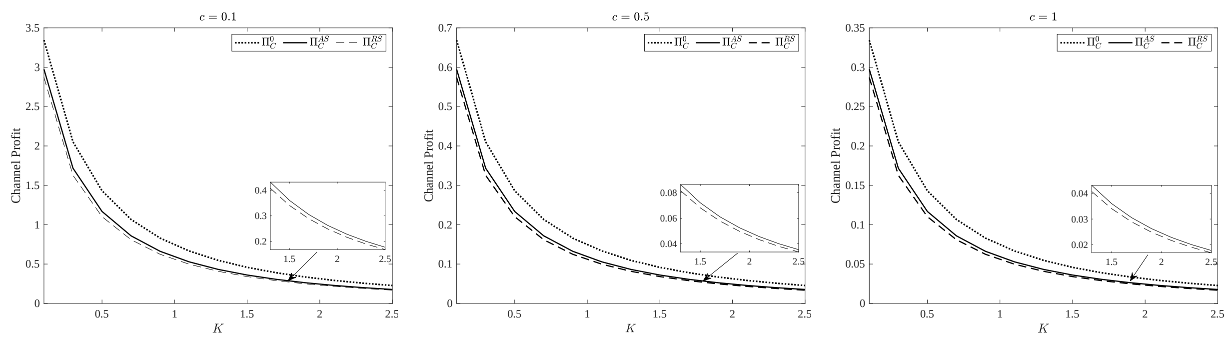


Figure D.2: Channel profit comparison with the same K in three channels (parameter values: $K \in [0.1, 2.5]$ and $c \in \{0.1, 0.5, 1\}$)

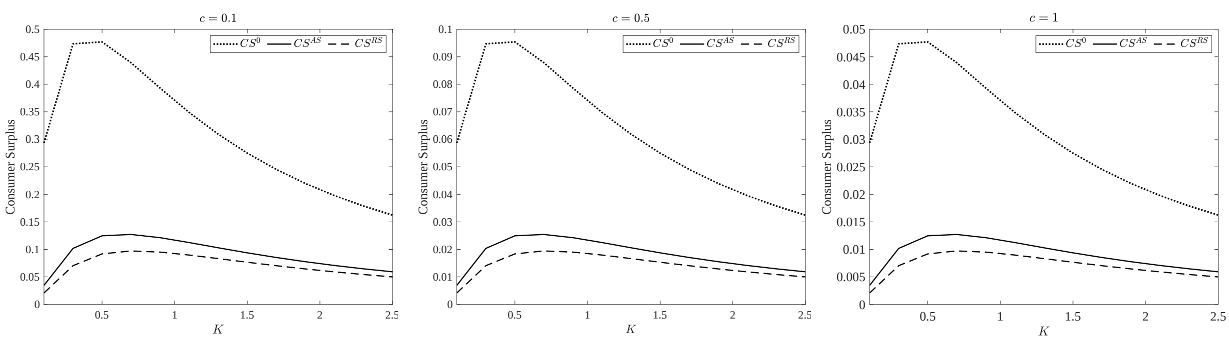


Figure D.3: Consumer surplus comparison with the same K in three channels (parameter values: $K \in [0.1, 2.5]$ and $c \in \{0.1, 0.5, 1\}$)

When all three channel structures with the same K, our numerical examples shows robust results. Specifically, the profit of each channel structure is decreasing in K. In addition, both the largest channel profit and consumer surplus can be preserved in a centralized distribution channel, followed by the agency selling and reselling channels (see Figures D.2 and D.3 for an illustration). This is because vertical integration can efficiently incorporate a pricing strategy that promotes both profit and consumer surplus maximization. Moreover, the consumer surplus is concave in K, suggesting that a lower K (approximately 0.5) can promote higher consumer surplus. Leveraging the driving force of

K, we also find that agency selling format always generates more consumer surplus than the reselling format. The above results can be sustained in either cost reduction or escalation scenario.

Proof of Proposition D.1

By propositions 1, 2, and 3, we have equilibrium product qualities and retail prices of three channels: $q^0 = \frac{2}{(2+K)c}$, $p^0 = \frac{4}{(2+K)^2c}$, $q^{AS} = \frac{\Delta}{c}$, $p^{AS} = \frac{\Delta(2-2\gamma^*+\Delta K)}{2c(1+K)(1-\gamma^*)}$, $q^{RS} = \frac{2}{(2+K)c}$, and $p^{RS} = \frac{4+6K}{(1+K)(2+K)^2c}$, where $\Delta = \frac{(1-\gamma^*)\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)}-(1-K)(1-\gamma^*)^2}{K(1+\gamma^*K)}$. (i) Given the same K, we have $q^0 = q^{RS}$; we compare q^{AS} and q^{RS}: $q^{AS} - q^{RS} = \frac{(2+K)(1+\gamma^*)\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)}-(2+K)(1-K)(1-\gamma^*)^2-2K(1+\gamma^*K)}{cK(2+K)(1+\gamma^*K)}$; as $(2+K)(1+\gamma^*)\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)} > (2+K)(1+K) > (2+K)(1-K)+2K(1+\gamma^*K) > (2+K)(1-K)(1-\gamma^*)^2 + 2K(1+\gamma^*K)$, we can show $q^{AS} > q^{RS} = q^0$. (ii) By comparing p^{RS} and p^0, we have $\frac{p^{RS}}{p^0} = \frac{4+6K}{4+4K} \geqslant 1$ since $K > 0$; that is, $p^{RS} \geqslant p^0$. We then compare p^{AS} and p^{RS}: $\frac{p^{AS}}{p^{RS}} = \frac{(\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)}-(1-K)(1-\gamma^*))(2-2\gamma^*+\Delta K)(2+K)^2}{2K(1+\gamma^*K)(4+6K)} > \frac{8K(2+K)^2}{2K(1+\gamma^*K)(4+6K)} > 1$; this shows that $p^{AS} > p^{RS}$. Thus, we have $p^{AS} > p^{RS} > p^0$. (iii) By comparing $1 - \frac{p^0}{q^0}$ and $1 - \frac{p^{RS}}{p^{RS}}$, we obtain $\left(1 - \frac{p^0}{q^0}\right) / \left(1 - \frac{p^{RS}}{q^{RS}}\right) = 1 + \frac{1}{K} > 1$; that is, we have $Q^0 > Q^{RS}$. We then compare $\frac{p^0}{q^0}$ and $\frac{p^{AS}}{p^{AS}}$; as $\left(\frac{p^{AS}}{q^{AS}}\right) / \left(\frac{p^0}{p^0}\right) = \frac{2+K(\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)}-(1-K)(1-\gamma^*))}{2(1+K)} \cdot \frac{2+K}{2} > \frac{2+K((1+K)-(1-K))}{2(1+K)} \cdot \frac{2+K}{2} > 1$, we show $\frac{p^{AS}}{q^{AS}} > \frac{p^0}{q^0}$; that is, we have $Q^0 > Q^{AS}$. Next, by comparing $\left(\frac{p^{AS}}{q^{AS}}\right)$ and $\left(\frac{p^{RS}}{q^{RS}}\right)$, we have $\left(\frac{p^{AS}}{q^{AS}}\right) / \left(\frac{p^{RS}}{p^{RS}}\right) = \frac{(2+K)(2+K(\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)})}{4+6K} > 1$ if $K > \eta^{-1}(1)$, where $\eta = \frac{(2+K)(2+K(\sqrt{(1+K)(1-\gamma^*)(1+K-\gamma^*+3\gamma^*K)})-6K}{4}$; otherwise, we have $\frac{q^{AS}}{q^{RS}} \leqslant 1$ if $K \leqslant \eta^{-1}(1)$; that is, we can obtain $Q^{AS} < Q^{RS}$ if $K > \eta^{-1}(1)$; $Q^{AS} \geqslant Q^{RS}$ if $K \leqslant \eta^{-1}(1)$. Therefore, by comparing demand Q with different K in each retailing channel, we can show Q^0 is the largest one; Q^{AS} is either greater or smaller than Q^{RS} depending on the threshold. Q.E.D.

Reference

Forbes (2023). How Nike Learned A Wholesale Lesson. www.forbes.com/sites/retailwire/2023/06/16/how-nike-learned-a-wholesale-lesson/?sh=48b079aa4618

Appendix E. Prioritize Agency Fee

In this appendix, we examine a different sequence under an agency selling agreement in this section. Specifically, the intermediary is the Stackelberg leader and the manufacturer is the Stackelberg follower. The sequence of events is described as follows. First, the intermediary announces the agency fee, which is a fraction of the manufacturer's retail price. Second, given the observed agency fee, the manufacturer determines the retail price and the quality of the product. Following the standard backward induction approach, we begin by looking at the manufacturer's decisions given the agency fee, and we then analyze the intermediary's strategic decision. This model setting is similar to the agency model proposed by Tan and Carrillo (2017). However, the revenue-sharing proportion, i.e., the agency fee, is exogenous in their work; in contrast, this is an endogenous variable in our model.

By backward induction, the first step is to examine the manufacturer's strategic production and sales decisions. Given agency fee γ, the manufacturer aims to maximize its expected profit by choosing the optimal retail price and product quality. We use the symbol '$\check{}$' to describe the payoff functions and analytical results in this case. The manufacturer's problem is formulated as follows:

$$\max_{p,q\geqslant 0} \check{\Pi}_M(p, q|\gamma) = \left[(1-\gamma)p - \frac{1}{2}cq^2\right] Q, \tag{E.1}$$

where $Q = (1 - \frac{p}{q})^K$, and $(1-\gamma)p$ is the marginal revenue. Following this agreement, the manufacturer can sell the product by accessing consumers through the channel/platform operated by the intermediary. Thus, the manufacturer needs to share a fraction of its revenue, i.e., γpQ, with the intermediary. Solving this problem yields the following results.

Lemma E.1. *Given agency fee γ, the manufacturer's best response for the retail price and the product quality are $p^* = \frac{4(1-\gamma)}{(2+K)^2 c}$ and $q^* = \frac{2(1-\gamma)}{(2+K)c}$.*

Lemma E.1 characterizes the manufacturer's optimal decisions given the agency fee of the intermediary. As a strategic selling agreement, sharing partial revenue with the intermediary could induce her to choose this agreement, and this strategy could also assist the manufacturer in expending market demand, which may be profitable as well. This lemma shows that the manufacturer's best response for the retail price and the product quality is to decrease the agency fee. Having understood the manufacturer's strategic decisions as shown in Lemma E.1, we are now ready to study the intermediary's decision. The intermediary's problem is formulated as follows:

$$\max_{0\leqslant\gamma\leqslant 1} \quad \check{\Pi}_{INT}(\gamma|p,q) = \gamma p Q \tag{E.2}$$

$$\text{s.t.} \quad p = \frac{4(1-\gamma)}{(2+K)^2 c}, \tag{E.3}$$

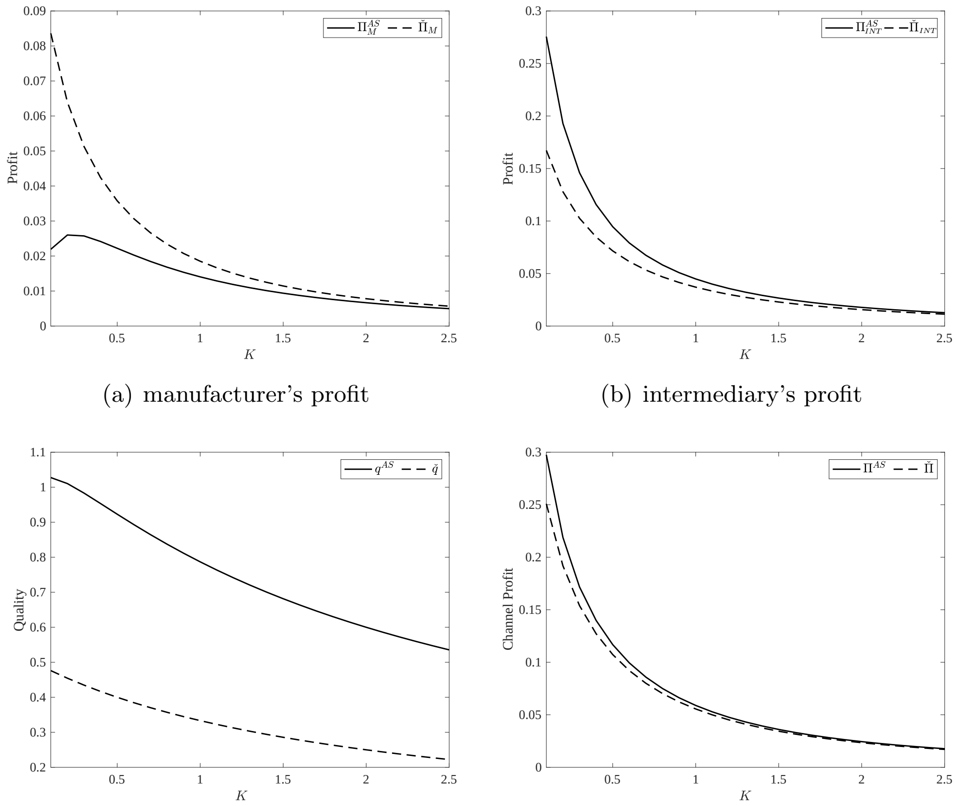
$$q = \frac{2(1-\gamma)}{(2+K)c}. \tag{E.4}$$

where $Q = (1-p/q)^K$. We now explore the property of the intermediary's profit function in the above problem. The equilibrium outcomes of stakeholders are summarized in the following proposition.

Proposition E.1. *When the intermediary acts as a leader under the agency selling agreement, (i) the intermediary's equilibrium agency fee is $\check{\gamma} = \frac{1}{2}$; (ii) the manufacturer's equilibrium retail price and product quality are $\check{p} = \frac{2}{(2+K)^2 c}$ and $\check{q} = \frac{1}{(2+K)c}$, respectively.*

Proposition E.1 characterizes the equilibrium outcomes when the intermediary acts as the leader under the agency selling format. Surprisingly, it is noted that the equilibrium agency fee is independent of K, the retail price, and product quality. That is, irrespective of whether the market demand is high or low, the intermediary always prefers to choose a revenue-sharing scheme characterized by a fixed proportion of 0.5. The manufacturer is unconcerned about establishing a higher retail price that could potentially result in market erosion. Accordingly, the manufacturer always needs to pay 50% of revenue as the agency fee to the intermediary for accessing consumers and obtaining a stable market basis.

Compared to the centralized distribution channel, both the retail price and product quality are affected by the agency fee in the agency selling channel. Thus, the manufacturer's retail price should remain lower to attract consumers and sustain market demand. In addition, consumers have lower price sensitivity if they prefer to buy an exclusive product from the prestigious manufacturer. Consumers are less sensitive to the price if there is no significant quality difference compared with other alternative products on existing features. Given different decision sequences under the agency selling agreement, we also compare the profits of the manufacturer and the intermediary. An interesting result can be observed from Figure E.1; the first-mover advantage does not exist under the agency selling agreement. That is, the leader is unable to secure a greater profit, but the follower can obtain a higher profit in such a selling agreement. In addition, there is another interesting result: when the intermediary acts as a leader, the manufacturer benefits the most by offering a low-quality product (see Figures E.1 (a) and (c) for illustration). This occurs because, when confronted with an elevated agency fee, the manufacturer has to compromise on quality and set a lower retail price to secure the highest possible market share. In short, as we can see from Figure E.1 (d), when $K < 1$, offering a high-quality product increases the overall channel profit. Conversely, when $K > 1$, there is no significant difference in channel profits between offering the high-quality product and setting a higher agency fee ($\gamma^* = 0.5$).



(a) manufacturer's profit

(b) intermediary's profit

(c) product quality

(d) channel profit

Figure E.1: The profits and product quality under the agency selling format with different decision sequences (parameter values: $c = 1$, $K \in (0, 2.5]$, and $\gamma^* = 0.5$).

Proof of Lemma E.1

To obtain the equilibrium retail price and product quality, we first analyze the manufacturer's best response problem. Given the agency fee γ, the optimization problem of the manufacturer is formulated as follows: $\max \check{\Pi}_M(p, q|\gamma) = ((1-\gamma)p - cq^2/2)Q$, subject to $p > 0$ and $q > 0$, where $Q = (1 - p/q)^K$. The Hessian matrix of $\check{\Pi}_M$ can be developed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \check{\Pi}_M}{\partial p^2} & \frac{\partial^2 \check{\Pi}_M}{\partial p \partial q} \\ \frac{\partial^2 \check{\Pi}_M}{\partial q \partial p} & \frac{\partial^2 \check{\Pi}_M}{\partial p^q} \end{bmatrix}. \tag{E.5}$$

Let H_i be the leading principal minor of order i; we have $H_1 = -\frac{1}{2}cK(1+K)\left(\frac{K}{2+K}\right)^{-2+K} < 0$, and $H_2 = \frac{1}{4}c^2K(2+K)\left(\frac{K}{2+K}\right)^{2(-1+K)} > 0$. Thus, H is negative definite, implying that $\check{\Pi}_M$ is jointly concave in (p, q). From the first-order conditions, we have

$$\frac{\partial \check{\Pi}_M}{\partial p} = (1-\gamma)\left(1 - \frac{p}{q}\right)^K - \frac{1}{q}\left((1-\gamma)p - \frac{1}{2}cq^2\right)K\left(1 - \frac{p}{q}\right)^{K-1} = 0, \tag{E.6}$$

$$\frac{\partial \check{\Pi}_M}{\partial q} = -cq\left(1 - \frac{p}{q}\right)^K + \frac{p}{q^2}\left((1-\gamma)p - \frac{1}{2}cq^2\right)K\left(1 - \frac{p}{q}\right)^{K-1} = 0. \tag{E.7}$$

From Equation (E.6), we obtain $(1-\gamma)\left(1 - \frac{p}{q}\right)^K = \frac{1}{q}\left((1-\gamma)p - \frac{1}{2}cq^2\right)K\left(1 - \frac{p}{q}\right)^{K-1}$. Substituting $(1-\gamma)\left(1-\frac{p}{q}\right)^K$ into Equation (E.7) yields $p=\frac{cq^2}{1-\gamma}$. Then, substituting $p=\frac{cq^2}{1-\gamma}$ back into Equation (E.6) yields $\check{q}=\frac{2(1-\gamma)}{(2+K)c}$; accordingly, we have $\check{p}=\frac{4(1-\gamma)}{(2+K)^2c}$. Q.E.D.

Proof of Proposition E.1

By Lemma E.1, we have $q^*=\frac{2(1-\gamma)}{(2+K)c}$ and $p^*=\frac{4(1-\gamma)}{(2+K)^2c}$. Substituting $p=p^*$, $q=q^*$, $Q=\left(1-\frac{p^*}{q^*}\right)^K$ into the retailer's payoff function developed in Equation (E.2), we obtain $\check{\Pi}_R=\gamma\frac{4(1-\gamma)}{(2+K)^2c}\left(1-\frac{2}{2+K}\right)^K$.
(i) The existence and uniqueness of the agency fee holds because $\frac{\mathrm{d}^2\check{\Pi}_R}{\mathrm{d}\gamma^2}=\frac{-8}{(4+K)^2c}\left(1-\frac{2(2+K)}{(4+K)^2}\right)^K<0$; thus, the equilibrium agency fee, $\check{\gamma}$, is 1/2. (ii) The manufacturer's equilibrium retail price and product quality follow directly from the expression for $\check{\gamma}$ and the results of Lemma E.1. Q.E.D.

Reference

Tan, Y., J., Carrillo (2017). Strategic analysis of the agency model for digital goods. *Production and Operations Management*, **26**(4), 724-741.

Appendix F. Numerical Illustration of Result 2

In this appendix, the graphical representation of Result 2 is shown in the following figures.

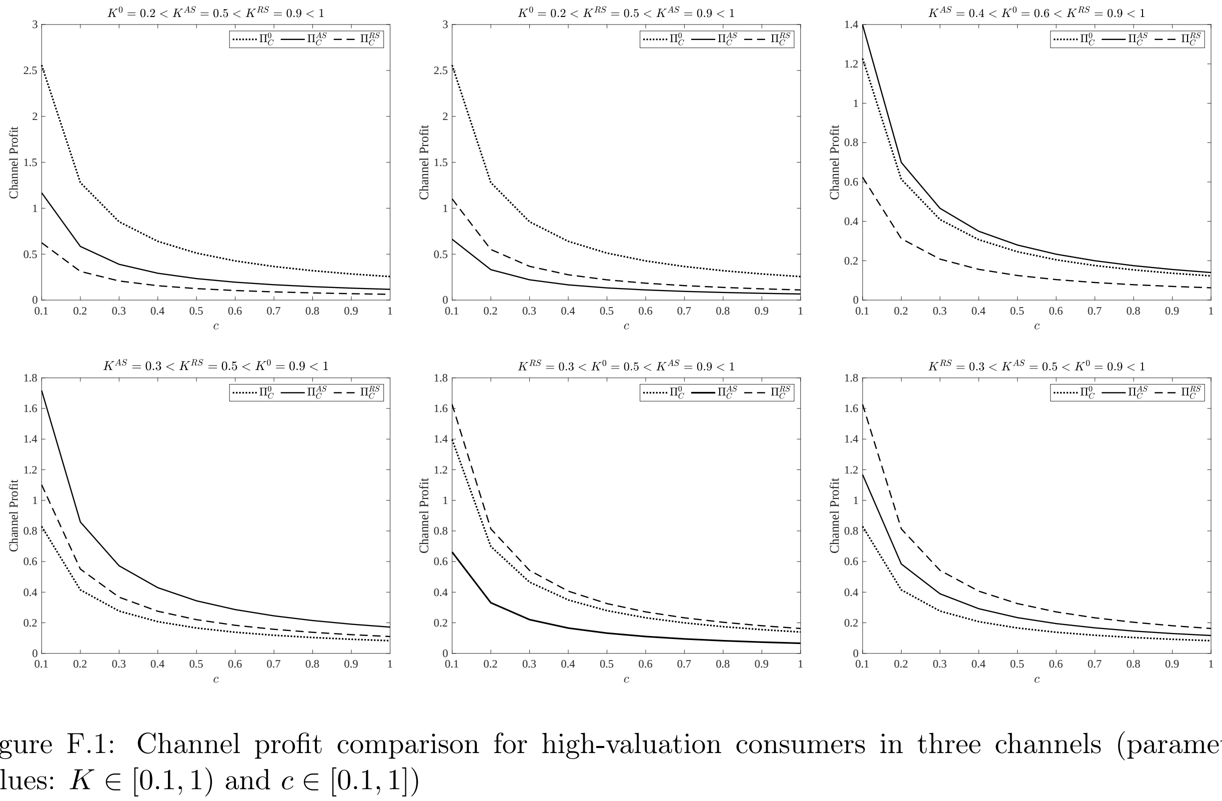


Figure F.1: Channel profit comparison for high-valuation consumers in three channels (parameter values: $K \in [0.1, 1)$ and $c \in [0.1, 1]$)

The numerical results depicted in Figures F.1 and F.2 demonstrate that as product cost increases, interesting outcomes are observed when consumer valuations are either relatively high (i.e., $K < 1$) or low (i.e., $K > 1$). Specifically, in scenarios where $K < 1$ or $K > 1$, the optimum preservation of the largest channel profit can be achieved with the minimum value of a channel-specific K. The underlying principle is that within each retailing channel, profitability hinges on offering a specific product quality category tailored to meet the needs of consumers.

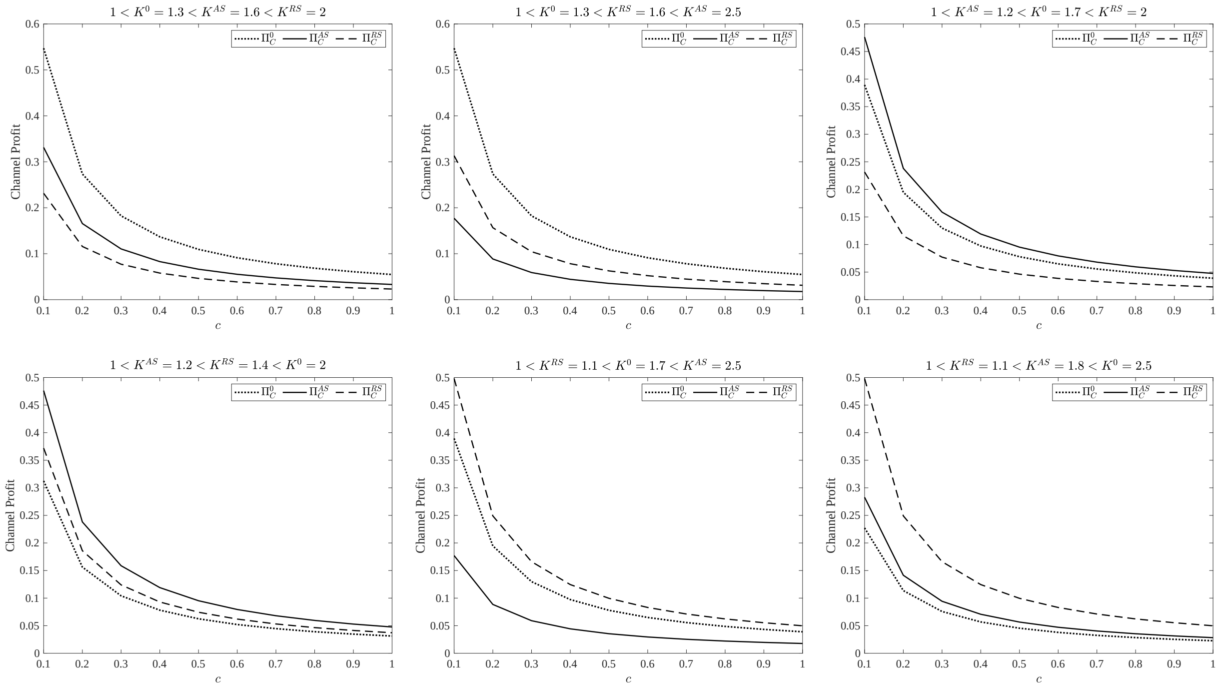


Figure F.2: Channel profit comparison for low-valuation consumers in three channels (parameter values: $K \in (1, 2.5]$ and $c \in [0.1, 1]$)

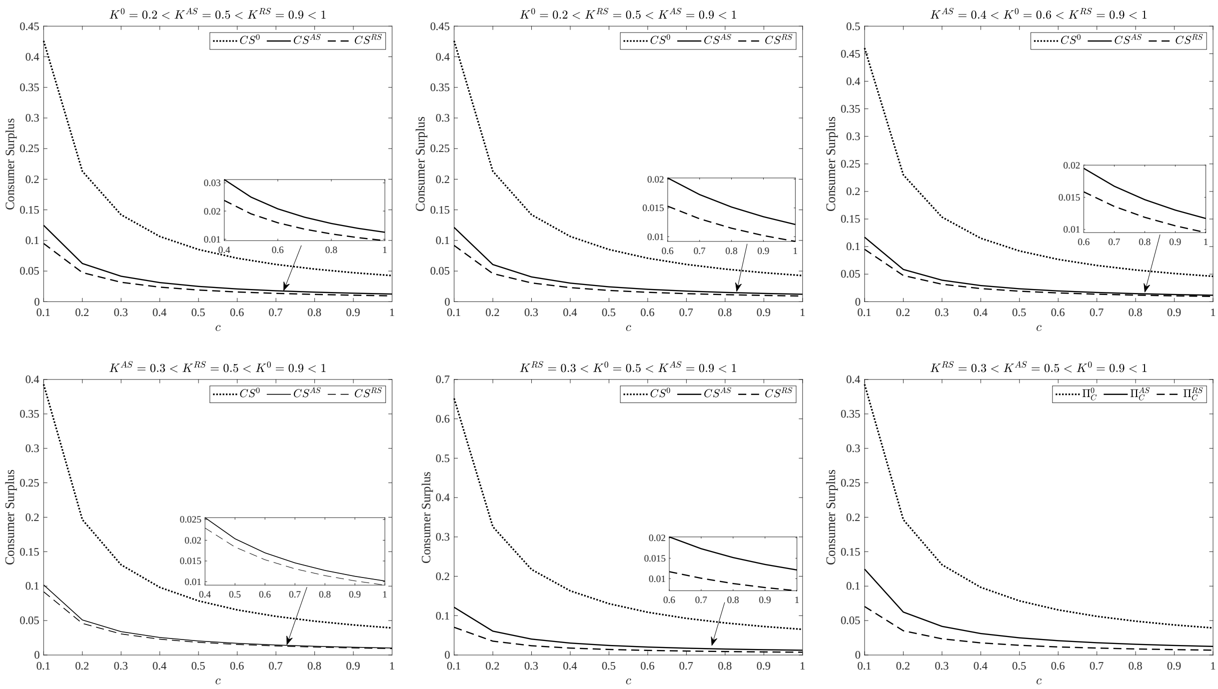


Figure F.3: Consumer surplus comparison for low-valuation consumers in three channels (parameter values: $K \in [0.1, 1)$ and $c \in [0.1, 1]$)

Moreover, as we can see from Figures F.3 and F.4, the centralized distribution channel can yield a higher consumer surplus compared to decentralized channels. This is because an increase in the wholesale price can exert downward pressure on the intermediary's pricing decision in the reselling format. Additionally, in the agency selling format, a rise in the agency fee may influence the manufacturer's retail pricing decision. A higher consumer surplus can be attained in an agency selling channel compared

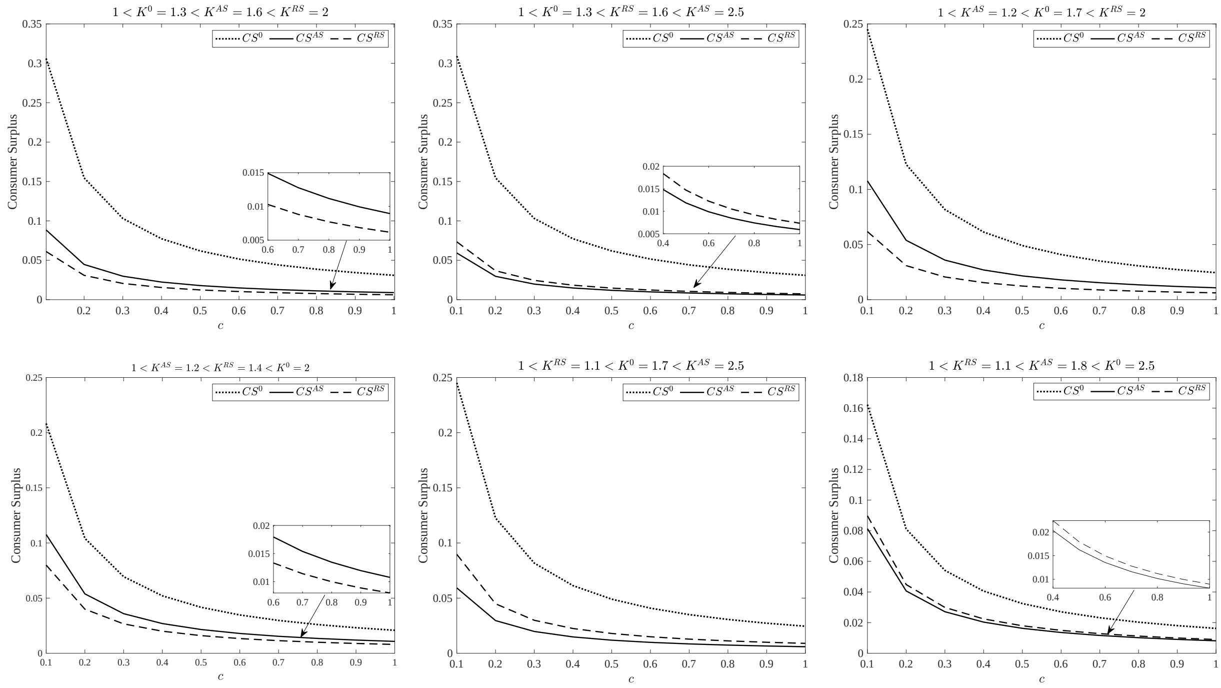


Figure F.4: Consumer surplus comparison for low-valuation consumers in three channels (parameter values: $K \in (1, 2.5]$ and $c \in [0.1, 1]$)

to a reselling channel in markets with a mass of high-end consumers. However, this advantage is not sustainable in markets predominantly composed of high-end consumers; in such a scenario, consumer surplus becomes more sensitive to the value of a channel-specific K.