

Online Technical Appendix to “Welfare Impact of Bilateral Rating Display Systems on Ride-Sharing Platforms”

Appendix A: Proofs of the Main Results

Proof of Lemma 1. By Definition of inactive players, we know that all riders are active in URDS, i.e., $\pi_r = 1$. Therefore, we only need to consider $\pi_d = 1$ and $\pi_d < 1$ for drivers. It is obvious that a necessary condition for drivers to be active is $h \geq w$.

Case 1: $\pi_d = 1$. In this case, the driver with type $-t_0$ will be active. This happens if and only if $v - t_0 - p \geq 0$.

Case 2: $0 < \pi_d < 1$. In this case, we have $\mathbb{P}\{T_d \geq \underline{t}_d\} = \pi_d$ and $C(\pi_r, \pi_d) = \gamma$. Hence, we have:

$$\begin{cases} v + \underline{t}_d - \gamma - p = 0 & \Leftrightarrow \underline{t}_d = p + \gamma - v, \\ \underline{t}_d \in (-t_0, t_0) & \Leftrightarrow v - \gamma - t_0 < p < v - \gamma + t_0. \end{cases} \Rightarrow \pi_d = \mathbb{P}\{T_d \geq \underline{t}_d\} = 1 - \frac{\underline{t}_d + t_0}{2t_0} = \frac{v + t_0 - p - \gamma}{2t_0}.$$

The results are, thus, summarized in Table 1. \square

Proof of Proposition 2. We solve the revenue maximization problem of the platform analytically. Then this proposition directly follows from the platform’s optimal solution. Based on the equilibrium in Lemma 1, we discuss the following cases.

Case 1: $(\pi_r, \pi_d) = (1, 1)$. The platform’s optimization problem becomes:

$$\begin{aligned} \max_{p \geq h \geq w} \quad & p - h \\ \text{s.t.} \quad & v - t_0 - p \geq 0. \end{aligned}$$

Let $(\tilde{p}_1, \tilde{h}_1, \tilde{\Pi}_1)$ be the optimal price, driver’s revenue share and platform’s revenue in Case 1. It is clearly that $\tilde{p}_1 = v - t_0$, $\tilde{h}_1 = w$, $\tilde{\Pi}_1 = v - t_0 - w$. It is obvious that for the platform to obtain positive revenue, we need $v > w + t_0$.

Case 2: $(\pi_r, \pi_d) = (1, \frac{v+t_0-p-\gamma}{2t_0})$. The platform’s optimization problem becomes:

$$\begin{aligned} \max_{p \geq h \geq w} \quad & (p - h) \left(\frac{v + t_0 - p - \gamma}{2t_0} \right) \\ \text{s.t.} \quad & v - \gamma - t_0 < p < v - \gamma + t_0. \end{aligned}$$

Let $(\tilde{p}_2, \tilde{h}_2, \tilde{\Pi}_2)$ be the optimal price, driver’s revenue share and platform’s revenue in Case 2. It is clearly that $\tilde{h}_2 = w$. To ensure the feasibility of this optimization problem, we must have $v - \gamma + t_0 \geq w$. As the objective function is a quadratic function, we have:

$$(\tilde{p}_2, \tilde{h}_2, \tilde{\Pi}_2) = \begin{cases} \left(\frac{v+t_0-\gamma+w}{2}, w, \frac{(v-w+t_0-\gamma)^2}{8t_0} \right), & \text{if } w + \gamma - t_0 < v < w + \gamma + 3t_0, \\ (v - t_0 - \gamma, w, v - t_0 - w - \gamma), & \text{if } v \geq w + \gamma + 3t_0. \end{cases}$$

Note that if $v - \gamma - t_0 - p = 0$, then we must have $v - t_0 - p > 0$, which implies $\pi_d = 1$. In addition, $w + \gamma - t_0 < w + t_0 \Leftrightarrow \gamma < 2t_0$. Hence, when $\gamma \geq 2t_0$, we compare $v - w - t_0$ and $\frac{(v-w+t_0-\gamma)^2}{8t_0}$. Note that we define:

$$L(v) \triangleq v - w - t_0 - \frac{(v - w + t_0 - \gamma)^2}{8t_0} \Rightarrow \frac{dL(v)}{dv} \propto 1 - \frac{v - w + t_0 - \gamma}{4t_0}.$$

It is easy to verify that $L(v)$ is increasing in v for $w + \gamma - t_0 < v < w + \gamma + 3t_0$ and $L(v) \geq 0$ for $w + \gamma - t_0 < v \leq w + \gamma + 3t_0$. When $\gamma < 2t_0$, we have:

$$\frac{(v - w + t_0 - \gamma)^2}{8t_0} - (v - w - t_0) > 0 \Leftrightarrow \frac{1}{8t_0} [v - (w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0})][v - (w + \gamma + 3t_0 + 2\sqrt{2\gamma t_0})] > 0.$$

Note that:

$$\begin{cases} w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} \geq w + t_0 & \Leftrightarrow (\sqrt{\gamma} - \sqrt{2t_0})^2 \geq 0, \\ w + \gamma + 3t_0 + 2\sqrt{2\gamma t_0} > w + \gamma + 3t_0. \end{cases}$$

Hence, we have:

$$\frac{(v - w + t_0 - \gamma)^2}{8t_0} > v - w - t_0, \text{ if } w + \gamma - t_0 \leq v < w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}.$$

Therefore, combining Case 1 and Case 2, we have, if $0 < t_0 \leq \gamma/2$, $(p^*, h^*, \Pi^*) = (v - t_0, w, v - t_0 - w)$; if $t_0 > \gamma/2$,

$$(p^*, h^*, \Pi^*) = \begin{cases} \left(\frac{v+t_0-\gamma+w}{2}, w, \frac{(v-w+t_0-\gamma)^2}{8t_0} \right), & \text{if } w + \gamma - t_0 < v \leq w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}, \\ (v - t_0, w, v - t_0 - w), & \text{if } v > w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}. \end{cases}$$

Let $\bar{v}_U \triangleq w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$. Thus, the result follows immediately. \square

Proof of Lemma 2. Based on the Definition of active players, it is clearly that $\pi_r > 0$ only if $h > w - t_0$.

Case 1: $w + t_0 \leq h$. In this case, we have $\pi_r = 1$, and as $h - w > 0$, it implies that this is the same situation as in the proof of Lemma 1.

Case 2: $w - t_0 < h < w + t_0$. In this case, we have $\underline{t}_r = w - h$ and $\pi_r = 1 - F_{T_r}(\underline{t}_r) = 1 - \frac{w-h+t_0}{2t_0}$. Then \underline{t}_d is determined by $v + t_0 - 2t_0\pi_d - \gamma \mathbb{1}_{\{\pi_r > \pi_d\}} - p = 0$. Note that $\underline{t}_d = 2t_0(1 - \pi_d) - t_0$.

Case 2.1: $\pi_d < 1 - \frac{w-h+t_0}{2t_0}$. In this case, we have $v + t_0 - 2t_0\pi_d - \gamma - p = 0 \Leftrightarrow \pi_d = \frac{v+t_0-p-\gamma}{2t_0}$. We require $0 < \frac{v+t_0-p-\gamma}{2t_0} < 1 - \frac{w-h+t_0}{2t_0}$.

Case 2.2: $1 - \frac{w-h+t_0}{2t_0} \leq \pi_d < 1$. In this case, we have $v + t_0 - 2t_0\pi_d - p = 0 \Leftrightarrow \pi_d = \frac{v+t_0-p}{2t_0}$. We require $1 - \frac{w-h+t_0}{2t_0} \leq \frac{v+t_0-p}{2t_0} < 1$.

Case 2.3: $\pi_d = 1$. In this case, we require $v - t_0 - p \geq 0$.

The results are summarized in Table 2. \square

Proof of Proposition 3. Part (i). We show that the optimal pricing policy must induce $\pi_r^* = \pi_d^*$.

Show by contradiction. Assume $\pi_r^* < \pi_d^*$. In this case, the platform's revenue is $(p^* - h^*)\pi_r^*$. The boundary driver is the one with $\underline{t}_d^* = \max\{-t_0, p^* - v\}$. The platform could keep h^* fix and increase p^* to $p' = p^* + \epsilon$ such that π_d^* reduces to $\pi_d' > \pi_r^*$. Then the platform's revenue becomes $(p' - h^*)\pi_r^*$, which is strictly higher than $(p^* - h^*)\pi_r^*$. Hence, a contradiction.

Assume $\pi_r^* > \pi_d^*$. In this case, the platform's revenue is $(p^* - h^*)\pi_d^*$. The boundary rider is the one with $\underline{t}_r^* = \max\{-t_0, w - h^*\}$. The platform could keep p^* fix and decrease h^* to $h' = h^* - \epsilon$ such that π_r^* reduces to $\pi_r' > \pi_d^*$. Then the platform's revenue becomes $(p^* - h')\pi_d^*$, which is strictly higher than $(p^* - h^*)\pi_d^*$. Hence, a contradiction.

Part (ii) and (iii). Similar as the proof of Proposition 2, we solve the revenue maximization problem of the platform analytically. Then this proposition directly follows from the optimal solution.

Case 1: $\pi_d = \pi_r = 1$. In this case, the platform's optimization problem becomes:

$$\begin{aligned} & \max_{p \geq h \geq 0} p - h \\ & \text{s.t. } p \leq v - t_0, \quad h \geq w + t_0. \end{aligned}$$

To ensure the feasibility of this optimization problem, one needs $v - t_0 \geq w + t_0 \Leftrightarrow v \geq w + 2t_0$. The optimal solution is $(\tilde{p}_1, \tilde{h}_1, \tilde{\Pi}_1) = (v - t_0, w + t_0, v - w - 2t_0)$.

Case 2: $\pi_d = \pi_r < 1$. In this case, the platform's optimization problem becomes:

$$\begin{aligned} \max_{p \geq h \geq 0} & (p - h) \left(1 - \frac{w - h + t_0}{2t_0} \right) \\ \text{s.t.} & \quad w - t_0 < h \leq w + t_0, \quad w + v = p + h. \end{aligned}$$

As we have $p = w + v - h \geq h \Leftrightarrow h \leq \frac{v+w}{2}$. So the optimization problem becomes:

$$\begin{aligned} \max_h & (w + v - 2h) \left(1 - \frac{w - h + t_0}{2t_0} \right) \\ \text{s.t.} & \quad w - t_0 < h \leq \min \left\{ \frac{v+w}{2}, w + t_0 \right\}. \end{aligned}$$

To ensure feasibility, we require $\frac{v+w}{2} > w - t_0 \Leftrightarrow v > w - 2t_0$.

When $\frac{v+w}{2} \leq w + t_0$ (which is equivalent to $v \leq w + 2t_0$), we have:

$$(\tilde{p}_2, \tilde{h}_2, \tilde{\Pi}_2) = \left(\frac{2t_0 + 3v + w}{4}, \frac{3w + v - 2t_0}{4}, \frac{(2t_0 + v - w)^2}{16t_0} \right).$$

When $\frac{v+w}{2} > w + t_0$ (which is equivalent to $v > w + 2t_0$), we have:

$$(\tilde{p}_2, \tilde{h}_2, \tilde{\Pi}_2) = \begin{cases} \left(\frac{2t_0 + 3v + w}{4}, \frac{3w + v - 2t_0}{4}, \frac{(2t_0 + v - w)^2}{16t_0} \right), & \text{if } w + 2t_0 < v < w + 6t_0, \\ (v - t_0, w + t_0, v - w - 2t_0), & \text{if } v \geq w + 6t_0. \end{cases}$$

Note that $\frac{(2t_0 + v - w)^2}{16t_0} - (v - w - 2t_0) = \frac{1}{16t_0}(6t_0 - v + w)^2 \geq 0$. Hence, combining Case 1 and Case 2 together, we have:

$$(p^*, h^*, \pi_r^*, \pi_d^*, \Pi^*) = \begin{cases} \left(\frac{2t_0 + 3v + w}{4}, \frac{3w + v - 2t_0}{4}, \frac{2t_0 + v - w}{8t_0}, \frac{2t_0 + v - w}{8t_0}, \frac{(2t_0 + v - w)^2}{16t_0} \right), & \text{if } w - 2t_0 < v < w + 6t_0, \\ (v - t_0, w + t_0, 1, 1, v - w - 2t_0), & \text{if } v \geq w + 6t_0. \end{cases}$$

We define $\bar{v}_B \triangleq w + 6t_0$. Thus, the result follows immediately. \square

Proof of Theorem 1. Given the results in Proposition 2 and 3:

Part (i). The comparison between BRDS and URDS in terms of the drivers' revenue share is straightforward.

We define $\tilde{v}_1 \triangleq w + 2t_0$. It is obvious that \tilde{v}_1 is increasing in w and t_0 , and BRDS generates higher drivers' revenue share than URDS if and only if $v > \tilde{v}_1$.

Part (ii). The comparison between BRDS and URDS in terms of the platform's revenue share is straightforward. We define $\tilde{v}_2 \triangleq w + 4t_0$. It is obvious that \tilde{v}_2 is increasing in w and t_0 , and BRDS generates higher platform's revenue share than URDS if and only if $v < \tilde{v}_2$.

Part (iii). The comparison between URDS and BRDS in terms of the average quality of drivers (and riders) are equivalent to the comparison in terms of the fraction of active drivers (and riders). It is obvious that the riders' average quality in BRDS is no less than that in URDS, since in URDS, all riders are active ($\pi_r^U = 1$).

For average quality of drivers, let π_d^B and π_d^U be the fraction of active drivers in BRDS and URDS. When $\gamma \geq 2t_0$, then $\pi_d^U = 1$, in which case $\pi_d^B \leq \pi_d^U$. Hence, the average drivers' quality in BRDS is always greater (at least weakly) than that in URDS. When $\gamma < 2t_0$, we have $w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} < w + 6t_0$, since it is equivalent to $\gamma - 3t_0 - 2\sqrt{2\gamma t_0} < 0$. It is easy to verify that $\gamma - 3t_0 - 2\sqrt{2\gamma t_0}$ is decreasing in γ for $\gamma \in [0, 2t_0]$ and hence,

$\gamma - 3t_0 - 2\sqrt{2\gamma t_0} < 0$ for $\gamma \in [0, 2t_0]$. Therefore, when $w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} < v < w + 6t_0$, we have $\pi_d^B < \pi_d^U = 1$, i.e., the average drivers' quality in BRDS is always greater than that in URDS. Note that when $\pi_d^B < 1$ and $\pi_d^U < 1$, we have $\frac{2t_0 + v - w}{8t_0} > \frac{v - w + t_0 - \gamma}{4t_0} \Leftrightarrow v < w + 2\gamma$. Hence, we define $\tilde{v}_3 \triangleq \min\{w + 2\gamma, w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}\}$. Then we have $\pi_d^B > \pi_d^U$ if and only if $v < \tilde{v}_3$. Note that $3t_0 - 2\sqrt{2\gamma t_0}$ is increasing in t_0 for $t_0 \geq 2\gamma/9$. As we require $t_0 > \gamma/2$ in this case, thus, \tilde{v}_3 is increasing in t_0 . \square

Proof of Theorem 2. Given the results in Proposition 2 and 3:

Part (i). Driver's welfare in URDS is zero and in BRDS is positive.

Part (ii). Let W_r^B and W_r^U be riders' welfare under BRDS and URDS. Let $v_1 \triangleq w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma$ and $v_2 \triangleq w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$. First, note that

$$v_1 < v_2 \Leftrightarrow w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma < w + 3t_0 + \gamma - 2\sqrt{2\gamma t_0} \Leftrightarrow (1 + \sqrt{2})\gamma + 2\sqrt{2\gamma t_0} - (3 - \sqrt{2})t_0 < 0. \quad (\text{A.1})$$

This is because it is easy to verify that $(1 + \sqrt{2})\gamma + 2\sqrt{2\gamma t_0} - (3 - \sqrt{2})t_0$ is increasing in γ for $\gamma \geq 0$. It is also easy to verify that when $\gamma = t_0(\sqrt{2} - 1)^2$, we have $(1 + \sqrt{2})\gamma + 2\sqrt{2\gamma t_0} - (3 - \sqrt{2})t_0 = 0$. This property will be used later in the proof.

In URDS, we have:

$$\begin{aligned} W_r^U &= \frac{1}{2t_0} \int_{t_d^*}^{t_0} (v + t - \gamma \mathbb{1}_{\{\pi_r^* > \pi_d^*\}} - p^*) dt \\ &= \begin{cases} t_0, & \text{if } \gamma \geq 2t_0 \text{ or } \gamma < 2t_0 \text{ but } v \geq w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}, \\ \frac{(v - w + t_0 - \gamma)^2}{16t_0}, & \text{if } \gamma < 2t_0 \text{ and } w + \gamma - t_0 \leq v < w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}. \end{cases} \end{aligned}$$

In BRDS, we have:

$$W_r^B = \frac{1}{2t_0} \int_{t_d^*}^{t_0} (v + t - p^*) dt = \begin{cases} \frac{(2t_0 + v - w)^2}{64t_0}, & \text{if } w - 2t_0 \leq v < w + 6t_0, \\ t_0, & \text{if } v \geq w + 6t_0. \end{cases}$$

Note that we have $\frac{(2t_0 + v - w)^2}{64t_0} < t_0 \Leftrightarrow v < w + 6t_0$. Hence, the case of $\gamma \geq 2t_0$ is proved. When $\gamma < 2t_0$, it is obvious that if $v \geq w + 6t_0$, then $W_r^U = W_r^B$ and if $v \in [w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}, w + 6t_0)$, then $W_r^U > W_r^B$. Now if $v \in (w + \gamma - t_0, w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0})$, we have $\frac{(2t_0 + v - w)^2}{64t_0} > \frac{(v - w + t_0 - \gamma)^2}{16t_0} \Leftrightarrow v < w + 2\gamma$. Hence, $\tilde{v}_4 \triangleq (w + t_0) \mathbb{1}_{\{t_0 \leq \gamma/2\}} + \tilde{v}_3 \mathbb{1}_{\{t_0 > \gamma/2\}}$. It is clear that \tilde{v}_4 is increasing in t_0 and w as in the proof of Theorem 1.

Part (iii). Given the optimal solution of the platform's revenue maximization problem in URDS and BRDS, we have

$$w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} < w + 2t_0 \Leftrightarrow \gamma + t_0 - 2\sqrt{2\gamma t_0} < 0 \Leftrightarrow (\sqrt{2t_0} - \sqrt{\gamma})^2 - t_0 < 0 \Leftrightarrow \gamma > t_0(\sqrt{2} - 1)^2. \quad (\text{A.2})$$

This property will be used later in the proof. It is easy to verify that $v_1 > w + \gamma - t_0$. Note that when $w + t_0 \leq v \leq w + 6t_0$:

$$\frac{(2t_0 + v - w)^2}{16t_0} > v - w - t_0 \Leftrightarrow \frac{1}{16t_0}(v - w - 2t_0)(v - w - 10t_0) > 0.$$

Hence, when $w - 2t_0 < v < w + 2t_0$, we have $\frac{(2t_0 + v - w)^2}{16t_0} > v - w - t_0$ and the case of $0 < t_0 \leq \gamma/2$ is proved.

Now we consider the case when $t_0 > \gamma/2$. Note that when $w + \gamma - t_0 \leq v \leq w + 6t_0$:

$$\begin{aligned} \frac{(2t_0 + v - w)^2}{16t_0} &> \frac{(v - w + t_0 - \gamma)^2}{8t_0} \Leftrightarrow (2t_0 + v - w) > \sqrt{2}(v - w + t_0 - \gamma) \\ &\Rightarrow v < w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma \triangleq v_1. \end{aligned}$$

By (A.1), we know that when $2t_0 > \gamma \geq t_0(\sqrt{2}-1)^2$ (or equivalently $\gamma/2 < t_0 \leq \gamma(\sqrt{2}-1)^{-2}$), we have:

$$\begin{cases} w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma > w + 3t_0 + \gamma - 2\sqrt{2\gamma t_0}, \\ w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} < w + 2t_0, \text{ by (A.2).} \end{cases}$$

As a result, when $v < w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$, $\Pi^B = \frac{(2t_0+v-w)^2}{16t_0} > \frac{(v-w+t_0-\gamma)^2}{8t_0} = \Pi^U$. When $w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} \leq v < w + 2t_0$, we have $\Pi^U = v - w - t_0$ and $\Pi^B = \frac{(2t_0+v-w)^2}{16t_0}$. We still have $\Pi^B > \Pi^U$. Consequently, only when $v > w + 2t_0$, we have $\Pi^B < \Pi^U$.

When $\gamma < t_0(\sqrt{2}-1)^2$ (or equivalently $t_0 > \gamma(\sqrt{2}-1)^{-2}$), we have:

$$\begin{cases} w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma < w + 3t_0 + \gamma - 2\sqrt{2\gamma t_0}, \\ w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} > w + 2t_0, \text{ by (A.2).} \end{cases}$$

As a result, when $v < w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma$, $\Pi^B = \frac{(2t_0+v-w)^2}{16t_0} > \frac{(v-w+t_0-\gamma)^2}{8t_0} = \Pi^U$. When $w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma \leq v \leq w + 6t_0$, we still have $\Pi^B = \frac{(2t_0+v-w)^2}{16t_0}$. In this case, no matter if $\Pi^U = \frac{(v-w+t_0-\gamma)^2}{8t_0}$ or $\Pi^U = v - w - t_0$, we all have $\Pi^U > \Pi^B$, as $w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} > w + 2t_0$. To sum up, we define

$$\tilde{v}_5 \triangleq \begin{cases} w + 2t_0, & \text{if } t_0 \leq \gamma(\sqrt{2}-1)^{-2}, \\ w + \sqrt{2}t_0 + (2 + \sqrt{2})\gamma, & \text{if } t_0 > \gamma(\sqrt{2}-1)^{-2}. \end{cases}$$

Then, BRDS generates higher platform's revenue per unit of time than URDS if and only if $v < \tilde{v}_5$ and \tilde{v}_5 is clearly increasing in t_0 and w .

Part (iv). Let W^B and W^U be the social welfare under BRDS and URDS. Note that social welfare is the sum of riders' welfare, drivers' welfare and the platform's revenue. It is clearly that in URDS, the drivers' welfare is zero. Thus, we denote drivers' welfare in BRDS by W_d^B .

$$\begin{aligned} W_d^B &= \Lambda_d \int_{t_d^*}^{t_0} \min \left\{ \frac{\pi_r^*}{\pi_d^*}, 1 \right\} \left(\int_{t_r^*}^{t_0} (h^* + t - w) f_{T_r}(t|t \geq t_r^*) dt \right) f_{T_d}(t_d) dt_d \\ &= \pi_d^* \frac{1/(2t_0)}{1 - F_{T_r}(t_r^*)} \int_{t_r^*}^{t_0} (h^* + t - w) dt, \text{ note in BRDS } \pi_d^* = \pi_r^* \text{ and we assume } \Lambda_r = \Lambda_d = 1 \\ &= \begin{cases} \frac{1}{2t_0} \int_{\frac{2t_0-v+w}{4}}^{t_0} \left(\frac{3w+v-2t_0}{4} + t - w \right) dt = \frac{(2t_0+v-w)^2}{64t_0}, & \text{if } w - 2t_0 \leq v < w + 6t_0, \\ \frac{1}{2t_0} \int_{-t_0}^{t_0} (w + t_0 + t - w) dt = t_0, & \text{if } v \geq w + 6t_0. \end{cases} \end{aligned}$$

In URDS, we have:

$$\begin{cases} W^U = W_r^U + \Pi^U = t_0 + v - w - t_0 = v - w, & \text{if } \gamma \geq 2t_0, \\ W^U = W_r^U + \Pi^U = \begin{cases} v - w, & \text{if } v \geq w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}, \\ \frac{3(v-w+t_0-\gamma)^2}{16t_0}, & \text{if } w + \gamma - t_0 \leq v \leq w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}. \end{cases} & \text{if } \gamma < 2t_0. \end{cases}$$

In BRDS, we have:

$$W^B = W_r^B + W_d^B + \Pi^B = \begin{cases} \frac{3(2t_0+v-w)^2}{32t_0}, & \text{if } w - 2t_0 \leq v < w + 6t_0, \\ v - w, & \text{if } v \geq w + 6t_0. \end{cases}$$

Note that when $w + t_0 < v < w + 6t_0$, we have:

$$\frac{3(2t_0+v-w)^2}{32t_0} < v - w \Leftrightarrow \frac{3}{32t_0}(v-w-2t_0/3)(v-w-6t_0) < 0.$$

Hence, the case of $0 < t_0 \leq \gamma/2$ is proved.

Note that when $v > w + \gamma - t_0$, we have:

$$\frac{3(2t_0 + v - w)^2}{32t_0} < \frac{3(v - w + t_0 - \gamma)^2}{16t_0} \Leftrightarrow 2t_0 + v - w < \sqrt{2}(v - w + t_0 - \gamma) \Leftrightarrow v > v_1.$$

Note that when $v_1 < w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$, if $w + \gamma - t_0 < v < v_1$, then we have $W^B > W^U$; if $v_1 \leq v < w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$, then $W^U > W^B$; if $w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} < v < w + 6t_0$, although $W^U = v - w$, we still have $W^U > W^B$. If $v_1 > w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$, if $w + \gamma - t_0 < v < w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0}$, then we have $W^B > W^U$; if $w + \gamma + 3t_0 - 2\sqrt{2\gamma t_0} < v < w + 6t_0$, we have $W^U > W^B$.

To sum up, we know that when $t_0 \leq \gamma/2$, social welfare in URDS is no less than that in BRDS; when $t_0 > \gamma/2$, we define $\tilde{v}_6 \triangleq \min\{v_1, v_2\}$, then BRDS generates higher social welfare if and only if $v \leq \tilde{v}_6$. Note that $\tilde{v}_5 > \tilde{v}_4$. This is equivalent to $\tilde{v}_5 - w > \min\{2\gamma, \gamma + 3t_0 - 2\sqrt{2\gamma t_0}\}$. When $t_0 > \gamma(\sqrt{2} - 1)^{-2}$, $\tilde{v}_5 - w = t_0\sqrt{2} + \gamma(2 + \sqrt{2}) > 2\gamma$. Hence, $\tilde{v}_5 - w > \min\{2\gamma, \gamma + 3t_0 - 2\sqrt{2\gamma t_0}\}$. When $t_0 \leq \gamma(\sqrt{2} - 1)^{-2}$, $\tilde{v}_5 - w = 2t_0 > \gamma + 3t_0 - 2\sqrt{2\gamma t_0} \Leftrightarrow \gamma + t_0 < 2\sqrt{2\gamma t_0} \Leftrightarrow (1 + y)^2 < 8y$, where $y \triangleq \gamma/t_0$. Note that $(1 + y)^2 < 8y \Leftrightarrow y \in ((\sqrt{2} - 1)^2, (\sqrt{2} + 1)^2)$. Note that $\gamma/2 < t_0 \leq \gamma(\sqrt{2} - 1)^{-2} \Leftrightarrow (\sqrt{2} - 1)^2 \leq y < 2 < (1 + \sqrt{2})^2$. Hence, $\tilde{v}_5 - w > \min\{2\gamma, \gamma + 3t_0 - 2\sqrt{2\gamma t_0}\}$. Therefore, when v is less than \tilde{v}_4 , BRDS generates higher surplus for all stakeholders (drivers, riders and the platform) than URDS. \square

Appendix B: Value of Carpooling Service

PROPOSITION B.1. In URDS, $h^* = w$.

(i) When $\Lambda_r > \Lambda_d k$, the optimal p^* and Π^* are

$$(p^*, \Pi^*) = \begin{cases} \text{unprofitable}, & \text{if } v \leq w + \gamma - t_0, \\ \left(\frac{v + t_0 - \gamma + w}{2}, \frac{\Lambda_d k (v - w + t_0 - \gamma)^2}{8t_0} \right), & \text{if } w + \gamma - t_0 < v < w + \gamma + 3t_0, \\ (v - t_0 - \gamma, \Lambda_d k (v - t_0 - w - \gamma)), & \text{if } v \geq w + \gamma + 3t_0. \end{cases}$$

(i) When $\Lambda_r \leq \Lambda_d k$,

(1) if $\Lambda_d k \gamma \geq 2t_0 \Lambda_r$,

$$(p^*, \Pi^*) = \begin{cases} \text{unprofitable}, & \text{if } v \leq w - t_0 + \frac{2t_0 \Lambda_r}{\Lambda_d k}, \\ \left(v + t_0 - \frac{2t_0 \Lambda_r}{\Lambda_d k}, \left(v + t_0 - \frac{2t_0 \Lambda_r}{\Lambda_d k} - w \right) \Lambda_r \right), & \text{if } v > w - t_0 + \frac{2t_0 \Lambda_r}{\Lambda_d k}. \end{cases}$$

(2) if $\Lambda_d k \gamma < 2t_0 \Lambda_r$,

$$(p^*, \Pi^*) = \begin{cases} \text{unprofitable}, & \text{if } v \leq w + \gamma - t_0, \\ \left(\frac{v + t_0 - \gamma + w}{2}, \frac{\Lambda_d k (v - w + t_0 - \gamma)^2}{8t_0} \right), & \text{if } w + \gamma - t_0 < v < \gamma + w - t_0 + \frac{1 - 2\sqrt{a\gamma}}{2a}, \\ \left(v + t_0 - \frac{2t_0 \Lambda_r}{\Lambda_d k}, \left(v + t_0 - \frac{2t_0 \Lambda_r}{\Lambda_d k} - w \right) \Lambda_r \right), & \text{if } v \geq \gamma + w - t_0 + \frac{1 - 2\sqrt{a\gamma}}{2a}, \end{cases}$$

where $a \triangleq \frac{\Lambda_d k}{8t_0 \Lambda_r}$.

Proof of Proposition B.1 In URDS, the drivers cannot observe riders' ratings. So a driver with K service capacity will accept all riders. Thus, a driver will take the match if and only if $h \geq w$. All riders are active and a rider will accept a match if and only if $v + t - \gamma \mathbb{1}_{\{\Lambda_r > \Lambda_d \pi_d k\}} - p \geq 0$. The platform solves

$$\max_{p \geq h > 0} (p - h) \min\{\Lambda_r, \Lambda_d \pi_d k\}.$$

Case 1: $\Lambda_r > \Lambda_d k$. To support the equilibrium $\pi_d = 1$, we need $v - t_0 - \gamma \geq p$. To support the equilibrium $\pi_d < 1$, we must have

$$\begin{cases} v + \underline{t}_d - \gamma - p = 0 & \Leftrightarrow \underline{t}_d = p + \gamma - v \\ \underline{t}_d > -t_0 & \Leftrightarrow p > v - \gamma - t_0. \end{cases} \Rightarrow \pi_d = \mathbb{P}\{T_d \geq \underline{t}_d\} = 1 - \frac{\underline{t}_d + t_0}{2t_0} = \frac{v + t_0 - p - \gamma}{2t_0} < 1.$$

Thus, in Case 1, each p will induce one unique equilibrium, i.e., $\pi_d = \min\left\{\frac{(v+t_0-p-\gamma)^+}{2t_0}, 1\right\}$.

Case 2: $\Lambda_r \leq \Lambda_d k$. To support the equilibrium $\pi_d = 1$, we just need $v - t_0 \geq p$. To support $\pi_d < 1$, we need to consider the following situations. First, note that as long as $\Lambda_r > \Lambda_d k \frac{v+t_0-p-\gamma}{2t_0} \Leftrightarrow p > v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - \gamma$, we have $\pi_d = \frac{v+t_0-p-\gamma}{2t_0}$. Also note that $\frac{v+t_0-p-\gamma}{2t_0} < 1 \Leftrightarrow p > v - \gamma - t_0$. And $v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - \gamma > v - \gamma - t_0$ is ensured by the condition in Case 2. And $v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - \gamma < v - \gamma + t_0$ holds obviously. Second,

Case 2.1: $\Lambda_r \leq \Lambda_d k \frac{v+t_0-p}{2t_0} \Leftrightarrow p \leq v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$. In this case, we have $\pi_d < 1 \Leftrightarrow v - t_0 < p < v + t_0$ and $\underline{t}_d = p - v$. The condition in Case 2 ensures $v - t_0 \leq v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$. It is obvious that if the optimal p^* is in in Case 2.1, then we must have $p^* = v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$.

Case 2.2: $\Lambda_r > \Lambda_d k \frac{v+t_0-p}{2t_0} \Leftrightarrow p > v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$. In this case, we have $\pi_d = \frac{v+t_0-p-\gamma}{2t_0} < 1$ and $\underline{t}_d = p + \gamma - v$. This is because, $\frac{v+t_0-p-\gamma}{2t_0} < 1 \Leftrightarrow p > v - \gamma - t_0$. And $v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} > v - \gamma - t_0 \Leftrightarrow 2t_0 \left(1 - \frac{\Lambda_r}{\Lambda_d k}\right) > -\gamma$ is implied by the condition in Case 2.

As a summary, in Case 2, i.e., $\Lambda_r \leq \Lambda_d k$, we have 2 classes of equilibria:

$$\begin{aligned} (\pi_d, \underline{t}_d) &= \begin{cases} \left(\frac{v+t_0-p-\gamma}{2t_0}, p + \gamma - v\right), & \text{if } v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - \gamma < p < v - \gamma + t_0, \\ (0, t_0), & \text{if } p \geq v - \gamma + t_0. \end{cases} \text{ and} \\ (\pi_d, \underline{t}_d) &= \begin{cases} (1, -t_0), & \text{if } p \leq v - t_0, \\ \left(\frac{v+t_0-p}{2t_0}, p - v\right), & \text{if } v - t_0 < p \leq v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}, \end{cases} \end{aligned}$$

Now we solve the platform's optimization problem. It is obvious that $h^* = w$.

In a market with $\Lambda_r > \Lambda_d k$, we have $\pi_d = \min\left\{\frac{(v+t_0-p-\gamma)^+}{2t_0}, 1\right\}$, the platform solves

$$\max_{p \geq h > 0} (p - h) \min\{\Lambda_r, \Lambda_d \pi_d k\}.$$

So the objective function of the platform becomes

$$\max_{p \geq w} (p - w) \min\{\Lambda_r, \Lambda_d \pi_d k\} = \Lambda_d k (p - w) \min\left\{\frac{(v + t_0 - p - \gamma)^+}{2t_0}, 1\right\}.$$

It is obvious that we only need to solve

$$\begin{aligned} \max_{p \geq w} \Lambda_d k (p - w) \frac{v + t_0 - p - \gamma}{2t_0} \\ \text{s.t. } v - \gamma - t_0 < p < v - \gamma + t_0. \end{aligned}$$

To ensure the feasibility, we need $v - \gamma + t_0 \geq w$; otherwise the platform will just set $p^* = w^*$ to get a 0 revenue, or simply just quit. So we have

$$(p^*, h^*, \Pi^*) = \begin{cases} \left(\frac{v+t_0-\gamma+w}{2}, w, \frac{\Lambda_d k (v-w+t_0-\gamma)^2}{8t_0}\right), & \text{if } w + \gamma - t_0 < v < w + \gamma + 3t_0, \\ (v - t_0 - \gamma, w, \Lambda_d k (v - t_0 - w - \gamma)), & \text{if } v \geq w + \gamma + 3t_0. \end{cases}$$

In a market with $\Lambda_r \leq \Lambda_d k$, we have: either $p = v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$, which leads to

$$\Pi_1 = \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - w\right) \Lambda_r.$$

Note that $v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} > w$ ensures a positive revenue; or we can have

$$\begin{aligned} \Pi_2 &= \max_{p \geq w} \Lambda_d k (p - w) \frac{v + t_0 - p - \gamma}{2t_0} \\ \text{s.t. } & v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - \gamma < p < v - \gamma + t_0. \end{aligned}$$

To ensure the feasibility, we require $v - \gamma + t_0 > w$. We also know that

$$\frac{v + t_0 - \gamma + w}{2} > v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - \gamma \Leftrightarrow v < w + \gamma - t_0 + \frac{4t_0\Lambda_r}{\Lambda_d k}.$$

So we have

$$(p^*, \Pi_2^*) = \begin{cases} \left(\frac{v + t_0 - \gamma + w}{2}, \frac{\Lambda_d k (v - w + t_0 - \gamma)^2}{8t_0} \right), & \text{if } w + \gamma - t_0 < v < w + \gamma - t_0 + \frac{4t_0\Lambda_r}{\Lambda_d k}, \\ \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}, \Lambda_d k (p^* - w) \frac{v + t_0 - p^* - \gamma}{2t_0} \right), & \text{if } v \geq w + \gamma - t_0 + \frac{4t_0\Lambda_r}{\Lambda_d k}. \end{cases}$$

So we need to compare Π_1 and Π_2^* . Note that when $v \geq w + \gamma - t_0 + \frac{4t_0\Lambda_r}{\Lambda_d k}$, it is obvious that $\Pi_1 > \Pi_2^*$. This is because, $\Pi_2^* = \Lambda_d k (\hat{p} - w) \frac{v + t_0 - \hat{p} - \gamma}{2t_0}$, where $\hat{p} = v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$. And $\Pi_1 = \Lambda_d k (\hat{p} - w) \frac{v + t_0 - \hat{p}}{2t_0}$.

• **When $w - t_0 + \frac{2t_0\Lambda_r}{\Lambda_d k} < w - t_0 + \gamma \Leftrightarrow \gamma > \frac{2t_0\Lambda_r}{\Lambda_d k}$** , we have

$$L(v) \triangleq \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - w \right) \Lambda_r - \frac{\Lambda_d k (v - w + t_0 - \gamma)^2}{8t_0} \Rightarrow \frac{L(v)}{dv} \propto \Lambda_r - \frac{\Lambda_d k (v - w + t_0 - \gamma)}{4t_0}.$$

It can be verified that $L(v)$ is increasing in $v \in [w + \gamma - t_0, w + \gamma - t_0 + \frac{4t_0\Lambda_r}{\Lambda_d k}]$ and $L(v) \geq 0$, $\forall w + \gamma - t_0 < v < w + \gamma - t_0 + \frac{4t_0\Lambda_r}{\Lambda_d k}$. Hence, when $\gamma \geq \frac{2t_0\Lambda_r}{\Lambda_d k}$, we always have the global optimal is $p^* = v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}$, $t_d = p^* - v$, $\pi_d^* = \frac{v + t_0 - p^*}{2t_0}$, and $\Pi^* = \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - w \right) \Lambda_r$ for $v > w - t_0 + \frac{2t_0\Lambda_r}{\Lambda_d k}$; otherwise, the platform chooses not to do the business and get zero revenue.

• **When $\gamma < \frac{2t_0\Lambda_r}{\Lambda_d k}$** , we have

$$\begin{aligned} & \frac{\Lambda_d k (v - w + t_0 - \gamma)^2}{8t_0\Lambda_r} - \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - w \right) > 0 \\ \Leftrightarrow & v < \gamma + w - t_0 + \frac{1 - 2\sqrt{a\gamma}}{2a} \text{ or } v > \gamma + w - t_0 + \frac{1 - 2\sqrt{a\gamma}}{2a}, \end{aligned}$$

where $a \triangleq \frac{\Lambda_d k}{8t_0\Lambda_r}$.

Hence, in this case

$$(p^*, \Pi^*) = \begin{cases} \left(\frac{v + t_0 - \gamma + w}{2}, \frac{\Lambda_d k (v - w + t_0 - \gamma)^2}{8t_0} \right), & \text{if } w + \gamma - t_0 < v < \gamma + w - t_0 + \frac{1 - 2\sqrt{a\gamma}}{2a}, \\ \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k}, \left(v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - w \right) \Lambda_r \right), & \text{if } v \geq \gamma + w - t_0 + \frac{1 - 2\sqrt{a\gamma}}{2a}. \end{cases} \quad \square$$

PROPOSITION B.2. *In BRDS, the transaction volume under the platform's optimal pricing strategy is*

$$q^* = \begin{cases} \frac{(2t_0 + v - w)\Lambda_r \Lambda_d k}{4t_0(\Lambda_r + \Lambda_d k)}, & \text{if } v < 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0, \\ \min\{\Lambda_r, \Lambda_d k\}, & \text{if } v \geq 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0. \end{cases}$$

The optimal price (h^*, p^*) are given by

$$\begin{cases} \frac{q^*}{\Lambda_r} = \frac{t_0 - w + h^*}{2t_0}, \\ \frac{q^*}{\Lambda_d k} = \frac{v + t_0 - p^*}{2t_0}. \end{cases}$$

Proof of Proposition B.2 In BRDS, drivers can reject riders based on riders' ratings. Because drivers and riders are homogenous in their v and w , then, a rider with rating t_r is accepted by a driver if and only if $h - w + t_r > 0$. Hence, it is clear that $\pi_r > 0$ if and only if $h - w + t_0 > 0 \Leftrightarrow h > w - t_0$.

Case 1: $w + t_0 \leq h$. In this case, we have $\pi_r = 1$, i.e., all riders will be accepted and since this case implies $h > w$, so this is the same situation as URDS.

Case 2: $w - t_0 < h < w + t_0$. In this case, we have $\underline{t}_r = w - h$ and $\pi_r = 1 - F_r(\underline{t}_r) = \frac{t_0 - w + h}{2t_0}$. Then \underline{t}_d is determined by

$$\begin{cases} v + \underline{t}_d - \gamma \mathbb{1}_{\{\Lambda_r \pi_r > \Lambda_d \pi_d k\}} - p = 0, \\ \pi_d = 1 - \frac{\underline{t}_d + t_0}{2t_0} \Leftrightarrow \underline{t}_d = t_0(1 - 2\pi_d) \end{cases} \Rightarrow v + t_0(1 - 2\pi_d) - \gamma \mathbb{1}_{\{\Lambda_r \pi_r > \Lambda_d \pi_d k\}} - p = 0.$$

Case 2.1: $\Lambda_d \pi_d k < \frac{\Lambda_r(t_0 - w + h)}{2t_0}$. In this case, we have $v + t_0(1 - 2\pi_d) - \gamma - p = 0 \Leftrightarrow \pi_d = \frac{v + t_0 - p - \gamma}{2t_0}$. We require

$$0 < \Lambda_d \pi_d k < \frac{\Lambda_r(t_0 - w + h)}{2t_0} \Leftrightarrow v + t_0 - \gamma - \frac{\Lambda_r(t_0 - w + h)}{\Lambda_d k} < p < v + t_0 - \gamma.$$

Note that $\pi_d < 1 \Leftrightarrow p > v - \gamma - t_0$ and we have

$$v - \gamma - t_0 > v + t_0 - \gamma - \frac{\Lambda_r(t_0 - w + h)}{\Lambda_d k} \Leftrightarrow \Lambda_d k < \frac{\Lambda_r(t_0 - w + h)}{2t_0}.$$

- If $\Lambda_d k < \frac{\Lambda_r(t_0 - w + h)}{2t_0}$, then we require $v - \gamma - t_0 < p < v + t_0 - \gamma$.
- If $\Lambda_d k \geq \frac{\Lambda_r(t_0 - w + h)}{2t_0}$, then we require $v + t_0 - \gamma - \frac{\Lambda_r(t_0 - w + h)}{\Lambda_d k} < p < v + t_0 - \gamma$.

Case 2.2: $\Lambda_d \pi_d k \geq \frac{\Lambda_r(t_0 - w + h)}{2t_0}$. In this case, we have $v + t_0(1 - 2\pi_d) - p = 0 \Leftrightarrow \pi_d = \frac{v + t_0 - p}{2t_0}$. We require

$$\frac{\Lambda_r(t_0 - w + h)}{2t_0 \Lambda_d k} \leq \pi_d < 1.$$

- If $\frac{\Lambda_r(t_0 - w + h)}{2t_0} > \Lambda_d k$, then it means that even if all drivers are active, still cannot meet the demand.

So in this case, there will be inconvenience cost. Then this means, Case 2.2. is not applicable.

- If $\frac{\Lambda_r(t_0 - w + h)}{2t_0} \leq \Lambda_d k$, then

$$\Lambda_d \pi_d k \geq \frac{\Lambda_r(t_0 - w + h)}{2t_0} \Leftrightarrow p \leq v + t_0 - \frac{\Lambda_r(t_0 - w + h)}{\Lambda_d k}. \text{ and } \pi_d < 1 \Leftrightarrow p > v - t_0.$$

$$v - t_0 \leq v + t_0 - \frac{\Lambda_r(t_0 - w + h)}{\Lambda_d k} \Leftrightarrow \frac{\Lambda_r(t_0 - w + h)}{2t_0} \leq \Lambda_d k.$$

Note that here, $p \leq v - t_0$ just means $\pi_d = 1$. The platform's optimization problem is:

$$\max_{p \geq h > 0} (p - h) \min\{\Lambda_r \pi_r, \Lambda_d \pi_d k\}.$$

Now we show that under the optimal pricing strategy (p^*, h^*) , we have $\Lambda_r \pi_r = \Lambda_d \pi_d k$.

If $\Lambda_r \pi_r < \Lambda_d \pi_d k$, then the platform's revenue is $(p - h)\Lambda_r \pi_r$. The boundary driver is the one with $\underline{t}_d = \max\{-t_0, p - v\}$. Then the platform could keep h fix while increasing p by ϵ such that $\Lambda_r \pi_r < \Lambda_d \pi'_d k$ still hold, where π'_d is under $p + \epsilon$. The resulting platform's revenue is $(p + \epsilon - h)\Lambda_r \pi_r > (p - h)\Lambda_r \pi_r$.

If $\Lambda_r \pi_r > \Lambda_d \pi_d k$, then the platform's revenue is $(p - h)\Lambda_d \pi_d k$. The boundary rider is the one with $\underline{t}_r = \max\{-t_0, w - h\}$. The platform could keep p fix and decrease h by ϵ such that $\Lambda_r \pi'_r > \Lambda_d \pi_d k$ still hold, where π'_r is under $h - \epsilon$. The resulting platform's revenue is $(p - h + \epsilon)\Lambda_d \pi_d k > (p - h)\Lambda_d \pi_d k$.

The maximum transaction volume is $\bar{q} = \min\{\Lambda_r, \Lambda_d k\}$. Note that $\forall q \in [0, \bar{q}]$, we have $q = \min\{\Lambda_r \pi_r, \Lambda_d k \pi_d\}$. The platform solves for the revenue maximizing $[p(q) - h(q)]q$, where $p(q)$ and $h(q)$ are induced by q :

$$\begin{cases} h(q) \triangleq \inf\{h : \Lambda_r \pi_r \geq q\}, \\ p(q) \triangleq \sup\{p : \Lambda_d \pi_d k \geq q\}. \end{cases}$$

Note that because we proved that optimal (h^*, p^*) will induce supply equal demand, i.e., $\Lambda_r \pi_r = \Lambda_d k \pi_d$, so based on the previous analysis, we only need to consider $\pi_r = \frac{t_0 - w + h}{2t_0}$ and $\pi_d = \frac{v + t_0 - p}{2t_0}$. Also, it suffices to consider $h \leq w + t_0$, as when $h > w + t_0$, $\pi_r = 1$, then under supply equal demand, the platform's revenue is $(p - h)\Lambda_r$. Thus, we have

$$\begin{cases} h(q) \triangleq \inf\{h : \Lambda_r \pi_r \geq q\} = \inf\left\{h : \frac{\Lambda_r(t_0 - w + h)}{2t_0} \geq q\right\} = \frac{2qt_0}{\Lambda_r} + w - t_0, \\ p(q) \triangleq \sup\{p : \Lambda_d \pi_d k \geq q\} = \max\left\{p : \frac{\Lambda_d(v + t_0 - p)k}{2t_0} \geq q\right\} = v + t_0 - \frac{2t_0 q}{\Lambda_d k}. \end{cases}$$

Note that this $(h(q), p(q))$ ensures that they will make supply equal demand, i.e., $\Lambda_r \pi_r = \Lambda_d k \pi_d$ by construction. Hence, the platform solves

$$\max_{q \in [0, \bar{q}]} [p(q) - h(q)]q = \max_{q \in [0, \bar{q}]} \left[v + 2t_0 - w - 2t_0 q \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \right] q.$$

The function is quadratic in q with interior solution

$$q^* = \frac{2t_0 + v - w}{4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right)}.$$

We only need to compare q^* with the boundar 0 and $\bar{q} = \min\{\Lambda_r, \Lambda_d k\}$.

Note that in order to have the platform being profitable, we need $q^* > 0 \Leftrightarrow v > w - 2t_0$. Then, we have

$$q^* < \bar{q} \Leftrightarrow \frac{2t_0 + v - w}{4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right)} < \min\{\Lambda_r, \Lambda_d k\} \Leftrightarrow v < 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0. \quad \square$$

Proof of Theorem 3 We will compare the two systems in URDS: pooling is allowed vs. pooling is not allowed. Note that in both systems all riders are active, i.e., $\pi_r^* = 1$, and $h^* = w$, which implies drivers have zero surplus.

When pooling is not allowed, the riders' welfare is

$$\begin{aligned} W_r^U &= \min\{\Lambda_r \pi_r, \Lambda_d \pi_d\} \int_{\underline{t}_d^*}^{t_0} (v + t - p^* - \gamma \mathbb{1}_{\{\Lambda_r \pi_r > \Lambda_d \pi_d^*\}}) f(t | t \geq \underline{t}_d) dt. \\ &= \frac{\min\{\Lambda_r, \Lambda_d \pi_d\}}{2t_0 \pi_d} \int_{\underline{t}_d^*}^{t_0} (v + t - p^* - \gamma \mathbb{1}_{\{\Lambda_r > \Lambda_d \pi_d^*\}}) dt \end{aligned}$$

Based on Proposition B.1, we have:

- **When $\Lambda_r > \Lambda_d$.**

$$(\pi_d^*, \Pi^*, W_r^U) = \begin{cases} \text{platform quits,} & \text{if } v \leq w + \gamma - t_0, \\ \left(\frac{v + t_0 - \gamma - w}{4t_0}, \frac{\Lambda_d(v - w + t_0 - \gamma)^2}{8t_0}, \frac{\Lambda_d(v - w + t_0 - \gamma)^2}{16t_0} \right), & \text{if } w + \gamma - t_0 < v < w + \gamma + 3t_0, \\ (1, \Lambda_d(v - t_0 - w - \gamma), \Lambda_d t_0), & \text{if } v \geq w + \gamma + 3t_0. \end{cases} \quad (\text{B.1})$$

- **When $\Lambda_r \leq \Lambda_d$.**

If $\gamma \geq \frac{2t_0\Lambda_r}{\Lambda_d}$, then

$$(\pi_d^*, \Pi^*, W_r^U) = \begin{cases} \text{platform quits,} & \text{if } v \leq w - t_0 + \frac{2t_0\Lambda_r}{\Lambda_d}, \\ \left(\frac{\Lambda_r}{\Lambda_d}, (v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d} - w)\Lambda_r, \frac{\Lambda_d^2 t_0}{\Lambda_d} \right), & \text{if } v > w - t_0 + \frac{2t_0\Lambda_r}{\Lambda_d}. \end{cases} \quad (\text{B.2})$$

When pooling is allowed, the riders' welfare is

$$\begin{aligned} W_r^U &= \min\{\Lambda_r\pi_r, \Lambda_d k\pi_d\} \int_{\underline{t}_d^*}^{t_0} (v + t - p^* - \gamma \mathbb{1}_{\{\Lambda_r\pi_r > \Lambda_d\pi_d^* k\}}) f(t|t \geq \underline{t}_d) dt \\ &= \frac{\min\{\Lambda_r, \Lambda_d k\pi_d\}}{2t_0\pi_d} \int_{\underline{t}_d^*}^{t_0} (v + t - p^* - \gamma \mathbb{1}_{\{\Lambda_r > \Lambda_d\pi_d^* k\}}) dt \end{aligned}$$

• When $\Lambda_r > \Lambda_d k$,

$$(\pi_d^*, \Pi^*, W_r^U) = \begin{cases} \text{platform quits,} & \text{if } v \leq w + \gamma - t_0, \\ \left(\frac{v+t_0-\gamma-w}{4t_0}, \frac{\Lambda_d k(v-w+t_0-\gamma)^2}{8t_0}, \frac{\Lambda_d k(v-w+t_0-\gamma)^2}{16t_0} \right), & \text{if } w + \gamma - t_0 < v < w + \gamma + 3t_0, \\ (1, \Lambda_d k(v - t_0 - w - \gamma), \Lambda_d k t_0), & \text{if } v \geq w + \gamma + 3t_0. \end{cases} \quad (\text{B.3})$$

• When $\Lambda_r \leq \Lambda_d k$

If $\gamma \geq \frac{2t_0\Lambda_r}{\Lambda_d k}$, then

$$(\pi_d^*, \Pi^*, W_r^U) = \begin{cases} \text{platform quits,} & \text{if } v \leq w - t_0 + \frac{2t_0\Lambda_r}{\Lambda_d k}, \\ \left(\frac{\Lambda_r}{\Lambda_d k}, (v + t_0 - \frac{2t_0\Lambda_r}{\Lambda_d k} - w)\Lambda_r, \frac{\Lambda_r^2 t_0}{\Lambda_d(\delta k + 1 - \delta)} \right), & \text{if } v > w - t_0 + \frac{2t_0\Lambda_r}{\Lambda_d k}. \end{cases} \quad (\text{B.4})$$

The theorem directly follows from the comparison between (B.1) and (B.3), (B.2) and (B.4). \square

Proof of Theorem 4 By the proof of Proposition B.2, we know that the platform's optimal pricing strategy induces $\Lambda_r\pi_r = \Lambda_d\pi_d k$. Hence, the riders' welfare is

$$W_r^B = \min\{\Lambda_r\pi_r, \Lambda_d k\pi_d\} \int_{\underline{t}_d}^{t_0} (v + t - p^*) f(t|t \geq \underline{t}_d) dt = \frac{\Lambda_r\pi_r}{2t_0\pi_d} \int_{\underline{t}_d}^{t_0} (v + t - p^*) dt.$$

The drivers' welfare is

$$W_d^B = \min\{\Lambda_d k\pi_d, \Lambda_r\pi_r\} \int_{\underline{t}_r}^{t_0} (h + t - w) f(t|t \geq \underline{t}_r) dt = \frac{\Lambda_d k\pi_d}{2t_0\pi_r} \int_{\underline{t}_r}^{t_0} (h + t - w) dt.$$

In BRDS, we have supply equal demand and $\pi_r = \frac{t_0 - w + h}{2t_0} = \frac{q}{\Lambda_r}$ and $\pi_d = \frac{v + t_0 - p}{2t_0} = \frac{q}{\Lambda_d k}$. So $\underline{t}_r = w - h$ and $\underline{t}_d = p - v$. Note that if $v \leq w - 2t_0$, then the platform quits.

• When pooling is allowed:

When $w - 2t_0 < v < 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0$,

$$\begin{cases} \Pi^* = \frac{(2t_0 + v - w)^2 \Lambda_r \Lambda_d k}{8t_0(\Lambda_d k + \Lambda_r)}, \\ \pi_r^* = \frac{(2t_0 + v - w)\Lambda_d k}{4t_0(\Lambda_d k + \Lambda_r)}, \\ \pi_d^* = \frac{(2t_0 + v - w)\Lambda_r}{4t_0(\Lambda_d k + \Lambda_r)}, \\ W_r^B = \frac{\Lambda_d \Lambda_d^2 k (2t_0 + v - w)^2}{16t_0(\Lambda_d k + \Lambda_r)^2}, \\ W_d^B = \frac{\Lambda_d^2 k^2 \Lambda_r (2t_0 + v - w)^2}{16t_0(\Lambda_r + \Lambda_d k)^2}. \end{cases}$$

When $v \geq 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0$,

$$\begin{cases} \Pi^* = \left[v + 2t_0 - w - 2t_0 \min\{\Lambda_r, \Lambda_d k\} \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \right] \min\{\Lambda_r, \Lambda_d k\}, \\ \pi_r^* = \frac{\min\{\Lambda_r, \Lambda_d k\}}{\Lambda_r}, \\ \pi_d^* = \frac{\min\{\Lambda_r, \Lambda_d k\}}{\Lambda_d k}, \\ W_r^B = \frac{t_0 \min\{\Lambda_r, \Lambda_d k\}^2}{\Lambda_d k}, \\ W_d^B = \frac{t_0 \min\{\Lambda_r, \Lambda_d k\}^2}{\Lambda_r}. \end{cases}$$

• When pooling is NOT allowed:

When $w - 2t_0 < v < 4t_0 \left(\frac{1}{\Lambda_d} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d\} + w - 2t_0$,

$$\begin{cases} \Pi^* = \frac{(2t_0+v-w)^2 \Lambda_r \Lambda_d}{8t_0(\Lambda_d + \Lambda_r)}, \\ \pi_r^* = \frac{(2t_0+v-w)\Lambda_d}{4t_0(\Lambda_d + \Lambda_r)}, \\ \pi_d^* = \frac{(2t_0+v-w)\Lambda_r}{4t_0(\Lambda_d + \Lambda_r)}, \\ W_r^B = \frac{\Lambda_d \Lambda_r^2 (2t_0+v-w)^2}{16t_0(\Lambda_d + \Lambda_r)^2}, \\ W_d^B = \frac{\Lambda_d^2 \Lambda_r (2t_0+v-w)^2}{16t_0(\Lambda_d + \Lambda_r)^2}. \end{cases}$$

When $v \geq 4t_0 \left(\frac{1}{\Lambda_d} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d\} + w - 2t_0$,

$$\begin{cases} \Pi^* = \left[v + 2t_0 - w - 2t_0 \min\{\Lambda_r, \Lambda_d\} \left(\frac{1}{\Lambda_d} + \frac{1}{\Lambda_r} \right) \right] \min\{\Lambda_r, \Lambda_d\}, \\ \pi_r^* = \frac{\min\{\Lambda_r, \Lambda_d\}}{\Lambda_r}, \\ \pi_d^* = \frac{\min\{\Lambda_r, \Lambda_d\}}{\Lambda_d}, \\ W_r^B = \frac{t_0 \min\{\Lambda_r, \Lambda_d\}^2}{\Lambda_d}, \\ W_d^B = \frac{t_0 \min\{\Lambda_r, \Lambda_d\}^2}{\Lambda_r}. \end{cases}$$

1. When Λ_r is sufficiently large, i.e., first, it has to be $\Lambda_r > \Lambda_d k$ at least, then $v < 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0 \Leftrightarrow \frac{4t_0 \Lambda_d k}{\Lambda_r} > v - w - 2t_0$

If $v - w - 2t_0 < 0$, i.e., v is small, then, the above inequality is automatically true. So, we are comparing:

$$\text{No Pooling: } \begin{cases} \Pi^* = \frac{(2t_0+v-w)^2 \Lambda_r \Lambda_d}{8t_0(\Lambda_d + \Lambda_r)}, \\ \pi_r^* = \frac{(2t_0+v-w)\Lambda_d}{4t_0(\Lambda_d + \Lambda_r)}, \\ \pi_d^* = \frac{(2t_0+v-w)\Lambda_r}{4t_0(\Lambda_d + \Lambda_r)}, \\ W_r^B = \frac{\Lambda_d \Lambda_r^2 (2t_0+v-w)^2}{16t_0(\Lambda_d + \Lambda_r)^2}, \\ W_d^B = \frac{\Lambda_d^2 \Lambda_r (2t_0+v-w)^2}{16t_0(\Lambda_d + \Lambda_r)^2}. \end{cases} \quad \text{Pooling: } \begin{cases} \Pi^* = \frac{(2t_0+v-w)^2 \Lambda_r \Lambda_d k}{8t_0(\Lambda_d k + \Lambda_r)}, \\ \pi_r^* = \frac{(2t_0+v-w)\Lambda_d k}{4t_0(\Lambda_d k + \Lambda_r)}, \\ \pi_d^* = \frac{(2t_0+v-w)\Lambda_r}{4t_0(\Lambda_d k + \Lambda_r)}, \\ W_r^B = \frac{\Lambda_d \Lambda_r^2 k (2t_0+v-w)^2}{16t_0(\Lambda_d k + \Lambda_r)^2}, \\ W_d^B = \frac{\Lambda_d^2 k^2 \Lambda_r (2t_0+v-w)^2}{16t_0(\Lambda_d k + \Lambda_r)^2}. \end{cases} \quad (\text{B.5})$$

If $v - w - 2t_0 > 0$, i.e., v is small, then, we should be looking at $v > 4t_0 \left(\frac{1}{\Lambda_d k} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d k\} + w - 2t_0 \Leftrightarrow \frac{4t_0 \Lambda_d k}{\Lambda_r} < v - w - 2t_0 \Leftrightarrow \Lambda_r > \frac{4t_0 \Lambda_d k}{v - w - 2t_0}$. Then, we are comparing

$$\text{No Pooling: } \begin{cases} \Pi^* = \left(v - w - \frac{2t_0 \Lambda_d}{\Lambda_r} \right) \Lambda_d, \\ \pi_r^* = \frac{\Lambda_d}{\Lambda_r}, \\ \pi_d^* = 1, \\ W_r^B = t_0 \Lambda_d, \\ W_d^B = \frac{t_0 \Lambda_d^2}{\Lambda_r}. \end{cases} \quad \text{Pooling: } \begin{cases} \Pi^* = \left(v - w - \frac{2t_0 \Lambda_d k}{\Lambda_r} \right) \Lambda_d k, \\ \pi_r^* = \frac{\Lambda_d k}{\Lambda_r}, \\ \pi_d^* = 1, \\ W_r^B = t_0 \Lambda_d k, \\ W_d^B = \frac{t_0 \Lambda_d^2 k^2}{\Lambda_r}. \end{cases} \quad (\text{B.6})$$

2. When Λ_r is sufficiently small, i.e., first, it has to be $\Lambda_r < \Lambda_d$ at least, then $v \geq 4t_0 \left(\frac{1}{\Lambda_d} + \frac{1}{\Lambda_r} \right) \min\{\Lambda_r, \Lambda_d\} + w - 2t_0 \Leftrightarrow \frac{4t_0 \Lambda_r}{\Lambda_d} < v - w - 2t_0$

If $v - w - 2t_0 > 0$, i.e., v is large, then that's it, we are looking at $\Lambda_r < \frac{\Lambda_d(v-w-2t_0)}{4t_0}$, and we are comparing

$$\text{No Pooling: } \begin{cases} \Pi^* = \left(v - w - \frac{2t_0 \Lambda_r}{\Lambda_d} \right) \Lambda_r, \\ \pi_r^* = 1, \\ \pi_d^* = \frac{\Lambda_r}{\Lambda_d}, \\ W_r^B = \frac{t_0 \Lambda_r^2}{\Lambda_d}, \\ W_d^B = t_0 \Lambda_r. \end{cases} \quad \text{Pooling: } \begin{cases} \Pi^* = \left(v - w - \frac{2t_0 \Lambda_r}{\Lambda_d k} \right) \Lambda_r, \\ \pi_r^* = 1, \\ \pi_d^* = \frac{\Lambda_r}{\Lambda_d k}, \\ W_r^B = \frac{t_0 \Lambda_r^2}{\Lambda_d k}, \\ W_d^B = t_0 \Lambda_r. \end{cases} \quad (\text{B.7})$$

If $v - w - 2t_0 < 0$, i.e., v is small, then we are comparing

$$\text{No Pooling: } \begin{cases} \Pi^* = \frac{(2t_0+v-w)^2 \Lambda_r \Lambda_d}{8t_0(\Lambda_d + \Lambda_r)}, \\ \pi_r^* = \frac{(2t_0+v-w)\Lambda_d}{4t_0(\Lambda_d + \Lambda_r)}, \\ \pi_d^* = \frac{(2t_0+v-w)\Lambda_r}{4t_0(\Lambda_d + \Lambda_r)}, \\ W_r^B = \frac{\Lambda_d \Lambda_r^2 (2t_0+v-w)^2}{16t_0(\Lambda_d + \Lambda_r)^2}, \\ W_d^B = \frac{\Lambda_d^2 \Lambda_r (2t_0+v-w)^2}{16t_0(\Lambda_d + \Lambda_r)^2}. \end{cases} \quad \text{Pooling: } \begin{cases} \Pi^* = \frac{(2t_0+v-w)^2 \Lambda_r \Lambda_d k}{8t_0(\Lambda_d k + \Lambda_r)}, \\ \pi_r^* = \frac{(2t_0+v-w)\Lambda_d k}{4t_0(\Lambda_d k + \Lambda_r)}, \\ \pi_d^* = \frac{(2t_0+v-w)\Lambda_r}{4t_0(\Lambda_d k + \Lambda_r)}, \\ W_r^B = \frac{\Lambda_d \Lambda_r^2 k (2t_0+v-w)^2}{16t_0(\Lambda_d k + \Lambda_r)^2}, \\ W_d^B = \frac{\Lambda_d^2 k^2 \Lambda_r (2t_0+v-w)^2}{16t_0(\Lambda_d k + \Lambda_r)^2}. \end{cases} \quad (\text{B.8})$$

The theorem directly follows from the comparison within (B.5), (B.6), (B.7), (B.8). \square