Online Supplement

S.1 Theorems, Proofs and Solution Algorithms

S.1.1 Proof of Theorem 1

This result follows by setting $A(t) = 0$ in the proof of Theorem 2. ■

S.1.2 Proof of Proposition 1

This result follows by setting $A(t) = 0$ in the proof of Proposition 3. ■

S.1.3 Proof of Proposition 2

We have $u^{\text{aw}}(\alpha(t)) = \alpha(t)\,[\,b_1 + b_2\,\alpha(t) - b_3\,\alpha(t)\,\beta I(t)\,]$. We omit t in α^*_{aw} and I for simplicity. We write $\widehat{b}_2 \equiv b_2 - b_3\beta I$. Then the function $u^{\text{aw}}(\alpha)$ can be written as $u^{\text{aw}}(\alpha) = \alpha(b_1 + \widehat{b}_2\alpha)$.

If $\widehat{b}_2 \geq 0$, or equivalently $I \leq \frac{b_2}{b_3\beta}$, $u^{\text{aw}}(\alpha)$ is increasing in $\alpha \in [0, 1]$ and thus $\alpha^*_{\text{aw}} = 1 > \alpha^*_B$.

If $\widehat{b}_2 < 0$, or equivalently $I > \frac{b_2}{b_3\beta}$, it follows from the first order condition of $u^{\text{aw}}(\alpha)$ and $\alpha \leq 1$ that $\alpha^*_{\text{aw}} = \min(-\frac{b_1}{2\widehat{b}_2}, 1)$. When $\frac{b_2}{b_3\beta} < I \leq \frac{b_2+0.5b_1}{b_3\beta}$, it follows that $-\frac{b_1}{2\widehat{b}_2} \geq 1$ and $\alpha^*_{\text{aw}} = 1 > \alpha^*_B$. When $I > \frac{b_2+0.5b_1}{b_3\beta}$, it follows that $-\frac{b_1}{2\widehat{b}_2} < 1$ and $\alpha^*_{\text{aw}} = -\frac{b_1}{2\widehat{b}_2}$. We next find conditions under which $\alpha^*_{\text{aw}} = -\frac{b_1}{2\widehat{b}_2} < \alpha^*_B$ in this case.

We define function $h(\alpha)$ as $h(\alpha) \equiv \log\left(\frac{\alpha}{1-\alpha}\right) - b_1 - \widehat{b}_2\alpha$. Because $h'(\alpha) = \frac{1}{\alpha(1-\alpha)} - \widehat{b}_2 > 0$ for $\alpha \in (0, 1)$ and $\widehat{b}_2 < 0$, $h(\alpha)$ increases in α. To show that $\alpha^*_{\text{aw}} = -\frac{b_1}{2\widehat{b}_2} < \alpha^*_B$, we only need to show $h(\alpha^*_{\text{aw}}) < h(\alpha^*_B)$. One may verify that α^*_B defined in Eq.(2) satisfies $h(\alpha^*_B) = 0$. Therefore, $\alpha^*_{\text{aw}} < \alpha^*_B$ if and only if $h(\alpha^*_{\text{aw}}) < 0$, which is equivalent to

$$h(\alpha^*_{\text{aw}}) = \log\left(\frac{\alpha^*_{\text{aw}}}{1-\alpha^*_{\text{aw}}}\right) - b_1 - \widehat{b}_2\alpha^*_{\text{aw}} = \log\left(\frac{-b_1}{b_1 + 2\widehat{b}_2}\right) - \frac{b_1}{2} < 0.$$

The above condition can be reduced to $I > \frac{1}{b_3\beta}\left[0.5b_1 + \frac{0.5b_1}{\exp(0.5b_1)} + b_2\right]$.

Combining all conditions, $\alpha^*_{\text{aw}} < \alpha^*_B$ if and only if $I > \frac{1}{b_3\beta}\left[0.5b_1 + \frac{0.5b_1}{\exp(0.5b_1)} + b_2\right]$.

S.1.4 Additional Remarks on Proposition 2

We note that when $I(t)$ is small, specifically when $I(t) < \frac{1}{b_3\beta}\left[0.5b_1 + \frac{0.5b_1}{\exp(0.5b_1)} + b_2\right]$ (approximately $I(t) < 0.016$ based on our estimated parameters in Section 5.1), we could have $\alpha^*_{aw}(t) > \alpha^*_B(t)$. On such low infection levels, the aggregate individual utility gain derived from a higher social activity level could outweigh the disutility brought about by a small number of infections. Notably, the socially optimal activity level peaks at $\alpha^*_{\text{aw}} = 1$ (the maximum activity level) when $I \leq \frac{b_2+0.5b_1}{b_3\beta}$, as demonstrated in the proof of Proposition 2. In contrast, the individual equilibrium activity level $\alpha^*_B(t)$ at such low $I(t)$ values might be lower than $\alpha^*_{aw}(t)$ for the following two reasons: First, because of the randomness in individual utility, $\alpha^*_B(t) \to 1$ but is not equal to 1 as $I \to 0$, whereas $\alpha^*_{\text{aw}} = 1$ when I is small (precisely, $I \leq \frac{b_2+0.5b_1}{b_3\beta}$). Second, when the infection risk is significantly lower, the positive externality that individuals contribute towards enhancing the aggregate individual welfare becomes more pronounced, yet individuals might overlook this positive externality when determining their own activity level, resulting in a smaller individual activity level compared to the social optimum.

S.1.5 Proof of Theorem 2

We only need to show that for any $t = 1, \ldots, T-1$, given $I(t)$ and $A(t)$, there exists a unique equilibrium activity level $\alpha^*_A(t)$. If it holds, there are only finite possible values of $\boldsymbol{A}$. We can obtain that there must exist one optimal solution $\boldsymbol{A}^*$ of the lockdown model. Thus, given any $\boldsymbol{A}$ and initial states $S(0)$, $I(0)$ and $R(0)$, we can recursively compute $\alpha(t)$ based on Eq.(5), and then compute $S(t+1)$, $I(t+1)$ and $R(t+1)$ from the dynamics Eq.(1). By doing this we could obtain $\alpha^*_A(t)$ for all $t = \{1, \ldots, T-1\}$.

Consider any $t \in \{1, \ldots, T\}$. We write $\alpha(t)$, $I(t)$ and $A(t)$ as α, I and A for simplicity. Define $\widehat{b}_2 \equiv b_2(1-A) - b_3\beta I$. Based on Eq.(5), we define

$$h(\alpha) \equiv \log\left(\frac{\alpha}{1-\alpha}\right) - b_1 - \widehat{b}_2\alpha. \tag{S.1}$$

Obviously, $h(\alpha)$ is continuous on $(0,1)$. We also have $\lim_{\alpha\downarrow 0} h(\alpha) = -\infty - b_1 = -\infty$ and $\lim_{\alpha\uparrow 1} h(\alpha) = +\infty - b_1 - \widehat{b}_2 = +\infty$. It then follows from the intermediate value theorem that there must exist $\alpha_A^* \in (0,1)$ such that $h(\alpha_A^*) = 0$ and α_A^* is a solution to Eq.(5). Next, we prove the uniqueness of the solution for Eq.(5). We consider two cases.
Case I: $\widehat{b}_2 < 4$. We then have $h'(\alpha) = \frac{1}{\alpha(1-\alpha)} - \widehat{b}_2 > 0$, i.e., $h(\alpha)$ is strictly increasing on $(0,1)$. Hence, there exists at most one α_A^* such that $h(\alpha_A^*) = 0$.
Case II: $\widehat{b}_2 \geq 4$. The values of α satisfying $h'(\alpha) = 0$ are $\overline{\alpha}_+ = \frac{1}{2}\left(\sqrt{1 - 4\widehat{b}_2^{-1}} + 1\right) \in (0,1)$ and $\overline{\alpha}_- = \frac{1}{2}\left(-\sqrt{1 - 4\widehat{b}_2^{-1}} + 1\right) \in (0,1)$. We observe the following two facts: 1) $h(\alpha)$ is strictly decreasing on $[\overline{\alpha}_-, \overline{\alpha}_+]$ and strictly increasing on $(0, \overline{\alpha}_-] \cup [\overline{\alpha}_+, 1)$; 2) $h(\bar{\alpha}_-) < 0$ and $\lim_{\alpha\downarrow 0} h(\alpha) = -\infty$. Putting them together one can obtain that $h(\alpha) < 0$ for all $\alpha \in (0, \overline{\alpha}_+]$, which further implies that there exists at most one α_A^* such that $h(\alpha_A^*) = 0$.

Since t is arbitrarily chosen from $\{1, \ldots, T-1\}$, the proof of this theorem is thus completed. ■

S.1.6 Proof of Proposition 3

Following the notation in the proof of Theorem 2, we consider any $t \in \{1, \ldots, T\}$ and write $\widehat{b}_2 \equiv b_2(1-A) - b_3\beta I$ without explicitly spelling out t. To establish the stated results in the proposition, it suffices to show that α_A^* is increasing in both $\widehat{b}_2$ and b_1. It is derived from the proof of Theorem 2 that $h'(\alpha_A^*) = \frac{1}{\alpha_A^*(1-\alpha_A^*)} - \widehat{b}_2 > 0$. Therefore, it follows from the implicit function theorem that α_A^* satisfying Eq.(5) can be taken as a single-valued function of b_1 and $\widehat{b}_2$ such that

$$\begin{pmatrix} \frac{\partial \alpha_A^*}{\partial b_1} \\ \frac{\partial \alpha_A^*}{\partial \widehat{b}_2} \end{pmatrix} = \left[\frac{1}{\alpha_A^*(1-\alpha_A^*)} - \widehat{b}_2\right]^{-1} \begin{pmatrix} 1 \\ \alpha_A^* \end{pmatrix} > 0.$$

The above inequality shows that α_A^* is an increasing function in both b_1 and $\widehat{b}_2$. ■

S.1.7 Proof of Proposition 4

To prove this proposition, we first show the following lemma by treating lockdown decision A as a continuous variable $A \in [0,1]$.

Lemma S.1. *For $0 < I_1 < I_2 < 1$, the following two properties hold: (a) if $I_1 < I_2 \leq I_L < 1$, then $\frac{\partial \alpha_A^*(I_2)}{\partial A} < \frac{\partial \alpha_A^*(I_1)}{\partial A} < 0$ at any $A \in [0,1]$ when $b_3\beta I_L + \frac{2-3\alpha_{A=1}^*(I_L)}{\alpha_{A=1}^*(I_L)(1-\alpha_{A=1}^*(I_L))^2} \leq 0$; (b) if $0 < I_H \leq I_1 < I_2$, then $\frac{\partial \alpha_A^*(I_1)}{\partial A} < \frac{\partial \alpha_A^*(I_2)}{\partial A} < 0$ at any $A \in [0,1]$ when $b_3\beta I_H - b_2 + \frac{2-3\alpha_{A=0}^*(I_H)}{\alpha_{A=0}^*(I_H)(1-\alpha_{A=0}^*(I_H))^2} \geq 0$.*

Proof. Based on Eq.(5), the equilibrium activity levels $\alpha_A^*(I) \in (0,1)$ at given lockdown decisions A are solutions to the following equation:

$$b_1 + b_2(1-A)\alpha_A^*(I) = \log(\alpha_A^*(I)) - \log(1 - \alpha_A^*(I)) + b_3\beta I \alpha_A^*(I).$$

From Proposition 3, we have $\alpha_{A=0}^*(I) > \alpha_{A=1}^*(I)$ and $\frac{\partial \alpha_A^*(I)}{\partial A} < 0$.

Next, we would like to establish conditions under which $\frac{\partial \alpha_A^*(I)}{\partial A}$ is monotone in $I \in [I_1, I_2]$ and thus to establish the relationship between $\frac{\partial \alpha_A^*(I_1)}{\partial A}$ and $\frac{\partial \alpha_A^*(I_2)}{\partial A}$. Recall that $\alpha_A^*(I)$ is decreasing in I. Thus, the monotonicity of $\frac{\partial \alpha_A^*(I)}{\partial A}$ in $I \in [I_1, I_2]$ can be established by showing that $\frac{\partial \alpha_A^*(I)}{\partial A}$ is monotone in $\alpha_A^*(I) \in [\alpha_A^*(I_2), \alpha_A^*(I_1)]$. It follows from the implicit function theorem that

$$\frac{\partial \alpha_A^*(I)}{\partial A} = -\frac{b_2 \alpha_A^*(I)}{b_3\beta I + b_2 A - b_2 + (\alpha_A^*(I))^{-1} + (1-\alpha_A^*(I))^{-1}}.$$

Denoting $g(\alpha) \equiv -\frac{b_2\alpha}{b_3\beta I + b_2(A-1) + \alpha^{-1} + (1-\alpha)^{-1}}$, it follows that

$$g'(\alpha) = -b_2\left[\frac{b_3\beta I + b_2(A-1) + (2-3\alpha)\alpha^{-1}(1-\alpha)^{-2}}{(b_3\beta I + b_2(A-1) + \alpha^{-1} + (1-\alpha)^{-1})^2}\right].$$

For statement (a), if $b_3\beta I_L + \frac{2-3\alpha^*_{A=1}(I_L)}{\alpha^*_{A=1}(I_L)(1-\alpha^*_{A=1}(I_L))^2} \leq 0$, we have for any $A \in [0,1]$ and $I \in [I_1, I_2]$,

$$\begin{aligned} b_3\beta I + b_2(A-1) + \frac{2-3\alpha_A^*(I)}{\alpha_A^*(I)(1-\alpha_A^*(I))^2} &\leq b_3\beta I_2 + b_2(A-1) + \frac{2-3\alpha_A^*(I_2)}{\alpha_A^*(I_2)(1-\alpha_A^*(I_2))^2} \\ &\leq b_3\beta I_L + \frac{2-3\alpha^*_{A=1}(I_L)}{\alpha^*_{A=1}(I_L)(1-\alpha^*_{A=1}(I_L))^2} \leq 0. \end{aligned}$$

The first two inequalities hold because $\alpha_A^*(I)$ is decreasing in I from Proposition 3 and the function $\frac{2-3x}{x(1-x)^2}$ is decreasing in x for $x \in (0,1)$ and $A \in [0,1]$. Therefore, $g'(\alpha) \geq 0$ for $\alpha \in [\alpha_A^*(I_2), \alpha_A^*(I_1)]$ at any $A \in [0,1]$, where the equality could only hold at $A = 1$ and $I_2 = I_L$. That is, $\frac{\partial \alpha_A^*(I)}{\partial A}$ is nondecreasing in $\alpha_A^*(I) \in [\alpha_A^*(I_2), \alpha_A^*(I_1)]$. Thus, $g(\alpha_A^*(I_2)) = \frac{\partial \alpha_A^*(I_2)}{\partial A} < \frac{\partial \alpha_A^*(I_1)}{\partial A} = g(\alpha_A^*(I_1))$ at any $A \in [0,1]$.

Similarly for statement (b), if $b_3\beta I_H - b_2 + \frac{2-3\alpha^*_{A=0}(I_H)}{\alpha^*_{A=0}(I_H)(1-\alpha^*_{A=0}(I_H))^2} \geq 0$, we have for any $A \in [0,1]$ and $I \in [I_1, I_2]$,

$$\begin{aligned} b_3\beta I + b_2(A-1) + \frac{2-3\alpha_A^*(I)}{\alpha_A^*(I)(1-\alpha_A^*(I))^2} &\geq b_3\beta I_1 + b_2(A-1) + \frac{2-3\alpha_A^*(I_1)}{\alpha_A^*(I_1)(1-\alpha_A^*(I_1))^2} \\ &\geq b_3\beta I_H - b_2 + \frac{2-3\alpha^*_{A=0}(I_H)}{\alpha^*_{A=0}(I_1)(1-\alpha^*_{A=0}(I_H))^2} \geq 0. \end{aligned}$$

Therefore, $g'(\alpha) \leq 0$ for $\alpha \in [\alpha_A^*(I_2), \alpha_A^*(I_1)]$ at any $A \in [0,1]$, where the equality could only hold at $A = 0$ and $I_1 = I_H$. Thus, $\frac{\partial \alpha_A^*(I_1)}{\partial A} < \frac{\partial \alpha_A^*(I_2)}{\partial A}$ at any $A \in [0,1]$. ■

Define

$$\gamma = \frac{1}{3}\left(\frac{\sqrt[3]{3\sqrt{3}\sqrt{-b_3^5\beta^5 - 9b_3^4\beta^4 - 27b_3^3\beta^3} - \beta^3 b_3^3}}{\beta b_3} + \frac{\beta b_3 + 9}{\sqrt[3]{3\sqrt{3}\sqrt{-b_3^5\beta^5 - 9b_3^4\beta^4 - 27b_3^3\beta^3} - \beta^3 b_3^3}} + 2\right),$$

and

$$\delta = \frac{1}{3}\left(\frac{\sqrt[3]{3\sqrt{3}\sqrt{b_2^5 - 9b_2^4 + 27b_2^3} - b_2^3}}{b_2} + \frac{9 - b_2}{\sqrt[3]{\sqrt[3]{3\sqrt{3}\sqrt{b_2^5 - 9b_2^4 + 27b_2^3} - b_2^3}}} + 2\right).$$

Also, we define

$$\underline{I} \equiv \frac{b_1 - \log(\frac{\gamma}{1-\gamma})}{b_3\beta\gamma} \quad \text{and} \quad \overline{I} \equiv \frac{b_1 + b_2\delta - \log(\frac{\delta}{1-\delta})}{b_3\beta\delta}.$$

If the logarithm is not well defined, we set the values of $\underline{I}$ and $\overline{I}$ to be $\pm\infty$. We show $\underline{I}$ and $\overline{I}$ satisfy the condition in Lemma S.1.

Lemma S.2. *If $\underline{I} \in (0,1)$, we have*

$$b_3\beta\underline{I} + \frac{2-3\alpha^*_{A=1}(\underline{I})}{\alpha^*_{A=1}(\underline{I})(1-\alpha^*_{A=1}(\underline{I}))^2} \leq 0.$$

If $\overline{I} \in (0,1)$, we have

$$b_3\beta\overline{I} - b_2 + \frac{2-3\alpha^*_{A=0}(\overline{I})}{\alpha^*_{A=0}(\overline{I})(1-\alpha^*_{A=0}(\overline{I}))^2} \geq 0.$$

Proof: First, according to Eq.(5), we have $\log(\frac{\alpha^*_{A=1}(I)}{1-\alpha^*_{A=1}(I)}) = b_1 - b_3\beta I\alpha^*_{A=1}(I)$ if a lockdown is implemented and $\log(\frac{\alpha^*_{A=0}(I)}{1-\alpha^*_{A=0}(I)}) = b_1 + b_2\alpha^*_{A=0}(I) - b_3\beta I\alpha^*_{A=0}(I)$ if no lockdown is implemented. Therefore, according to the definition of γ and δ and the uniqueness of equilibrium in Theorem 2, if $\underline{I} \in (0,1)$, we have that $\gamma = \alpha^*_{A=1}(\underline{I})$; if $\overline{I} \in (0,1)$, we have $\delta = \alpha^*_{A=0}(\overline{I})$.

Second, one can verify that γ is a real root of $b_3\beta + \frac{2-3\gamma}{\gamma(1-\gamma)^2} = 0$ and δ is a real root of $-b_2 + \frac{2-3\delta}{\delta(1-\delta)^2} = 0$. Therefore, we have

$$\begin{aligned} b_3\beta + \frac{2-3\alpha^*_{A=1}(\underline{I})}{\alpha^*_{A=1}(\underline{I})(1-\alpha^*_{A=1}(\underline{I}))^2} &= 0 \\ -b_2 + \frac{2-3\alpha^*_{A=0}(\overline{I})}{\alpha^*_{A=0}(\overline{I})(1-\alpha^*_{A=0}(\overline{I}))^2} &= 0, \end{aligned}$$

Third, since $\underline{I}, \overline{I} \in (0,1)$, we have

$$\begin{aligned} b_3\beta\underline{I} + \frac{2-3\alpha^*_{A=1}(\underline{I})}{\alpha^*_{A=1}(\underline{I})(1-\alpha^*_{A=1}(\underline{I}))^2} &\leq b_3\beta + \frac{2-3\alpha^*_{A=1}(\underline{I})}{\alpha^*_{A=1}(\underline{I})(1-\alpha^*_{A=1}(\underline{I}))^2} = 0 \\ b_3\beta\overline{I} - b_2 + \frac{2-3\alpha^*_{A=0}(\overline{I})}{\alpha^*_{A=0}(\overline{I})(1-\alpha^*_{A=0}(\overline{I}))^2} &\geq -b_2 + \frac{2-3\alpha^*_{A=0}(\overline{I})}{\alpha^*_{A=0}(\overline{I})(1-\alpha^*_{A=0}(\overline{I}))^2} = 0. \end{aligned}$$

■

Lemma S.3. *$\overline{I} > \underline{I}$.*

Proof: Based on our definition, $\alpha^*_{A=1}(\underline{I})$ and $\alpha^*_{A=0}(\overline{I})$ are solutions to the following equations: $b_3\beta + \frac{2-3\alpha^*_{A=1}(\underline{I})}{\alpha^*_{A=1}(\underline{I})(1-\alpha^*_{A=1}(\underline{I}))^2} = 0$ and $-b_2 + \frac{2-3\alpha^*_{A=0}(\overline{I})}{\alpha^*_{A=0}(\overline{I})(1-\alpha^*_{A=0}(\overline{I}))^2} = 0$. Since $\frac{2-3x}{x(1-x)^2}$ is decreasing for $x \in (0,1)$, we have $\alpha^*_{A=0}(\overline{I}) < \alpha^*_{A=1}(\underline{I})$. Since $\alpha^*_{A=1}(\underline{I}) < \alpha^*_{A=0}(\underline{I})$ and thus $\alpha^*_{A=0}(\overline{I}) < \alpha^*_{A=0}(\underline{I})$, we have $\overline{I} > \underline{I}$ by Proposition 3. ■

Proof of Proposition 4:

It follows Lemma S.2 that $b_3\beta\underline{I} + \frac{2-3\alpha^*_{A=1}(\underline{I})}{\alpha^*_{A=1}(\underline{I})(1-\alpha^*_{A=1}(\underline{I}))^2} \leq 0$. From Lemma S.1, we know that $\frac{\partial\alpha^*_A(I_2)}{\partial A} < \frac{\partial\alpha^*_A(I_1)}{\partial A} < 0$ at any $A \in [0,1]$ for $I_1 < I_2 \leq \underline{I}$,

which yields that

$$\int_{A=0}^{A=1} \frac{\partial\alpha^*_A(I_2)}{\partial A} dA < \int_{A=0}^{A=1} \frac{\partial\alpha^*_A(I_1)}{\partial A} dA.$$

Hence,

$$\alpha^*_{A=1}(I_2) - \alpha^*_{A=0}(I_2) < \alpha^*_{A=1}(I_1) - \alpha^*_{A=0}(I_1) < 0.$$

Rearranging the terms in the above inequality, we obtain the first statement of this proposition. The proof of the second statement can be derived in a similar fashion. ■

S.1.8 Proof of Proposition 5

Recall that the size of the susceptible and infected populations at the beginning of day t are S_t and I_t, respectively. We compare two lockdown strategies: lockdown at day t and lockdown at day $t+1$.

For the strategy where lockdown is implemented at day $t+1$, based on Eq.(1), we have

$$\begin{aligned}
S_{t+1} =& S_t[1-\beta\alpha^*_{A=0}(I_t)I_t] \\
I_{t+1} =& I_t[1-r+\beta\alpha^*_{A=0}(I_t)S_t] \\
S_{t+2} =& S_{t+1}[1-\beta\alpha^*_{A=1}(I_{t+1})I_{t+1}] = S_t[1-\beta\alpha^*_{A=0}(I_t)I_t][1-\beta\alpha^*_{A=1}(I_{t+1})I_{t+1}],
\end{aligned}$$

where S_{t+2} is the size of the susceptible population at the beginning of day $t+2$. Note that $\alpha^*_{A=0}(I)$ and $\alpha^*_{A=1}(I)$ indicate the equilibrium activity level at disease prevalence level I without and with lockdown, respectively. Based on Eqs.(4) and (5), $\alpha^*_{A=0}(I)$ and $\alpha^*_{A=1}(I)$ satisfy the following equations.

$$\begin{aligned}
\log\left(\frac{\alpha^*_{A=0}(I)}{1-\alpha^*_{A=0}(I)}\right) =& b_1 + b_2\alpha^*_{A=0}(I) - b_3\beta\alpha^*_{A=0}(I)I \\
\log\left(\frac{\alpha^*_{A=1}(I)}{1-\alpha^*_{A=1}(I)}\right) =& b_1 - b_3\beta\alpha^*_{A=1}(I)I
\end{aligned} \tag{S.2}$$

Similarly, for the strategy where lockdown is implemented at day t, we have

$$\begin{aligned}
S'_{t+1} =& S_t[1-\beta\alpha^*_{A=1}(I_t)I_t] \\
I'_{t+1} =& I_t[1-r+\beta\alpha^*_{A=1}(I_t)S_t] \\
S'_{t+2} =& S'_{t+1}[1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}] = S_t[1-\beta\alpha^*_{A=1}(I_t)I_t][1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}],
\end{aligned}$$

where S'_{t+2} is the corresponding size of the susceptible population at the beginning of day $t+2$.

Note that a larger susceptible population at the beginning of day $t+2$ implies fewer new infections in day t and $t+1$. Therefore, comparing S_{t+2} and S'_{t+2} is equivalent to the comparison of new infections in day t and $t+1$ as a result of these two policies. Moreover, since lockdown is only implemented for one day in each case, comparing S_{t+2} and S'_{t+2} is also equivalent to comparing the total cost in Eq.(3). Next we compare the value of S_{t+2} and S'_{t+2}.

$$\begin{aligned}
S'_{t+2} - S_{t+2} = & S_t[1-\beta\alpha^*_{A=1}(I_t)I_t - \beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1} + \beta^2\alpha^*_{A=1}(I_t)I_t\alpha^*_{A=0}(I'_{t+1})I'_{t+1}] \\
& - S_t[1-\beta\alpha^*_{A=0}(I_t)I_t - \beta\alpha^*_{A=1}(I_{t+1})I_{t+1} + \beta^2\alpha^*_{A=0}(I_t)I_t\alpha^*_{A=1}(I_{t+1})I_{t+1}]
\end{aligned}$$

Thus, we have

$$\begin{aligned}
& \frac{S'_{t+2} - S_{t+2}}{\beta S_t} = I_t[\alpha^*_{A=0}(I_t) - \alpha^*_{A=1}(I_t)] + [\alpha^*_{A=1}(I_{t+1})I_{t+1} - \alpha^*_{A=0}(I'_{t+1})I'_{t+1}] \\
& \quad + \beta I_t[\alpha^*_{A=1}(I_t)\alpha^*_{A=0}(I'_{t+1})I'_{t+1} - \alpha^*_{A=0}(I_t)\alpha^*_{A=1}(I_{t+1})I_{t+1}] \\
=& I_t\big[\alpha^*_{A=0}(I_t) - \alpha^*_{A=1}(I_t)\big] + \big[\alpha^*_{A=1}(I_{t+1})I_{t+1} - \alpha^*_{A=0}(I'_{t+1})I'_{t+1}\big] \\
& \quad + \beta I_t\Big[\alpha^*_{A=0}(I'_{t+1})I'_{t+1}\big[\alpha^*_{A=1}(I_t) - \alpha^*_{A=0}(I_t)\big] - \alpha^*_{A=0}(I_t)\big[\alpha^*_{A=1}(I_{t+1})I_{t+1} - \alpha^*_{A=0}(I'_{t+1})I'_{t+1}\big]\Big] \\
=& I_t\big[\alpha^*_{A=0}(I_t) - \alpha^*_{A=1}(I_t)\big]\big[1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}\big] + \big[\alpha^*_{A=1}(I_{t+1})I_{t+1} - \alpha^*_{A=0}(I'_{t+1})I'_{t+1}\big]\big[1-\beta I_t\alpha^*_{A=0}(I_t)\big] \\
=& \big[1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}\big]\big[1-\beta I_t\alpha^*_{A=0}(I_t)\big]\left\{\frac{I_t\alpha^*_{A=0}(I_t) - I_t\alpha^*_{A=1}(I_t)}{1-\beta I_t\alpha^*_{A=0}(I_t)} - \frac{\alpha^*_{A=0}(I'_{t+1})I'_{t+1} - \alpha^*_{A=1}(I_{t+1})I_{t+1}}{1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}}\right\}
\end{aligned}$$

Note that $1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1} = \frac{S'_{t+2}}{S'_{t+1}} > 0$ and $1-\beta I_t\alpha^*_{A=0}(I_t) = \frac{S_{t+1}}{S_t} > 0$. That is, the sign of $S'_{t+2} - S_{t+2}$ will be the same as

$$\left\{\frac{I_t\alpha^*_{A=0}(I_t) - I_t\alpha^*_{A=1}(I_t)}{1-\beta I_t\alpha^*_{A=0}(I_t)} - \frac{\alpha^*_{A=0}(I'_{t+1})I'_{t+1} - \alpha^*_{A=1}(I_{t+1})I_{t+1}}{1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}}\right\} \tag{S.3}$$

It follows from Eq.(S.2) that $\alpha^*_{A=0}(I)I = \frac{1}{b_3\beta}[b_1 + b_2\alpha^*_{A=0}(I) - \log(\frac{\alpha^*_{A=0}(I)}{1-\alpha^*_{A=0}(I)})]$ and $\alpha^*_{A=1}(I)I = \frac{1}{b_3\beta}[b_1 - \log(\frac{\alpha^*_{A=1}(I)}{1-\alpha^*_{A=1}(I)})]$. Therefore, by substituting these terms, expression (S.3) can be written as follows:

$$
\begin{aligned}
&\frac{I_t\alpha^*_{A=0}(I_t) - I_t\alpha^*_{A=1}(I_t)}{1-\beta I_t\alpha^*_{A=0}(I_t)} - \frac{\alpha^*_{A=0}(I'_{t+1})I'_{t+1} - \alpha^*_{A=1}(I_{t+1})I_{t+1}}{1-\beta\alpha^*_{A=0}(I'_{t+1})I'_{t+1}} \\
=&\frac{\frac{1}{b_3\beta}\big[b_1 + b_2\alpha^*_{A=0}(I_t) - \log(\frac{\alpha^*_{A=0}(I_t)}{1-\alpha^*_{A=0}(I_t)}) - b_1 + \log(\frac{\alpha^*_{A=1}(I_t)}{1-\alpha^*_{A=1}(I_t)})\big]}{1-\frac{1}{b_3}\big[b_1 + b_2\alpha^*_{A=0}(I_t) - \log(\frac{\alpha^*_{A=0}(I_t)}{1-\alpha^*_{A=0}(I_t)})\big]} \\
&-\frac{\frac{1}{b_3\beta}\big[b_1 + b_2\alpha^*_{A=0}(I'_{t+1}) - \log(\frac{\alpha^*_{A=0}(I'_{t+1})}{1-\alpha^*_{A=0}(I'_{t+1})}) - b_1 + \log(\frac{\alpha^*_{A=1}(I_{t+1})}{1-\alpha^*_{A=1}(I_{t+1})})\big]}{1-\frac{1}{b_3}\big[b_1 + b_2\alpha^*_{A=0}(I'_{t+1}) - \log(\frac{\alpha^*_{A=0}(I'_{t+1})}{1-\alpha^*_{A=0}(I'_{t+1})})\big]}.
\end{aligned}
$$

We define

$$
g(x,y) \equiv \frac{\big[b_2x - \log(\frac{x}{1-x}) + \log(\frac{y}{1-y})\big]}{\beta\big[b_3 - b_1 - b_2x + \log(\frac{x}{1-x})\big]}.
$$

Let $x_1 = \alpha^*_{A=0}(I_t)$, $x_2 = \alpha^*_{A=0}(I'_{t+1})$, $y_1 = \alpha^*_{A=1}(I_t)$ and $y_2 = \alpha^*_{A=1}(I_{t+1})$. Expression (S.3) can be written as

$$
g(\alpha^*_{A=0}(I_t), \alpha^*_{A=1}(I_t)) - g(\alpha^*_{A=0}(I'_{t+1}), \alpha^*_{A=1}(I_{t+1})) = g(x_1,y_1) - g(x_2,y_2).
$$

Next, we analyze the performance of $g(x,y)$ and derive sufficient conditions in the following lemma to compare S_{t+2} and S'_{t+2}.

Lemma S.4. *(a). We have $g(x_1,y_1) \geq g(x_2,y_2)$ under the following Case 1-4.*
Case 1: $b_2 \leq 4,\ x_1 \leq x_2,\ y_1 \geq y_2$
Case 2: $b_2 > 4,\ 0 < x_1 \leq x_2 \leq 0.5 - 0.5\sqrt{1-\frac{4}{b_2}},\ y_1 \geq y_2$
Case 3: $b_2 > 4,\ 0.5 + 0.5\sqrt{1-\frac{4}{b_2}} \leq x_1 \leq x_2 < 1,\ y_1 \geq y_2$
Case 4: $b_2 > 4,\ 0.5 - 0.5\sqrt{1-\frac{4}{b_2}} \leq x_2 \leq x_1 \leq 0.5 + 0.5\sqrt{1-\frac{4}{b_2}},\ y_1 \geq y_2$
(b). We have $g(x_1,y_1) \leq g(x_2,y_2)$ under the following Case 5.
Case 5: $b_2 > 4,\ 0.5 - 0.5\sqrt{1-\frac{4}{b_2}} \leq x_1 \leq x_2 \leq 0.5 + 0.5\sqrt{1-\frac{4}{b_2}},\ y_1 \leq y_2$

Proof: First, we derive partial derivative of $g(x,y)$. We define $f_1(x) \equiv b_3 - b_1 - b_2x + \log(\frac{x}{1-x})$ and $f_2(y) \equiv b_3 - b_1 + \log(\frac{y}{1-y})$. Thus, we have $f'_1(x) = -b_2 + \frac{1}{x(1-x)}$ and $f'_2(y) = \frac{1}{y(1-y)}$.

$$
\begin{aligned}
\frac{\partial g}{\partial y} &= \frac{1}{\beta\big[b_3 - b_1 - b_2x + \log(\frac{x}{1-x})\big]}\frac{1}{y(1-y)} = \frac{f'_2(y)}{\beta f_1(x)} \\
\frac{\partial g}{\partial x} &= \frac{\big[b_3 - b_1 - b_2x + \log(\frac{x}{1-x})\big]\big[b_2 - \frac{1}{x(1-x)}\big] - \big[b_2x - \log(\frac{x}{1-x}) + \log(\frac{y}{1-y})\big]\big[\frac{1}{x(1-x)} - b_2\big]}{\beta\big[b_3 - b_1 - b_2x + \log(\frac{x}{1-x})\big]^2} \\
&= \frac{\big[b_2 - \frac{1}{x(1-x)}\big]\big[b_3 - b_1 + \log(\frac{y}{1-y})\big]}{\beta\big[b_3 - b_1 - b_2x + \log(\frac{x}{1-x})\big]^2} = -\frac{f'_1(x)f_2(y)}{\beta f_1^2(x)}
\end{aligned}
$$

Next, we derive conditions to guarantee that $g(x,y)$ is monotone in x between x_1 and x_2 and monotone in y between y_1 and y_2.

First, we analyze properties of $f_1(x)$. We have $f_1(x_1) = b_3\big[1 - \beta I_t\alpha^*_{A=0}(I_t)\big] = b_3\frac{S_{t+1}}{S_t} > 0$ and $f_1(x_2) = b_3\big[1-\beta I'_{t+1}\alpha^*_{A=0}(I'_{t+1})\big] = b_3\frac{S'_{t+2}}{S'_{t+1}} > 0$. Also, we have $f'_1(x) = -b_2 + \frac{1}{x(1-x)} = -b_2 + \frac{1}{0.25-(x-0.5)^2}$.

(i). If $b_2 \le 4$, $f_1'(x) \ge 0$ and thus $f_1(x)$ is increasing for $x \in (0,1)$. Therefore, we have $f_1(x) > 0$ between x_1 and x_2.

(ii). If $b_2 > 4$, we have $f_1'(x) < 0$ when $x \in (0.5 - 0.5\sqrt{1-\frac{4}{b_2}}, 0.5 + 0.5\sqrt{1-\frac{4}{b_2}})$, and $f_1'(x) \ge 0$ when $x \in (0, 0.5 - 0.5\sqrt{1-\frac{4}{b_2}}] \cup [0.5 + 0.5\sqrt{1-\frac{4}{b_2}}, 1)$. Therefore, if $b_2 > 4$, $f_1(x)$ is first increasing, then decreasing and finally increasing in $x \in (0,1)$.

Second, we analyze properties of $f_2(y)$. We have $f_2(y_1) = b_3\big[1 - \beta I_1 \alpha^*_{A=1}(I_1)\big] = b_3 \frac{S_2'}{S_1} > 0$ and $f_2(y_2) = b_3\big[1 - \beta I_1 \alpha^*_{A=1}(I_2)\big] = b_3 \frac{S_3}{S_2} > 0$. Also, we have $f_2'(y) = \frac{1}{y(1-y)} > 0$ and thus $f_2(y)$ is increasing for $y \in (0,1)$. Therefore, we have $f_2(y) > 0$ between y_1 and y_2.

Given the properties of $f_1(x)$ and $f_2(y)$ and $\frac{\partial g}{\partial y} = \frac{f_2'(y)}{\beta f_1(x)}$ and $\frac{\partial g}{\partial x} = -\frac{f_1'(x) f_2(y)}{\beta f_1^2(x)}$, we can study the monotonicity of $g(x,y)$ between x_1 and x_2 and between y_1 and y_2.

Under Case 1-3, we have $f_1'(x) \ge 0$ between x_1 and x_2 and thus $\frac{\partial g}{\partial x} \le 0$ and $\frac{\partial g}{\partial y} \ge 0$. Since $x_1 \le x_2$ and $y_1 \ge y_2$, we have $g(x_1,y_1) \ge g(x_2,y_2)$.

Under Case 4, we have $f_1'(x) \le 0$ between x_1 and x_2 and thus $\frac{\partial g}{\partial x} \ge 0$ and $\frac{\partial g}{\partial y} \ge 0$. Since $x_1 \ge x_2$ and $y_1 \ge y_2$, we have $g(x_1,y_1) \ge g(x_2,y_2)$.

Under Case 5, we have $f_1'(x) \le 0$ between x_1 and x_2 and thus $\frac{\partial g}{\partial x} \ge 0$ and $\frac{\partial g}{\partial y} \ge 0$. Since $x_1 \le x_2$ and $y_1 \le y_2$, we have $g(x_1,y_1) \le g(x_2,y_2)$. ■

We note that $x_1 > x_2$ and $y_1 \le y_2$ cannot hold at the same time. This is because $\alpha^*_{A=0}(I)$ and $\alpha^*_{A=0}(I)$ are decreasing in I as shown in Proposition 3. Therefore, based on the definition of x_1, x_2, y_1, and y_2, it follows that $x_1 \ge x_2$ is equivalent to $I_t \le I'_{t+1}$ and $y_1 \ge y_2$ is equivalent to $I_t \le I_{t+1}$, and vice versa. Moreover, since we have $I_{t+1} > I'_{t+1}$ according to Proposition 3, inequalities $x_1 > x_2$ and $y_1 \le y_2$ cannot hold at the same time.

It follows from Eq.(5) that $\log(\frac{\alpha^*_{A=0}(I)}{1-\alpha^*_{A=0}(I)}) = b_1 + b_2\alpha^*_{A=0}(I) - b_3\beta I \alpha^*_{A=0}(I)$ and thus we have $I = \frac{b_1 + b_2\alpha^*_{A=0}(I) - \log(\frac{\alpha^*_{A=0}(I)}{1-\alpha^*_{A=0}(I)})}{b_3\beta\alpha^*_{A=0}(I)}$. Define $\Phi_{A=0}(x) = \frac{b_1 + b_2 x - \log(\frac{x}{1-x})}{b_3\beta x}$. Because of the uniqueness of equilibrium as shown in Theorem 2, the conditions above on x_1, x_2, y_1, and y_2 (recall that they are activity levels) can be converted to conditions on the corresponding disease prevalence based on Eq.(5). Moreover, it follows from the property in Proposition 3 that $\alpha^*_{A=1}(I)$ are decreasing in I. According to Expression (S.3), $S'_{t+2} - S_{t+2}$ has the same sign as $g(x_1,y_1) - g(x_2,y_2)$. Therefore, we can simplify conditions in Lemma S.4 to

$$S'_{t+2} - S_{t+2} \begin{cases} \ge 0 & \text{if } b_2 \le 4,\ I_{t+1} \ge I_t \ge I'_{t+1} \\ \ge 0 & \text{if } b_2 > 4,\ I_{t+1} \ge I_t \ge I'_{t+1} \ge \Phi_{A=0}(0.5 - 0.5\sqrt{1-\frac{4}{b_2}}) \\ \ge 0 & \text{if } b_2 > 4,\ I_{t+1} \ge I_t \ge I'_{t+1},\ I_t \le \Phi_{A=0}(0.5 + 0.5\sqrt{1-\frac{4}{b_2}}) \\ \ge 0 & \text{if } b_2 > 4,\ I_{t+1} \ge I_t,\ \Phi_{A=0}(0.5 + 0.5\sqrt{1-\frac{4}{b_2}}) \le I_t \le I'_{t+1} \le \Phi_{A=0}(0.5 - 0.5\sqrt{1-\frac{4}{b_2}}) \\ \le 0 & \text{if } b_2 > 4,\ I_t \ge I_{t+1},\ \Phi_{A=0}(0.5 + 0.5\sqrt{1-\frac{4}{b_2}}) \le I'_{t+1} \le I_t \le \Phi_{A=0}(0.5 - 0.5\sqrt{1-\frac{4}{b_2}}) \end{cases}$$

We note that $I'_{t+1} = I_t[1 - r + \beta\alpha^*_{A=1}(I_t)S_t]$. Moreover, we have $\log(\frac{\alpha^*_{A=1}(I)}{1-\alpha^*_{A=1}(I)}) = b_1 - b_3\beta I\alpha^*_{A=1}(I)$ and thus $I = \frac{b_1 - \log(\frac{\alpha^*_{A=1}(I)}{1-\alpha^*_{A=1}(I)})}{b_3\beta\alpha^*_{A=1}(I)}$ from Eq.(5). If we define $\Phi_{A=1}(x) = \frac{b_1 - \log(\frac{x}{1-x})}{b_3\beta x}$ if $0 < x < 1$ and $\Phi_{A=1}(x) = -\infty$ if $x \ge 1$, based on Proposition 3, we have $I_t \le I'_{t+1}$ equivalent to $I_t \le \Phi_{A=1}(\frac{r}{\beta S_t})$ and vice versa.

Therefore, we have $S'_{t+2} \ge S_{t+2}$ if

Condition (a): $b_2 \le 4$ and $I_{t+1} \ge I_t \ge \Phi_{A=1}(\frac{r}{\beta S_t})$,

Condition (b): $b_2 > 4,\ I_{t+1} \ge I_t \ge \Phi_{A=1}(\frac{r}{\beta S_t})$, and $I'_{t+1} \ge \Phi_{A=0}(0.5 - 0.5\sqrt{1-\frac{4}{b_2}})$,

Condition (c): $b_2 > 4,\ I_{t+1} \geq I_t,$ and $\Phi_{A=1}(\frac{r}{\beta S_t}) \leq I_t \leq \Phi_{A=0}(0.5 + 0.5\sqrt{1 - \frac{4}{b_2}}),$

Condition (d): $b_2 > 4,\ I_{t+1} \geq I_t,\ \Phi_{A=0}(0.5 + 0.5\sqrt{1 - \frac{4}{b_2}}) \leq I_t \leq \Phi_{A=1}(\frac{r}{\beta S_t}),$ and $I'_{t+1} \leq \Phi_{A=0}(0.5 - 0.5\sqrt{1 - \frac{4}{b_2}}).$

We have $S'_{t+2} \leq S_{t+2}$ if

Condition (e): $b_2 > 4,\ I_t \geq I_{t+1},\ \Phi_{A=1}(\frac{r}{\beta S_t}) \leq I_t \leq \Phi_{A=0}(0.5 - 0.5\sqrt{1 - \frac{4}{b_2}}),$ and $I'_{t+1} \geq \Phi_{A=0}(0.5 + 0.5\sqrt{1 - \frac{4}{b_2}}),$

where $\Phi_{A=1}(\cdot)$, $\Phi_{A=0}(\cdot)$ and I'_{t+1} are functions of known parameters S_t, I_1, b_1, b_2, b_3, β and r.

Under conditions (a)-(d), implementing lockdown at day t results in weakly fewer infections. Under condition (e), implementing lockdown at day $t+1$ results in weakly fewer infections. Because the total cost includes both policy cost and disease burden of infection, for the same number of days of lockdown, the strategy that results in more infections leads to a higher total cost. ■

S.1.9 Proof of Theorem 3

To show Theorem 3, we show in the following lemma that when the social distancing policy is stringent and requires an activity level lower than individual baseline equilibrium activity level (i.e., $1 - \kappa(t) < \alpha^*_B(t)$) on a given day t, one symmetric equilibrium activity level $\alpha^*(t)$ is exactly $1 - \kappa(t)$.

Define $\alpha_i(t)$ as the activity level of individual i and $\alpha_{-i}(t)$ as the activity level of all other individuals except individual i. We first derive individual i's activity level given $\alpha_{-i}(t) = 1 - \kappa(t)$.

Lemma S.5. *Suppose that the social distancing policy $\kappa(t)$ on day t satisfies $1 - \kappa(t) < \alpha^*_B(t)$, and $\alpha_{-i}(t) = 1 - \kappa(t)$. Then, individual i's activity level under social distancing policy $\kappa(t)$ is $\alpha^*_i(t) = 1 - \kappa(t)$.*

Proof: Recall that

$$u_{SD}(\text{go out}, \alpha_\kappa(t)) = b_1 + b_2\alpha_\kappa(t) - b_3\alpha_\kappa(t)\beta I(t) + \epsilon,$$

where $\alpha_\kappa(t)$ is the average activity level of all individuals. Suppose there are N individuals in the population and N is sufficiently large, then $\alpha_\kappa(t) = \lim_{N\to\infty} \frac{\alpha_i(t)+(N-1)\alpha_{-i}(t)}{N} = \alpha_{-i}(t)$.

Thus, we can write $u_i(\text{go out}, \alpha_{-i}(t))$ as the utility gain for the individual i that chooses to *go out* when all others hold the same activity level $\alpha_{-i}(t)$, i.e.,

$$u_i(\text{go out}, \alpha_{-i}(t)) = b_1 + b_2\alpha_{-i}(t) - b_3\alpha_{-i}(t)\beta I(t) + \epsilon. \tag{S.4}$$

By definition, the feasible region of $\alpha_i(t)$ is $\alpha_i(t) \in [0, 1 - \kappa(t)]$.

Denote

$$F_i(\alpha_{-i}(t)) \equiv \frac{\exp(b_1 + \widehat{b}_2\alpha_{-i}(t))}{1 + \exp(b_1 + \widehat{b}_2\alpha_{-i}(t))}, \tag{S.5}$$

where $\widehat{b}_2 \equiv b_2 - b_3\beta I(t)$ and it follows from Eq.(2) that $\alpha_i(t) = F_i(\alpha_{-i}(t))$. We note that $F_i(\alpha_{-i}(t))$ could be outside the feasible region of $\alpha_i(t)$. It can be verified that $F_i(\cdot)$ is increasing in $\widehat{b}_2$. Moreover, $F_i(\cdot)$ is increasing in α_{-i} if $\widehat{b}_2 \geq 0$, and decreasing in α_{-i} if $\widehat{b}_2 < 0$.

We have $F_i(\alpha^*_B(t)) = \alpha^*_B(t)$ by the definition of $\alpha^*_B(t)$, which satisfies that $\alpha^*_B(t) > 1 - \kappa(t)$ by the assumption in this lemma.

To proceed, we consider a variation of Eq.(S.1) as follows:

$$h(\alpha_{-i}(t)) = \log\left(\frac{\alpha_{-i}(t)}{1 - \alpha_{-i}(t)}\right) - b_1 - \widehat{b}_2\alpha_{-i}(t). \tag{S.6}$$

From the proof of Theorem 2, we have $h(\alpha_{-i}(t)) < 0$ and thus $\alpha_{-i}(t) < F_i(\alpha_{-i}(t)) = \alpha_i(t)$ for any $\alpha_{-i}(t) \in [0, \alpha^*_B(t))$. That is, individual i's activity level $\alpha_i(t)$ when $\alpha_{-i}(t) = 1 - \kappa(t)$ is larger than

$1-\kappa(t)$ (i.e., $\alpha_i(t)=F_i(1-\kappa(t))>1-\kappa(t)$). Thus, when $\alpha_B^*(t)>1-\kappa(t)$, under social distancing policy $\kappa(t)$, the individual i's activity level $\alpha_i^*(t)$ given $\alpha_{-i}(t)=1-\kappa(t)$ is $1-\kappa(t)$. ■

Proof of Theorem 3: We consider the following two scenarios.

When $\alpha_B^*(t)\le 1-\kappa(t)$, since $\alpha_B^*(t)\in[0,1-\kappa(t)]$, it follows from Theorem 1 that $\alpha_B^*(t)$ is unique.

When $\alpha_B^*(t)>1-\kappa(t)$, it follows from Lemma S.5 that $\alpha_\kappa(t)=1-\kappa(t)$ is a symmetric equilibrium solution. Suppose there exists another symmetric equilibrium solution $\alpha_\kappa^{'}(t)$ such that $0<\alpha_\kappa^{'}(t)<1-\kappa(t)$. Set $\alpha_{-i}(t)=\alpha_\kappa^{'}(t)$. Following the proof of Lemma S.5, we have $F_i(\alpha_\kappa^{'}(t))>\alpha_\kappa^{'}(t)$. Thus, the best response of individual i is $\min(F(\alpha_\kappa^{'}(t)),1-\kappa(t))>\alpha_\kappa^{'}(t)$. This contradicts the definition of a symmetric equilibrium. Thus, $\alpha_\kappa(t)=1-\kappa(t)$ is the unique symmetric equilibrium. ■

S.1.10 Proposition S.1 and Proofs of Proposition 6 and Corollary 1

We first show that CP shall either implement a stringent social distancing policy that affects individual behavior or no social distancing policy at all in Proposition S.1.

Proposition S.1. *Given disease prevalence I on a day, the optimal distancing policy κ^* should satisfy one of the following conditions: 1) $1-\alpha_B^*(I)<\kappa^*\le\kappa_{\max}$, or 2) $\kappa^*=0$.*

Proof: Given a disease prevalence I, a social distancing policy $\kappa\le\kappa_{\max}$ can affect the disease dynamics on a day if $\kappa>1-\alpha_B^*(I)$ and cannot if $\kappa\le 1-\alpha_B^*(I)$. Moreover, if $\kappa\le 1-\alpha_B^*(I)$, social distancing policy does not change the disease burden of infection but may incur the policy cost $q(\kappa)$. Given $q(\kappa)$ is an increasing function in κ, setting $\kappa=0$ instead of $0<\kappa\le 1-\alpha_B^*(I)$ resulting in a lower total cost. Therefore, the optimal social distancing policy is either 1) $1-\alpha_B^*(I)<\kappa^*\le\kappa_{\max}$, or 2) $\kappa^*=0$. ■

Proof of Proposition 6 From Eq.(16), we have $\alpha^*(\kappa,I)=\min(1-\kappa,\alpha_B^*(I))$. Based on Proposition 3 (by setting $A=0$), we can derive that $\alpha_B^*(I)$ is decreasing in I and thus $\alpha_B^*(I_1)>\alpha_B^*(I_2)$. Therefore,

$$\alpha_B^*(I_1)-\alpha^*(\kappa,I_1)=\max(\alpha_B^*(I_1)-1+\kappa,0)\ge\max(\alpha_B^*(I_2)-1+\kappa,0)=\alpha_B^*(I_2)-\alpha^*(\kappa,I_2).$$

Since $\alpha_B^*(I_1)>\alpha_B^*(I_2)$ and $\alpha_B^*(I_1)-\alpha^*(\kappa_1,I_1)=\alpha_B^*(I_2)-\alpha^*(\kappa_2,I_2)\ge 0$, we have $\alpha^*(\kappa_1,I_1)>\alpha^*(\kappa_2,I_2)$. If $\alpha_B^*(I_1)-\alpha^*(\kappa_1,I_1)=\alpha_B^*(I_2)-\alpha^*(\kappa_2,I_2)>0$, from Theorem 3, we have $\kappa_2-\kappa_1=[1-\alpha^*(\kappa_2,I_2)]-[1-\alpha^*(\kappa_1,I_1)]>0$. If $\alpha_B^*(I_1)-\alpha^*(\kappa_1,I_1)=\alpha_B^*(I_2)-\alpha^*(\kappa_2,I_2)=0$, we have $\kappa_2=\kappa_1$. Given that $\kappa_2\ge\kappa_1$ and $q(\kappa)$ is increasing in κ, we have $q(\kappa_1)\le q(\kappa_2)$. ■

Proof of Corollary 1 By plugging $\overline{I}_\kappa$ into Eq.(2), one can verify that $\alpha_B^*(\overline{I}_\kappa)=1-\kappa_{\max}$. Thus, the result follows from Proposition S.1. ■

S.1.11 Solution Algorithm for the *Lockdown Model*

We reformulate the *Lockdown Model* into a dynamic programming problem and solve it via backward induction. We note that $d_3[S(1)-S(T)]$ in objective Eq.(6) can be written as $d_3\sum_{t=1}^{T}[S(t-1)-S(t)]+d_3[S(1)-S(0)]$, in which we define $S(0)=0$ and thus $d_3[S(1)-S(0)]$ is a constant and can be omitted in the optimization problem. We define the state on day t as $O(t)\equiv(S(t),I(t),z(t),S(t-1),A(t-1))$, which contains information about both day t and day $t-1$ including the lockdown decision $A(t-1)$.

Due to Constraint (11), if $z(t)=L$ and $A(t-1)=0$, the admissible action set on day t is $\widehat{A}_t=\{A(t)=0\}$; otherwise, $\widehat{A}_t=\{A(t)\in\{0,1\}\}$. Given an action $A(t)\in\widehat{A}_t$, the state transition is determined by constraints in the *Lockdown Model*. The immediate reward is $G(O(t+1),A(t),O(t))=d_1A(t)+d_3[S(t)-S(t-1)]$. Define $V_T(O(T))=0$ for any terminal state $O(T)$. Thus, the Bellman Equation can be written as follows.

$$V_t(O(t))=\min_{A(t)\in\widehat{A}_t}\Big(G(O(t+1),A(t),O(t))+V_{t+1}(O(t+1))|O(t),A(t)\Big).$$

For computational tractability, we discretize $S(t),I(t)\in[0,1]$ into N intervals $\left\{\frac{i}{N-1},i=0,1,\ldots,N-1\right\}$ and solve the problem using backward induction. In our experiments in Section 5, we choose $N=5000$ to derive the optimal lockdown policy.

S.1.12 Solution Algorithm for the *Social Distancing Model*

We propose a gradient descent algorithm (Algorithm 1) to solve the *Social Distancing Model*. Specifically, we derive the gradient of the objective function regarding $\kappa(t)$ recursively through $t \in \{T-1, T-2, \ldots, 1\}$ using backpropagation, in which Eq.(19) is treated as an activation function.

Due to the nonconvexity of the *Social Distancing Model*, we use the classic multistart gradient descent algorithm (e.g., Jain and Agogino 1993) to find the best solution when starting/restarting the search from randomly-generated feasible solutions. In our pilot experiment, we use the number of restarts $M = 5000$ in Algorithm 1 and we observe that the best objective value has little change after 1000-2500 restarts (see an example in Figure S.1). Therefore, we choose to use $M = 5000$ random restarts in our numerical experiments.

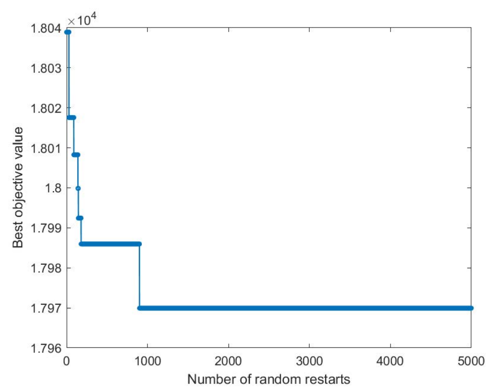
Algorithm 1: Gradient descent algorithm for the *Social Distancing Model*

Data: Initial disease status $(S(1), I(1), R(1))$; parameters b_1, b_2, b_3, d_3 β, r, ϕ, $q_{\max}$, $\kappa_{\max}$; convergence criteria $\epsilon_0 > 0$; iteration limit N; step size λ; number of restarts M

Result: Optimized social distancing policy $\boldsymbol{\kappa}^*$

```
for m ← 1 to M do
    Randomly select initial feasible κ_1; iterator i ← 1;
    while i < N and ||κ_i − κ_{i−1}|| > ε_0 do
        Simulate the pandemic with policy κ_i;
        for t_1 ← T − 1 to 1 do
            for t_2 ← t_1 to T − 1 do
                Compute (∂S(t_2+1)/∂κ_i(t_1), ∂I(t_2+1)/∂κ_i(t_1), ∂R(t_2+1)/∂κ_i(t_1), ∂α(t_2+1)/∂κ_i(t_1)) based on
                (∂S(j)/∂κ_i(t_1), ∂I(j)/∂κ_i(t_1), ∂R(j)/∂κ_i(t_1)), ∂α(j)/∂κ_i(t_1)) where j ∈ {t_1 + 1, ..., t_2} ;   /* derivatives
                derived from Eq.(1) */
            end
            Compute ∂g(κ_i)/∂κ_i(t_1) = Σ_{j=t_1}^{T−1} ∂q(κ_i(j))/∂κ_i(t_1) − d_3 ∂S(T)/∂κ_i(t_1);
        end
        κ_{i+1} = κ_i − λ∇_{κ_i} g(κ_i) where ∇_{κ_i} g(κ_i) = (∂g(κ_i)/∂κ_i(1), ∂g(κ_i)/∂κ_i(2), .., ∂g(κ_i)/∂κ_i(T−1))^T ;
        i = i + 1;
    end
    κ* = arg min(g(κ*), g(κ_i));   /* Update κ* as the best solution */
end
```

Figure S.1: The best objective value changes as the number of restarts increases in the pilot experiment.



S.2 Model Parameterization for Section 5

In this section, we detail our estimation of parameters on disease dynamics, disease burden of infection, and policy cost based on Minnesota data and previous literature. The key data we use in the parameterization process is summarized in Table S.1.

S.2.1 Disease Dynamics Parameters

First, we obtain the per-contact transmission rate $\beta_0 = 0.0295$ and recovery rate $r = 0.1282$ from a validated micro-simulation model for Minnesota (MDH 2020). We aggregate the contact information from (MDH 2020) to estimate the contact matrix C_{ij} between the youth, adult, and elderly groups. Based on the contact matrix between age groups and the Minnesota age distribution, we can find the population average daily contacts $\overline{C} = 12$. Thus, $\beta = \beta_0 \overline{C} = 0.3540$.

We obtain the mortality rate (μ_i) by taking the multiplication of the symptomatic mortality rate of (0.01%, 0.55%, 5.61%) and the symptomatic infection ratio of (30.3%, 67.5%, 42.8%) for the youth, adult and elderly group, respectively (Zhang and Enns 2022). The population average mortality rate is estimated as 0.75% based on Minnesota age distribution.

S.2.2 Disease Burden of Infection $b_{3,i}$ and $d_{3,i}$

Following previous literature, we calculate the overall disease burden of COVID-19 infection considering both fatal and non-fatal cases based on the following equation.

$$\text{Disease burden of infection } b_{3,i} = \big(\overbrace{\text{treatment} + \text{temporary productivity loss} + \text{years of healthy life lost due to disability}}^{\text{Cost per non-fatal case}}\big)$$
$$\times \text{ symptomatic rate} \times \text{clinical confirmation rate} + \big(\overbrace{\text{years of life lost}}^{\text{Cost per fatal case}}\big) \times \text{mortality rate}. \tag{S.7}$$

To calculate the cost per non-fatal case, we consider the treatment cost, temporary productivity loss, and the years of healthy life lost due to disability (YLD) resulting from COVID-19 infection. To calculate the cost per fatal case, we consider the years of life lost (YLL) due to premature mortality associated with COVID-19. We detail our calculation in the following paragraphs.

WTP To estimate the cost associated with YLL, temporary productivity loss, and YLD, we use the classical approach in health economics to compute their Net Monetary Benefit by multiplying an appropriate willingness-to-pay (WTP) threshold (WHO 2012, Garber and Phelps 1997). WTP is typically estimated to be 1-3 times of GDP per capita (WHO 2001) and in our case we set the WTP to be 1× U.S. GDP per-capita in 2019: $65118 (World Bank 2020).

Cost per fatal cases The YLL due to COVID-19 death is estimated by the difference between life expectancy and the average age of each age group in Minnesota. Given Minnesota age composition (MDH 2020), we calculate the mean age to be 9.58, 39.35, and 71.01 years old for the youth, adult, and elderly group, respectively; and the population mean age as 39.22 years old. Based on a population average life expectancy of 78.5 years (WHO 2020), we calculate YLL due to COVID-19 death as 68.92, 39.15, and 7.49 for the youth, adult, and elderly group; the population average YLL is 39.27 based on population mean age. The corresponding cost per fatal case of COVID-19 is calculated as WTP × the corresponding YLL (in Table S.1). The mortality rate is calculated in Appendix S.2.1.

Cost per non-fatal cases From previous literature, we obtain average treatment cost of a clinical confirmed and symptomatic COVID-19 case as $3045 (Bartsch et al. 2020); and obtain the symptomatic infection rate of (30.3%, 67.5%, 42.8%) and the the clinical confirmation rate of (9.5%, 81.9%, 100%) for the youth, adult and elderly group from (Zhang and Enns 2022). The population average symptomatic infection ratio and clinical confirmation rate are estimated as 52.2% and 67.7% based on Minnesota age distribution.

Following previous literature, we assume the number of lost workdays to be 60 days (Mohanty et al. 2020). Following the same estimation procedure as in (Zhang and Enns 2022), we calculate the temporary productivity loss as follows.

$$\text{Temporary productivity loss} = \text{WTP per day} \times \text{average number of lost workdays}$$

$\times$ the age-specific productivity weight,

where the productivity weight is 0.15 for youth, 0.8 for adult, 0.1 for elderly, and 0.469 for the population average. To calculate YLD due to COVID-19 infection, we follow the standard approach in medical literature (e.g., Nurchis et al. 2020) and adopt a disability weight of 0.133 to account for lower respiratory tract infection. Note that we only consider clinically confirmed cases (with moderate to severe symptoms) when estimating this cost, as undetected mild cases are assumed to result in negligible treatment cost, temporary productivity loss, and YLD.

Combining the preceding calculation, we calculate the overall disease burden of COVID-19 infection using Eq.(S.7). Moreover, we assume that individuals and the CP have the same perceived disease burden of infection. Therefore, we estimate $b_{3,1} = d_{3,1} = \$1740.189$, $b_{3,2} = d_{3,2} = \$46640.377$, $b_{3,3} = d_{3,3} = \$39638.333$ for the youth, adult, and elderly group, respectively, and the population average disease burden of infection $b_3 = d_3 = 24844.44$.

S.2.3 Socioeconomic Loss: q_{max} and d_1

Next, we estimate the policy cost of social distancing and lockdown policies. In the U.S., different control measures were implemented simultaneously, which makes it difficult to estimate the policy cost of lockdown and social distancing. China, in contrast, implemented a city-wide lockdown in the epicenter Wuhan, and strict social distancing in other regions in 2020 Q1 during the COVID-19 outbreak. The duration of lockdown in Wuhan and social distancing in other regions was 69 days (Zhang and Enns 2022). Wuhan saw a 40.5% GDP decline in 2020 Q1 compared to 2019 Q1, which we assume is a result of lockdown. Meanwhile, GDP loss in China was 12.8% in 2020 Q1, which we assume is mainly due to the mandated strict social distancing (Zhang and Enns 2022). We also assume that implementing lockdown and social distancing would cause a similar GDP decline percentage in the state of Minnesota as in China. Therefore, we estimate that the lockdown institution causes a 0.5869% daily loss in the quarterly GDP, and a strict social distancing with a maximum possible contact reduction $\kappa_{\max} = 0.5$ (Wu et al. 2020) causes a 0.1855% daily loss in the quarterly GDP. Therefore, given Minnesota 2019 Q1 GDP of 378,047.3 million (U.S. Department of Commerce 2020), we estimate the policy cost of social distancing as $q_{\max} = 123.25$ per person per day for a strict social distancing with $\kappa_{\max} = 0.5$. Similarly, we estimate the policy cost of lockdown as $d_1 = \$389.929$ per person per day.

We understand that the socioeconomic differences between the U.S. and China may result in a biased estimate of the policy cost of these control measures. Therefore, we confirm the validity of our estimates with existing literature. Vardavas et al. (2020) evaluated the socioeconomic impact of non-pharmaceutical interventions (NPI), ranging from the least strict social distancing policy to the most strict lockdown measure. They predicted that 30 days of NPI implementation in September 2020 would result in a reduction of 0.9% to 2% in Minnesota's annual gross state income, that is, one day of NPI implementation in September 2020 would roughly cause a $0.12\% - 0.267\%$ loss in the quarterly gross state income. We note that our estimated policy cost of social distancing is aligned with their prediction whereas our estimated policy cost of lockdown is higher. In Section 5.2, we study an alternative case with a relatively lower policy cost of lockdown policies.

S.2.4 Utility parameters

Lastly, we estimate parameters b_1 and b_2 in the utility functions. We use the annual average income in Minnesota in 2019 (Cubit Planning Inc 2021) to estimate the total economic gain of going out and having normal social activities, which is $b_1 + b_2 = \$187.43$ per day per person. Since we estimate that lockdown results in a daily decline of 0.5869% in the quarterly GDP (roughly 52.83% in the daily GDP), we estimate that b_2 contributes to 52.83% of this daily economic gain. Therefore, we estimate $b_2 = \$99.02$ and $b_1 = \$88.41$.

S.3 Alternative Social Distancing Cost Function

Our social distancing cost function Eq.(13) in Section 4.2 depends on the CP's social distancing policy decision. To test the robustness of our results, we consider an alternative form of the social distancing

Table S.1: Key data used for parameterization.

Variable	Description	Value	Source
Minnesota Age distribution			
	Total population	5.69 million	
	Youth	25.49%	(MDH 2020)
	Adult	50.93%	
	Elderly	23.58%	
β_0	Per-contact transmission rate	2.95%	(MDH 2020)
r^{-1}	Average days in the infectious state	7.8	
Contact pattern			
C_{11}, C_{12}, C_{13}	Youth to (Youth, Adult, Elderly)	8.74, 4.48, 0.74	
C_{21}, C_{22}, C_{23}	Adult to (Youth, Adult, Elderly)	2.77, 9.46, 0.86	(MDH 2020)
C_{31}, C_{32}, C_{33}	Elderly to (Youth, Adult, Elderly)	0.99, 2.55, 1.52	
Daily mortality rate			
μ_1	Youth	0.003%	
μ_2	Adult	0.37%	Estimated in Appendix S.2.1
μ_3	Elderly	2.40%	
Disease burden of infection			
$b_{3,1}$, $d_{3,1}$	Youth	\$1740.189	
$b_{3,2}$, $d_{3,2}$	Adult	\$46640.377	Estimated in Appendix S.2.2
$b_{3,3}$, $d_{3,3}$	Elderly	\$39638.333	
Lockdown and social distancing policies			
ϕ	Shape parameter	1	(Rowthorn and Maciejowski 2020)
$\kappa_{\max}$	Maximum activity reduction	50%	(Wu et al. 2020)
$q_{\max}$	Policy cost of strict social distancing per person per day	\$123.25	Estimated in Appendix S.2.3
d_1	Policy cost of lockdown per person per day	\$389.929	Estimated in Appendix S.2.3
b_1	Individual utility	\$88.41	Estimated in Appendix S.2.4
b_2		\$99.02	

cost function:

$$q(\kappa(t)) = q_{\max} \left(\frac{[\alpha_B^*(t) - (1 - \kappa(t))]^+}{\kappa_{\max}} \right)^{1+\phi},$$

Note that the individual activity level under a social distancing policy $\kappa(t)$ is determined by $\alpha_\kappa^*(t) = \min(1 - \kappa(t), \alpha_B^*(t))$ where $\alpha_B^*(t)$ is the individual equilibrium activity level. Therefore, the cost of social distancing policies depends on the actual activity reduction $[\alpha_B^*(t) - (1 - \kappa(t))]^+$ instead of social distancing policy $\kappa(t)$ in this alternative cost function. We revise the social distancing optimization model and the gradient descent algorithm in Section 4.2.3 to optimize the social distancing decision.

We repeat the experiment in Section 5.2 with this new cost function. Figure S.2 shows that CP should implement social distancing at the initial and later stages of the pandemic to effectively reduce individual activity level, which is consistent with our previous findings.

S.3.1 Alignment of Social Distancing and Lockdown Policies

To align the most stringent social distancing policy with a lockdown policy in terms of resulting individual activity levels, one may solve for the equilibrium individual activity level $\alpha_{A=1}^*(t)$ under lockdown $A(t) = 1$, and then make $\kappa_{\max} = \alpha_{A=1}^*(t)$ so that the feasible region of a social distancing decision is $\kappa(t) \in [0, 1 - \alpha_{A=1}^*(t)]$.

S.4 Disease Prevalence with and without Public Health Interventions

We include a direct comparison of disease prevalence with vs. without lockdown and social distancing implementations in Figures S.3 and S.4.

Figure S.2: Social distancing results with a new social distancing cost function

(a) Social distancing policy when $d_3 = b_3$. **(b)** Social distancing policy when $d_3 = 20 \times b_3$.

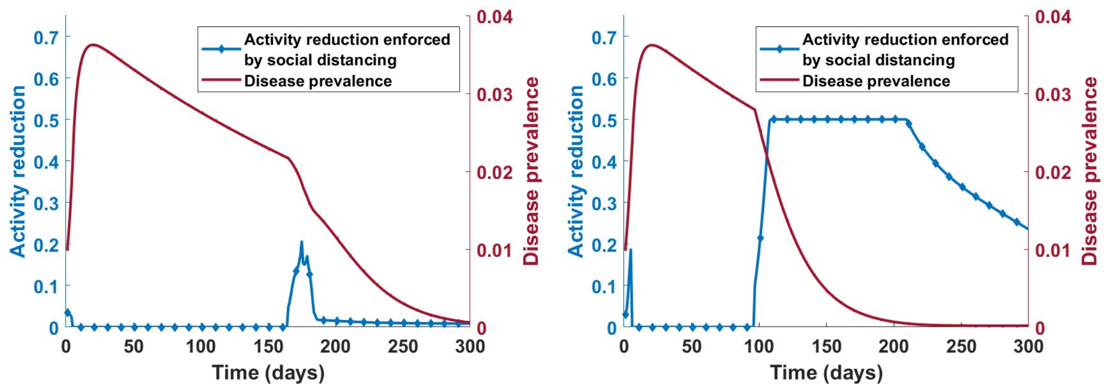


Figure S.3: Disease prevalence with vs. without lockdown when $d_3 = 20b_3$.

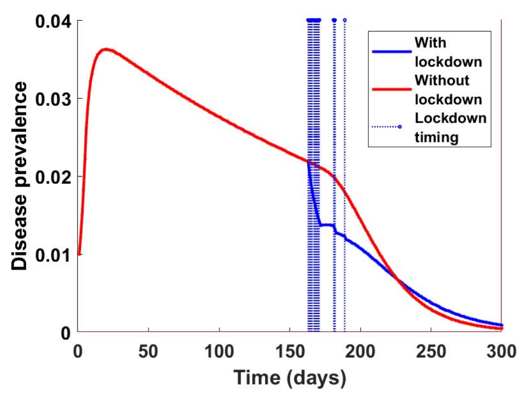
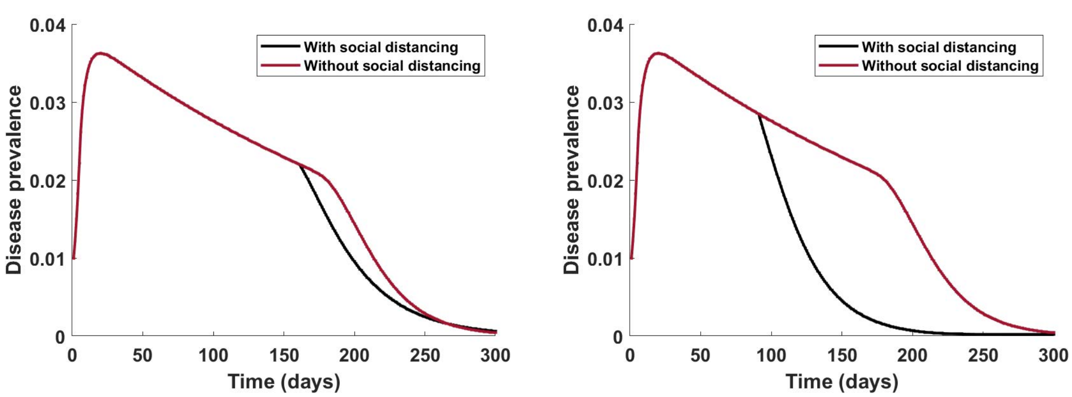


Figure S.4: Disease prevalence with vs. without social distancing.

(a) $d_3 = b_3$ **(b)** $d_3 = 20b_3$



S.5 Joint Implementation of Lockdown and Social Distancing Policies

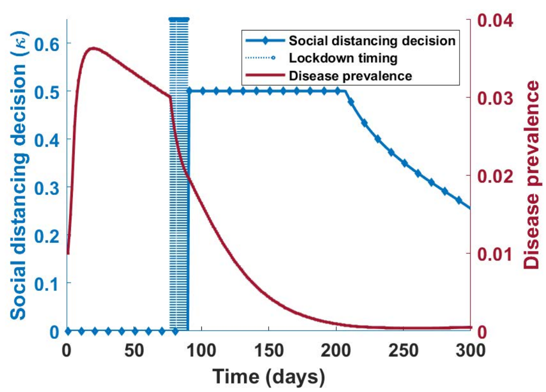
We study the timing of additional lockdown when the social distancing policy has already been determined. Our results based on the estimated parameters in Section 5 suggest that no lockdown should be implemented in addition to social distancing in the scenario of $d_3 = b_3$. However, when $d_3 = 20 \times b_3$, we find that additional lockdown is beneficial (see Table S.2) and the timing of such lockdown implementation is consistent with our previous findings (See Figure S.5).

Table S.2: **Cost decomposition for lockdown and social distancing policies.**

Policy	Cost scenarios	Cost per person		
		CP's perceived disease burden of infection	**Policy cost**	**Total**
Lockdown+social distancing*	$d_3 = 20 \times b_3$	\$229403.5	\$26797.7	\$256201.2

*Lockdown is optimized after the social distancing policy is determined.

Figure S.5: Optimal lockdown timing in addition to social distancing implementation when $d_3 = 20 \times b_3$.



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