Electronic Companion

EC.1. Proofs

EC.1.1. Proof of Proposition 1

Note that $\sum_{j\in J}(d_j-\mu_j)/\kappa_n$ and $(d_j-\mu_j)/\sigma_j$ for $j\in J$ are all increasing in d_j for $j\in J$. Thus, by (11), (12), and (13), it can be seen that $\mathcal{U}_{\mathrm{LC}}$ satisfies both (i) and (ii) specified in Proposition 1.

EC.1.2. Proof of Theorem 1

To prove Theorem 1, consider any $\mathbf{x}\in\mathcal{X}$ and $\mathbf{y}=\mathbf{y}(\mathbf{x})$. We need to show that $f(\mathbf{y})=\phi(\mathbf{x})$, for which, due to $f(\mathbf{y})=\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d})$ and $\phi(\mathbf{x})=\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}}\phi(\mathbf{x},\mathbf{d})$, we only need to prove that

$$\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d})=\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}}\phi(\mathbf{x},\mathbf{d}). \tag{EC.1}$$

To prove (EC.1), define $\hat{J}(\mathbf{d})$ as the set of services $i\in J$ with $f_i(\mathbf{y},\mathbf{d})=1$ but facing no conflict for $\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}$, which can be formulated as follows:

$$\hat{J}(\mathbf{d}):=\Big\{i\in J:\sum_{j\in J} I(e_i+d_i\geq a_j)y_{ij}=1 \text{ and } \phi_i(\mathbf{x},\mathbf{d})=0\Big\}.$$

If $\hat{J}(\mathbf{d})$ is not empty, then for each service $i\in\hat{J}(\mathbf{d})$, we know that $f_i(\mathbf{y},\mathbf{d})=1>0=\phi_i(\mathbf{x},\mathbf{d})$. However, for such $\hat{J}(\mathbf{d})$, Lemma EC.1 below indicates that one can reduce delays for some services without reducing the total conflict number $\phi(\mathbf{x},\mathbf{d})$ or changing the value of $f(\mathbf{y},\mathbf{d})$.

LEMMA EC.1. *For any $\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}$ with $\hat{J}(\mathbf{d})$ not empty, there exists $\tilde{\mathbf{d}}$ with $\mathbf{0}\leq\tilde{\mathbf{d}}\leq\mathbf{d}$ and $\sum_{j\in J}\tilde{d}_j<\sum_{j\in J}d_j$ such that $\phi(\mathbf{x},\tilde{\mathbf{d}})\geq\phi(\mathbf{x},\mathbf{d})$ and $f(\mathbf{y},\tilde{\mathbf{d}})=f(\mathbf{y},\mathbf{d})$.*

Proof. Consider any $\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}$ with $\hat{J}(\mathbf{d})$ not empty. Then, there exists $i\in\hat{J}(\mathbf{d})$ such that $f_i(\mathbf{y},\mathbf{d})=1$ and $\phi_i(\mathbf{x},\mathbf{d})=0$, implying that service i faces no conflict. Since $f_i(\mathbf{y},\mathbf{d})=1$, there must exist $j'\in J$ such that $y_{ij'}=1$ and $e_i+d_i\geq a_{j'}$. Since $\phi_i(\mathbf{x},\mathbf{d})=0$, we know that i does not face any conflict with the schedule of its succeeding service j'. Thus, services i and j' satisfy that

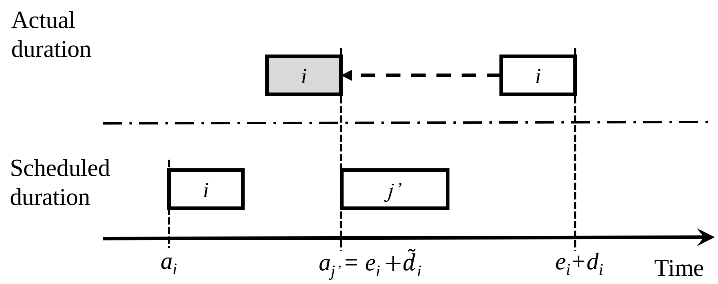
$$y_{ij'}=1 \text{ and } e_i+d_i>a_{j'}. \tag{EC.2}$$

From $y_{ij'}=1$ we know $a_i<e_i<a_{j'}$, which, together with $e_i+d_i>a_{j'}$ in (EC.2), implies that $0<a_{j'}-e_i<d_i$.

Accordingly, as illustrated in Figure EC.1, we can first keep delays of services other than i unchanged, and then reduce delay d_i to $\tilde{d}_i:=a_{j'}-e_i$ for service i, so that $\tilde{d}_i<d_i$ and $a_i+\tilde{d}_i=a_i+a_{j'}-e_i<a_{j'}=e_i+\tilde{d}_i$, which implies that the actual completion time of service i now equals the scheduled starting time of its succeeding service j'. Accordingly, we have $I(a_i+\tilde{d}_i<a_{j'}=e_i+\tilde{d}_i)=1$, implying that under $\tilde{\mathbf{d}}$, service i faces a conflict with the schedule of service j', and thus,

$$\phi_i(\mathbf{x},\tilde{\mathbf{d}})=1>\phi_i(\mathbf{x},\mathbf{d})=0. \tag{EC.3}$$

Figure EC.1 **Setting new delay $\tilde{d}_i := a_{j'} - e_i$ for service i in the proof of Lemma EC.1.**



Moreover, by $e_i + \tilde{d}_i = a_{j'}$ and $y_{ij'} = 1$ we obtain from (9) that $f_i(\mathbf{y}, \tilde{\mathbf{d}}) = 1$, which, together with $f_i(\mathbf{y}, \mathbf{d}) = 1$, implies that

$$f_i(\mathbf{y}, \tilde{\mathbf{d}}) = f_i(\mathbf{y}, \mathbf{d}) = 1. \tag{EC.4}$$

Next, let $L(i)$ indicate a subset of services in J that use the same resource as i (which includes i). Let $L_1(i) \subseteq L(i)$ indicate a subset of services $l \in L(i) \setminus \{i\}$ such that service l faces conflicts with the actual duration of service i but faces no conflicts with the schedules of any services other than l under $\mathbf{y}$ and $\mathbf{d}$. Accordingly, $L_1(i)$ consists of all the services $l \in L(i) \setminus \{i\}$ such that

$$a_i + d_i \leq a_l + d_l \leq e_i + d_i, \text{ and} \tag{EC.5}$$

$$a_l + d_l > e_j \text{ or } e_l + d_l < a_j \text{ for all } j \in L(i) \setminus \{l\}. \tag{EC.6}$$

Thus, for each $l \in L_1(i)$, we have $\phi_l(\mathbf{x}, \mathbf{d}) = 1$ and $d_l > 0$. Since service i faces no conflict with the schedule of service l, and since (EC.5) implies that $a_l < a_l + d_l \leq e_i + d_i$, we obtain that

$$a_l < a_i + d_i, \text{ for each } l \in L_1(i). \tag{EC.7}$$

If $L_1(i)$ is empty, then since delays of all services other than i are unchanged, and since our change of the delay of service i ensures $\phi_i(\mathbf{x}, \tilde{\mathbf{d}}) = 1$ but does not change the values of the crossing function or the conflict function of services other than i, we can obtain that $f_j(\mathbf{y}, \tilde{\mathbf{d}}) = f_j(\mathbf{y}, \mathbf{d})$ and $\phi_j(\mathbf{x}, \tilde{\mathbf{d}}) = \phi_j(\mathbf{x}, \mathbf{d})$ for $j \in J \setminus \{i\}$. This, together with (EC.3) and (EC.4) for service i, implies that $f(\mathbf{y}, \tilde{\mathbf{d}}) = f(\mathbf{y}, \mathbf{d})$ and $\phi(\mathbf{x}, \tilde{\mathbf{d}}) \geq \phi(\mathbf{x}, \mathbf{d})$. Moreover, by $\tilde{d}_i < d_i$ and $\tilde{d}_j = d_j$ for $j \in J \setminus \{i\}$, we have that $\sum_{j \in J} \tilde{d}_j < \sum_{j \in J} d_j$ and $\mathbf{0} \leq \tilde{\mathbf{d}} \leq \mathbf{d}$, which completes the proof of Lemma EC.1.

Accordingly, in the remainder of the proof of Lemma EC.1, we only need to consider the situation where $L_1(i)$ is not empty. For this situation, let l' indicate the service in $L_1(i)$ with the latest scheduled starting time $a_{l'}$. We can adjust the delays for all the services in $L_1(i)$ other than l' so that their adjusted actual arrival times all equal the scheduled arrival time $a_{l'}$ of service l'. In particular, consider each service $l \in L_1(i) \setminus \{l'\}$ if $L_1(i) \setminus \{l'\}$ is not empty. We have $a_l < a_{l'}$, which, together with (EC.7) and (EC.5), implies that $a_l < a_{l'} < a_i + d_i \leq a_l + d_l$. Thus, as illustrated in

Figure EC.2 **Setting new delay $\tilde{d}_l := a_{l'} - a_l$ for each service $l \in L_1(i) \setminus \{l'\}$ in the proof of Lemma EC.1.**

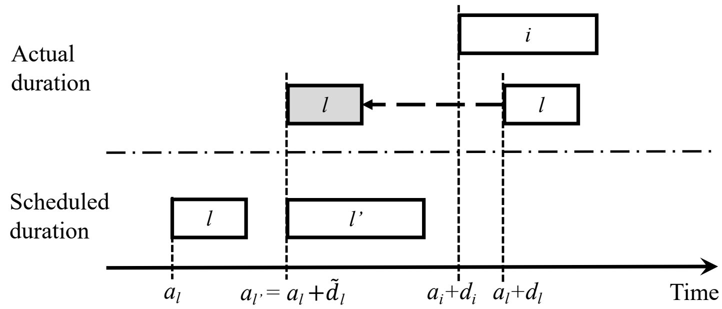


Figure EC.2, we can reduce d_l to $\tilde{d}_l := a_{l'} - a_l > 0$ for service l, so that $\tilde{d}_l = a_{l'} - a_l < d_l$, and that $a_l + \tilde{d}_l = a_{l'} < a_{l'} + e_l - a_l = e_l + \tilde{d}_l$, which implies that the actual starting time of service l is now equal to the scheduled starting time of service l'. Accordingly, $I(a_l + \tilde{d}_l = a_{l'} < e_l + \tilde{d}_l) = 1$, implying that under $\tilde{\mathbf{d}}$, service l faces a conflict with the schedule of service l', (instead of with the actual duration of service i), and thus,

$$\phi_l(\mathbf{x}, \tilde{\mathbf{d}}) = 1 = \phi_l(\mathbf{x}, \mathbf{d}), \text{ for } l \in L_1(i) \setminus \{l'\}. \tag{EC.8}$$

Moreover, by $a_l + \tilde{d}_l = a_{l'} < e_l + \tilde{d}_l$, we have $e_l + \tilde{d}_l > a_{l'}$, implying that the actual completion time of service l is now later than the scheduled starting time of l'. Thus, for any $j \in J$ with $y_{lj} = 1$, since $a_{l'} > a_l$, we have $a_{l'} \geq a_j$, which implies that $e_l + \tilde{d}_l > a_j$. Hence, $\sum_{j \in J} I(e_l + \tilde{d}_l \geq a_j) y_{lj} = 1$. This, together with $\tilde{d}_l < d_l$, implies that $\sum_{j \in J} I(e_l + d_l \geq a_j) y_{lj} = 1$, and thus, by (9),

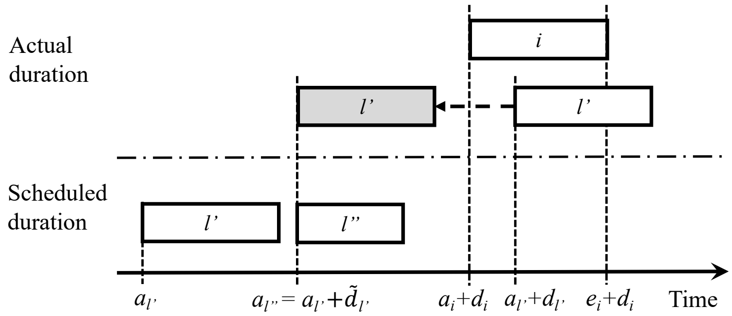
$$f_l(\mathbf{y}, \tilde{\mathbf{d}}) = f_l(\mathbf{y}, \mathbf{d}) = 1, \text{ for } l \in L_1(i) \setminus \{l'\}. \tag{EC.9}$$

It can also be seen that with the above changes in delays of services $l \in L_1(i) \setminus \{l'\}$, since the schedule of service j' (which is the succeeding service of service i) is not changed, (EC.3) and (EC.4) still hold for service i.

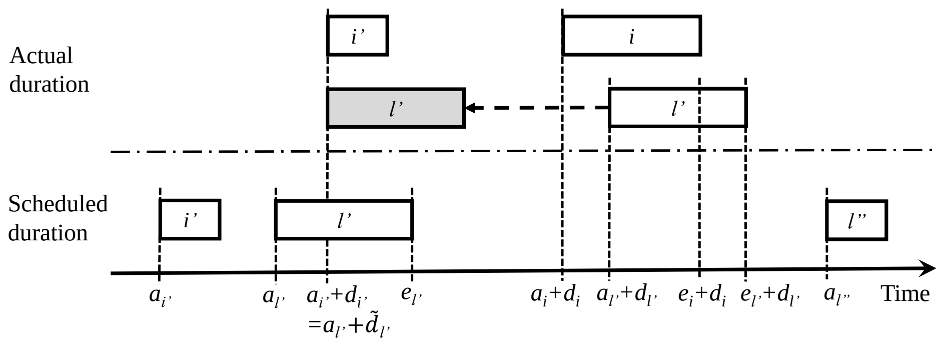
Now, we are going to consider the following three cases, so as to adjust the delay of service l' which, as defined above, is the service in $L_1(i)$ with the latest scheduled starting time.

- Case 1: If there exists a service $l'' \in L(i)$ with $a_{l'} < a_{l''} \leq e_{l'} + d_{l'}$, so that the scheduled starting time of service l'' is within the time interval $(a_{l'}, e_{l'} + d_{l'}]$, then we can adjust the delay of service l' so that the actual starting time of service l' equals the scheduled starting time of service l''. In particular, from $a_{l'} < a_{l''}$ we know that $l' \neq l''$. From $l' \in L_1(i)$, we know that service l' faces a conflict with the actual duration of service i but does not face any conflict with the schedule of service l''. Thus, we have that $a_i + d_i \leq a_{l'} + d_{l'} \leq e_i + d_i$, which, together with $a_{l''} \leq e_{l'} + d_{l'}$, $l' \neq l''$, and (EC.6) for l', implies that $a_{l''} < e_{l''} < a_{l'} + d_{l'} \leq e_i + d_i$. Then, by the argument below, we can obtain that $a_{l''} < a_i + d_i$:

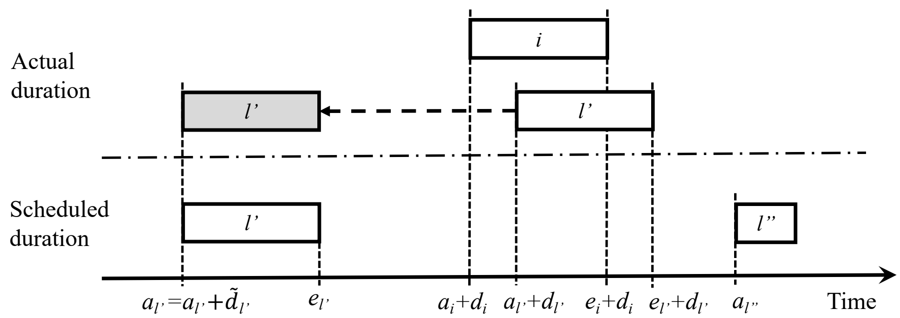
Figure EC.3 Illustration of three cases for setting new delay $\tilde{d}_{l'}$ for service l' in the proof of Lemma EC.1.



(a) For Case 1, set $\tilde{d}_{l'} := a_{l''} - a_{l'}$.



(b) For Case 2, set $\tilde{d}_{l'} := a_{i'} + d_{i'} - a_{l'}$.



(c) For Case 3, set $\tilde{d}_{l'} := 0$.

—If $l'' \neq i$, we know that service i does not face any conflict with the schedule of service l'', which, together with $a_{l''} < e_i + d_i$ and $d_i > 0$, implies that $a_{l''} < a_i + d_i$;

—Otherwise, $l'' = i$, and then since $d_i > 0$, we have that $a_{l''} = a_i < a_i + d_i$.

Accordingly, by $a_{l''} < a_i + d_i$, we have that $a_{l'} < a_{l''} < a_i + d_i \leq a_{l'} + d_{l'} \leq e_i + d_i$. Thus, as illustrated in Figure EC.3(a), we can reduce delay $d_{l'}$ to $\tilde{d}_{l'} := a_{l''} - a_{l'} > 0$ for service l', so that $\tilde{d}_{l'} = a_{l''} - a_{l'} < d_{l'}$ and $a_{l''} = a_{l'} + \tilde{d}_{l'} < e_{l'} + \tilde{d}_{l'}$. Accordingly, the actual starting time of service l' is now equal to the scheduled starting time of service l'', and we have $I(a_{l'} + \tilde{d}_{l'} = a_{l''} < e_{l'} + \tilde{d}_{l'}) = 1$, implying that under $\tilde{\mathbf{d}}$, service l' faces a conflict with the schedule of service l'', and thus,

$$\phi_{l'}(\mathbf{x},\tilde{\mathbf{d}}) = 1 = \phi_{l'}(\mathbf{x},\mathbf{d}). \tag{EC.10}$$

Moreover, note that $a_{l'} < a_{l''} < e_{l'} + \tilde{d}_{l'}$. Thus, for any $j \in J$ with $y_{l'j} = 1$, since $a_{l''} > a_{l'}$, we have $a_{l''} \geq a_j$, which implies that $e_{l'} + \tilde{d}_{l'} > a_j$. Hence, we obtain that $\sum_{j\in J} I(e_{l'} + \tilde{d}_{l'} \geq a_j) y_{l'j} = 1$, which, together with $\tilde{d}_{l'} < d_{l'}$ and (9), implies that

$$f_{l'}(\mathbf{y},\tilde{\mathbf{d}}) = f_{l'}(\mathbf{y},\mathbf{d}) = 1. \tag{EC.11}$$

- Case 2: If there exists no service $l'' \in L(i)$ with $a_{l'} < a_{l''} \leq e_{l'} + d_{l'}$, but there exists a service $i' \in L(i) \setminus \{i\}$ with $\phi_{i'}(\mathbf{x},\mathbf{d}) = 0$ and with $a_{l'} \leq a_{i'} + d_{i'} \leq e_{l'}$, so that service i' faces no conflict and its actual starting time is within the scheduled duration $[a_{l'}, e_{l'}]$ of service l', then we can adjust the delay of service l' so that the actual starting time of service l' equals the actual starting time of service i'. To be more specific, since $\phi_l(\mathbf{x},\mathbf{d}) = 1$ for $l \in L_1(i)$, we have that $i' \in L(i) \setminus \{i\} \setminus L_1(i)$, which, together with $l' \in L_1(i)$, implies that $l' \neq i'$ and $a_{l'} \neq a_{i'}$. Thus, since service i' does not satisfy $a_{l'} < a_{i'} \leq e_{l'} + d_{l'}$, and since $a_{l'} \leq a_{i'} + d_{i'} \leq e_{l'}$ implies that $a_{i'} \leq a_{i'} + d_{i'} \leq e_{l'} \leq e_{l'} + d_{l'}$, we obtain that $a_{i'} < a_{l'} \leq a_{i'} + d_{i'} \leq e_{l'}$. Moreover, since service i' does not face any conflict with the actual duration of service l', by $a_{i'} + d_{i'} \leq e_{l'} \leq e_{l'} + d_{l'}$ and $a_{i'} < a_{l'}$, we have $a_{i'} + d_{i'} \leq a_{l'} + d_{l'}$. Since service i' does not face any conflict with the schedule of service l', by $a_{l'} \leq a_{i'} + d_{i'}$, we have $a_{l'} < a_{i'} + d_{i'}$. Thus, as illustrated in Figure EC.3(b), we can reduce delay $d_{l'}$ to $\tilde{d}_{l'} := a_{i'} + d_{i'} - a_{l'} > 0$ for service l', so that $\tilde{d}_{l'} = a_{i'} + d_{i'} - a_{l'} \leq d_{l'}$, and that $a_{l'} + \tilde{d}_{l'} = a_{i'} + d_{i'} \leq e_{i'} + d_{i'}$. Accordingly, we have $I(a_{i'} + d_{i'} = a_{l'} + \tilde{d}_{l'} \leq e_{i'} + d_{i'}) = 1$. Since $i' \in L(i) \setminus \{i\} \setminus L_1(i)$, we know that the delay of service i' is not changed. Thus, $d_{i'} = \tilde{d}_{i'}$, which implies that $I(a_{i'} + \tilde{d}_{i'} = a_{l'} + \tilde{d}_{l'} \leq e_{i'} + \tilde{d}_{i'}) = 1$. The actual starting time of service l' is now equal to the actual starting time of service i'. Thus, by $a_{i'} < a_{l'}$ we obtain that under $\mathbf{y}$ and $\tilde{\mathbf{d}}$, service l' still faces a conflict, but with respect to the actual duration of service i', (instead of the actual duration of service i.) This implies that

$$\phi_{l'}(\mathbf{x},\tilde{\mathbf{d}}) = 1 = \phi_{l'}(\mathbf{x},\mathbf{d}). \tag{EC.12}$$

Moreover, since there exists no service $l'' \in L(i)$ with $a_{l'} < a_{l''} \leq e_{l'} + d_{l'}$, every service $l'' \in L(i)$ with $a_{l'} < a_{l''}$ must satisfy that $a_{l''} > e_{l'} + d_{l'}$, which together with (EC.5) for l' and $d_i > 0$, implies that $a_{l''} > e_{l'} + d_{l'} > a_{l'} + d_{l'} \geq a_i + d_i > a_i$. Thus, $l'' \neq i$, and since service i faces no conflict with the schedule of l'' under $\mathbf{d}$, from $a_{l''} > a_i + d_i$ we obtain that $a_{l''} > e_i + d_i$, which together with (EC.5) for l', implies that $a_{l''} > e_i + d_i \geq a_{l'} + d_{l'}$. Now, consider any $j \in J$ with $y_{l'j} = 1$. Since $a_{l'} < a_j$, and since j uses the same resource as l' and i, we have $j \in L(i)$. Hence, by following the above argument for l'' to service j, we have $a_j > e_{l'} + d_{l'}$. Thus, $\sum_{j\in J} I(e_{l'} + d_{l'} \geq a_j) y_{l'j} = 0$, which, together with $\tilde{d}_{l'} \leq d_{l'}$ and (9), implies that

$$f_{l'}(\mathbf{y},\tilde{\mathbf{d}}) = f_{l'}(\mathbf{y},\mathbf{d}) = 0. \tag{EC.13}$$

- Case 3: If there exists no service $l'' \in L(i)$ with $a_{l'} < a_{l''} \le e_{l'} + d_{l'}$, and there exists no service $i' \in L(i) \setminus \{i\}$ with $\phi_{i'}(\mathbf{x}, \mathbf{d}) = 0$ and with $a_{l'} \le a_{i'} + d_{i'} \le e_{l'}$, then as illustrated in Figure EC.3(c), we can reduce delay $d_{l'} > 0$ to $\tilde{d}_{l'} := 0$ for service l', so that service l' faces no conflict, implying that

$$\phi_{l'}(\mathbf{x}, \tilde{\mathbf{d}}) = 0 < 1 = \phi_{l'}(\mathbf{x}, \mathbf{d}). \tag{EC.14}$$

Since there exists no service $i' \in L(i) \setminus \{i\}$ with $\phi_{i'}(\mathbf{x}, \mathbf{d}) = 0$ and with $a_{l'} \le a_{i'} + d_{i'} \le e_{l'}$, we know that for each service $i' \in L(i) \setminus \{i\}$ with no conflict under $\mathbf{d}$, it still has no conflict under $\tilde{\mathbf{d}}$. Note that service i faces a conflict under $\mathbf{d}$, as shown above in (EC.3). Moreover, since there exists no service $l'' \in L(i)$ with $a_{l'} < a_{l''} \le e_{l'} + d_{l'}$, by following the same argument as that for deriving (EC.13) in Case 2, we can also obtain that

$$f_{l'}(\mathbf{y}, \tilde{\mathbf{d}}) = f_{l'}(\mathbf{y}, \mathbf{d}) = 0. \tag{EC.15}$$

For the three cases above, it can also be seen that the change made in the delay of service l' does not affect the schedules of service j' and l', implying that under $\tilde{\mathbf{d}}$, service i and services $l \in L_1(i) \setminus \{l'\}$ still face conflicts with the schedules of service j' and service l', respectively, and thus (EC.3), (EC.4), (EC.8), and (EC.9) still hold. Since these changes of the delays also do not affect the schedules or actual duration of services not in $L(i)$, we have $\phi_{i''}(\mathbf{x}, \tilde{\mathbf{d}}) = \phi_{i''}(\mathbf{x}, \mathbf{d})$ and $f_{i''}(\mathbf{y}, \tilde{\mathbf{d}}) = f_{i''}(\mathbf{y}, \mathbf{d})$ for $i'' \in J \setminus L(i)$.

Accordingly, for both Case 1 and Case 2, it can be seen that $\phi(\mathbf{x}, \tilde{\mathbf{d}}) = \phi(\mathbf{x}, \mathbf{d}) + 1 \ge \phi(\mathbf{x}, \mathbf{d})$. For Case 3, it can be seen that $\phi(\mathbf{x}, \tilde{\mathbf{d}}) = \phi(\mathbf{x}, \mathbf{d})$. For all three cases, it can be seen that $f(\mathbf{y}, \tilde{\mathbf{d}}) = f(\mathbf{y}, \mathbf{d})$, and it can also be seen that $\tilde{d}_i < d_i$ and $\tilde{d}_j \le d_j$ for $j \in J \setminus \{i\}$, which implies that $\sum_{j \in J} \tilde{d}_j < \sum_{j \in J} d_j$, and that $\mathbf{0} \le \tilde{\mathbf{d}} \le \mathbf{d}$. Hence, Lemma EC.1 is proved. □

Based on Lemma EC.1, we now prove that $\max_{\mathbf{d} \in \mathcal{U}_{\text{LC}}} f(\mathbf{y}, \mathbf{d}) \ge \max_{\mathbf{d} \in \mathcal{U}_{\text{LC}}} \phi(\mathbf{x}, \mathbf{d})$. Among all the maximizers of $\max_{\mathbf{d} \in \mathcal{U}_{\text{LC}}} \phi(\mathbf{x}, \mathbf{d})$, let $\mathbf{d}^*$ denote the one with $\sum_{j \in J} d_j^*$ minimized. Thus, $\phi(\mathbf{x}, \mathbf{d}^*) = \max_{\mathbf{d} \in \mathcal{U}_{\text{LC}}} \phi(\mathbf{x}, \mathbf{d})$. It can be seen that $\hat{J}(\mathbf{d}^*)$ must be empty, because otherwise, by Lemma EC.1 there must exist $\tilde{\mathbf{d}}$ with $\mathbf{0} \le \tilde{\mathbf{d}} \le \mathbf{d}^*$ and $\sum_{j \in J} \tilde{d}_j < \sum_{j \in J} d_j^*$ such that $\phi(\mathbf{x}, \tilde{\mathbf{d}}) \ge \phi(\mathbf{x}, \mathbf{d}^*) = \max_{\mathbf{d} \in \mathcal{U}_{\text{LC}}} \phi(\mathbf{x}, \mathbf{d})$, which, together with $\tilde{\mathbf{d}} \in \mathcal{U}_{\text{LC}}$ (due to $\mathbf{0} \le \tilde{\mathbf{d}} \le \mathbf{d}^*$ and Proposition 1), leads to a contradiction with the definition of $\mathbf{d}^*$.

Therefore, for each $i \in J$, if $\sum_{j \in J} I(e_i + d_i^* \ge a_j) y_{ij} = 1$, then $f_i(\mathbf{y}, \mathbf{d}^*) = 1$, and since $\hat{J}(\mathbf{d}^*)$ is empty, we have $\phi_i(\mathbf{x}, \mathbf{d}^*) = 1$, implying that $f_i(\mathbf{y}, \mathbf{d}^*) = \phi_i(\mathbf{x}, \mathbf{d}^*) = 1$. Otherwise, $\sum_{j \in J} I(e_i + d_i^* \ge a_j) y_{ij} = 0$, and thus $f_i(\mathbf{y}, \mathbf{d}^*) = 0$. For this case, by establishing Lemma EC.2 below, we have that $f_i(\mathbf{y}, \mathbf{d}^*) = \phi_i(\mathbf{x}, \mathbf{d}^*) = 0$.

Lemma EC.2. *For each $i \in J$, if $\sum_{j \in J} I(e_i + d_i^* \ge a_j) y_{ij} = 0$, then $\phi_i(\mathbf{x}, \mathbf{d}^*) = 0$.*

Proof. Consider any $i \in J$ with $\sum_{j\in J} I(e_i + d_i^* \geq a_j)y_{ij} = 0$. By contradiction, suppose $\phi_i(\mathbf{x},\mathbf{d}^*) = 1$. Then, since $\sum_{j\in J} I(e_i + d_i^* \geq a_j)y_{ij} = 0$, we know that service i has no conflict with any schedules of other services under $\mathbf{d}^*$, which implies that there exists a service $i' \in J\setminus\{i\}$ with $\phi_{i'}(\mathbf{x},\mathbf{d}^*) = 0$ such that service i faces a conflict with the actual duration of service i', implying that service i is assigned the same resource as service i' with $a_{i'} + d_{i'}^* \leq a_i + d_i^* \leq e_{i'} + d_{i'}^*$. Thus, we have that $e_i + d_i^* \geq a_i + d_i^* \geq a_{i'} + d_{i'}^* \geq a_{i'}$. Therefore, it can be seen that $a_{i'} < a_i$, because otherwise, we have $a_{i'} > a_i$, which implies that there exists $j \in J$ with $y_{ij} = 1$, so that $a_j \leq a_{i'} \leq e_i + d_i^*$, which contradicts that $\sum_{j\in J} I(e_i + d_i^* \geq a_j)y_{ij} = 0$. Therefore, since $a_{i'} < a_i$ and service i is assigned the same resource as service i', we have that $a_{i'} < e_{i'} < a_i$. Thus, since $e_{i'} + d_{i'}^* \geq a_i + d_i^* \geq a_i > a_{i'}$, we have that $\sum_{j'\in J} I(e_{i'} + d_{i'}^* \geq a_{j'})y_{i'j'} = 1$. This implies that $i' \in \hat{J}(\mathbf{d}^*)$, contradicting the fact that $\hat{J}(\mathbf{d}^*)$ is empty. Hence, $\phi_i(\mathbf{x},\mathbf{d}^*) = 0$. Thus, we obtain that if $\sum_{j\in J} I(e_i + d_i^* \geq a_j)y_{ij} = 0$, then $f_i(\mathbf{y},\mathbf{d}^*) = \phi_i(\mathbf{x},\mathbf{d}^*) = 0$, which completes the proof of Lemma EC.2. □

Following the argument above, together with Lemma EC.2, we obtain that $f_i(\mathbf{y},\mathbf{d}^*) = \phi_i(\mathbf{x},\mathbf{d}^*)$ for each $i \in J$, implying that $f(\mathbf{y},\mathbf{d}^*) = \phi(\mathbf{x},\mathbf{d}^*)$. Thus, we obtain that

$$\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d}) \geq f(\mathbf{y},\mathbf{d}^*) = \phi(\mathbf{x},\mathbf{d}^*) = \max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} \phi(\mathbf{x},\mathbf{d}). \tag{EC.16}$$

We next show that $\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} \phi(\mathbf{x},\mathbf{d}) \geq \max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d})$. Among all the maximizers of $\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d})$, let $\mathbf{d}^\#$ denote the one with $\sum_{j\in J} d_j^\#$ minimized. Thus, $f(\mathbf{y},\mathbf{d}^\#) = \max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d})$. Similar to $\hat{J}(\mathbf{d}^*)$, it can also be seen that $\hat{J}(\mathbf{d}^\#)$ must be empty, because otherwise, by Lemma EC.1 there exists $\tilde{\mathbf{d}}$ with $\mathbf{0} \leq \tilde{\mathbf{d}} \leq \mathbf{d}^\#$ and $\sum_{j\in J} \tilde{d}_j < \sum_{j\in J} d_j^\#$ such that $f(\mathbf{y},\tilde{\mathbf{d}}) = f(\mathbf{y},\mathbf{d}^\#) = \max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d})$, which, together with $\tilde{\mathbf{d}} \in \mathcal{U}_{\mathrm{LC}}$ (due to $\mathbf{0} \leq \tilde{\mathbf{d}} \leq \mathbf{d}^\#$ and Proposition 1), leads to a contradiction with the definition of $\mathbf{d}^\#$.

Since $\hat{J}(\mathbf{d}^\#)$ is an empty set, we know that $\phi_i(\mathbf{x},\mathbf{d}^\#) = 1$ for each $i \in J$ with $f_i(\mathbf{y},\mathbf{d}^\#) = \sum_{j\in J} I(e_i + d_i^\# \geq a_j)y_{ij} = 1$. Moreover, for each $i \in J$ with $f_i(\mathbf{y},\mathbf{d}^\#) = \sum_{j\in J} I(e_i + d_i^\# \geq a_j)y_{ij} = 0$, we have $f_i(\mathbf{y},\mathbf{d}^\#) = 0 \leq \phi_i(\mathbf{x},\mathbf{d}^\#)$. Hence, we obtain that $f_i(\mathbf{y},\mathbf{d}^\#) \leq \phi_i(\mathbf{x},\mathbf{d}^\#)$ for each $i \in J$, which implies that $\phi(\mathbf{x},\mathbf{d}^\#) \geq f(\mathbf{y},\mathbf{d}^\#)$. Thus, we obtain that

$$\max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} \phi(\mathbf{x},\mathbf{d}) \geq \phi(\mathbf{x},\mathbf{d}^\#) \geq f(\mathbf{y},\mathbf{d}^\#) = \max_{\mathbf{d}\in\mathcal{U}_{\mathrm{LC}}} f(\mathbf{y},\mathbf{d}). \tag{EC.17}$$

By (EC.16) and (EC.17), (EC.1) is proved. Thus, $f(\mathbf{y}) = \phi(\mathbf{x})$. Based on the preceding proof, this finding extends beyond the specific uncertainty set $\mathcal{U}_{\mathrm{LC}}$ and applies more generally to any uncertainty set that satisfies the down-monotonicity property delineated in Proposition 1. Consequently, Theorem 1 is established.

EC.1.3. Proof of Corollary 1

To prove Corollary 1, consider any given $\mathbf{x} \in \mathcal{X}$ and $\mathbf{y} = \mathbf{y}(\mathbf{x})$. Consider any delays $\mathbf{d}^{\#} \in \mathcal{U}_{\text{LC}}$ that maximizes $f(\mathbf{y}, \mathbf{d})$. We have that

$$f(\mathbf{y}, \mathbf{d}^{\#}) = \max_{\mathbf{d} \in \mathcal{U}_{\text{LC}}} f(\mathbf{y}, \mathbf{d}) = f(\mathbf{y}). \tag{EC.18}$$

We now transform delays $\mathbf{d}^{\#}$ to new delays $\mathbf{d}'$ as follows, so as to satisfy the condition specified in Corollary 1. For each service $i \in J$, if there exists $j \in J$ with $y_{ij} = 1$ (i.e., service j succeeds service i in using the same resource), and if $e_i + d_i^{\#} \geq a_j$ (i.e., service i's actual completion time exceeds or equals service j's scheduled starting time), then we reduce the delay $d_i^{\#}$ of service i by $e_i + d_i^{\#} - a_j \geq 0$ to obtain a new delay $d_i' := d_i^{\#} - (e_i + d_i^{\#} - a_j) = a_j - e_i$, so that service i's actual completion time is advanced to be exactly equal to service j's scheduled starting time (i.e., $e_i + d_i' = a_j$). Otherwise, we know that either service i is the one with the latest scheduled starting time among those assigned to the same resource as i, or its actual completion time is still earlier than its succeeding service j (i.e., $e_i + d_i^{\#} < a_j$), and accordingly, we shorten the delay $d_i^{\#}$ of service i to zero by setting $d_i' := 0$.

It can be seen that the new delays $\mathbf{d}'$ constructed above from $\mathbf{d}^{\#}$ satisfies the condition specified in Corollary 1 that for each service $i \in I$, $d_i' = 0$ or $d_i' = a_j - e_i$ for some $j \in J$ with $y_{ij} = 1$. For the new delays $\mathbf{d}'$, we also have $\mathbf{0} \leq \mathbf{d}' \leq \mathbf{d}^{\#}$, which, together with the down monotonicity of $\mathcal{U}_{\text{LC}}$ (shown in Proposition 1), implies that $\mathbf{d}' \in \mathcal{U}_{\text{LC}}$. It can also be seen that $I(e_i + d_i^{\#} \geq a_j) y_{ij} = I(e_i + d_i' \geq a_j) y_{ij}$ for each service $i \in J$, which, together with (10), implies that the crossing functions $f(\mathbf{y}, \mathbf{d}^{\#})$ and $f(\mathbf{y}, \mathbf{d}')$ have equal values, i.e., $f(\mathbf{y}, \mathbf{d}^{\#}) = f(\mathbf{y}, \mathbf{d}')$.

Moreover, for any two services $i \in J$ and $j' \in J$ with $y_{ij'} = 0$, if service i and service j' are assigned to the same resource, then either $e_i < a_{j'}$ or $e_{j'} < a_i$, and thus, under delays $\mathbf{d}'$, since for both service i and service j', their actual completion times are before or equal to the scheduled starting times of their succeeding services (if existing), it can be seen from $y_{ij'} = 0$ that service i cannot face a conflict with the actual duration of service j'. Therefore, it can also be seen that under delays $\mathbf{d}'$, service i faces a conflict with the schedule or with the actual duration of service j if and only if $y_{ij} = 1$ and $e_i + d_i' = a_j$. This implies that the crossing function $f(\mathbf{y}, \mathbf{d}')$ and conflict function $\phi(\mathbf{x}, \mathbf{d}')$ have equal values, which, together with $f(\mathbf{y}, \mathbf{d}^{\#}) = f(\mathbf{y}, \mathbf{d}')$, (EC.18), and $f(\mathbf{y}) = \phi(\mathbf{x})$ (by Theorem 1), implies that

$$\phi(\mathbf{x}) = f(\mathbf{y}) = f(\mathbf{y}, \mathbf{d}^{\#}) = f(\mathbf{y}, \mathbf{d}') = \phi(\mathbf{x}, \mathbf{d}'),$$

which completes the proof of Corollary 1.

EC.1.4. Proof of Theorem 2

For any $\mathbf{P}$,

$$\mathbf{E_P}(\phi(\mathbf{x},\mathbf{d})) := \sum_{\mathbf{d}} \phi(\mathbf{x},\mathbf{d})\mathbf{P}(\mathbf{d})$$

Construct the uncertainty set $\mathcal{U}_{\text{LC}}(\mathbf{d})$ as follows:

$$\mathcal{U}_{\text{LC}}(\mathbf{d}) := \{\mathbf{d}' : \mathbf{d}' \le \mathbf{d}\}$$

We have

$$\begin{aligned}\sum_{\mathbf{d}} \phi(\mathbf{x},\mathbf{d})\mathbf{P}(\mathbf{d}) &\le \sum_{\mathbf{d}} \max_{\mathbf{d}'\in\mathcal{U}_{\text{LC}}(\mathbf{d})} \phi(\mathbf{x},\mathbf{d}')\mathbf{P}(\mathbf{d}) \\ &= \sum_{\mathbf{d}} \max_{\mathbf{d}'\in\mathcal{U}_{\text{LC}}(\mathbf{d})} f(\mathbf{y}(\mathbf{x}),\mathbf{d}')\mathbf{P}(\mathbf{d})\end{aligned}$$

The first inequality follows from the fact that $\mathbf{d}'$ attains the maximum number of conflicts for any down monotone set dominated by $\mathbf{d}$, whereas the second follows from the equivalence of worst-case crossings and conflicts for down-monotone set established in Theorem 1.

Define $g(\mathbf{d})$ to be the worst-case crossing delay realization for the uncertainty set $\mathcal{U}_{\text{LC}}(\mathbf{d})$, and we have

$$g(\mathbf{d}) \le \mathbf{d} \ \forall \ \mathbf{d}$$

By definition,

$$\begin{aligned}\sum_{\mathbf{d}} \max_{\mathbf{d}'\in\mathcal{U}_{\text{LC}}(\mathbf{d})} f(\mathbf{y}(\mathbf{x}),\mathbf{d}'))\mathbf{P}(\mathbf{d}) &= \sum_{\mathbf{d}} f(\mathbf{y}(\mathbf{x}),g(\mathbf{d}))\mathbf{P}(\mathbf{d}) \\ &= \sum_{\mathbf{d}'} f(\mathbf{y}(\mathbf{x}),\mathbf{d}')\mathbf{P}_g(\mathbf{d}') \\ &\le \max_{\mathbf{P}\in\mathcal{P}} \mathbf{E_P}(f(\mathbf{y}(\mathbf{x}),\mathbf{d}))\end{aligned}$$

Hence, we have

$$\max_{\mathbf{P}\in\mathcal{P}} \mathbf{E_P}(\phi(\mathbf{x},\mathbf{d})) \le \max_{\mathbf{P}\in\mathcal{P}} \mathbf{E_P}(f(\mathbf{y}(\mathbf{x}),\mathbf{d})), \ \forall \mathbf{x}\in\mathcal{X} \tag{EC.19}$$

Next, we have

$$\begin{aligned}\sum_{\mathbf{d}} f(\mathbf{y}(\mathbf{x}),\mathbf{d})\mathbf{P}(\mathbf{d}) &\le \sum_{\mathbf{d}} \max_{\mathbf{d}'\in\mathcal{U}_{\text{LC}}(\mathbf{d})} f(\mathbf{y}(\mathbf{x}),\mathbf{d}')\mathbf{P}(\mathbf{d}) \\ &= \sum_{\mathbf{d}} \max_{\mathbf{d}'\in\mathcal{U}_{\text{LC}}(\mathbf{d})} \phi(\mathbf{x},\mathbf{d}')\mathbf{P}(\mathbf{d})\end{aligned}$$

The first equality follows from the fact that $\mathbf{d}'$ attains the maximum number of crossings for any down monotone set dominated by $\mathbf{d}$, whereas the second follows from the equivalence of worst-case conflicts and crossings for down-monotone set established in Theorem 1.

For some $g(\cdot)$, define $g(\mathbf{d})$ as the worst-case conflict delay realization for the uncertainty set $\mathcal{U}_{\mathrm{LC}}(\mathbf{d})$, and we have

$$g(\mathbf{d}) \le \mathbf{d} \ \forall \ \mathbf{d}$$

By definition,

$$\begin{aligned}\sum_{\mathbf{d}} \max_{\mathbf{d}' \in \mathcal{U}_{\mathrm{LC}}} \phi(\mathbf{x}, \mathbf{d}')\mathbf{P}(\mathbf{d}) &= \sum_{\mathbf{d}} \phi(\mathbf{x}, g(\mathbf{d}))\mathbf{P}(\mathbf{d}) \\ &= \sum_{\mathbf{d}'} \phi(\mathbf{x}, \mathbf{d}')\mathbf{P}_g(\mathbf{d}') \\ &\le \max_{\mathbf{P} \in \mathcal{P}} \mathbf{E}_{\mathbf{P}}(\phi(\mathbf{x}, \mathbf{d}))\end{aligned}$$

Hence, we have

$$\max_{\mathbf{P} \in \mathcal{P}} \mathbf{E}_{\mathbf{P}}(\phi(\mathbf{x}, \mathbf{d})) \ge \max_{\mathbf{P} \in \mathcal{P}} \mathbf{E}_{\mathbf{P}}(f(\mathbf{y}(\mathbf{x}), \mathbf{d})), \ \forall \mathbf{x} \in \mathcal{X} \tag{EC.20}$$

Combining (EC.19) and (EC.20), we have

$$\max_{\mathbf{P} \in \mathcal{P}} \mathbf{E}_{\mathbf{P}}(\phi(\mathbf{x}, \mathbf{d})) = \max_{\mathbf{P} \in \mathcal{P}} \mathbf{E}_{\mathbf{P}}(f(\mathbf{y}(\mathbf{x}), \mathbf{d})), \ \forall \mathbf{x} \in \mathcal{X} \tag{EC.21}$$

EC.1.5. BDRAP

We derive a nontrivial special case of the CROP that can be solved in polynomial time. Consider a basic case of the DRAP, which is referred to as BDRAP for short, where resources in R are identical for all services $j \in J$, and only the two basic constraints (1) and (2) are taken into account. Moreover, in the BDRAP, for every pair of services $i \in J$ and $j \in J$, if service j uses the same resource following service i, then there is a changeover cost denoted by c_{ij}. For each service $i \in J$, if i is not assigned any resource, there is a non-assignment penalty denoted by p_i. The BDRAP aims to minimize the total cost of changeover and non-assignment. Since resources are identical, each resource $r \in R$ has the same set of feasible service sequences, which is denoted by S. Thus, the BDRAP can be formulated as:

$$\begin{aligned}\text{(BDRAP)} \ \min \ & \sum_{i \in J} \sum_{j \in J} \sum_{r \in R} \sum_{s \in S} c_{ij} b_{rsij} x_{rs} + \sum_{i \in J} p_i (1 - \sum_{r \in R} \sum_{s \in S} h_{si} x_{rs}) \\ \text{s.t.} \ & \sum_{r \in R} \sum_{s \in S} h_{sj} x_{rs} \le 1, \ \text{for all } j \in J, \\ & \sum_{s \in S} x_{rs} \le 1, \ \text{for all } r \in R, \\ & x_{rs} \in \{0, 1\}, \ \text{for } r \in R \text{ and } s \in S.\end{aligned}$$

From Lemma EC.3 below we know that the BDRAP can be solved to optimum in polynomial time.

LEMMA EC.3. *The BDRAP can be transformed equivalently to a minimum cost network flow problem, and thus can be solved to optimum in polynomial time.*

Notice that SAA model (22), the arctangent model (EC.22) and the weighted SAA using k-nearest-neighbors model (referred to as "wSAA-kNN" model) are variants of the BDRAP. To solve these models, we replace changeover costs c_{ij} by $\hat{\mathbb{P}}(d_i \geq a_j - e_i)$, $\mathbb{P}_0(a_j - e_i)$ and $\hat{\mathbb{P}}^{\text{kNN}}(d_i \geq a_j - e_i)$ respectively for $i \in J$ and $j \in J$. By Lemma EC.3 we know that if the BDRAP can be solved to optimum in polynomial time, then the three models can be solved to optimum in polynomial time.

EC.1.6. Proof of Lemma EC.3

To prove Lemma EC.3, consider any instance of the BDRAP with n services in J and a set R of m resources. We can transform it equivalently to a minimum cost network flow problem as follows. Let $G = (V, A)$ denote an acyclic directed network, where the node set $V = \{0, n+1, 1', 1'', 2', 2'', \ldots, n', n''\}$, and the arc set $A = \{(0, i') : i \in J\} \cup \{(i', i'') : i \in J\} \cup \{(i'', j') : e_i < a_j, i \in J, j \in J\} \cup \{(i'', n+1) : i \in J\} \cup \{(0, n+1)\}$. Node 0 is the source node of the network with a supply $b(0) = m$, node $n+1$ is the sink node of the network with a supply $b(n+1) = -m$ (i.e., a demand of m), and other nodes $v \in V \setminus \{s, t\}$ are transshipment nodes with $b(v) = 0$. For each arc $(u, v) \in A \setminus \{(0, n+1)\}$, it has a capacity of 1, and for $(0, n+1)$, it has a capacity of infinity. For each arc $(u, v) \in A$, it has a cost w_{uv} defined as follows:

$$w_{uv} = \begin{cases} c_{ij}, & \text{if } (u,v) = (i'', j') \text{ with } e_i < a_j,\ i \in J, \text{ and } j \in J \\ -p_i, & \text{if } (u,v) = (i', i'') \text{ with } i \in J \\ 0, & \text{otherwise.} \end{cases}$$

The minimum cost network flow problem (MCNFP) on this acyclic network $G = (V, A)$ can be formulated as follows, where ζ_{uv} denotes the flow variable for each arc $(u, v) \in A$:

$$\begin{aligned} \text{(MCNFP)} \ \min \quad & \sum_{(u,v)\in A} w_{uv}\zeta_{uv} \\ \text{s.t.} \quad & \sum_{v:(u,v)\in A} \zeta_{uv} - \sum_{v:(v,u)\in A} \zeta_{vu} = b(u), \ \text{for all } u \in V, \\ & 0 \leq \zeta_{uv} \leq 1, \ \text{for all } (u,v) \in A \setminus \{(0, n+1)\}, \\ & \zeta_{uv} \in \mathbb{Z}_+, \ \text{for all } (u,v) \in A. \end{aligned}$$

Next, we are going to prove that to solve the BDRAP, it is equivalent to solving the MCNFP. Let $\text{OPT}_{\text{BDRAP}}$ and $\text{OPT}_{\text{MCNFP}}$ denote the optimal objective values of the BDRAP and problem

MCNFP, respectively. On the one hand, consider any optimal solution ζ^* to the MCNFP. Since $b(0)=m$, we have that $\zeta_{uv}\in\{0,1\}$ for $(u,v)\in A\setminus\{(0,n+1)\}$, and $\zeta_{0,n+1}\in\{0,1,\ldots,m\}$. Since arc $(0,n+1)$ has an infinite capacity and $b(0)=m$, it can be seen that $m=\sum_{\forall v:(0,v)\in A}\zeta^*_{0v}=\sum_{i\in J}\zeta^*_{0i'}+\zeta^*_{0,n+1}$. Let $m'=m-\zeta^*_{0,n+1}$. It can be seen that arcs $(u,v)\in A\setminus\{(0,n+1)\}$ with $\zeta^*_{uv}=1$ form m' edge-disjoint paths that start from s and end at t, each of which can be denoted by $\langle 0,i'_{r1},i''_{r1},i'_{r2},i''_{r2},\ldots,i'_{r,n_r},i''_{r,n_r},n+1\rangle$ for $r\in\{1,2,\ldots,m'\}$, where $n_r\ge 0$, and $i_k\in J$ for $1\le k\le n_r$. With these paths, we can obtain m' service sequences, denoted by $s_r=\langle i_{r1},i_{r2},\ldots,i_{r,n_r}\rangle$, for $r\in\{1,2,\ldots,m'\}$. Since for each arc $(i'',j')\in A$ with $i\in J$ and $j\in J$, it satisfies $e_i<a_j$, service sequence s_r does not contain any two consecutive services $i\in J$ and $j\in J$ that have an overlap on their scheduled durations, implying that s_r is a feasible service sequence in S. Accordingly, for each $r\in R$ and $s\in S$, we set $x_{rs}=1$ if $r\in\{1,2,\ldots,m'\}$ and $s=s_r$, and $x_{rs}=0$, otherwise. Since arc (i',i'') for each service $i\in J$ has a capacity of 1, there is at most one sequence s_r with $r\in\{1,2,\ldots,m'\}$ that contains service i. Hence, $\mathbf{x}$ is a feasible solution to the BDRAP. Moreover, it can be seen that

$$\begin{aligned}
\mathrm{OPT}_{\mathrm{BDRAP}} &\le \sum_{i\in J}\sum_{j\in J}\sum_{r\in R}\sum_{s\in S}c_{ij}b_{rsij}x_{rs}+\sum_{i\in J}p_i(1-\sum_{r\in R}\sum_{s\in S}h_{si}x_{rs})\\
&=\sum_{r=1}^{m'}\sum_{k=1}^{n_r-1}c_{i_{rk},i_{r,k+1}}+\sum_{i\in J}p_i-\sum_{r=1}^{m'}\sum_{k=1}^{n_r}p_{i_{rk}}\\
&=\sum_{r=1}^{m'}\sum_{k=1}^{n_r-1}w_{i''_{rk},i'_{r,k+1}}\zeta^*_{i''_{rk},i'_{r,k+1}}+\sum_{r=1}^{m'}\sum_{k=1}^{n_r}w_{i'_{rk},i''_{rk}}\zeta^*_{i'_{rk},i''_{rk}}+\sum_{i\in J}p_i\\
&=\sum_{(u,v)\in A}w_{uv}\zeta^*_{uv}+\sum_{i\in J}p_i=\mathrm{OPT}_{\mathrm{MCNFP}}+\sum_{i\in J}p_i.
\end{aligned}$$

On the other hand, consider any optimal solution $\mathbf{x}^*$ to the BDRAP. Without loss of generality, we assume $\sum_{s\in S}x^*_{rs}=1$ for $r=1,2,\ldots,m''$, and $\sum_{s\in S}x^*_{rs}=0$ for $r=m''+1,m''+2,\ldots,m$, where $0\le m''\le m$. For $1\le r\le m''$, let s_r denote the unique feasible sequence that r is assigned to, and represent it as $\langle i_{r1},i_{r2},\ldots,i_{r,n_r}\rangle$. Accordingly, we set $\zeta_{uv}=1$ for each arc (u,v) of paths $\langle 0,i'_{r1},i''_{r1},i'_{r2},i''_{r2},\ldots,i'_{r,n_r},i''_{r,n_r},n+1\rangle$ of network G for $1\le r\le m'$, set $\zeta_{0,n+1}=m-m''$ for arc $(0,n+1)$, and set $\zeta_{uv}=0$ for other arcs of network G. Since each service can be assigned to at most one resource, we have that the obtained ζ is a feasible solution to the MCNFP without exceeding any arc capacities. Moreover, it can be seen that

$$\begin{aligned}
\mathrm{OPT}_{\mathrm{MCNFP}}+\sum_{i\in J}p_i &\le \sum_{(u,v)\in A}w_{uv}\zeta_{uv}+\sum_{i\in J}p_i=\sum_{r=1}^{m''}\sum_{k=1}^{n_r-1}w_{i''_{rk},i'_{r,k+1}}+\sum_{r=1}^{m''}\sum_{k=1}^{n_r}w_{i'_{rk},i''_{rk}}+\sum_{i\in J}p_i\\
&=\sum_{r=1}^{m''}\sum_{k=1}^{n_r-1}c_{i_{rk},i_{r,k+1}}-\sum_{r=1}^{m''}\sum_{k=1}^{n_r}p_{i_{rk}}+\sum_{i\in J}p_i\\
&=\sum_{i\in J}\sum_{j\in J}\sum_{r\in R}\sum_{s\in S}c_{ij}b_{rsij}x^*_{rs}+\sum_{i\in J}p_i(1-\sum_{r\in R}\sum_{s\in S}h_{si}x^*_{rs})=\mathrm{OPT}_{\mathrm{BDRAP}}.
\end{aligned}$$

Therefore, we obtain that $\text{OPT}_{\text{MCNFP}} + \sum_{i\in J} p_i = \text{OPT}_{\text{BDRAP}}$, which also implies that $\mathbf{x}$ constructed above from ζ^* is an optimal solution to the BDRAP, and that ζ constructed above from $\mathbf{x}^*$ is an optimal solution to the MCNFP.

Hence, to solve the BDRAP, it is equivalent to solving the MCNFP on the acyclic network $G = (V, A)$ constructed earlier, which is known to have a polynomial-time algorithm (see, e.g., Ahuja et al. 1993). This completes the proof of Lemma EC.3.

EC.2. Case Study: Gate Assignment in San Francisco (SFO) Airport

The benchmark **Arctangent Model** is defined as follows:

$$\text{(Arctangent Model)} \quad \min_{\mathbf{y}\in\mathcal{Y}} \sum_{i\in J}\sum_{j\in J} \mathbb{P}_0(a_j - e_i) y_{ij} \tag{EC.22}$$

where

$$\mathbb{P}_0(s) = \begin{cases} \arctan\Big(0.21(5-s)\Big) + \frac{\pi}{2}, & \text{if } s \geq \tau, \\ +\infty, & \text{otherwise.} \end{cases} \tag{EC.23}$$

τ is a threshold for minimum buffer space allowed, and is taken to be 20 minutes in this paper. $\mathbb{P}_0(s)$ is a monotone (decreasing) penalty function to"grade" a buffer space of s between successive services assigned to a common resource. This penalty function is pre-determined, and does not depend on the historical data sample $\{\mathbf{d}_t\}$. Arctangent model is used to solve DRO minimizing worst-case number of conflicts approximately. As shown in Appendix, it can be cast as a variant of BDRAP, and therefore, can be solved in polynomial time.

In this section, we apply our method to a real-world gate assignment problem. The data set comes from open source, from which we extract the SFO flight inbound and outbound data including daily flight timetable in 2008 (scheduled arrival/departure time, actual arrival/departure time), carrier code, origin-destination information. We randomly divided the 366-day flight dataset into two sets: one containing 306 days as the **In-sample** dataset, and the other with 60 days as the **Out-of-sample** dataset. Flights conducted by the same carrier on the same OD route with identical scheduled arrival and departure times were considered as a single scheduled event. Importantly, it should be noted that flight schedules can vary daily. Within the in-sample dataset, there were a total of 3,310 different scheduled events. To maintain consistency in performance evaluation, flights within these 3,310 scheduled events were reserved in the out-of-sample set, resulting in the elimination of only a small portion of flights. Figure EC.4 illustrates the fluctuating flight volume in both datasets, while Figure EC.5 displays the daily average delay duration in terms of arrival time. In our experiment, we only considered arrival delay uncertainty and assumed that service time was fixed (i.e., the service time of each scheduled event equaled the scheduled departure time

Figure EC.4 Daily Flight Volume

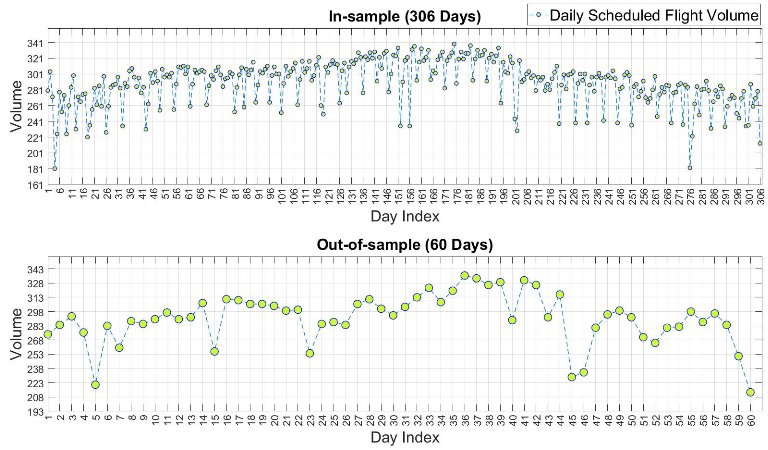
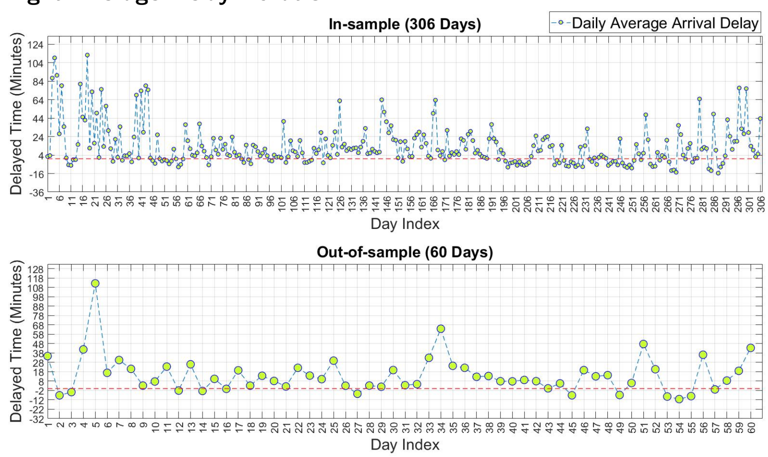


Figure EC.5 Daily Flight Average Delay Duration

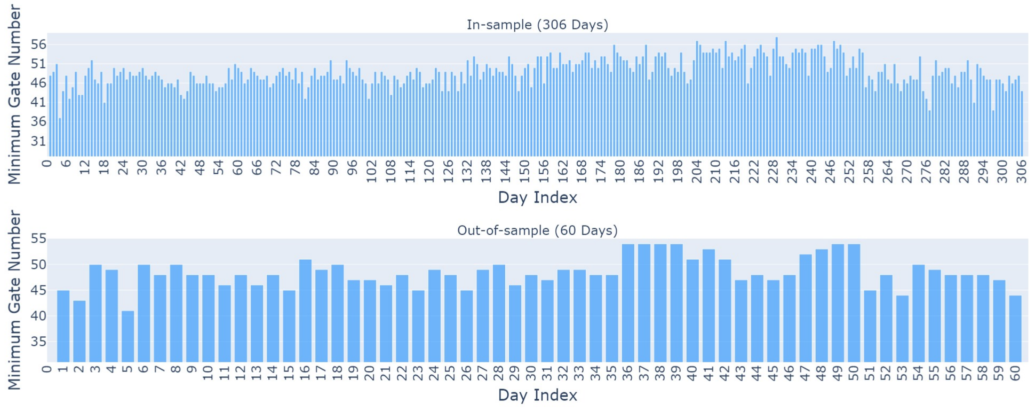


minus the scheduled arrival time, and consequently, the actual departure time was equal to the realized arrival time plus the fixed service time).

By solving a special case of BDRAP with all service time extended by 20 minutes, we could determine the minimum number of gates required daily by conducting binary search. This is used as the number of gates available each day for the experiment. It facilitates fair comparison between different methods, and captures the actual conflict without the disruption from un-gated flights. Figure EC.6 shows the daily minimum gate number we used to optimize gate assignment plan.

While we solved the model aiming to minimize the expected total number of crossings, we examine the solution's performance by using the actual conflicts realized over the out-of-sample test data. Under such a performance measurement, we observed that the solution $\mathbf{x}^*_{\text{RC}}$ of CROP based on the ambiguity set that uses only mean and variance information does not have good out-of-sample performance. This is mainly because such an ambiguity set cannot reflect the characteristics of the real-world data. In contrast, as we will show in detail below, the SAA model has significantly better

Figure EC.6 Daily Minimum Gate Number Requirement



out-of-sample performance, which also dominates the performance of the arctangent method. We also observed that the wSAA-kNN model outperforms all the other methods, as it is capable to synthesize the value of the training data as well as benefit the design of resilient gate assignments for the testing data. Both SAA model and wSAA-kNN model reduce the objective of (dynamic) conflicts minimization to (static) crossings minimization. To apply SAA model, we estimate the marginal crossing probability $\hat{\mathbb{P}}(d_i \geq a_j - e_i)$ for each scheduled flight using in-sample data set, and thus obtain the reference distribution. Then we directly minimize the total crossing probability under the reference distribution to obtain $\mathbf{x}^*_{\text{SAA}}$. wSAA-kNN model is adopted by exploiting the feature information in the training data set and re-characterize the empirical crossing probability $\hat{\mathbb{P}}(d_i \geq a_j - e_i)$ in the SAA model. In the experiment, we set the neighbor number to be 30 and the features we use include the month information, the origin of flight, the destination of flight, the carrier name, the tail number, the scheduled arrival time as well as the scheduled departure time.

Table EC.1 presents the actual crossing number statistics for the three models under consideration. Consistent with the analysis of the actual conflict number statistics, the wSAA-kNN model demonstrates superior performance compared to the other two models in both the in-sample and out-of-sample datasets. It is important to highlight that the primary goals of the wSAA-kNN and SAA models are to minimize the expected crossing number. This, in effect, reduces the worst-case conflict for the down-monotone set. Consequently, the consistent performance magnitude order among arctangent, SAA, and wSAA-kNN models is maintained in Tables 1 and EC.1. Figure EC.7 compares the histogram of daily conflict number differences between $N(\mathbf{x}^*_{\text{Arctangent}}, t) - N(\mathbf{x}^*_{\text{SAA}}, t)$ and $N(\mathbf{x}^*_{\text{Arctangent}}, t) - N(\mathbf{x}^*_{\text{wSAA-kNN}}, t)$, where $N(\mathbf{x}^*, t)$ denotes the conflict number on day t using gate assignment solution $\mathbf{x}^*$. We could observe that wSAA-kNN method exhibits better performance than SAA approach for in-sample data set. Figure EC.8 compares the histogram of daily conflict number differences between $N(\mathbf{x}^*_{\text{Arctangent}}, t) - N(\mathbf{x}^*_{\text{SAA}}, t)$ and $N(\mathbf{x}^*_{\text{Arctangent}}, t) - N(\mathbf{x}^*_{\text{wSAA-kNN}}, t)$. We could observe that in terms of conflicts, the wSAA-kNN method dominates

Table EC.1 Comparing the In-sample (306-day) and Out-of-sample (60-day) worst-case, standard deviation and average total number of crossings (max, std and avg of realized crossings).

Data set	In-sample (306 days)			Out-of-sample (60 days)			
Methodology	$\mathbf{x}^*_{\text{Arctangent}}$	$\mathbf{x}^*_{\text{SAA}}$	$\mathbf{x}^*_{\text{wSAA-kNN}}$	$\mathbf{x}^*_{\text{Arctangent}}$	$\mathbf{x}^*_{\text{SAA}}$	$\mathbf{x}^*_{\text{wSAA}-\text{kNN}}$	$\mathbf{x}^{*\text{Hindsight}}_{\text{SAA}}$
Mean	22.47	17.08	**15.66**	24.10	19.55	**18.45**	13.97
Std	24.18	18.91	**17.92**	22.60	18.35	**17.79**	14.84
Max	104	87	**84**	104	87	**84**	66

the arctangent method in almost all the out-of-sample data set, and enhances the resilience of gate assignment solution against the SAA approach with larger reduction in the number of conflicts. Finally, in terms of running time, the wSAA-kNN approach takes almost the same time as the deterministic model (i.e., SAA model and arctangent model).

Figure EC.7 Histogram of Conflict Number Differences using Different Models for In-sample Data Set

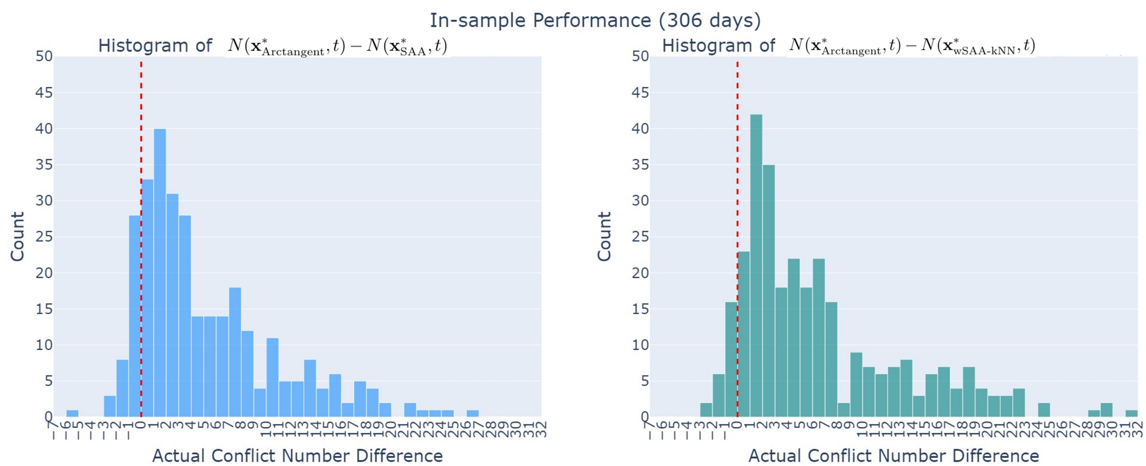


Figure EC.8 Histogram of Conflict Number Differences using Different Models for Out-of-sample Data Set