# Online Appendix to "Battery as a Service: Flexible Electric Vehicle Battery Leasing"

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*Proof of Lemma 1.* The total cost of leasing high-capacity batteries in two periods is

$$p_h + \rho(d - r_h)^+ + (1-\rho)\mathbb{E}[(D - r_h)^+] = p_h + \rho(d - r_h)^+ + (1-\rho)(a - r_h)^2/(2a),$$

and the total cost of leasing low-capacity batteries is $\rho d + (1-\rho)\mathbb{E}[D] = \rho d + a(1-\rho)/2$.
We compare the total costs of leasing high- and low-capacity batteries. It is trivial to find that all customers lease high- or low-capacity batteries when $d \geq r_h$. So we focus on $d < r_h$, where the total cost of leasing high-capacity batteries is $p_h + (1-\rho)(a - r_h)^2/(2a)$.

$$p_h + (1-\rho)(a-r_h)^2/(2a) > \rho d + a(1-\rho)/2 \Longleftrightarrow d < [2ap_h - (1-\rho)r_h(2a - r_h)]/[2a\rho].$$

So the low-capacity battery lease volume is $[2ap_h - (1-\rho)r_h(2a - r_h)]/[2a\rho]$. We have $[(1-\rho)r_h - (1-\rho)r_h^2/(2a)] \leq p_h \leq [(1-\rho)r_h - (1-\rho)r_h^2/(2a) + \rho]$; a lower primary lease rate would lead to the trivial case of all customers leasing high-capacity batteries, and a higher primary lease rate would lead to the trivial case of all customers leasing low-capacity batteries. □

*Proof of Proposition 1.* In the simple battery lease program, the manufacturer's problem is

$$\max_{p_h} \quad \{(p_h - c_h)(1-\tau): \quad \tau = [2ap_h - (1-\rho)r_h(2a - r_h)]/[2a\rho]\}.$$

which is concave in $p_h$ as the second-order derivative is negative. So the first-order condition yields the optimal price $p_h^b = [(1-\rho)(2ar_h - r_h^2) + 2a(c_h + \rho)]/(4a)$, and the maximized profit is $\Pi^b = [(1-\rho)(2ar_h - r_h^2) - 2a(c_h - \rho)]^2/(16\rho a^2)$.

$$\partial p_h^b/\partial a = (1-\rho)r_h^2/(4a^2) > 0,\ \partial p_h^b/\partial r_h = (1-\rho)(a - r_h)/(2a) > 0,$$
$$\partial p_h^b/\partial c_h = 1/2,\ \partial p_h^b/\partial \rho = (2a - 2ar_h + r_h^2)/(4a).$$

So $p_h^b$ increases in $a,\ r_h,\ c_h$, and increases in $\rho$ if $2a - 2ar_h + r_h^2 > 0$ but decreases otherwise.

Accordingly, the low-capacity battery lease volume is $\tau^b = [2a(c_h+\rho)-(1-\rho)(2ar_h - r_h^2)]/(4a\rho)$. We have $(1-\rho)(2ar_h-r_h^2)/(2a)-\rho < c_h < (1-\rho)(2ar_h-r_h^2)/(2a)+\rho$, otherwise all customers lease low- or high-capacity batteries.

$$\partial\tau^b/\partial a = -(1-\rho)r_h^2/(4\rho a^2) < 0,\ \partial\tau^b/\partial r_h = -(1-\rho)(a-r_h)/(2a\rho) < 0,$$
$$\partial\tau^b/\partial c_h = 1/2(\rho) > 0,\ \partial\tau^b/\partial\rho = -(2ac_h - 2ar_h + r_h^2)/(4a\rho^2).$$

So $\tau^b$ decreases in $a$, $r_h$, increases in $c_h$, and increases in $\rho$ if $2ac_h - 2ar_h + r_h^2 < 0$ but decreases otherwise. Recall that we assume $r_l = 0$. So the total battery capacity is

$$BT^b = (1-\tau^b)r_h = [2a(\rho - c_h) + (1-\rho)(2ar_h - r_h^2)]r_h/(4a\rho).$$

We then compute total customer cost. A customer's total cost of leasing low-capacity batteries is

$$CT_l^b = \rho\mathbb{E}[d] + (1-\rho)\mathbb{E}[D] = \rho\tau^b/2 + a(1-\rho)/2.$$

A customer's total cost of leasing high-capacity batteries is

$$\begin{aligned} CT_h^b &= \rho(\mathbb{E}[(d-r_h)^+] + p_h^b) + (1-\rho)(\mathbb{E}[(D-r_h)^+] + p_h^b) \\ &= \rho(1-r_h)^2/(2(1-\tau^b)) + (1-\rho)(a-r_h)^2/(2a) + p_h^b \end{aligned}$$

when $r_h \le 1$, and $CT_h^b = (1-\rho)(a-r_h)^2/(2a) + p_h^b$ when $r_h > 1$.

Then, the total customer cost is $CT^b = \tau^b CT_l^b + (1-\tau^b)CT_h^b$, which is $[16\rho(1-\rho)a^3 - r_h^4(1-\rho)^2 + 4ar_h^2(1-\rho)(r_h - c_h + \rho(1-r_h)) + 4a^2((7-3(2-r_h)r_h)\rho^2 - (c_h-r_h)^2 + 2\rho(c_h-r_h)(1-r_h))]/(32\rho a^2)$ for $r_h \le 1$, and $[16\rho(1-\rho)a^3 - (r_h^4(1-\rho)^2 + 4a^2(c_h - r_h(1-\rho)+\rho)(r_h - c_h + \rho(3-r_h)) + 4ar_h^2(1-\rho)(r_h - c_h + (1-r_h)\rho)]/(32\rho a^2))$ for $r_h > 1$. □

*Proof of Lemma 2.* We first solve for the peak-period upgrade volume. Recall customers' peak-period costs in Table 1. A customer leasing low-capacity batteries decides to upgrade if and only if $(D-r_h)^+ + \bar{p}_h \le D$.

- $D \ge r_h$. The peak-period cost is $D - r_h + \bar{p}_h$ for high-capacity batteries, and $D$ for low-capacity batteries. A customer chooses high-capacity batteries if $D - r_h + \bar{p}_h \le D \Longleftrightarrow \bar{p}_h \le r_h$, and low-capacity batteries otherwise.
- $D < r_h$. The peak-period cost is $\bar{p}_h$ for high-capacity batteries, and $D$ for low-capacity batteries. When $\bar{p}_h > r_h$, all customers choose low-capacity batteries as $D < r_h < \bar{p}_h$. When $\bar{p}_h \le r_h$, a customer chooses low-capacity batteries if $D < \bar{p}_h$ and high-capacity batteries otherwise.

To summarize, no upgrade happens when $\bar{p}_h > a$. Otherwise, a customer leasing low-capacity batteries upgrades if $D \geq \bar{p}_h$, i.e., $\mathbb{P}(D \geq \bar{p}_h) = (a - \bar{p}_h)/a$. Given the regular-period low-capacity battery lease volume $\tau$, the peak-period upgrade volume is $\tau(a - \bar{p}_h)/a$.

Similarly, we can solve for the peak-period downgrade volume. From Table 1, a customer leasing high-capacity batteries downgrades if and only if $(D - r_h)^+ + p_h > D + \bar{p}_l$.

- $D \geq r_h$. A customer chooses low-capacity batteries if $D - r_h + p_h > D + \bar{p}_l \Longleftrightarrow \bar{p}_l < p_h - r_h$, and high-capacity batteries otherwise.
- $D < r_h$. A customer chooses low-capacity batteries if $p_h > D + \bar{p}_l \Longleftrightarrow D < p_h - \bar{p}_l$.

To summarize, no downgrade happens if $\bar{p}_l > p_h$. If $\bar{p}_l < p_h - a$, all customers downgrade. If $p_h - a \leq \bar{p}_l \leq p_h$, a customer leasing high-capacity batteries downgrades if $D < p_h - \bar{p}_l$, i.e., is $\mathbb{P}(D < p_h - \bar{p}_l) = (p_h - \bar{p}_l)/a$. Given the regular-period high-capacity battery lease volume $1 - \tau$, the peak-period downgrade volume is $(1 - \tau)(p_h - \bar{p}_l)/a$. $\square$

*Proof of Lemma 3.* The peak up/downgrade volumes imply an additional acquisition of high-capacity batteries $Q_h = \tau(a - \bar{p}_h)/a - (1 - \tau)(p_h - \bar{p}_l)/a$ to meet the peak demand. We insert this equality to the manufacturer's peak-period problem (1), and obtain $\bar{p}_h^f(p_h) = [a + c_h/(1 - \rho)]/2$, $\bar{p}_l^f(p_h) = p_h - c_h/(2 - 2\rho)$. Accordingly, $Q_h^f(p_h) = [\tau - c_h/(a(1 - \rho))]/2$ and the peak-period profit is $[c_h^2/(a(1 - \rho)^2) + a\tau - 2\tau c_h/(1 - \rho) + 4(1 - \tau)p_h]/4$. $\square$

*Proof of Lemma 4.* We first derive the expected peak-period costs. A customer leasing low-capacity batteries upgrades if $D \geq \bar{p}_h$ in the peak period and keeps low-capacity batteries otherwise. So we obtain the expected peak-period cost by breaking up the integrals into two parts, which differs on using high- or low-capacity batteries in peak period.

$$\begin{aligned}\mathbb{E}[c_l(D)] &= \frac{\bar{p}_h}{a}\int_0^{\bar{p}_h} Df(D)dD + \frac{a - \bar{p}_h}{a}\int_{\bar{p}_h}^{a}((D - r_h)^+ + \bar{p}_h)f(D)dD \\ &= \frac{a^2 - 2ar_h + r_h^2 + (2a - \bar{p}_h)\bar{p}_h}{2a}.\end{aligned}$$

Similarly, a customer leasing high-capacity batteries downgrades if $D < p_h - \bar{p}_l$ in the peak period and keeps high-capacity batteries otherwise. The expected peak-period cost is

$$\begin{aligned}\mathbb{E}[c_h(D)] &= \frac{p_h - \bar{p}_l}{a}\int_0^{p_h - \bar{p}_l}(D + \bar{p}_l)f(D)dD + \frac{a - p_h + \bar{p}_l}{a}\int_{p_h - \bar{p}_l}^{a}((D - r_h)^+ + p_h)f(D)dD \\ &= \frac{a^2 - 2ar_h + r_h^2 - (p_h - \bar{p}_l)^2 + 2ap_h}{2a}.\end{aligned}$$

If a customer leases high- or low-capacity batteries in the regular period, the total cost in two periods is $\rho[p_h + (d - r_h)^+] + (1 - \rho)\mathbb{E}[c_h(D)]$ or $\rho d + (1 - \rho)\mathbb{E}[c_l(D)]$. The indifferent cost must happen when $d < r_h$, otherwise all customers lease high- or low-capacity batteries. A customer leases low-capacity batteries at the beginning if $\rho d < \rho p_h + (1-\rho)(\mathbb{E}[c_h(D)] - \mathbb{E}[c_l(D)])$ and high-capacity batteries otherwise. The low-capacity battery lease volume is

$$\tau = [2ap_h - (1-\rho)((p_h - \bar{p}_l)^2 + 2a\bar{p}_h - \bar{p}_h^2)]/[2a\rho],$$

and the high-capacity volume is

$$1-\tau = [2a(\rho - p_h) + (1-\rho)((p_h - \bar{p}_l)^2 + 2a\bar{p}_h - \bar{p}_h^2)]/[2a\rho].$$

Inserting $\bar{p}_h^f(p_h)$ and $\bar{p}_l^f(p_h)$, the low-capacity lease volume is $\tau^f(p_h) = [8p_h - 2c_h - 3a(1-\rho)]/(8\rho)$, and the high-capacity lease volume is $1-\tau^f(p_h) = [8\rho + 3a(1-\rho) + 2c_h - 8p_h]/(8\rho)$. □

*Proof of Proposition 2.* (i) With $\tau^f(p_h)$ in Lemma 4, we solve the manufacturer's regular-period problem (2). The second-order derivative of the regular-period problem wrt $p_h$ is $-2/\rho$, so the F.O.C yields the optimal $p_h^f = [6c_h + 5a(1-\rho) + 8\rho]/16$. The optimal up/downgrade rates and high-capacity battery acquisition quantity are

$$\bar{p}_h^f = [a + c_h/(1-\rho)]/2,\ \bar{p}_l^f = [c_h(6 - 8/(1-\rho)) + 5a(1-\rho) + 8\rho]/16,$$
$$Q_h^f = [a(1-\rho)(2c_h + \rho(8+a) - a) - 16\rho c_h]/[32a\rho(1-\rho)],$$

the low-capacity lease volume is $\tau^f = (2c_h + \rho(8+a) - a)/(16\rho)$, and the maximized profit is

$$\Pi^f = [a\rho(1-\rho)(2(40-a)(a-2c_h) + 64\rho - (80-a)a\rho) + a(1-\rho)(a-2c_h)^2 + 64\rho c_h^2]/[256a\rho(1-\rho)].$$

Here, $\bar{p}_h^f$,$\tau^f$ increases in $c_h$ and $Q_h^f$ decreases in $c_h$. Moreover, the upgrade volume $\tau^f(a-\bar{p}_h^f)/a$ and the proportion of additional battery acquisition over peak-period upgrade volume $Q_h^f/[\tau^f(a-\bar{p}_h^f)/a]$ decrease in $c_h$. We can check the conditions for $\bar{p}_h^f \leq a$ and $\bar{p}_l^f \geq p_h^f - a$. We then check the nonnegativity of high-capacity battery acquisition quantity.

$$Q_h^f > 0 \Longleftrightarrow c_h < a(1-\rho)/2.$$

The cost threshold increases in $a$ and decreases in $\rho$.

(ii) Next we optimize the manufacturer's problems with $Q_h = 0$ for $c_h \geq a(1-\rho)/2$.

From the supply and demand equation in the peak period, we have $\bar{p}_h^f(\bar{p}_l) = a - (1-\tau)(p_h - \bar{p}_l)/\tau$. Substituting $\bar{p}_h^f(\bar{p}_l)$ into the peak-period problem, the second-order derivative is negative, so the F.O.C yields the optimal $\bar{p}_l^f(p_h) = p_h - a\tau/2,\ \bar{p}_h^f(p_h) = a(1+\tau)/2$, and the corresponding profit is $(1-\tau)(a\tau + 4p_h)/4$.

Inserting $\bar{p}_h^f(p_h)$, $\bar{p}_l^f(p_h)$ into Lemma 4, the low-capacity battery lease volume becomes $\tau^f(p_h) = [8p_h - 3a(1-\rho)]/[2a(1-\rho)+8\rho]$. Then we solve the regular-period problem and obtain

$$p_h^f = \frac{13a^2(1-\rho)^2 + 32\rho(c_h+\rho) + 4a(1-\rho)(2c_h+9\rho)}{32a(1-\rho)+64\rho}.$$

The optimal up/downgrade rates are

$$\bar{p}_h^f = \frac{a[9a(1-\rho)+8(c_h+3\rho)]}{16a(1-\rho)+32\rho},\ \bar{p}_l^f = \frac{32\rho(c_h+\rho)+4a\rho(5-9\rho)-8ac_h(1+\rho)+a^2(1-\rho)(11-13\rho)}{32a(1-\rho)+64\rho},$$

the low-capacity battery lease volume is $\tau^f = [a - a\rho + 8(c_h+\rho)]/[8(a+2\rho-a\rho)]$, and the maximized profit is $\Pi^f = [8(c_h-\rho)+7a(1-\rho)]^2/[128(a(1-\rho)+2\rho)]$.

Here, $\bar{p}_h^f$, $\tau^f$ increases in $c_h$, and the upgrade volume $\tau^f(a-\bar{p}_h^f)/a$ decreases in $c_h$. We can similarly check the conditions for $\bar{p}_h^f \le a$ and $\bar{p}_l^f \ge p_h^f - a$.

(iii) The total battery capacity under flexible battery leasing is $BT^f = (1-\tau^f+Q_h^f)r_h$.

We then compute total customer cost. The total cost of leasing low-capacity batteries in two periods under flexible program is

$$CT_l^f = \rho\mathbb{E}[d] + (1-\rho)\mathbb{E}[c_l(D)] = \rho\tau^f/2 + (1-\rho)\frac{a^2-2ar_h+r_h^2+(2a-\bar{p}_h^f)\bar{p}_h^f}{2a},$$

The total cost of leasing high-capacity batteries in two periods under flexible program is

$$\begin{aligned} CT_h^f &= \rho\mathbb{E}[(d-r_h)^+ + p_h^f] + (1-\rho)\mathbb{E}[c_h(D)] \\ &= \rho((1-r_h)^2/(2(1-\tau^f)) + p_h^f) + (1-\rho)\frac{(a-r_h)^2+2ap_h^f-(p_h^f-\bar{p}_l^f)^2}{2a} \end{aligned}$$

when $r_h \le 1$, and $CT_h^f = p_h^f + (1-\rho)((a-r_h)^2 - (p_h^f-\bar{p}_l^f)^2)/(2a)$ when $r_h > 1$.

The total customer cost is $CT^f = \tau^f CT_l^f + (1-\tau^f)CT_h^f$. □

*Proof of Corollary 1.* We check the sign of $\bar{p}_l^f$ under two cases.

$c_h < a(1-\rho)/2$. We find that $\bar{p}_l^f \ge 0$ for $a \le 8\rho/(1+3\rho)$. When $a > 8\rho/(1+3\rho)$, $\bar{p}_l^f < 0 \Longleftrightarrow c_h > (1-\rho)(5a(1-\rho)+8\rho)/(2+6\rho)$. We can check that $(1-\rho)(5a(1-\rho)+8\rho)/(2+6\rho) < a(1-\rho)/2$.

$c_h \ge a(1-\rho)/2$. We find that $\bar{p}_l^f < 0 \Longleftrightarrow a > 4\rho/(1+\rho)$ and $c_h > [32\rho^2 + 4a\rho(5-9\rho) + a^2(11-24\rho+13\rho^2)]/[8(a-4\rho+a\rho)]$. We can check that $[32\rho^2+4a\rho(5-9\rho)+a^2(11-24\rho+13\rho^2)]/[8(a-4\rho+a\rho)] > a(1-\rho)/2$. □

*Proof of Proposition 3.* We compare manufacturer profit under simple and flexible battery leasing. Recall the profits in the proof of Propositions 1 and 2.

$c_h < a(1-\rho)/2$.

$$\Pi^f > \Pi^b \Longleftrightarrow g_1(r_h, c_h, a, \rho) = \rho a^2[2(40-a)(a-2c_h) + (a-2c_h)^2/\rho + \rho(64-(80-a)a)$$

$$+64c_h^2/(a-a\rho)]-16[r_h^2(1-\rho)+2(c_h-(1-\rho)r_h-\rho)a]^2>0.$$

We find that $g_1(r_h,c_h,a,\rho)$ is concave in $c_h$ for $\rho<15a/(15a+16)$, and $\partial g_1(r_h,c_h,a,\rho)/\partial c_h<0$ at $c_h=a(1-\rho)/2$ for $r_h>a/2$. So $g_1(r_h,c_h,a,\rho)$ is increasing in $c_h\in[0,a(1-\rho)/2]$. We can also check that $g_1(r_h,c_h=a(1-\rho)/2,a,\rho)>0$ for $r_h>a/2$. There exists $c_h=(1-\rho)[a(1-\rho)(32ar_h-a^2-16r_h^2)-8\rho a^2-4K]/[2a(15a(1-\rho)-16\rho)]$ such that $g_1(r_h,c_h=c_h^k,a,\rho)=0$, where $K=[a(a^5(1-\rho)^2+16\rho(1-\rho)r_h^4-a^4(1-\rho)(4r_h(1-\rho)-75\rho)+ar_h^2(r_h^2(1-\rho)^2-64\rho(1-\rho)r_h-64\rho^2)+a^3(3r_h^2(1-\rho)^2-76\rho(1-\rho)r_h-38\rho^2)-4a^2r_h(r_h^2(1-\rho)^2+35\rho(1-\rho)r_h-32\rho^2))]^{1/2}$. Therefore, $\Pi^f>\Pi^b$ holds for $(1-\rho)[a(1-\rho)(32ar_h-a^2-16r_h^2)-8\rho a^2-4K]^+/[2a(15a(1-\rho)-16\rho)]<c_h<a(1-\rho)/2$ for $\rho<15a/(15a+16)$ and $r_h>a/2$.

$\underline{c_h\geq a(1-\rho)/2}$.

$$\Pi^f>\Pi^b\Longleftrightarrow K^a-K^n<c_h<K^a+K^n,$$

where $K^a=[4ar_h(2a-r_h)-6\rho a^2+16\rho ar_h-8\rho r_h(a^2-r_h)+4\rho ar_h^2]/(8a^2)$ and $K^n=(7a^2-8ar_h+4r_h^2)\sqrt{2\rho(a(1-\rho)-2\rho)}/(8a^2)$. We can check that $K^a-K^n<a(1-\rho)/2$ and $K^a+K^n>a(1-\rho)/2$. Moreover, we have $K^a+K^n>a(1-\rho)$ for $r_h>a/2$. So $\Pi^f>\Pi^b$ always hold over $c_h\in[a(1-\rho)/2,a(1-\rho)]$ for $r_h>a/2$. □

*Proof of Proposition 4.* We compare the total customer cost under simple and flexible battery leasing. Note that $CT^b-CT^f$ for $r_h\leq 1$ and $r_h>1$ are the same, so it is enough to write out the comparing expression for $r_h>1$.

$\underline{c_h<a(1-\rho)/2}$. The total customer cost under flexible battery leasing for $r_h>1$ is

$$CT^f=[4a^2(1-\rho)^2(c_h+108\rho)-a^3(1-\rho)^3-64\rho(c_h^2-4r_h^2(1-\rho)^2)$$
$$-4a(1-\rho)(c_h^2-40\rho c_h-16\rho(3\rho-8(1-\rho)r_h))]/[512\rho(1-\rho)a].$$

Now compare it with the total customer cost under simple battery leasing.

$$CT^b>CT^f\Longleftrightarrow l_1(r_h,c_h,a,\rho)=(1-\rho)^3(16r_h^4-a^4)+4a^3(1-\rho)^2(c_h+44\rho)+64a(c_hr_h^2(1-\rho)^2-\rho c_h^2$$
$$+r_h^2(1-\rho)^2((3+r_h)\rho-r_h))+4a^2(1-\rho)(15c_h^2+8c_h(\rho-4(1-\rho)r_h)+16r_h(1-\rho)(r_h-(6+r_h)\rho))<0$$

When $r_h\to a$ and $c_h\to a(1-\rho)/2$, we have

$$l_1(r_h\to a,c_h\to a(1-\rho)/2,a,\rho)=-16\rho(1-\rho)^2a^3<0.$$

Because the function is continuous, $l_1(r_h,c_h,a,\rho)<0$ holds when $r_h$ and $c_h$ are sufficiently large. Note that $a$ is compared with $r_h$ and $a>r_h$, so $l_1(r_h,c_h,a,\rho)<0$ when $a$ is sufficiently low.

Similarly, we show that $l_1(r_h, c_h, a, \rho) > 0$ for sufficiently large $\rho$ as $l_1(r_h, c_h, a, \rho \to 1) = -64ac_h^2 < 0$.

$\underline{c_h \geq a(1-\rho)/2}$. The total customer cost under flexible battery leasing for $r_h > 1$ is

$$\begin{aligned} CT^f = &[463a^4(1-\rho)^3 + 1024(1-\rho)\rho^2 r_h^2 + 4a^3(1-\rho)^2(28c_h - 128(1-\rho)r_h + 499\rho) \\ &- 256a\rho(c_h - 2r_h(1-\rho)+\rho)(c_h + 2r_h - 3\rho - 2\rho r_h) - 64a^2(1-\rho)(c_h^2 - 4r_h^2 \\ &- 9\rho c_h + 8\rho r_h(4+r_h) - 4(10 + r_h(8+r_h))\rho^2)]/[512a(a(1-\rho)+2\rho)^2]. \end{aligned}$$

Now we compare it with the total customer cost under simple battery leasing.

$$\begin{aligned} CT^b > CT^f \Longleftrightarrow &\, l_2(r_h, c_h, a, \rho) = 16[(1-\rho)^2 r_h^4 - 16\rho(1-\rho)a^3 + 4a^2(c_h - (1-\rho)r_h \\ &+ \rho)(c_h - r_h - (3-r_h)\rho) + 4ar_h^2(1-\rho)(c_h - (1-\rho)r_h - \rho)]/\rho + a[463a^4(1-\rho)^3 + 1024(1-\rho)\rho^2 r_h^2 \\ &+ 4a^3(1-\rho)^2(28c_h - 128r_h(1-\rho) + 499\rho) - 256\rho a(c_h - 2r_h(1-\rho)+\rho)(c_h + 2r_h - 3\rho - 2\rho r_h) \\ &- 64a^2(1-\rho)(c_h^2 - 4r_h^2 - 9\rho c_h + 8\rho r_h(4+r_h) - 4(10+r_h(8+r_h))\rho^2)]/[(1-\rho)a+2\rho]^2 < 0 \end{aligned}$$

When $r_h \to a$ and $c_h \to a(1-\rho)/2$, we have

$$l_2(r_h \to a, c_h \to a(1-\rho)/2, a, \rho) = -\rho(1-\rho)^2(9(1-\rho)a + 20\rho)a^4 < 0.$$

Because the function is continuous, $l_2(r_h, c_h, a, \rho) < 0$ holds when $r_h$ is sufficiently large and $c_h$ is sufficiently low. And $l_2(r_h, c_h, a, \rho) < 0$ holds when $a$ is sufficiently low. $\square$

*Proof of Proposition 5.* We compare the total battery capacity under simple and flexible battery leasing.

$\underline{c_h < a(1-\rho)/2}$.

$$BT^f = (1-\tau^f)r_h + Q_h^f r_h \leq BT^b = (1-\tau^b)r_h \Longleftrightarrow \tau^f - Q_h^f \geq \tau^b,$$

which holds if and only if $\rho < 7a/(8+7a)$ and $c_h \leq (1-\rho)[(16ar_h - a^2 - 8r_h^2)(1-\rho) - 8a\rho]/[14a(1-\rho) - 16\rho)]$. We can show that $(1-\rho)[(16ar_h - a^2 - 8r_h^2)(1-\rho) - 8a\rho]/[14a(1-\rho) - 16\rho] < a(1-\rho)/2$ when $\rho < 7a/(8+7a)$.

$\underline{c_h \geq a(1-\rho)/2}$.

$$\begin{aligned} BT^f = (1-\tau^f)r_h \leq BT^b = (1-\tau^b)r_h \Longleftrightarrow \tau^f \geq \tau^b \\ \Longleftrightarrow c_h \leq [2ar_h(4\rho - r_h(1-\rho)) - 4\rho r_h^2 - a^2(3\rho - 4(1-\rho)r_h)]/(4a^2), \end{aligned}$$

which is $[2a(1-\rho)+\rho]/4$ as $r_h \to a$. So $BT^f \leq BT^b$ for $r_h \to a$ and $c_h \leq [2a(1-\rho)+\rho]/4$. $\square$

*Proof of Proposition 6.* We search for a set of sufficient conditions for a win-win-win outcome based on the results in Lemma 4 and 5.

$\underline{c_h < a(1-\rho)/2}$. From the proof of Lemma 4, the total customer cost is lower for flexible battery leasing when $r_h \to a$ and $c_h \to a(1-\rho)/2$. Now we compare total battery capacities under these conditions. When $r_h \to a$, we find

$$c_h \leq (1-\rho)[(16ar_h - a^2 - 8r_h^2)(1-\rho) - 8a\rho]/[14a(1-\rho) - 16\rho)] = a(1-\rho)/2,$$

which holds as $c_h \to a(1-\rho)/2$, and thus the total battery capacity is smaller for flexible than simple battery leasing as $r_h \to a$, $c_h \to a(1-\rho)/2$, and $\rho < 7a/(8+7a)$.

$\underline{c_h \geq a(1-\rho)/2}$. From the proof of Lemma 4, the total customer cost is lower for flexible battery leasing when $r_h \to a$ and $c_h \to a(1-\rho)/2$. Now we compare total battery capacities under these conditions. From the proof of Proposition 5, the total battery capacity is smaller for flexible battery leasing as $r_h \to a$, $c_h \to [2a(1-\rho)+\rho]/4$. Because the functions are continuous, the win-win-win outcomes occur when $r_h \to a$ and $c_h \to a(1-\rho)/2$.

Combining the two cases of $c_h < a(1-\rho)/2$ and $c_h \geq a(1-\rho)/2$, the win-win-win outcomes occur when $c_h$ is moderate, $r_h$ is sufficiently large, and $\rho < 7a/(8+7a)$. □

*Proof of Proposition 7.* With correlated regular and peak needs for range, a customer's peak need for range is $corD \doteq \lambda d + (1-\lambda)D$ for a realized $d$. We first solve for the peak-period upgrade volume. Recall customers' peak-period costs in Table 1. A customer leasing low-capacity batteries decides to upgrade if $\lambda d + (1-\lambda)D \geq \bar{p}_h$, i.e., $\mathbb{P}(D \geq (\bar{p}_h - \lambda d)/(1-\lambda)) = (a(1-\lambda) - \bar{p}_h + \lambda d)/(a(1-\lambda))$. Given the regular-period low-capacity battery lease volume $\tau$, the peak-period upgrade volume is $\tau\mathbb{E}_d[(a(1-\lambda) - \bar{p}_h + \lambda d)/(a(1-\lambda))] = \tau(2(1-\lambda)a - 2\bar{p}_h + \lambda)/(2(1-\lambda)a)$.

Similarly, we can solve for the peak-period downgrade volume. A customer leasing high-capacity batteries downgrades if $\lambda d + (1-\lambda)D < p_h - \bar{p}_l$, i.e., $\mathbb{P}(D < (p_h - \bar{p}_l - \lambda d)/(1-\lambda)) = (p_h - \bar{p}_l - \lambda d)/(a(1-\lambda))$. Given the regular-period high-capacity battery lease volume $1-\tau$, the peak-period downgrade volume is $(1-\tau)\mathbb{E}_d[(p_h - \bar{p}_l - \lambda d)/(a(1-\lambda))] = (1-\tau)(2p_h - 2\bar{p}_l - \lambda)/(2(1-\lambda)a)$.

(i) We next solve the peak-period problem (1) with additional battery acquisition, and obtain $\bar{p}_h^c(p_h) = [2c_h + 2(1-\rho)(1-\lambda)a + (1-\rho)\lambda]/(4-4\rho)$, $\bar{p}_l^c(p_h) = p_h - c_h/(2-2\rho) - \lambda/4$. Accordingly, $Q_h^c(p_h) = [(1-\rho)(2(1-\lambda)a\tau - \lambda) - 2c_h]/(4(1-\rho)(1-\lambda)a)$ and the peak-period profit is $[4c_h^2 - 4c_h(1-\rho)(2(1-\lambda)a\tau + \lambda) + (1-\rho)^2(4(1-\lambda)^2a^2\tau + \lambda^2 + 4a(1-\lambda)(4(1-\tau)p_h + \lambda\tau))]/[16a(1-\rho)^2(1-\lambda)]$.

Then we study the customers' regular decisions. We obtain the expected peak-period cost for customers leasing low-capacity batteries by breaking up the integrals into two parts, which differs on using high- or low-capacity batteries.

$$\mathbb{E}[c_l(corD)] = \frac{\bar{p}_h - \lambda d}{(1-\lambda)a}\int_{d\lambda}^{\bar{p}_h} corDf(corD)\ d\ corD$$
$$+\frac{(1-\lambda)a-\bar{p}_h+\lambda d}{(1-\lambda)a}\int_{\bar{p}_h}^{(1-\lambda)a+d\lambda}((corD-r_h)^+ + \bar{p}_h)f(corD)\ d\ corD$$
$$= \frac{(2\bar{p}_h-\lambda)(\bar{p}_h+d\lambda)+2(2(1-\lambda)a-2\bar{p}_h+\lambda)\bar{p}_h}{4(1-\lambda)a} \quad \text{for} \quad r_h \geq (1-\lambda)a+\lambda.$$

The cost of leasing low-capacity batteries in two periods is $\rho d + (1-\rho)\mathbb{E}[c_l(corD)]$.

Similarly, a customer leasing high-capacity batteries downgrades if $corD < p_h - \bar{p}_l$ in the peak period and keeps high-capacity batteries otherwise. The expected peak-period cost is

$$\mathbb{E}[c_h(corD)] = \frac{2p_h - 2\bar{p}_l - \lambda}{2(1-\lambda)a}\int_{d\lambda}^{p_h-\bar{p}_l}(corD+\bar{p}_l)f(corD)\ d\ corD$$
$$+\frac{2(1-\lambda)a-2p_h+2\bar{p}_l+\lambda}{2(1-\lambda)a}\int_{p_h-\bar{p}_l}^{(1-\lambda)a+d\lambda}((corD-r_h)^+ + p_h)f(corD)\ d\ corD$$
$$= \frac{4(1-\lambda)ap_h-(2p_h-2\bar{p}_l-\lambda)(p_h-\bar{p}_l-d\lambda)}{4(1-\lambda)a} \quad \text{for} \quad r_h \geq (1-\lambda)a+\lambda.$$

The cost of leasing high-capacity batteries in two periods is $\rho[(d-r_h)^+ + p_h] + (1-\rho)\mathbb{E}[c_h(D)]$. By comparing the costs of leasing high- and low-capacity batteries in two periods, a customer leases low-capacity batteries if $\rho d + (1-\rho)\mathbb{E}[c_l(corD)] \leq \rho[(d-r_h)^+ + p_h] + (1-\rho)\mathbb{E}[c_h(D)]$. So the low-capacity lease volume with correlated needs for range is

$$\tau = \frac{4a(1-\lambda)(p_h-\bar{p}_h(1-\rho))-(1-\rho)(p_h-\bar{p}_h-\bar{p}_l)(2(p_h+\bar{p}_h-\bar{p}_l)-\lambda)}{4\rho(1-\lambda)a-2\lambda(1-\rho)(p_h-\bar{p}_h-\bar{p}_l)}.$$

Inserting $\bar{p}_h^c(p_h)$, $\bar{p}_l^c(p_h)$, we have $\tau^c(p_h) = (2(4p_h-(1-\rho)\lambda-c_h)-3(1-\rho)(1-\lambda)a)/(8\rho+2(1-\rho)\lambda)$. Then we solve the manufacturer's regular-period problem and obtain $p_h^c = [6c_h+8\rho+5a(1-\rho)(1-\lambda)+6(1-\rho)\lambda]/16$. The optimal up/downgrade rates and additional battery quantity are

$$\bar{p}_h^c = [2c_h+(1+\rho)\lambda+2a(1-\rho)(1-\lambda)]/[4(1-\rho)],$$
$$\bar{p}_l^c = [c_h(6-8/(1-\rho))+5a(1-\rho)(1-\lambda)+8\rho+2(1-3\rho)\lambda]/16,$$
$$Q_h^c = [(1-\rho)(a(1-\lambda)(2+a-(a-2c_h+3a\rho)/(\rho(4-\lambda)+\lambda))+2\lambda)/2-2c_h]/[4a(1-\rho)(1-\lambda)],$$

the low-capacity lease volume is $\tau^c = (2+a)/4 + (2c_h-(1+3\rho)a)/(16\rho+4(1-\rho)\lambda)$, and the maximized profit is

$$\Pi^c = [2(10-a)(a-2c_h)+2(8-a(10+a))\rho]/64 + [(4+a((20-a)a)-20)(1-\rho)\lambda]/(64a)$$
$$+(a-2c_h+3a\rho)^2/(64\rho(4-\lambda)+\lambda)+(4(-1+2ch+rho)^2)/(64a(1-\rho)(1-\lambda)).$$

We now check $\bar{p}_h \le (1-\lambda)a$ and $p_h - \bar{p}_l \ge \lambda$ under $\lambda \le a/(a+3)$ at the optimal solutions.

$$\bar{p}_h^c \le (1-\lambda)a) \Longleftrightarrow c_h \le (1-\rho)(2(1-\lambda)a-\lambda)/2,$$

$$\bar{p}_l^c \le p_h^c - \lambda \Longleftrightarrow c_h \ge 3(1-\rho)\lambda/2.$$

The optimal solution satisfies the conditions under $3(1-\rho)\lambda/2 \le c_h \le (1-\rho)(2(1-\lambda))a-\lambda)/2$. We then check the nonnegativity of high-capacity battery acquisition quantity.

$$Q_h^c > 0 \Longleftrightarrow c_h < \frac{(1-\rho)[2(1-\lambda)(\rho(4-\lambda)+\lambda)a-(1-\rho)(1-\lambda)^2a^2+2\lambda(\rho(4-\lambda)+\lambda)]}{4(4-\lambda)\rho-2a(1-\rho)(1-\lambda)+4\lambda}.$$

The cost threshold increases in $a$ and decreases in $\rho$.

(ii) Next we optimize the manufacturer's problems with $Q_h = 0$ for $c_h \ge (1-\rho)[2(1-\lambda)(\rho(4-\lambda)+\lambda)a-(1-\rho)(1-\lambda)^2a^2+2\lambda(\rho(4-\lambda)+\lambda)]/[4(4-\lambda)\rho-2a(1-\rho)(1-\lambda)+4\lambda]$.

With the supply and demand equation in the peak period, we solve the manufacturer's peak-period problem, and obtain $\bar{p}_l^c(p_h) = p_h - (a\tau(1-\lambda)+\lambda)/2$, $\bar{p}_h^c(p_h) = (a(1+\tau)(1-\lambda)+\lambda)/2$, and the corresponding profit is $(1-\tau)(4p_h + a\tau(1-\lambda))/4$.

Inserting $\bar{p}_h^c(p_h)$, $\bar{p}_l^c(p_h)$, the low-capacity lease volume becomes $\tau^c(p_h) = [8p_h - 3(1-\rho)(\lambda+(1-\lambda)a)]/[2(1-\rho)(1-\lambda)a+2\rho(4-\lambda)+2\lambda]$. Then we solve the regular-period problem and obtain

$$p_h^c = \frac{13a^2(1-\rho)^2(1-\lambda)^2+2a(1-\rho)(1-\lambda)(4c_h+9\rho(2-\lambda)+9\lambda)-(\rho(4-\lambda)+\lambda)(8(c_h+\rho)+5(1-\rho)\lambda)}{16[\rho(4-\lambda)+2a(1-\rho)(1-\lambda)+\lambda]}.$$

The optimal up/downgrade rates are

$$\bar{p}_h^c = \frac{a(1-\lambda)[a^2(1-\rho)^2(9+4\lambda)+8\rho(4c_h+12\rho-(1-\rho)\lambda)+4a(1-\rho)(2c_h+3\rho(5+\lambda)-\lambda)]}{16(a(1-\rho)+2\rho)(\rho(4-\lambda)+a(1-\rho)(1-\lambda)+\lambda)} + \lambda/2,$$

$$\bar{p}_l^c = p_h^c - \bar{p}_h^c + a(1-\lambda)/2,$$

the lease volume is $\tau^c = [64\rho(c_h+\rho) - 48\rho(1-\rho)\lambda + 2a^2(1-\rho)^2(1+12\lambda) + 8(1-\rho)(2c_h+3\rho-3\lambda+9\rho\lambda)a]/[16(a(1-\rho)+2\rho)(\rho(4-\lambda)+a(1-\rho)(1-\lambda)+\lambda)]$, and the maximized profit is

$$\begin{aligned}\Pi^c = {} & [(a(1-\rho)+4\rho)(8(c_h-\rho)-7(1-\rho)a)/(4a(1-\rho)+8\rho)-5(1-a)(1-\rho)\lambda][(12a(2c_h-3\rho)(1-\rho) \\ & -13a^2(1-\rho)^2+32\rho(c_h-\rho))(\rho(4-\lambda)+\lambda)/(4a(1-\rho)+8\rho)+3a^2(1-\rho)^2(1-\lambda)^2-a(1-\rho) \\ & (1-\lambda)(13a^2(1-\rho)^2+4\rho(8\rho-3(1-\rho)\lambda)-2a(1-\rho)(4c_h+3\lambda-3(6+\lambda)\rho))/(2a(1-\rho)+4\rho)] \\ & /[16(\rho(4-\lambda)+a(1-\rho)(1-\lambda)+\lambda)^2].\end{aligned}$$

We can similarly check the conditions for $\bar{p}_h \le (1-\lambda)a$ and $p_h - \bar{p}_l \ge \lambda$.

The total battery capacity under flexible battery leasing is $BT^c = (1-\tau^c+Q_h^c)r_h$. We then compute total customer cost. The total cost of a customer leasing low-capacity batteries is $CT_l^c = \rho\mathbb{E}[d] +$

$(1-\rho)\mathbb{E}[c_l(corD)]$, and the total cost of a customer leasing high-capacity batteries is $CT_h^c = \rho p_h^c + (1-\rho)\mathbb{E}[c_h(corD)]$. So the total customer cost under flexible battery leasing with correlated regular and peak needs for range is $CT^c = \tau^c CT_l^c + (1-\tau^c)CT_h^c$. $\square$

*Proof of Lemma 5* We insert $c_h = \theta r_h^2$ into the maximized profit in the proof of Proposition 2 and optimize $r_h$. Note that $c_h < a(1-\rho)/2$ implies $r_h^2 < a(1-\rho)/(2\theta)$.

$r_h^2 < a(1-\rho)/(2\theta)$. In this case, the manufacturer's profit is

$$\Pi^a(r_h) = [a\rho(1-\rho)(\rho(64-(80-a)a)+2(40-a)(a-2\theta r_h^2))+64\rho\theta^2 r_h^2 + a(1-\rho)(a-2\theta r_h^2)^2]/[256a\rho(1-\rho)].$$

The FOC yields $r_h^0 = \sqrt{a(1-\rho)[a(1-\rho)+40\rho]/[2\theta(a(1-\rho)+16\rho)]}$. The second-order derivative is $\partial^2\Pi^a(r_h)/\partial r_h^2 = (6\theta r_h^2(1/\rho + 16/(a-a\rho)) - 40 - a(1-\rho)/\rho)/(32\theta)$. If $\theta < [a^2(1-\rho)^2 + 40a\rho(1-\rho)]/[(96\rho+6a(1-\rho))r_h^2]$, we have $\partial^2\Pi^a(r_h)/\partial r_h^2 < 0$, and $\partial^2\Pi^a(r_h)/\partial r_h^2 \geq 0$ otherwise. We find that $r_h^0 > \sqrt{a(1-\rho)/(2\theta)}$, so the profit is maximized at $\sqrt{a(1-\rho)/(2\theta)}$.

$r_h^2 \geq a(1-\rho)/(2\theta)$. In this case, the manufacturer's profit is

$$\Pi^n(r_h) = [8\theta r_h^2 - 7a(1-\rho) - 8\rho]^2/[128(a(1-\rho)+2\rho)].$$

We find that $\partial^2\Pi^n(r_h)/\partial r_h^2 < 0 \Longleftrightarrow 7a(1-\rho) + 8(\rho - 3\theta r_h^2) > 0$. We discuss three cases: (1) $\rho < 5a/(8+5a)$, we have $\partial^2\Pi^n(r_h)/\partial r_h^2 > 0$; (2) $\rho > 17a/(8+17a)$, we have $\partial^2\Pi^n(r_h)/\partial r_h^2 < 0$; (3) $5a/(8+5a) \leq \rho \leq 17a/(8+17a)$, we have $\partial^2\Pi^n(r_h)/\partial r_h^2 < 0$ if and only if $r_h^2 < (7a(1-\rho)+8\rho)/(24\theta)$.

- Case 1: $\rho < 5a/(8+5a)$. The profit is convex in $r_h$, and $\Pi^n(r_h = \sqrt{a(1-\rho)/(2\theta)} > \Pi^n(r_h = \sqrt{a(1-\rho)/\theta})$. So the profit is maximized at $\sqrt{a(1-\rho)/(2\theta)}$.
- Case 2: $\rho > 17a/(8+17a)$. The profit is concave in $r_h$, and $\partial\Pi^n(r_h)/\partial r_h|_{r_h=\sqrt{a(1-\rho)/\theta}} > 0$. So the profit is maximized at $\sqrt{a(1-\rho)/\theta}$. We can further check that $\Pi^n(r_h = \sqrt{a(1-\rho)/\theta}) < \Pi^a(r_h = \sqrt{a(1-\rho)/(2\theta)})$.
- Case 3: $5a/(8+5a) \leq \rho \leq 17a/(8+17a)$. The profit is concave in $r_h$ if $r_h^2 < (7a(1-\rho)+8\rho)/(24\theta)$ and convex otherwise. As $\Pi^n(r_h = \sqrt{(7a(1-\rho)+8\rho)/(24\theta)}) > \Pi^n(r_h = \sqrt{a(1-\rho)/\theta})$, the profit is maximized at $\sqrt{(7a(1-\rho)+8\rho)/(24\theta)}$. We then compare the profits under $r_h^2 < a(1-\rho)/(2\theta)$ and $r_h^2 \geq a(1-\rho)/(2\theta)$.

$$\Pi^n(r_h = \sqrt{(7a(1-\rho)+8\rho)/(24\theta)}) < \Pi^a(r_h = \sqrt{a(1-\rho)/(2\theta)}) \Longleftrightarrow \rho > 31a/(31a+6\sqrt{69}-2).$$

To summarize, the optimal range is $r_h^* = \sqrt{a(1-\rho)/(2\theta)}$ if $\rho \leq 5a/(8+5a)$ or $\rho \geq 31a/(31a+6\sqrt{69}-2)$, and is $r_h^* = \sqrt{(7a(1-\rho)+8\rho)/(24\theta)}$ if $5a/(8+5a) < \rho < 31a/(31a+6\sqrt{69}-2)$. We can further check that $\sqrt{a(1-\rho)/(2\theta)} < \sqrt{(7a(1-\rho)+8\rho)/(24\theta)}$. $\square$