Online Companion

In the online companion (OC), we present supplemental material to the paper in OC.1: general over-ordering results under TC are in OC.1.1, discussion of Figure 2a is in OC.1.2, and supplemental figures are in OC.1.3. Proofs for benchmark model results are in §OC.2. Important intermediate results that lead to the solution of the dynamic game in OC.3. Proofs for statements in §5 and OC.3 are collected in an Extended Technical Report Appendix (ExTRA) available at https://ssrn.com/abstract=3474543.

OC.1. Supplemental results

OC.1.1. Over-ordering under TC under general assumptions

This section relaxes the assumptions about the TC contract and the buyer's revenue function in the benchmark model in §4.1. The game chronology remains the same. The proof of all formal results in this subsection is in OC.2.2.

Let $\pi(q,z)$ denote the buyer's revenue under order quantity $q \geq 0$ and market condition z with $\Pi(q,z) \geq 0$ and $\pi(0,\cdot) = 0$. Assume that z is a random variable with probability density function $f(z)$ on $[\underline{z}, \bar{z}]$ at the time of the order. We make the following assumptions about π.

Assumption 4 *Revenue $\pi(q,\cdot)$ is an increasing function of z for all $q \geq 0$.*

Assumption 5 *Revenue $\pi(\cdot,z)$ is a concave function of q for all $z \in [\underline{z}, \bar{z}]$.*

For expository simplicity, assume that π is first-order differentiable with respect to q and denote the derivative by $\pi_q(q,z)$.

Consider two types of TC contract: the *amount contract* where the supplier stipulates a single interest amount r, and the *rate contract* where the supplier stipulates a single interest rate ρ. The TC repayment is $wq + r$ under the amount contract and is ρwq under the rate contract.

Let $\bar{Q}_A(r)$ and $\bar{Q}_R(\rho)$ denote the buyer's optimal order quantities under the amount and rate contracts with interest r and ρ. The fair pricing equations of TC are

$$\text{Amount contract:} \quad \int_{\underline{z}}^{\bar{z}} \min\left\{r + w\bar{Q}_A(r), \pi(\bar{Q}_A(r), z)\right\} f(z)dz = w\bar{Q}_A(r), \tag{OC.1a}$$

$$\text{Rate contract:} \quad \int_{\underline{z}}^{\bar{z}} \min\left\{\rho w\bar{Q}_R(\rho), \pi(\bar{Q}_R(\rho), z)\right\} f(z)dz = w\bar{Q}_R(\rho). \tag{OC.1b}$$

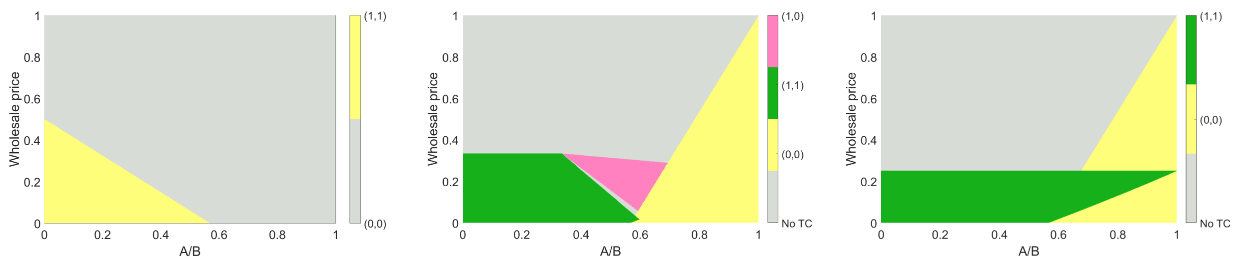
Let q_E^* denote the equilibrium order quantity if the buyer uses EC. Let q_{Ti}^* $(i = A, R)$ denote an equilibrium order quantity if the buyer uses TC with the amount and rate contract. The following lemma shows that TC leads to over-ordering under risky orders.

Lemma 5 *Under Assumptions 4 and 5, the buyer over-orders in equilibrium under TC financing: $q^*_{Ti} \geq q^*_C$ $(i = A, R)$. The equality holds if and only if the loan is risk-free, i.e., the buyer pays off the loan to the supplier with probability one.*

OC.1.2. Discussion of results in Figure 2a

To understand the result on regions (I)–(IV) in Figure 2a, it is useful to know the buyers' equilibrium order levels in different (x_1, x_2)-Cournot games ($x_i \in \{T, E\}$). Figure OC.1 presents the results under $p = 0.5$.

Figure OC.1 Equilibrium order levels in a (x_1, x_2)-Cournot game with fair interest under $p = 0.5$



(a) $(x_1, x_2) = (E, E)$ (b) $(x_1, x_2) = (T, E)$ (c) $(x_1, x_2) = (T, T)$

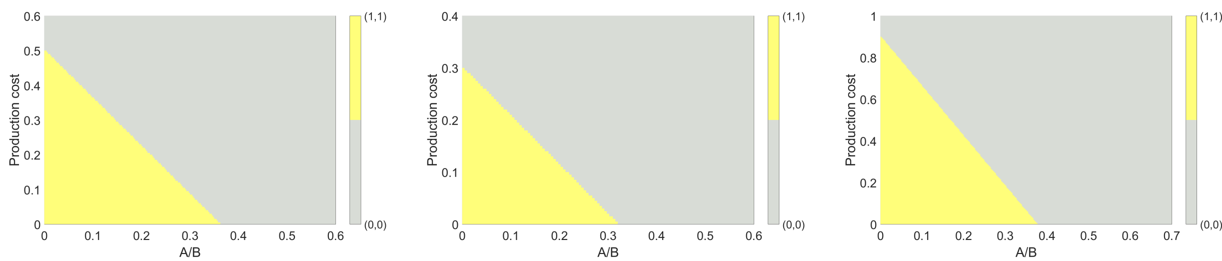
Note. B is normalized to one.

Comparing Figures OC.1a and 2a yields that the EC-financed buyers use the safe order level to the right of region (I) in Figure 2a. In this region, the buyers profit in both states A and B (Corollary 1) and their expected payoffs increase with respect to A. This implies that the advantage of TC over EC decreases with A, since the payoff from using TC occurs only in state B and is invariable with respect to A (Corollary 1). When the value of A is not sufficiently high and the wholesale price is high (region III), TC still has advantage over EC and using TC is the dominant equilibrium. A decrease in w boosts the payoff under EC more than under TC, as the buyers bear the full price under EC. Thus, there are multiple equilibria: the buyers both use TC or both use EC. Pareto-refinement yields that the buyers are better off under EC, eliminating the equilibrium with TC.

As A increases further to region (IV), however, the buyers both use TC again. This is due to a change of equilibrium order levels in the (T, E)-Cournot game. Both buyers use the risky order level to the left of region (IV) but starting from region (IV) and to its right, the EC-financed buyer switches to the safe order level (see Figure OC.1b). The use of the safe order level has two implications. On one hand, it yields revenues in both states and makes the EC-financed buyer's payoff increase in A. On the other hand, it yields a tepid response to the aggressive TC-financed competitor, causing the EC-financed buyer to lose more in the market than under the risky response. In region (VI) where A is not sufficiently high, the loss in market competition dominates. The safe order level weakens the equity value of the EC-financed buyer and elevates the competitor's equity value, thus enhancing the competitive benefit of TC. Consequently, (T,T) is the dominating equilibrium in region (IV). To the right of region (IV) as A further increases, there are multiple equilibria: the buyers both use TC or both use EC, making (E,E) the Pareto-dominant equilibrium.

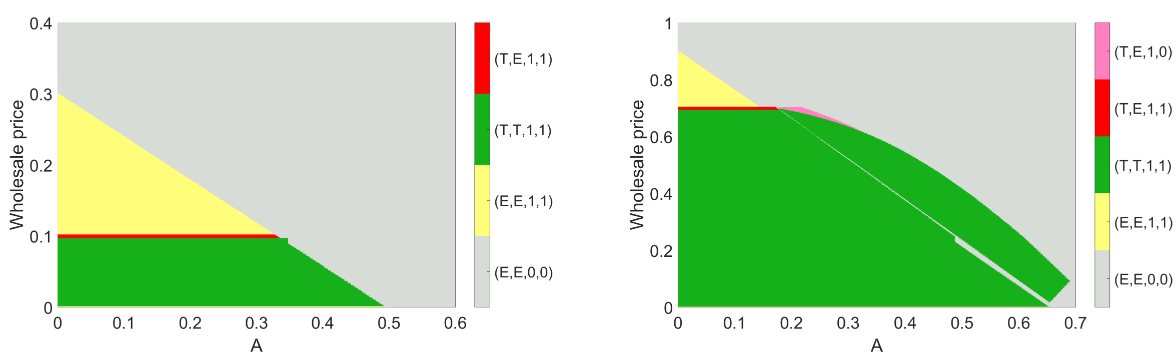
OC.1.3. Supplemental figures

Figure OC.2 Players' equilibrium order levels under endogenous wholesale price when buyers use EC



(a) $p=0.5$ (b) $p=0.3$ (c) $p=0.9$

Figure OC.3 Buyers' equilibrium decisions under exogenous wholesale price under $p=0.3$ and 0.9



(a) $p=0.3$ (b) $p=0.9$

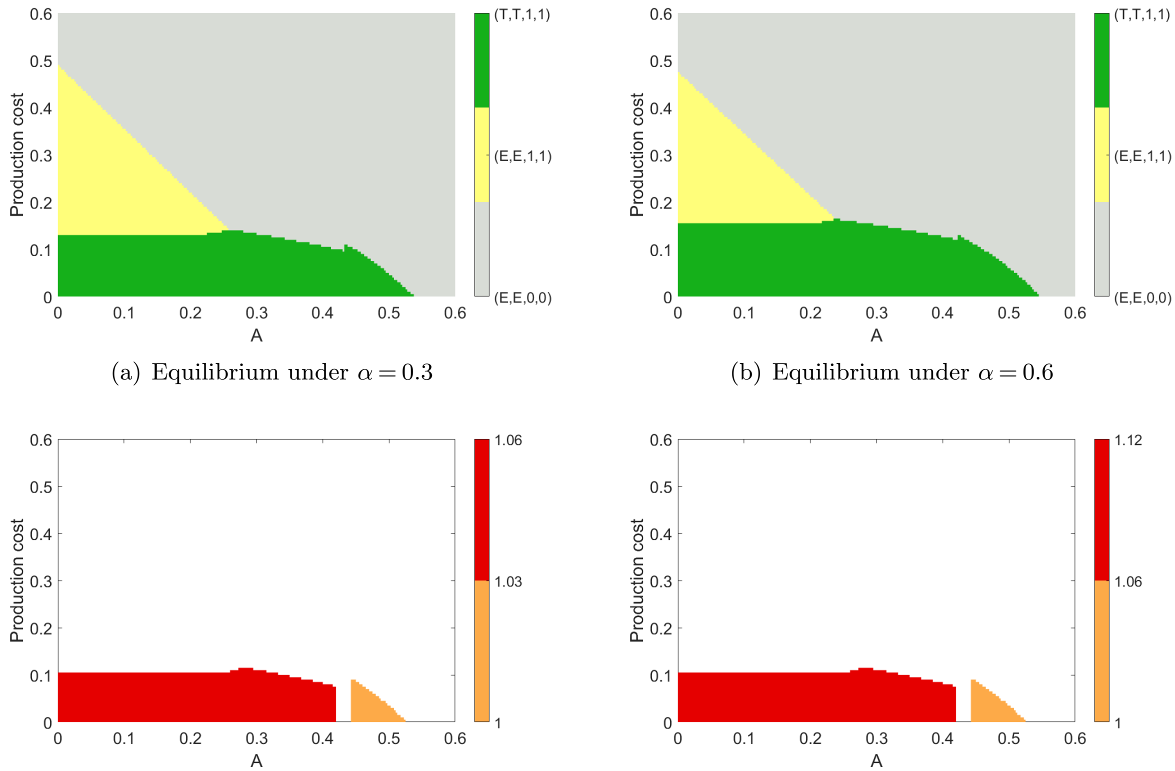
Note. $(T,E,1,1)$ and $(T,E,1,0)$ are shorthands for the multiple equilibria $\{(T,E,1,1),(E,T,1,1)\}$ and $\{(T,E,1,0),(E,T,0,1)\}$. B is normalized to one.

OC.2. Proofs of statements in the benchmark models

OC.2.1. Proofs of statements in §4.

Proof of Lemma 1. Under EC financing, the buyer's optimal order quantity q_E without competition is a special case of Lemma 7 with competition where $q=0$, i.e., $q_E=Q_E(0)$. From (OC.22), (9) is true.

Figure OC.4 Equilibrium result and relative wholesale price under costly EC with $p = 0.5$



(a) Equilibrium under $\alpha = 0.3$

(b) Equilibrium under $\alpha = 0.6$

(c) Relative wholesale price: $\alpha = 0.3$

(d) Relative wholesale price: $\alpha = 0.6$

Note. B is normalized to one in all panels. Comparing panels (a) and (b) with Fig. 3a under perfect EC ($\alpha = 0$) yields that the region with TC usage expands as α increases. Thus, the buyers are more likely to use TC under rising EC cost. Because the region with TC usage expands as α increases, the relative wholesale price is defined only in the TC region under perfect EC ($\alpha = 0$). Panels (c) and (d) show the relative wholesale price in this region.

Under TC financing, let $\tilde{q}_T(r)$ denote the buyer's optimal order quantity given TC interest r. Then $\tilde{q}_T(r)$ is the special case of the buyer's best response function $Q_T(q)$ in (OC.23) with $q = 0$, i.e.,

$$\tilde{q}_T(r) = Q_T(0) = \begin{cases} q_T^0 & \text{if } w \leq A - \sqrt{4r + p(B-A)^2}, \\ q_T^1 & \text{if } w \in (A - \sqrt{4r + p(B-A)^2}, B - 2\sqrt{r}]. \end{cases} \tag{OC.2}$$

If $w > B - 2\sqrt{r}$, the buyer is bankrupt for all order quantity $q \geq 0$.

Because the supplier prices TC fairly, anticipating the buyer's order quantity $\tilde{q}_T(r)$, in equilibrium r satisfies the following fair pricing equation:

$$(1-p)\min\left\{w\tilde{q}_T(r) + r, \, \tilde{q}_T(r)[A - \tilde{q}_T(r)]^+\right\} + p\min\left\{w\tilde{q}_T(r) + r, \, \tilde{q}_T(r)[B - \tilde{q}_T(r)]^+\right\} = w\tilde{q}_T(r). \tag{OC.3}$$

From Corollary 1, the loan is risk-free if $\tilde{q}_T(r) = q_T^0$, implying that the associated fair interest is $r = 0$. If $\tilde{q}_T(r) = q_T^1$, then the buyer repays the loan only in state B. Thus, the fair pricing equation becomes $p(wq_T^1 + r) = wq_T^1$, implying that the associated fair interest is $r = \frac{1-p}{p} wq_T^1$.

Therefore, in equilibrium

$$(r, q_T) = \begin{cases} (0, q_T^0) & \text{if } w \leq A - \sqrt{4r + p(B-A)^2}, \\ (\frac{1-p}{p} w q_T^1, q_T^1) & \text{if } w \in (A - \sqrt{4r + p(B-A)^2}, B - 2\sqrt{r}]. \end{cases}$$
$$= \begin{cases} (0, q_T^0) & \text{if } w \leq A - \sqrt{p}(B-A), \\ (r_1, q_T^1) & \text{if } w \in (A - \sqrt{4r_1 + p(B-A)^2}, B - 2\sqrt{r_1}] \text{ and } r_1 = \frac{1-p}{p} w q_T^1. \end{cases} \quad \text{(OC.4)}$$

If the fair interest r_1 for q_T^1 does not satisfy the condition in (OC.4), i.e., $w > B - 2\sqrt{r_1}$, then the buyer is bankrupt in both states. In this case, the supplier never offers TC and credit rationing occurs.

$$w > B - 2\sqrt{r_1} \quad \Longleftrightarrow \quad w > B - 2\sqrt{\frac{1-p}{p} w \frac{B-w}{2}} \quad \Longleftrightarrow \quad w > \frac{p}{2-p} B,$$

completing the proof. □

Proof of Corollary 1. This corollary is a special case of Corollaries 3 and 4 with $q_x^k = Q_x^k(0)$ where $x \in \{T, E\}$ and $k \in \{0, 1\}$. □

Proof of Proposition 1. **"if":** From (10) and (12), $q_T^1 > q_E^0$ and $q_T^1 > q_E^1$. Because $q_E \in \{q_E^0, q_E^1\}$, $q_T > q_E$ if $q_T = q_T^1$.

From (11), $q_T = q_T^0$ if $w \leq (1+\sqrt{p})A - \sqrt{p}B$. Because $w \geq 0$, this condition implies that $(1 + \sqrt{p})A - \sqrt{p}B \geq 0$ when $q_T = q_T^0$. Meanwhile, $(1+\sqrt{p})A - \sqrt{p}B \geq 0$ implies that $pB - \sqrt{p}(1+\sqrt{p})A \leq 0$, which implies $q_E = q_E^0$ from (9). Therefore, $q_T = q_T^0$ implies $q_E = q_E^0$. Because $q_T^0 = q_E^0$ (see (10) and (12)), $q_T = q_E$ if $q_T = q_T^0$.

"only if": From the arguments above, $q_T = q_E$ if $q_T = q_T^0$. Thus, $q_T > q_E$ only if $q_T \neq q_T^0$, i.e., $q_T = q_T^1$. Similarly, $q_T > q_E$ if $q_T = q_T^1$. Thus, $q_T = q_E$ only if $q_T \neq q_T^1$, i.e., $q_T = q_T^0$, completing the proof. □

Proof of Corollary 2. Follows from the Tie-breaking Assumption and Proposition 1. □

Proof of Proposition 2. The proposition follows from the following lemma, whose proof is in ExTRA §G.

Lemma 6 *In equilibrium, the buyer's expected equity value under EC and TC financing is:*

$$\tilde{v}_E = \begin{cases} (\bar{z} - w)^2/4 & \text{if } q_E = q_E^0, \\ p(B - \frac{w}{p})^2/4 & \text{if } q_E = q_E^1. \end{cases}, \qquad \tilde{v}_T = \begin{cases} (\bar{z} - w)^2/4 & \text{if } q_T = q_T^0, \\ p(B-w)(B - \frac{2-p}{p} w)/4 & \text{if } q_T = q_T^1. \end{cases} \quad \text{(OC.5)}$$

□

Proof of Lemma 2. We first prove that $q_E = q_E^1$ implies $q_T = q_T^1$, provided that credit rationing does not occur. From Lemma 1, $q_E = q_E^1$ iff $w < pB - \sqrt{p}(1 + \sqrt{p})A$. Because $w \geq 0$, condition $w < pB - \sqrt{p}(1+\sqrt{p})A$ implies that $\sqrt{p}B > (1+\sqrt{p})A$. Thus, if $q_E = Q_E^1$, then $w > (1+\sqrt{p})A - \sqrt{p}B$ for all $w \geq 0$. Because $q_T = q_T^0$ iff $w \leq (1+\sqrt{p})A - \sqrt{p}B$, $q_E = q_E^1$ implies that $q_T \neq q_T^0$. Thus, if credit rationing does not occur, then $q_E = q_E^1$ implies that $q_T = q_T^1$.

From Lemma 6, in this case $\tilde{v}_E - \tilde{v}_T = p(B - \frac{w}{p})^2/4 - p(B-w)(B - \frac{2-p}{p}w)/4 = \frac{(1-p)^2}{4p}w^2$, completing the proof. □

OC.2.2. Proof of the general model in OC.1.1

Proof of Lemma 5. We first derive the equilibrium order quantity under EC, q_E^*. When the buyer uses EC, the optimal order quantity solves the optimization $\max_q \mathbb{E}[\pi(q,\boldsymbol{z})] - wq$. Concavity of the revenue function (Assumption 5) implies that q_E^* satisfies the following first-order condition (FOC):

$$\int_{\underline{z}}^{\bar{z}} \pi_q(q_E^*, z) f(z) dz = w. \tag{OC.6}$$

Order quantity under amount contract q_{TA}^*. When the buyer uses TC with interest amount r, the optimal order quantity $\bar{Q}_A(r)$ is

$$\bar{Q}_A(r) = \arg\max_q \int_{\underline{z}}^{\bar{z}} [\pi(q,z) - wq - r]^+ f(z) dz. \tag{OC.7}$$

Let $\hat{z}_A(q,r)$ denote the bankruptcy state such that $\pi(q,z) - wq - r \gtreqless 0$ if and only if $z \gtreqless \hat{z}_A(q,r)$ (such a $\hat{z}(q,r)$ exists because of the monotonicity of π in z):

$$\pi\big(q, \hat{z}_A(q,r)\big) - wq - r = 0. \tag{OC.8}$$

For expository simplicity, the arguments of $\hat{z}_A$ and $\bar{Q}_A$ are suppressed in the proof. Using $\hat{z}_A$, optimization (OC.7) is $\max_q \int_{\hat{z}_A}^{\bar{z}} [\pi(q,z) - wq - r] f(z) dz$. FOC for the optimal q is:

$$\int_{\hat{z}_A}^{\bar{z}} [\pi_q(q,z) - w] f(z) dz - [\pi(q,\hat{z}_A) - wq - r] f(\hat{z}_A) \frac{d\hat{z}_A}{dq} = 0. \tag{OC.9}$$

Using (OC.8) in (OC.9) yields

$$\int_{\hat{z}_A}^{\bar{z}} [\pi_q(q,z) - w] f(z) dz = 0 \iff \int_{\hat{z}_A}^{\bar{z}} \pi_q(q,z) f(z) dz = w[1 - F(\hat{z}_A)], \tag{OC.10}$$

where $F(x) = \int_{\underline{z}}^{x} f(z) dz$. Thus, given r, the optimal order quantity $\bar{Q}_A$ solves (OC.10).

The fair interest r_A^* satisfies (OC.1a), namely,

$$\int_{\underline{z}}^{\hat{z}_A} \pi(\bar{Q}_A, z) f(z) dz + [r_A^* + w\bar{Q}_A][1 - F(\hat{z}_A)] = w\bar{Q}_A$$

$$\iff \int_{\underline{z}}^{\hat{z}_A} \pi(\bar{Q}_A, z) f(z) dz = w\bar{Q}_A F(\hat{z}_A) - r_A^*[1 - F(\hat{z}_A)]. \tag{OC.11}$$

Because $\pi(0,\cdot) = 0$, for any $x > 0$,

$$\pi(x,z) = \int_0^x \pi_q(q,z) dq > \int_0^x \pi_q(x,z) dq = \pi_q(x,z)x, \tag{OC.12}$$

where the inequality follows from the concavity of $\pi(\cdot, z)$.

Inequality (OC.12) implies that $\pi(\bar{Q}_A, z) > \pi_q(\bar{Q}_A, z)\bar{Q}_A$ for all r such that $\bar{Q}_A(r) > 0$. Now we show that at the fair interest $r = r_A^*$, $\bar{Q}_A(r_A^*) > 0$ by contradiction. Suppose that $\bar{Q}_A(r_A^*) = 0$. Then the fair pricing equation (OC.1a) requires $r_A^* = 0$, which implies $\bar{Q}_A(0) = 0$. By definition, $\bar{Q}_A(0)$ solves (OC.10). Thus, $\bar{Q}_A(0) = 0$ implies that $q = 0$ is the solution to (OC.10) at $r = 0$. At $(q = 0, r = 0)$, $\hat{z}_A(0,0) = \underline{z}$ since there is no bankruptcy risk and, thus, (OC.10) becomes $\int_{\underline{z}}^{\bar{z}} \pi_q(0,z) f(z)dz = w$. But this cannot be true because $q_0^* > 0$ is the unique solution to $\int_{\underline{z}}^{\bar{z}} \pi_q(q,z) f(z)dz = w$. Therefore, $\bar{Q}_A(r_A^*) > 0$.

From (OC.11), at $r = r_A^*$, $\bar{Q}_A(r_A^*) = q_{TA}^*$ and

$$wq_{TA}^* F(\hat{z}_A) - r_A^*[1 - F(\hat{z}_A)] = \int_{\underline{z}}^{\hat{z}_A} \pi(q_{TA}^*, z) f(z)dz \geq \int_{\underline{z}}^{\hat{z}_A} [\pi_q(q_{TA}^*, z) q_{TA}^*] f(z)dz$$

$$\iff \qquad wq_{TA}^* F(\hat{z}_A) - r_A^*[1 - F(\hat{z}_A)] \geq q_{TA}^* \int_{\underline{z}}^{\hat{z}_A} \pi_q(q_{TA}^*, z) f(z)dz. \tag{OC.13}$$

Because $q_{TA}^* > 0$, equality in (OC.13) holds if and only if (iff) $\hat{z}_A(q_{TA}^*, r_A^*) = \underline{z}$ (in which case $r_A^* = 0$), namely, there is no bankruptcy risk. Condition (OC.13) implies that $wq_{TA}^* F(\hat{z}_A) \geq q_{TA}^* \int_{\underline{z}}^{\hat{z}_A} \pi_q(q_{TA}^*, z) f(z)dz$, which reduces to, using $q_{TA}^* > 0$,

$$wF(\hat{z}_A) \geq \int_{\underline{z}}^{\hat{z}_A} \pi_q(q_{TA}^*, z) f(z)dz, \tag{OC.14}$$

The equality in (OC.14) is true iff $\hat{z}_A(q_{TA}^*, r_A^*) = \underline{z}$.

From (OC.10) and (OC.14), at r_A^*,

$$\begin{aligned}\int_{\underline{z}}^{\bar{z}} \pi_q(q_{TA}^*, z) f(z)dz &= \int_{\underline{z}}^{\hat{z}_A} \pi_q(q_{TA}^*, z) f(z)dz + \int_{\hat{z}_A}^{\bar{z}} \pi_q(q_{TA}^*, z) f(z)dz \\ &\leq wF(\hat{z}_A) + w[1 - F(\hat{z}_A)] \\ &= w. \end{aligned} \tag{OC.15}$$

Let $H(q) = \mathbb{E}[\pi_q(q, \boldsymbol{z})] = \int_{\underline{z}}^{\bar{z}} \pi_q(q,z) f(z)dz$. Then (OC.15) states that $H(q_{TA}^*) \leq w$. From (OC.6), $H(q_E^*) = w$. The concavity of π in q yields that H decreases in q. Thus, $q_{TA}^* \geq q_E^*$ at r_A^* with the equality holds iff the bankruptcy risk is zero.

Order quantity under the rate contract q_{TR}^*. When the buyer uses TC with interest rate ρ, the optimal order quantity $\bar{Q}_R(\rho) = \arg\max_q \int_{\underline{z}}^{\bar{z}} [\pi(q,z) - \rho w q]^+ f(z)dz$. Analysis similar to the amount contract yields the following FOC for $\bar{Q}_R(\rho)$:

$$\int_{\hat{z}_R}^{\bar{z}} [\pi_q(\bar{Q}_R, z) - \rho w] f(z)dz = 0, \tag{OC.16}$$

where $\hat{z}_R$ is the threshold bankruptcy state which depends on q and ρ so that $\pi(q, \hat{z}_R) - \rho w q = 0$.

Inequality (OC.12) implies that $\pi(\bar{Q}_R, z) > \pi_q(\bar{Q}_R, z)\bar{Q}_R$ for all ρ such that $\bar{Q}_R(\rho) > 0$. Similar reasoning as before yields that at $\rho = \rho_R^*$, $\bar{Q}_R(\rho_R^*) > 0$. Using $q_{TR}^* = \bar{Q}_R(\rho_R^*)$, the fair interest ρ_R^* satisfies (OC.1b), namely,

$$\begin{aligned} &\int_{\underline{z}}^{\hat{z}_R} \pi(q_{TR}^*, z) f(z) dz + \rho_R^* w q_{TR}^* [1 - F(\hat{z}_R)] = w q_{TR}^* \\ \iff \quad &\int_{\underline{z}}^{\hat{z}_R} \pi(q_{TR}^*, z) f(z) dz = w q_{TR}^* \big(1 - \rho_R^* [1 - F(\hat{z}_R)]\big). \end{aligned} \tag{OC.17}$$

Thus,

$$\begin{aligned} w q_{TR}^* \big(1 - \rho_R^* [1 - F(\hat{z}_R)]\big) = \int_{\underline{z}}^{\hat{z}_R} \pi(q_{TR}^*, z) f(z) dz \geq \int_{\underline{z}}^{\hat{z}_R} [\pi_q(q_{TR}^*, z) q_{TR}^*] f(z) dz \\ \iff \quad w\big(1 - \rho_R^* [1 - F(\hat{z}_R)]\big) \geq \int_{\underline{z}}^{\hat{z}} \pi_q(q_{TR}^*, z) f(z) dz. \end{aligned} \tag{OC.18}$$

The second line uses $q_{TR}^* > 0$ and the equality in (OC.18) holds iff $\hat{z}_R = \underline{z}$ (in which case $\rho_R^* = 1$), namely, there is no bankruptcy risk.

From (OC.16) and (OC.18),

$$\begin{aligned} \int_{\underline{z}}^{\bar{z}_R} \pi_q(q_{TR}^*, z) f(z) dz &= \int_{\underline{z}}^{\hat{z}_R} \pi_q(q_{TR}^*, z) f(z) dz + \int_{\hat{z}}^{\bar{z}_R} \pi_q(q_{TR}^*, z) f(z) dz \\ &\leq w\big(1 - \rho [1 - F(\hat{z}_R)]\big) + \rho w [1 - F(\hat{z}_R)] \\ &= w. \end{aligned} \tag{OC.19}$$

Using function H defined earlier, (OC.19) states that $H(q_{TR}^*) \leq w = H(q_E^*)$. Thus, $q_{TR}^* \geq q_E^*$ at ρ_R^*, with the equality holds iff there is no bankruptcy risk. □

OC.3. Solution for the dynamic game

OC.3.1. Buyers' best response functions

In the Cournot game, let $Q_x(q)$ denote a buyer's best response to its competitor's order quantity q when using TC ($x = T$) or EC ($x = B$). Define

$$Q_E^0(q) = \frac{(\bar{z} - q - w)^+}{2}, \qquad Q_E^1(q) = \frac{(B - q - w/p)^+}{2}, \tag{OC.20a}$$

$$Q_T^0(q) = \frac{(\bar{z} - q - w)^+}{2}, \qquad Q_T^1(q) = \frac{(B - q - w)^+}{2}. \tag{OC.20b}$$

Using indicator function $1(\cdot)$ that equals one if the argument is valid and zero otherwise, define

$$\mathcal{I}_E^0(q) = 1(w \geq p(B - A)) + 1(w < p(B - A)) \times 1(q \leq (1 + \sqrt{p})A - \sqrt{p}B + \frac{w}{\sqrt{p}}), \tag{OC.21a}$$

$$\mathcal{I}_T^0(q) = 1(q \leq A - w - \sqrt{p(B - A)^2 + 4r}), \tag{OC.21b}$$

$$\mathcal{I}_E^1(q) = 1 - \mathcal{I}_E^0(q), \qquad \mathcal{I}_T^1(q) = \big[1 - \mathcal{I}_T^0(q)\big] \times 1(q < B - w - 2\sqrt{r}). \tag{OC.21c}$$

Thus, $\mathcal{I}_E^0(q)$, $\mathcal{I}_T^0(q)$, $\mathcal{I}_E^1(q)$ and $\mathcal{I}_T^1(q)$ are indicator functions.

The following lemmas and corollaries show that a buyer's best response Q_x takes the form of either Q_x^0 or Q_x^1. As in the benchmark models in §4: $k=0$ represents the safe order level and $k=1$ the risky order level. The proofs are in ExTRA §B.

Lemma 7 *The best response of a EC-financed buyer to order quantity q, $Q_E(q)$, satisfies:*

$$Q_E(q) = \begin{cases} Q_E^0(q) & \text{if } \mathcal{I}_E^0(q) = 1, \\ Q_E^1(q) & \text{if } \mathcal{I}_E^0(q) = 0. \end{cases} \tag{OC.22}$$

Corollary 3 *If $Q_E(q) = Q_E^1(q) > 0$, then the market price is zero in state A and strictly positive in state B. If $Q_E(q) = Q_E^0(q) > 0$, then the market price is strictly positive in both states.*

Lemma 8 *The best response of a TC-financed buyer to order quantity q, $Q_T(q)$, satisfies:*

$$Q_T(q) = \begin{cases} Q_T^0(q) & \text{if } \mathcal{I}_T^0(q) = 1, \\ Q_T^1(q) & \text{if } \mathcal{I}_T^0(q) = 0 \text{ and } q \le B - w - 2\sqrt{r}. \end{cases} \tag{OC.23}$$

If $q > B - w - 2\sqrt{r}$, then the buyer defaults in both states A and B regardless of its own order quantity, implying that $Q_T(q) \in [0, \infty)$.

Corollary 4 *If $Q_T(q) = Q_T^1(q)$, then the TC-financed buyer defaults in state A and is solvent in state B. If $Q_T(q) = Q_T^0(q)$, then the TC-financed buyer does not default.*

Corollary 5 *Given the competitor's order quantity q, TC yields over-ordering, i.e., $Q_T(q) > Q_E(q)$, if and only if $Q_T(q) = Q_T^1(q)$, and yields the EC order quantity, i.e., $Q_T(q) = Q_E(q)$, if and only if $Q_T(q) = Q_T^0(q)$.*

It is useful to define a notation system to represent the intersection of best response functions in an (x_1, x_2)-Cournot game. Let $(q_{x_1x_2}^{k_1k_2}, q_{x_2x_1}^{k_2k_1})$ denote the intersection of the best responses of buyers 1 and 2 who use financing channels (x_1, x_2) and order levels (k_1, k_2):

$$q_{x_1x_2}^{k_1k_2} = Q_{x_1}^{k_1}(q_{x_2x_1}^{k_2k_1}), \qquad q_{x_2x_1}^{k_2k_1} = Q_{x_2}^{k_2}(q_{x_1x_2}^{k_1k_2}). \tag{OC.24}$$

The following corollary specifies the order quantities at selected intersection points.

Corollary 6 *Intersections of $Q_{x_1}^{k_1}$ and $Q_{x_2}^{k_2}$ ($x_1, x_2 \in \{T, E\}$, $k_1, k_2 \in \{0, 1\}$) satisfy:*

$$q_{EE}^{00} = \frac{(\bar{z} - w)^+}{3}, \qquad q_{TT}^{00} = \frac{(\bar{z} - w)^+}{3}, \qquad (q_{ET}^{00}, q_{TE}^{00}) = \left(\frac{(\bar{z} - w)^+}{3}, \frac{(\bar{z} - w)^+}{3} \right), \tag{OC.25a}$$

$$q_{EE}^{11} = \frac{(B - \frac{w}{p})^+}{3}, \qquad q_{TT}^{11} = \frac{B - w}{3}, \qquad (q_{ET}^{11}, q_{TE}^{11}) = \left(\frac{(B - \frac{2-p}{p} w)^+}{3}, \frac{B - w - q_{ET}^{11}}{2} \right), \tag{OC.25b}$$

$$(q_{ET}^{01}, q_{TE}^{10}) = \left(\frac{(2\bar{z} - B - w)^+}{3}, \frac{B - w - q_{ET}^{01}}{2} \right). \tag{OC.25c}$$

OC.3.2. Equilibrium of (x_1,x_2)-Cournot games given (w,r)

The following lemma presents the equilibrium results of (x_1,x_2)-Cournot games $(x_1,x_2\in\{T,E\})$. (See ExTRA §C for the proof.)

Lemma 9 *Given wholesale price w and TC interest r,*

1. *The (E,E)-Cournot game has a unique Pareto-dominant equilibrium with:*
   (a) $(K_1,K_2)\in\{(0,0)\}$ *iff* $\mathcal{I}_E^0(q_{EE}^{00})\times\big[\big(1-\mathcal{I}_E^1(q_{EE}^{11})\big)+\mathcal{I}_E^1(q_{EE}^{11})\times 1(p(q_{EE}^{11})^2\le(q_{EE}^{00})^2)\big]=1.$
   (b) $\big(K_1,K_2\big)\in\{(1,1)\}$ *iff* $\mathcal{I}_E^1(q_{EE}^{11})\times[\big(1-\mathcal{I}_E^0(q_{EE}^{00})\big)+\mathcal{I}_E^0(q_{EE}^{00})\times 1\big(p(q_{EE}^{11})^2>(q_{EE}^{00})^2\big)\big]=1.$
2. *The (T,T)-Cournot game has:*
   (a) *A unique Pareto-dominant equilibrium with* $\big(K_1,K_2\big)\in\{(0,0)\}$ *iff* $\mathcal{I}_T^0(q_{TT}^{00})\times\big[1-\mathcal{I}_T^1(q_{TT}^{11})+\mathcal{I}_T^1(q_{TT}^{11})\times 1(p[(q_{TT}^{11})^2-r]\le(q_{TT}^{00})^2-r)\big]=1.$
   (b) *A unique Pareto-dominant equilibrium with* $\big(K_1,K_2\big)\in\{(1,1)\}$ *iff* $\mathcal{I}_T^1(q_{TT}^{11})\times\big[1-\mathcal{I}_T^0(q_{TT}^{00})+\mathcal{I}_T^0(q_{TT}^{00})\times 1\big(p[(q_{TT}^{11})^2-r]>(q_{TT}^{00})^2-r\big)\big]=1.$
   (c) *At least one buyer who defaults with probability one iff conditions in 2a and 2b are violated.*
3. *The (T,E)-Cournot game has:*
   (a) *A unique Pareto-dominant equilibrium with* $\big(K_1,K_2\big)\in\{(1,0)\}$ *iff* $\mathcal{I}_T^1(q_{ET}^{01})\times\mathcal{I}_E^0(q_{TE}^{10})=1.$
   (b) *A unique Pareto-dominant equilibrium with* $\big(K_1,K_2\big)\in\{(1,1)\}$ *iff* $\mathcal{I}_T^1(q_{ET}^{11})\times\mathcal{I}_E^1(q_{TE}^{11})\times[1-\mathcal{I}_T^0(q_{ET}^{00})\times\mathcal{I}_E^0(q_{TE}^{00})]=1.$
   (c) *A unique Pareto-dominant equilibrium with* $\big(K_1,K_2\big)\in\{(0,0)\}$ *iff* $\mathcal{I}_T^0(q_{ET}^{00})\times\mathcal{I}_E^0(q_{TE}^{00})\times\big[\big(1-\mathcal{I}_T^1(q_{ET}^{11})\times\mathcal{I}_E^1(q_{TE}^{11})\big)+\mathcal{I}_T^1(q_{ET}^{11})\times\mathcal{I}_E^1(q_{TE}^{11})\times 1(p[(q_{TE}^{11})^2-r]\le(q_{TE}^{00})^2-r)\big]=1.$
   (d) *Multiple equilibria with* $\big(K_1,K_2\big)\in\{(1,1),(0,0)\}$ *iff* $\mathcal{I}_T^0(q_{ET}^{00})\times\mathcal{I}_E^0(q_{TE}^{00})\times\mathcal{I}_T^1(q_{ET}^{11})\times\mathcal{I}_E^1(q_{TE}^{11})\times 1\big(p[(q_{TE}^{11})^2-r]>(q_{TE}^{00})^2-r\big)=1.$
   (e) *The TC-financed buyer defaults with probability one iff conditions in 3a–3d are violated.*

OC.3.3. Equilibrium of the financing game given (w,r)

For expository simplicity, let $V_{x_1x_2}=v_{x_1}(Q_{x_1x_2}(w,r),Q_{x_2x_1}(w,r))$ denote the expected payoff of buyer 1 in equilibrium with order quantities $(Q_{x_1x_2}(w,r),Q_{x_2x_1}(w,r))$ in an (x_1,x_2)-Cournot game $(x_1,x_2\in\{T,E\})$. Then buyer 2's expected payoff in the same equilibrium is denoted by $V_{x_2x_1}$. The following lemma presents the financing game equilibrium, the proof of which is in ExTRA §D.

Lemma 10 *Given wholesale price w and TC interest r, the financing game has*

1. *A unique Pareto-dominant equilibrium* $(X_1(w,r),X_2(w,r))\in\{(E,E)\}$ *iff* $1(V_{TE}\le V_{EE})\times\big[1(V_{TT}\le V_{ET})+1(V_{TT}>V_{ET})\times 1(V_{TT}\le V_{EE})\big]=1.$
2. *A unique Pareto-dominant equilibrium* $(X_1(w,r),X_2(w,r))\in\{(T,T)\}$ *iff* $1(V_{TT}>V_{ET})\times\big[1(V_{TE}>V_{EE})+1(V_{TE}\le V_{ET})\times 1(V_{TT}>V_{EE})\big]=1.$
3. *Multiple equilibria* $(X_1(w,r),X_2(w,r))\in\{(T,E),(E,T)\}$ *iff* $1(V_{TT}\le V_{ET})\times 1(V_{TE}>V_{EE})=1.$

OC.3.4. Fair interest

Define the following TC interests using notation $q_{x_1x_2}^{k_1k_2}$:

$$r_{TT} = \frac{1-p}{p} w q_{TT}^{11}, \qquad r_{TE}^{11} = \frac{1-p}{p} w q_{TE}^{11}, \qquad r_{TE}^{10} = \frac{1-p}{p} w q_{TE}^{10}. \tag{OC.26}$$

The following lemma states that if the TC interest is sufficiently high, the buyers use EC in equilibrium in the financing game. (See ExTRA §E for the proof.)

Lemma 11 *For each parameter combination (w,p,A,B) under which a financing game exists, there exists $r_0 < \infty$ such that (E,E) is an equilibrium of the financing game only if $r \geq r_0$.*

The following lemma specifies the fair interest in different equilibrium scenarios, the proof of which in ExTRA §E.

Lemma 12 *If TC is used in equilibrium, then the fair TC interest is*

1. *Zero for equilibria $(T,T,0,0)$, $(T,E,0,0)$, and $(E,T,0,0)$.*
2. *r_{TT} for equilibrium $(T,T,1,1)$.*
3. *r_{TE}^{11} for equilibria $(T,E,1,1)$ and $(E,T,1,1)$.*
4. *r_{TE}^{10} for equilibria $(T,E,1,0)$ and $(E,T,0,1)$.*

OC.3.5. Exact solution of subgame SG_1

Proposition 5 *The following two-step procedure yields the exact solution of subgame SG_1. Given (w,p,A,B),*

Step 1. Check conditions (i)–(vii) in Table OC.1 to obtain the set of fair interests $\mathcal{S}(w)$ and, for each fair interest $\tilde{r} \in \mathcal{S}(w)$, the subsequent equilibrium decisions (X_1,X_2,K_1,K_2) and supplier's revenue.

Step 2. The fair interest in $\mathcal{S}(w)$ that yields the highest supplier's revenue is $R(w)$. The SGP NE of SG_1 comprises of $R(w)$ and its subsequent equilibrium (X_1,X_2,K_1,K_2).

Proof of Proposition 5 is in ExTRA §F.

OC.3.6. Solution procedure for results in §5

Given $p \in (0,1)$, equilibrium results in §5 are obtained by normalizing $B = 1$ and following the procedure below.

Equilibrium results of subgame SG_1. Given $(A,w) \in [0,1] \times [0,1]$, we compute the *exact* equilibrium solution of subgame SG_1 following Proposition 5. In the calculation, we set $(A,w) = (\frac{x}{400}, \frac{y}{1000})$ where $x = 0,\ldots,400$ and $y = 0,\ldots,1000$.

Table OC.1 Values of $(\tilde{r}, X_1, X_2, K_1, K_2)$, associated conditions, and supplier's revenue

Condition	$\tilde{r}$	(X_1, X_2, K_1, K_2)	Equilibrium (q_1, q_2)	**Supplier's revenue**
(i) Lemma 9 case 2b, and Lemma 10 case 2.	r_{TT}	$(T,T,1,1)$	$(q^{11}_{TT}, q^{11}_{TT})$	$2wq^{11}_{TT}$
(ii) Lemma 9 case 3a, and Lemma 10 case 3.	r^{10}_{TE}	$(T,E,1,0)$ $(E,T,0,1)$	$(q^{10}_{TE}, q^{01}_{ET})$ $(q^{01}_{ET}, q^{10}_{TE})$	$w(q^{10}_{TE} + q^{01}_{ET})$
(iii) Lemma 9 case 3b, and Lemma 10 case 3.	r^{11}_{TE}	$(T,E,1,1)$ $(E,T,1,1)$	$(q^{11}_{TE}, q^{11}_{ET})$ $(q^{11}_{ET}, q^{11}_{TE})$	$w(q^{11}_{TE} + q^{11}_{ET})$
(iv) Lemma 9 case 1a.	∞	$(E,E,0,0)$	$(q^{00}_{EE}, q^{00}_{EE})$	$2wq^{00}_{EE}$
(v) Lemma 9 case 1b.	∞	$(E,E,1,1)$	$(q^{11}_{EE}, q^{11}_{EE})$	$2wq^{11}_{EE}$

Equilibrium results of full game SG_0. Given $(A,c) = (\frac{x}{400}, \frac{z}{200})$ ($x = 0, \ldots, 400$, $z = 0, \ldots, 200$), we compute the supplier's equilibrium profit under each wholesale price $w \in \{\frac{y}{1000} : y = 0, \ldots, 1000\}$ using equation (6). The equilibrium wholesale price is the value $\frac{y^\star}{1000}$ that leads to the highest profit for the supplier. The subsequent equilibrium financing choices and ordering decisions are the equilibrium of subgame SG_1 at $(A,w) = (\frac{x}{400}, \frac{y^\star}{1000})$.