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E-Companion: The Implications of Strategic Inventory for Short-term vs. Long-Term Supply Contracts in Non-Exclusive Reselling Environments

EC.1. Proof of Proposition 1

We prove this result using backward induction. In stage $2b$ of the sub-game for SS contracts, the reseller holds inventories I_A and I_B and must respond to the offered wholesale prices by choosing procurement quantities, $q_{A2} \geq 0$ and $q_{B2} \geq 0$, to maximize her second-period profits:

$$\pi_2^{SS}\left(q_{i2}, q_{j2}\right) = \sum_{i \in \{A,B\}, j \neq i} \left(a - \left(q_{i2} + I_i\right) - d\left(q_{j2} + I_j\right)\right)\left(q_{j2} + I_j\right) - w_{i2} q_{i2}, \tag{EC.1}$$

where, as before, we use lower-case π to indicate the *reseller*'s profit. This yields the following:

LEMMA EC.1. *The reseller's optimal purchase quantities in stage $2b$ is given by:*

$$q_{i2}^{SS}\left(I_i, I_j, w_{i2}, w_{j2}\right) = \begin{cases} \frac{a(1-d) - 2\left(1-d^2\right) I_i - w_{i2} + d w_{j2}}{2\left(1-d^2\right)} & w_{i2} \leq \overline{w}_{i2}\left(I_i, w_{j2}\right),\ w_{j2} \leq \overline{w}_{j2}\left(I_j, w_{i2}\right), \\ \frac{a - w_{i2} - 2\left(I_i + d I_j\right)}{2} & w_{i2} \leq \widehat{w}_{i2}\left(I_i, I_j\right),\ w_{j2} > \overline{w}_{j2}\left(I_j, w_{i2}\right), \\ 0 & \text{otherwise}, \end{cases} \tag{EC.2}$$

where $\overline{w}_{i2}\left(I_i, w_{j2}\right) = a(1-d) - 2\left(1-d^2\right) I_i + d w_{j2}$, and $\widehat{w}_{i2}\left(I_i, I_j\right) = a - 2\left(I_i + d I_j\right)$, for all $i, j \in \{A, B\}$, $j \neq i$.

Proof of Lemma EC.1. By applying first-order conditions (FOC) on (EC.1) with respect to (w.r.t.) q_{A2} and q_{B2}, we obtain $q_{i2FOC}^{SS} = \frac{a(1-d) - 2\left(1-d^2\right) I_i - w_{i2} + d w_{j2}}{2\left(1-d^2\right)}$ $\forall i, j \in \{A, B\}$, $j \neq i$, which is decreasing in w_{i2}. Since we need $q_{i2} \geq 0$, we solve $q_{i2FOC}^{SS} = 0$ to obtain $\overline{w}_{i2}\left(I_i, w_{j2}\right)$ such that $q_{j2}^{SS} = q_{j2FOC}^{SS} \geq 0$ only when $w_{i2} \leq \overline{w}_{i2}\left(I_i, w_{j2}\right)$ and $w_{j2} \leq \overline{w}_{j2}\left(I_j, w_{i2}\right)$. We now focus $w_{j2} > \overline{w}_{j2}\left(I_j, w_{i2}\right)$, such that $q_{j2FOC}^{SS} < 0$. Substituting $q_{j2} = 0$ in (EC.1) and applying FOC w.r.t. q_{i2}, we obtain $q_{i2Nj2}^{SS} = \frac{a - w_{i2} - 2\left(I_i + d I_j\right)}{2}$, $\forall i, j \in \{A, B\}$, $j \neq i$, which is decreasing in w_{i2}. Solving $q_{i2Nj2}^{SS} = 0$ for w_{i2}, we determine $\widehat{w}_{i2}\left(I_i, I_j\right)$ such that when $w_{j2} > \overline{w}_{j2}\left(I_j, w_{i2}\right)$, $q_{i2}^{SS} = q_{i2Nj2}^{SS} \geq 0$ only when $w_{i2} \leq \widehat{w}_{i2}\left(I_i, I_j\right)$. Therefore, when $w_{j2} > \overline{w}_{j2}\left(I_j, w_{i2}\right)$, if $w_{i2} > \widehat{w}_{i2}\left(I_i, I_j\right)$, the reseller purchases nothing from either supplier. □

If the wholesale prices (inventories) are not too high, then the reseller orders positive quantities from both suppliers, and the reseller's response is characterized by the upper branch of (EC.2). We show that in equilibrium, the wholesale prices (inventories) are low enough to ensure that the reseller orders from both suppliers in period 2. In stage $2a$, both suppliers anticipate the reseller's quantity response, and set w_{i2} to maximize:

$$\Pi_{i2}^{SS}(w_{i2}, w_{j2}, I_i, I_j) = w_{i2} \cdot q_{i2}^{SS}(I_i, I_j, w_{i2}, w_{j2}),\ \forall i, j \in \{A, B\}, j \neq i. \tag{EC.3}$$

Substituting q_{i2}^{SS} from (EC.2) in (EC.3), and simultaneously solving the FOC and boundary conditions for suppliers A and B, we obtain the following result.

LEMMA EC.2. *When both suppliers offer short-term contracts, if the reseller holds $I_i \geq \frac{a}{2(1+d)}$ for both products $i \in \{A, B\}$, then neither supplier can induce the reseller to purchase additional units at a non-negative wholesale price. Otherwise, if the reseller holds $I_i < \frac{a}{2(1+d)}$ for at least one product, then the suppliers' wholesale prices satisfy the following:*

$$w_{i2}^{SS}(I_i, I_j) = \begin{cases} \frac{(1-d)\left(a(2+d)-2(1+d)(2I_i+dI_j)\right)}{4-d^2} & I_i < \overline{I}_i(I_j),\ I_j < \overline{I}_j(I_i) \\ \frac{(1-d)\left(2(1+d)I_j-a\right)}{d} & I_i < \overline{I}_i(I_j),\ I_j \in \left[\overline{I}_j(I_i), \widehat{I}_j(I_i)\right) \\ \frac{a-2(I_i+dI_j)}{2} & I_i < \overline{I}_i(I_j),\ I_j \in \left[\widehat{I}_j(I_i), \widetilde{I}_j(I_i)\right) \\ 0 & \textit{otherwise}, \end{cases} \tag{EC.4}$$

$\forall i, j \in \{A, B\}$, $j \neq i$, where: $\overline{I}_j(I_i) = \frac{1}{4}\left(\frac{a(2+d)}{1+d} - 2dI_i\right)$, $\widehat{I}_j(I_i) = \frac{a(2-d)-2dI_i}{2\left(2-d^2\right)}$, $\widetilde{I}_j(I_i) = \frac{a-2I_i}{2d}$, and $\overline{I}_j(I_i) = \widehat{I}_j(I_i) = \widetilde{I}_j(I_i) = \frac{a}{2(1+d)}$ for $I_i = \frac{a}{2(1+d)}$.

Proof of Lemma EC.2. We prove the above result in the following steps. First, from (EC.2) we know that in order for $q_{i2}^{SS} > 0$, we need either (a) $w_{i2} < \overline{w}_{i2}(I_i, w_{j2})$ when $w_{j2} \leq \overline{w}_{j2}(I_j, w_{i2})$, or (b) $w_{i2} < \widehat{w}_{i2}(I_i, I_j)$ when $w_{j2} > \overline{w}_{j2}(I_j, w_{i2})$. It can be seen that when $I_i \geq \frac{a}{2(1+d)}$ $\forall i \in \{A, B\}$, neither of the above two conditions hold for $w_{i2} \geq 0$, $i \in \{A, B\}$.

Next, we consider the case where $I_i < \frac{a}{2(1+d)}$ for some $i \in \{A, B\}$. When both I_A and I_B are sufficiently low, q_{i2}^{SS} is given by the top branch of (EC.2). Anticipating this, suppliers A and B simultaneously choose $w_{A2} \geq 0$ and $w_{B2} \geq 0$, respectively, to maximize $\Pi_{i2}^{SS}(w_{i2}, w_{j2}, I_i, I_j)$, given by substituting the top branch of (EC.2) in (EC.3). Applying FOC, we obtain $w_{i2FOC}^{SS}(I_i, I_j) = (1-d)\left(a(2+d) - 2(1+d)(2I_i + dI_j)\right)/(4-d^2)$, $\forall i, j \in \{A, B\}$, $j \neq i$ Substituting in the top branch of (EC.2), we obtain $q_{i2FOC}^{SSI}(I_i, I_j)$. Both $w_{i2FOC}^{SS}(I_i, I_j)$ and $q_{j2FOC}^{SSI}(I_j, I_i)$ are decreasing in I_j. Setting either to zero and solving for I_j, we obtain $\overline{I}_j(I_i)$, which implies that supplier i sets $w_{i2}^{SS} = w_{i2FOC}^{SS}(I_i, I_j) > 0$ only when $I_i < \overline{I}_i(I_j)$ and $I_j < \overline{I}_j(I_i)$.

Now let us consider that $I_i < \overline{I}_i(I_j)$, but $I_j \geq \overline{I}_j(I_i)$. If supplier i anticipates that I_j to be so high that $q_{j2}^{SS} = 0$, $j \neq i$, for any $w_{j2} \geq 0$, then q_{i2}^{SS} is given by the middle branch of (EC.2). Substituting this q_{i2}^{SS} in (EC.3) and applying FOC w.r.t w_{i2}, we obtain: $w_{i2Nqj2}^{SS}(I_i, I_j) = \frac{a-2\left(I_i+dI_j\right)}{2}$, $\forall i, j \in \{A, B\}$, $j \neq i$, which is positive as long as $I_j < \widetilde{I}_j(I_i)$. Note that $\widetilde{I}_j(I_i) > \overline{I}_j(I_i)$ when $I_i < \frac{a}{2(1+d)}$. For $w_{i2Nqj2}^{SS}(I_i, I_j)$ to be offered in equilibrium, supplier i's anticipation has to be correct, implying that the reseller's purchase quantity from supplier j at supplier j's best response to this price $w_{j2BR}^{SS}\left(I_j, w_{i2Nqj2}^{SS}(I_i, I_j)\right) = \left(a(2-d) - 2(2-d^2)I_j - 2dI_i\right)/4$ cannot be a positive quantity. Thus, supplier i sets $w_{i2}^{SS} = w_{i2Nqj2}^{SS}(I_i, I_j) > 0$ only when $I_i < \overline{I}$ and $I_j \in \left[\widehat{I}_j(I_i), \widetilde{I}_j(I_i)\right)$, such that $q_{j2Nqj2}^{SS}(I_i, I_j)$ as obtained by substituting $w_{i2Nqj2}^{SS}(I_i, I_j)$ and $w_{j2BR}^{SS}(I_i, I_j)$ in the top branch of (EC.2) equals zero.

When $I_j \in \left[\overline{I}_j(I_i), \widehat{I}_j(I_i)\right)$, the inventory from j is too high for supplier i to offer the FOC price $w_{i2FOC}^{SS}(I_i, I_j)$, yet too low to offer the price $w_{i2Nqj2}^{SS}(I_i, I_j)$. Therefore, supplier i sets w_{i2} low enough such that the reseller purchases nothing from supplier j even when supplier j offers his best response to w_{i2}, given by $w_{j2BR}^{SS}(I_j, w_{i2}) = \left((1-d)\left(a - 2(1+d)I_j\right) + dw_{i2}\right)/2$, $\forall j, i \in$

$\{A,B\}$, $j \neq i$. This is achieved when supplier i offers $w^{SS}_{i2BND}(I_i,I_j) = (1-d)(2(1+d)I_j - a)/d$, $\forall i,j \in \{A,B\}$, $j \neq i$. As $I_j \to \widehat{I}_j(I_i)$, $w^{SS}_{i2BND}(I_i,I_j) \to w^{SS}_{i2Nqj2}(I_i,I_j)$. Thus, supplier i sets $w^{SS}_{i2} = w^{SS}_{i2BND}(I_i,I_j) > 0$ only when $I_i < \overline{I}$ and $I_j \in \left[\widehat{I}_j(I_i), \widetilde{I}_j(I_i)\right)$.

When $I_j \geq \widetilde{I}_j(I_i) = \frac{a-2I_i}{2d}$, $w^{SS}_{i2Nqj2}(I_i,I_j) = 0$ even when $I_i < \overline{I}_i(I_j)$. Finally, substituting $I_i = \frac{a}{2(1+d)}$ in $\overline{I}_j(I_i)$, $\widehat{I}_j(I_i)$, and $\widetilde{I}_j(I_i)$, we find that $\overline{I}_j(I_i) = \widehat{I}_j(I_i) = \widetilde{I}_j(I_i) = \frac{a}{2(1+d)}$ for $I_i = \frac{a}{2(1+d)}$. □

By substituting the results of Lemmas EC.2 and EC.1 back into (EC.1) and (EC.3) respectively, we have the sub-game-perfect equilibrium *(SPE)* profit of the reseller, $\pi^{SS*}_2(I_A,I_B)$, and the suppliers, $\Pi^{SS*}_{i2}(I_i,I_j)$ $\forall i \in \{A,B\}$, $j \neq i$, respectively. In stage $1b$, the reseller determines her first-period selling quantities and inventories to maximize her total profit:

$$\pi^{SS}(s_{A1},s_{B1},I_A,I_B) = \sum_{i,j\in\{A,B\},j\neq i} \left(p_{i1}s_{i1} - w_{i1}(s_{i1}+I_i) - hI_i\right) + \pi^{SS*}_2(I_A,I_B), \tag{EC.5}$$

where p_{it} is given by (1). Because each supplier earns nothing in period 1, if he cannot induce the reseller to procure a positive quantity from him, in the following lemma, we restrict our attention to wholesale price pairs in which neither supplier sets a price high enough relative to his rival that the reseller would order nothing from him in period 1.

LEMMA EC.3. *There exist thresholds on the first-period wholesale prices, $\overline{w}^{SS}_{i1}(w_{j1}) \leq \overline{\overline{w}}_{i1}(w_{j1})$, where $\overline{w}^{SS}_{i1}(w_{j1}) := ((3+d)(a(3-d)(1-d)-(2-d)^2h) + d(8-d^2)w_{j1})(12-d^2)$ and $\overline{\overline{w}}_{i1}(w_{j1}) := a(1-d) + dw_{j1}$, such that:*

(i) For any (w_{A1},w_{B1}) satisfying $w_{A1} \leq \overline{\overline{w}}_{A1}(w_{B1})$ and $w_{B1} \leq \overline{\overline{w}}_{B1}(w_{A1})$, the reseller's best response to the offered wholesale prices in period 1 is to sell:

$$s^{SS}_{i1}(w_{i1},w_{j1}) = \frac{a(1-d) - w_{i1} + dw_{j1}}{2(1-d^2)}, \forall i \in \{A,B\}, j \neq i, \tag{EC.6}$$

and to hold inventory that depends as follows on the two wholesale prices.

(ii) If $w_{A1} \leq \overline{w}^{SS}_{A1}(w_{B1})$ and $w_{B1} \leq \overline{w}^{SS}_{B1}(w_{A1})$, then the reseller holds a positive inventory of both products. If either of these inequalities is reversed, then the reseller holds no inventory of the product with the higher wholesale price, and holds inventory of the other one only if its wholesale price is sufficiently small.

Proof of Lemma EC.3. We first focus on the case where $I_i < \overline{I}(I_j)$ and $I_j < \overline{I}_j(I_i)$. We later show that, in equilibrium, the suppliers do not offer wholesale prices low enough that either of these two constraints is violated. Therefore, $w^{SS}_{i2}(I_i,I_j) = w^{SS}_{i2FOC}(I_i,I_j)$ and $q^{SS}_{i2}(I_i,I_j) = q^{SSI}_{i2FOC}(I_i,I_j)$ from Lemmas EC.2 and EC.1. Substituting these into (EC.1), we obtain $\pi^{SS}_{2FOC}(I_A,I_B)$. Further substituting these into (EC.5), we obtain the reseller's first-period profit $\pi^{SS}_{FOC}(s_{A1},s_{B1},I_A,I_B)$ as a function of her first-period selling quantities and inventories, which is jointly concave in s_{A1}, s_{B1}, I_A, and I_B.

Applying FOC on $\pi^{SS}_{FOC}(s_{A1},s_{B1},I_A,I_B)$ simultaneously w.r.t. s_{A1}, s_{B1}, I_A, and I_B, we obtain the first-period selling quantities and inventories held by the reseller from each supplier: $s^{SS}_{i1FOC}(w_{i1},w_{j1}) = \frac{a(1-d)-w_{i1}+dw_{j1}}{2(1-d^2)}$, and $I^{SS}_{iFOC}(w_{i1},w_{j1}) = \frac{a(3-d)(1-d)-(2-d)^2h}{2(3-d)(1-d^2)} - \frac{(12-d^2)w_{i1}-d(8-d^2)w_{j1}}{2(9-d^2)(1-d^2)}$.

Applying the boundary conditions $s_{i1} \geq 0$ and $I_i \geq 0$ for all $i \in \{A,B\}$, we obtain $\overline{w}_{i1}^{SS}(w_{j1}) < \overline{\overline{w}}_{i1}(w_{j1})$ such that $s_{i1}^{SS}(w_{i1},w_{j1}) = s_{i1FOC}^{SS}(w_{i1},w_{j1}) \geq 0$ when $w_{i1} \leq \overline{\overline{w}}_{i1}(w_{j1})$, and $I_i^{SS}(w_{i1},w_{j1}) = I_{iFOC}^{SS}(w_{i1},w_{j1}) \geq 0$ when $w_{j1} \leq \overline{w}_{j1}^{SS}(w_{i1})$ $\forall i \in \{A,B\}$, $j \neq i$. Therefore, only if $w_{i1} \leq \overline{w}_{i1}^{SS}(w_{j1})$ $\forall i \in \{A,B\}$, $j \neq i$ both $s_{i1}^{SS}(w_{i1},w_{j1}) = s_{i1FOC}^{SS}(w_{i1},w_{j1}) \geq 0$ and $I_i^{SS}(w_{i1},w_{j1}) = I_{iFOC}^{SS}(w_{i1},w_{j1}) \geq 0$ $\forall i \in \{A,B\}$, $j \neq i$.

When $w_{j1} \geq \overline{w}_{j1}^{SS}(w_{i1})$ for some $j \in \{A,B\}$ while $w_{i1} \leq \overline{w}_{i1}^{SS}(w_{j1}), i \in \{A,B\}, i \neq j$, then from the above analysis, we know that $I_j^{SS}(w_{i1},w_{j1}) = 0$. W.L.O.G., suppose $I_B = 0$. Substituting $I_B = 0$ in $\pi_{FOC}^{SS}(s_{A1},s_{B1},I_A,I_B)$ and applying FOC simultaneously w.r.t. s_{A1}, s_{B1}, and I_A, we obtain the first-period selling quantities, $s_{i1NIj}^{SS}(w_{i1},w_{j1}) = s_{i1FOC}^{SS}(w_{i1},w_{j1})$ $\forall i \in \{A,B\}$, $j \neq i$, and the inventory held by the reseller from supplier i when there is no inventory of product j, $I_{iNIj}^{SS}(w_{i1},w_{j1}) = \frac{(2+d)^2\left(a(3-d)(1-d)-(2-d)^2(h+w_{i1})\right)}{2\left(12-13d^2+d^4\right)}$. We observe that when $w_{j1} \geq \overline{w}_{j1}^{SS}(w_{i1})$ for some $j \in \{A,B\}$ while $w_{i1} \leq \overline{w}_{i1}^{SS}(w_{j1})$, $i \in \{A,B\}$, $i \neq j$, we have $s_{i1}^{SS}(w_{i1},w_{j1}) = s_{i1NIj}^{SS}(w_{i1},w_{j1}) \geq 0$ $\forall i \in \{A,B\}$, $j \neq i$, $I_j^{SS}(w_{i1},w_{j1}) = 0$, and $I_i^{SS}(w_{i1},w_{j1}) = I_{iNIj}^{SS}(w_{i1},w_{j1}) \geq 0$. □

In stage $1a$, the suppliers simultaneously choose their wholesale prices for period 1 to maximize their respective total profits over the two periods:

$$\Pi_i^{SS}(w_{i1},w_{j1}) = w_{i1}\left(s_{i1}^{SS}(w_{i1},w_{j1}) + I_i^{SS}(w_{i1},w_{j1})\right) + \Pi_{i2}^{SS*}\left(I_i^{SS}(w_{i1},w_{j1}), I_j^{SS}(w_{j1},w_{i1})\right), \quad \text{(EC.7)}$$

$\forall i,j \in \{A,B\}$, $j \neq i$, where s_{i1}^{SS} is given in (EC.6) and $I_i^{SS}(w_{i1},w_{j1})$ solves the reseller's FOC for (EC.5), with a boundary condition at zero. Now we prove the two parts of Proposition 1. Recall that (SSI^*) represents the equilibrium in which the reseller has a positive inventory, and $(SSNI^*)$ represents the equilibrium in which no inventory is held.

EC.1.1. Proposition 1, part *(i)*

We begin with a scenario SSI where the reseller holds inventory from both suppliers offering (S) contracts, under the conditions $0 < I_i < \overline{I}_i(I_j)$, and $0 < I_j < \overline{I}_j(I_i)$. Substituting $q_{i2}^{SS}(I_i,I_j) = q_{i2FOC}^{SSI}(I_i,I_j)$ in (EC.3), we obtain $\Pi_{i2}^{SSI}(I_i,I_j)$. Substituting $\Pi_{i2}^{SSI}(I_i,I_j)$, s_{i1}^{SS} from (EC.6), and $I_i^{SS}(w_{i1},w_{j1}) = I_{iFOC}^{SS}(w_{i1},w_{j1})$, we obtain $\Pi_i^{SSI}(w_{i1},w_{j1})$, which is jointly concave in w_{i1},w_{j1}. Applying FOC on $\Pi_i^{SSI}(w_{i1},w_{j1})$ w.r.t. w_{i1} and w_{j1}, we arrive at supplier i's best response to supplier j's wholesale price, $w_{i1BR}^{SSI}(w_{j1})$ $\forall i,j \in \{A,B\}$, $j \neq i$, which, when solved simultaneously for $i,j \in \{A,B\}, j \neq i$, yield the FOC wholesale prices for supplier $i \in \{A,B\}$, given by

$$w_{i1FOC}^{SSI} = \frac{2a(3-d)^2(3+d)(1-d) - (2-d)(6-9d+d^3)h}{102 - d\left(81 - d\left(9 + 2(4-d)d\right)\right)}, \ \forall i \in \{A,B\}. \quad \text{(EC.8)}$$

Substituting w_{i1FOC}^{SSI} in $\Pi_i^{SS}(w_{i1},w_{j1})$, we obtain Π_{iFOC}^{SS}. Furthermore, substituting w_{i1FOC}^{SSI} into $s_{i1}^{SS}(w_{i1},w_{j1})$ and $I_i^{SS}(w_{i1},w_{j1})$, we obtain

$$s_{i1FOC}^{SSI} = \frac{3a\left(16 - d(3+d)\right) - h(2-d)(6-9d+d^3)}{2(1+d)\left(102 - d\left(81 - d\left(9+2(4-d)d\right)\right)\right)}, \text{ and} \quad \text{(EC.9)}$$

$$I_{iFOC}^{SSI} = \frac{a(1-d)\left(30 - d(9+d)\right) - h(2-d)^2\left(30 - d\left(9 + (3-d)d\right)\right)}{2(1-d)(1+d)\left(102 - d\left(81 - d\left(9+2(4-d)d\right)\right)\right)}, \ \forall i \in \{A,B\}. \quad \text{(EC.10)}$$

We similarly obtain the other equilibrium values, as shown in Table 2.

We now describe how to derive $h_I^{SS}(d)$, and show that these values constitute the equilibrium, that is, $w_{i1FOC}^{SSI}, i \in \{A,B\}$ satisfies $w_{A1} \leq \overline{\overline{w}}_{A1}(w_{B1})$ and $w_{B1} \leq \overline{\overline{w}}_{B1}(w_{A1})$ for $h < h_I^{SS}(d)$, and neither of the two suppliers nor the reseller deviate from these equilibrium values. It can be seen that the reseller does not deviate from $I_{iFOC}^{SSI} > 0$ if both suppliers offer w_{i1FOC}^{SSI} as long as $h < \overline{h}^{SS}(d) := \frac{a(1-d)(30-d(9+d))}{(2-d)^2(30-d(9+d(3-d)))}$, which is the solution to $I_i^{SS}\left(w_{i1FOC}^{SSI}, w_{j1FOC}^{SSI}\right) = 0$. However, for some $h < \overline{h}^{SS}(d)$, a supplier may deviate and induce the reseller to hold zero inventory of his product. Without loss of generality, suppose supplier A deviates from the FOC price and offers $w_{A1} > \overline{w}_{A1}^{SS}(w_{B1FOC}^{SSI}) > w_{A1FOC}^{SSI}$ to induce $I_A = 0$ while supplier B offers w_{B1FOC}^{SSI}. The reseller chooses s_A, s_B, $I_A = 0$, and I_B to optimize her profit, yielding $s_{A1devNI_A}^{SSI}(w_{A1}) = \frac{a(1-d)-w_{A1}+dw_{B1FOC}^{SSI}}{2(1-d^2)}$, $s_{B1devNI_A}^{SSI}(w_{A1}) = \frac{a(1-d)-w_{B1FOC}^{SSI}+dw_{A1}}{2(1-d^2)}$, and $I_{BdevNI_A}^{SSI}(w_{A1}) = \frac{(2+d)^2\left(a(3-d)(1-d)-(2-d)^2\left(h+w_{B1FOC}^{SSI}\right)\right)}{2\left(12-d^2\right)\left(1-d^2\right)}$, using which we obtain supplier A's total profit $\Pi_{Adev}^{SSI}(w_{A1})$, while supplier B offers w_{i1FOC}^{SSI}. Applying FOC on $\Pi_{Adev}^{SSI}(w_{A1})$ with respect to w_{A1}, we obtain $w_{A1devNI_A}^{SSI} = \frac{a(1-d)(102-d(27+d(9-2d)))-(2-d)d(6-9d+d^3)h}{204-2d(81-d(9+2(4-d)d))}$. Substituting $w_{A1devNI_A}^{SSI}$ into $\Pi_{Adev}^{SSI}(w_{A1})$, we obtain $\Pi_{AdevNI_A}^{SSI} = \frac{a(1-d)(102-d(27+d(9-2d)))-(2-d)d(6-9d+d^3)h}{204-2d(81-d(9+2(4-d)d))}$.

For supplier A to successfully deviate, we need $w_{A1devNI_A}^{SSI} > \overline{w}_{A1}^{SS}(w_{B1FOC}^{SSI})$ and $\Pi_{AdevNI_A}^{SSI} > \Pi_{iFOC}^{SS}$. Observe that (a) $\Pi_{AFOC}^{SSI} - \Pi_{AdevNI_A}^{SSI}$ is convex in h; (b) $h = h_I^{SS}(d)$ solves $\Pi_{AFOC}^{SSI} - \Pi_{AdevNI_A}^{SSI} = 0$, and is the only solution to the above equation that satisfies $h < \overline{h}^{SS}(d)$, and (c) $\frac{\partial}{\partial h}\left(\Pi_{AFOC}^{SSI} - \Pi_{AdevNI_A}^{SSI}\right) < 0$ at $h = h_I^{SS}(d)$. Furthermore, $w_{A1devNI_A}^{SSI} > \overline{w}_{A1}^{SS}(w_{B1FOC}^{SSI})$ when $h > h_I^{SS}(d)$. Thus, when $h < h_I^{SS}(d)$, neither supplier deviates from w_{i1FOC}^{SSI} to induce the reseller to hold zero inventory of his own product, and $w_{i1}^{SS*} = w_{i1}^{SSI*} = w_{i1FOC}^{SSI}$ is an equilibrium. We similarly verify that other deviations, such as inducing the reseller to hold too much inventory of his product that results in zero inventory of the competitor's product, or zero purchase in the second period, are never beneficial to the supplier when $h < h_I^{SS}(d)$, and we omit the details for the sake of brevity.

EC.1.2. Proposition 1, part *(ii)*

Now we focus on the scenario $SSNI$ where no inventory is held from either supplier. When both suppliers anticipate the reseller not to hold inventory from either of them, the SS contracts converge to the LL contracts, and they set the wholesale price for the first period $w_{i1FOC}^{SSNI} = w^{LL*} = a\left(\frac{1-d}{2-d}\right)$ $\forall i \in \{A,B\}$. In response, the reseller holds $I_{iFOC}^{SSNI} = 0$ as long as $h \geq h_{NI}^{SS}(d) := \frac{a(1-d)}{(2-d)^2}$, where $h_{NI}^{SS}(d)$ solves $I_i^{SS}(w^{LL*}, w^{LL*}) = 0$. It can be seen that $h_{NI}^{SS}(d) \leq h_I^{SS}(d)$, $\forall d \in [0,1)$.

We now show that neither the reseller nor either supplier has an incentive to deviate and induce $I_i > 0$ for any $i \in \{A,B\}$ when $h \geq h_{NI}^{SS}(d)$, which implies that for $h \geq h_I^{SS}(d)$, there exists a unique equilibrium in which each supplier $i \in \{A,B\}$ sets his wholesale price to: $w_{i1}^{SS*} = w_1^{SSNI*} := \frac{a(1-d)}{2-d}$, and no inventory is held.

First, from Lemma EC.3, the reseller's best response is to hold inventory from a supplier i only if his wholesale price is sufficiently low, i.e., $w_{i1} \leq \overline{w}_{i1}^{SS}(w_{j1})$. When $h \geq h_I^{SS}(d)$ (therefore,

$h \geq h_{NI}^{SS}(d)$) and both suppliers offer w_{i1FOC}^{SSNI}, this condition is not met, and the reseller's best response is to not hold inventory. Secondly, it can be seen that when supplier j offers w_{j1FOC}^{SSNI}, supplier i cannot earn a higher profit by deviating and offering either (a) $w_{i1} \leq \overline{w}_{i1}^{SS}\left(w_{j1FOC}^{SSNI}\right)$, such that $I_i > 0$, or (b) $w_{i1} \geq \overline{w}_{j1}^{SS^{-1}}\left(w_{j1FOC}^{SSNI}\right)$, such that $I_j > 0$, thus neither supplier deviates.

Finally, because $h_{NI}^{SS}(d) \leq h_I^{SS}(d)$, there is an alternative equilibrium immediately below $h_I^{SS}(d)$ for a very small range of $h \in (\eta(d) \times h_I^{SS}(d), h_I^{SS}(d))$, for $\eta(d) > 0.95$.

EC.2. Proof of Proposition 2

We obtain the results in this proposition by substituting $d = 0$ in the results in Table 2, and then comparing the results under LL with those under SS.

EC.3. Proof of Proposition 3

We prove Proposition 3 in three parts.

EC.3.1. Proposition 3, part *(i)*

First, we focus on the SSI scenario, where the reseller holds inventory in equilibrium as long as $h < h_I^{SS}(d)$. Supplier i's equilibrium profit under SS contracts, $\Pi_i^{SS*} = \Pi^{SSI*} = \Pi_{iFOC}^{SSI}$, and that under LL contracts, Π_i^{LL*}, are given by Table 2. Observe that $\frac{\partial^2 \Pi_i^{SS*}}{\partial h^2} > 0$ for all $d \in [0,1)$, implying that Π_i^{SS*} is convex in h for all $i \in \{A,B\}$, while Π_i^{LL*} is independent of h. Therefore, $\Pi_i^{SS*} - \Pi_i^{LL*}$ is also convex in h for all $i \in \{A,B\}$. Solving $\Pi_i^{SS*} - \Pi_i^{LL*} = 0$ for h, we obtain the solution $\widehat{h}(d)$ such that $\widehat{h}(d) \leq h_I^{SS}(d)\ \forall d \in [0,1)$, and the inequality is strict for $d > 0$. Further, $\left.\frac{\partial(\Pi_i^{SS*} - \Pi_i^{LL*})}{\partial h}\right|_{h=\widehat{h}(d)} < 0$. Therefore, $\Pi_i^{SS*} > \Pi_i^{LL*}$ when $h < \widehat{h}(d)$. Next, it can be seen that $\frac{\partial \widehat{h}(d)}{\partial d}|_{d=0} < 0$, while $\widehat{h}(0) = h_I^{SS}(0) > 0$. Solving $\widehat{h}(d) = 0$ for d with the condition $d \in [0,1)$, we obtain the solution $\widehat{d} \in (0,1)$, such that $\widehat{h}(d) < 0$ when $d > \widehat{d}$. Therefore, when $d > \widehat{d}$, $\Pi_i^{SS*} < \Pi_i^{LL*}$ for any $0 \leq h < h_I^{SS}(d)$.

EC.3.2. Proposition 3, part *(ii)*

Secondly, focusing on $h < h_I^{SS}(d)$ again, the reseller's equilibrium profit under SS contracts, π^{SS*}, and that under LL contracts, π^{LL*}, are also given by Table 2. Again, observe that π^{SS*} is convex in h, while π^{LL*} is independent of h, thus $\pi^{SS*} - \pi^{LL*}$ is convex in h. Recall from Section EC.1 that $I_{iFOC}^{SS*} = 0$ at $h \geq \overline{h}^{SS}(d) := \frac{a(1-d)(30-d(9+d))}{(2-d)^2(30-d(9+d(3-d)))}$, where $\overline{h}^{SS}(d) \geq h_I^{SS}(d)\ \forall d \in [0,1)$. It can be confirmed that $\lim_{d\to 1}\left(\left.\frac{\partial}{\partial h}\left(\pi^{SS*} - \pi^{LL*}\right)\right|_{h=\overline{h}^{SS}(d)}\right) = -\frac{a}{36}$, that is, as d becomes sufficiently high, the reseller prefers SS contracts for all holding costs low enough to induce inventory to be held. Furthermore, by solving $\pi^{SS*} - \pi^{LL*} = 0$ for h, one can derive the boundary given by the two roots of the quadratic equation $\pi^{SS*} - \pi^{LL*} = 0$, which encloses the region where $\pi^{SS*} < \pi^{LL*}$.

EC.3.3. Proposition 3, part 3*(iii)*

Since in the $SSNI$ scenario (i.e., $h > h_I^{SS}(d)$), the reseller does not hold inventory from either supplier under SS contracts in equilibrium, the equilibrium is identical to that under LL contracts.

EC.4. Proof of Proposition 4

We prove this result using backward induction. First, let us derive the reseller's different purchasing quantities and inventories as the best responses to the suppliers' wholesale price offers.

LEMMA EC.4. *Under LS contracts, for any (w_{A1}, w_{B1}) satisfying $w_{i1} \le \overline{\overline{w}}_{i1}(w_{j1}) := a(1-d)+dw_{j1}$ for all $i \in \{A, B\}, j \ne i$, the reseller's best response to the offered wholesale prices in period 1 is to sell:*

$$s_{i1}^{LS}(w_{i1}, w_{j1}) = \frac{a(1-d) - w_{i1} + dw_{j1}}{2(1-d)(1+d)}, \text{ for } i \in \{A, B\}, j \ne i. \tag{EC.11}$$

The reseller's optimal choices of the second-period order from supplier A offering the long-term contract, and the amount of inventory to hold from supplier B offering a short-term contract, depend upon the period 2 wholesale price offered by A and period 1 wholesale price offered by B:

$$q_{A2}^{LS}(w_{A2}, w_{B1}) = \frac{a(1-d) + dh - w_{A2} + dw_{B1}}{2(1-d^2)}, \tag{EC.12}$$

$$I_B^{LS}(w_{A2}, w_{B1}) = \frac{3a(1-d) - h(4-d^2) - w_{B1}(4-d^2) + 3dw_{A2}}{6(1-d^2)}, \tag{EC.13}$$

and $I_B^{LS}(w_{A2}, w_{B1}) = 0$ for $w_{B1} \ge \overline{w}_{B1}^{LS}(w_{A2})$, where $\overline{w}_{B1}^{LS}(w_{A2}) := \frac{3a(1-d)+3dw_{A2}}{4-d^2} - h \le \overline{\overline{w}}_{B1}(w_{A1})$.

Proof of Lemma EC.4. Since in equilibrium, supplier A offers $w_{A2} \le w_{A1} + h$, the reseller does not hold inventory from A. Substituting $I_A = 0$ and (12) back into (10), we obtain:

$$q_{B2}^{LSI}(q_{A2}, I_B) = q_{B2}^{LS}\left(0, I_B, q_{A2}, w_{B2}^{LS*}\right) = \begin{cases} \frac{a-2(I_B+dq_{A2})}{4} & \text{if } I_B \le \frac{a-2dq_{A2}}{2}, \\ 0 & \text{otherwise.} \end{cases} \tag{EC.14}$$

The reseller's revenue-maximizing selling quantity of product B in the second period is $\frac{a-2d(q_{A2}+I_A)}{2}$. Since there is no value of leftover inventory at the end of period 2, and since $I_A = 0$, the reseller never holds inventory exceeding $\frac{a-2dq_{A2}}{2}$, and we focus on the upper branch of (EC.14). Substituting (12) and (EC.14) in (9), we obtain the reseller's second-period profit $\pi_2^{LSI}(q_{A2}, I_B)$. Substituting (1) and $\pi_2^{LSI}(q_{A2}, I_B)$ in (13), we obtain the reseller's total profit $\pi^{LSI}(s_{A1}, q_{A2}, s_{B1}, I_B)$, which is jointly concave in $s_{A1}, q_{A2}, s_{B1}, I_B$. Applying FOC simultaneously with respect to $s_{A1}, q_{A2}, s_{B1}, I_B$, we obtain: $s_{i1}^{LSI}(w_{i1}, w_{j1})$, $q_{A2}^{LSI}(w_{A2}, w_{B1}) = s_{A2}^{LSI}(w_{A2}, w_{B1})$, and $I_B^{LSI}(w_{A2}, w_{B1})$ as given by Lemma EC.4. Observe that $s_{i1}^{LSI}(w_{i1}, w_{j1})$ is decreasing in w_{i1}, and $s_{i1}^{LSI}(w_{i1}, w_{j1}) \ge 0$ as long as $w_{i1} \le \overline{\overline{w}}_{i1}(w_{j1})$. This proves the first part of Lemma EC.4.

Similarly, $q_{A2}^{LSI}(w_{A2}, w_{B1})$ and $I_B^{LSI}(w_{A2}, w_{B1})$ are decreasing in w_{A2} and w_{B1}, respectively. Setting $q_{A2}^{LSI}(w_{A2}, w_{B1}) = 0$ and $I_B^{LSI}(w_{A2}, w_{B1}) = 0$, we obtain upper bounds on w_{A2} and w_{B1}, $\overline{w}_{A2}^{LS}(w_{B1}) = a(1-d) + d(h + w_{B1})$, and $\overline{w}_{B1}^{LS}(w_{A2}) = \frac{3(a(1-d)+dw_{A2})}{4-d^2} - h$, such that $q_{A2}^{LSI}(w_{A2}, w_{B1}) \ge 0$ as long as $w_{A2} \le \overline{w}_{A2}^{LS}(w_{B1})$, and $I_B^{LSI}(w_{A2}, w_{B1}) \ge 0$ as long as $w_{B1} \le \overline{w}_{B1}^{LS}(w_{A2})$, or $w_{A2} \ge \underline{w}_{A2}^{LS}(w_{B1})$. Recall that $w_{A2} \le w_{A1} + h$. Using the above thresholds,

$$\begin{aligned} \overline{\overline{w}}_{B1}(w_{A1}) - \overline{w}_{B1}^{LS}(w_{A2}) &= (a(1-d) + dw_{A1}) - \left(\frac{3(a(1-d) + (dw_{A2}))}{4-d^2} - h\right) \\ &\ge \frac{a(1-d)^2(1+d) + (4-d^2)d(w_{A2} - h) - dw_{A2} + h(4-d^2)}{4-d^2}, \text{ since } w_{A2} \le w_{A1} + h \\ &= \frac{a(1-d)^2(1+d) + (3-d^2)dw_{A2} + h(1-d)(4-d^2)}{4-d^2} \ge 0, \text{ since } d \in [0,1), a \ge 0, h \ge 0. \end{aligned}$$

This proves the second part of Lemma EC.4. □

Substituting (EC.11), (EC.12), and (EC.13) into (13), we obtain the reseller's conditionally optimal profit as a function of the wholesale prices offered by the long-term (A) and short-term (B) contract suppliers, denoted by $\pi^{LS*}(w_{A1}, w_{A2}, w_{B1})$. Finally, in stage $1a$, supplier A chooses w_{A1} and w_{A2} while supplier chooses w_{B1}, to maximize their respective profit-to-go over two periods:

$$\Pi_A^{LS}(w_{A1}, w_{A2}, w_{B1}) = w_{A1} s_{A1}^{LS}(w_{A1}, w_{B1}) + w_{A2} q_{A2}^{LS}(w_{A1}, w_{A2}, w_{B1}), \text{ and} \tag{EC.15a}$$

$$\Pi_B^{LS}(w_{A1}, w_{A2}, w_{B1}) = w_{B1}\left(s_{B1}^{LS}(w_{B1}, w_{A1}) + I_B^{LS}(w_{A2}, w_{B1})\right) + \Pi_{B2}^{LS*}\left(q_{A2}^{LS}(w_{A2}, w_{B1}), I_B^{LS}(w_{A2}, w_{B1})\right). \tag{EC.15b}$$

Note that, if the suppliers' contract types are reversed, so that it is supplier A (B) who offers the short-term (long-term) contract, then we denote their total profits-to-go over two periods by $\Pi_A^{SL}(w_{A1}, w_{B2}, w_{B2})$ and $\Pi_B^{SL}(w_{A1}, w_{B2}, w_{B2})$ respectively. We now prove the three parts of Proposition 4, beginning with part *(i)*, followed by part *(iii)*, and finally part *(ii)*.

EC.4.1. Proposition 4, part *(i)*

First, we consider the scenario where the reseller holds $I_A = 0$ and $I_B > 0$ under LS contracts. We denote this scenario with the superscript LSI. By substituting $I_A = 0$ and (12) in (11), we obtain supplier B's second-period profit as a function of the second-period quantity committed to supplier A, and the inventory carried from supplier B:

$$\Pi_{B2}^{LSI}(q_{A2}, I_B) = \Pi_{B2}^{LS}\left(0, I_B, q_{A2}, w_{B2}^{LS*}\right) = \begin{cases} \frac{1}{8}\left(a - 2\left(I_B + dq_{A2}\right)\right)^2 & \text{if } I_B \le \frac{a-2dq_{A2}}{2}, \\ 0 & \text{otherwise.} \end{cases} \tag{EC.16}$$

Since the reseller never holds inventory from supplier B exceeding $\frac{a-2dq_{A2}}{2}$, we focus only on the upper branch of (EC.16). Therefore, $\Pi_{B2FOC}^{LSI} = \Pi_{B2}^{LSI}\left(q_{A2FOC}^{LS}(w_{A2}, w_{B1}), I_{BFOC}^{LS}(w_{A2}, w_{B1})\right) = \frac{2}{9}(w_{B1} + h)^2$. Substituting Π_{B2FOC}^{LSI} along with $s_{i1}^{LSI}(w_{i1}, w_{j1})$, $q_{A2}^{LSI}(w_{A2}, w_{B1})$, and $I_B^{LSI}(w_{A2}, w_{B1})$ from Lemma EC.4 in (EC.15b), we obtain $\Pi_B^{LSI}(w_{A1}, w_{A2}, w_{B1})$. Similarly, substituting $s_{i1}^{LSI}(w_{i1}, w_{j1})$, $q_{A2}^{LSI}(w_{A2}, w_{B1})$, and $I_B^{LSI}(w_{A2}, w_{B1})$ from Lemma EC.4 in (EC.15a) and (13), we obtain $\Pi_A^{LSI}(w_{A1}, w_{A2}, w_{B1})$ and $\pi^{LSI}(w_{A1}, w_{A2}, w_{B1})$. As long as the inventory holding cost is low, both suppliers will anticipate the reseller to hold $I_B > 0$. In stage $1a$, supplier A chooses w_{A1} and w_{A2} while supplier chooses w_{B1}, to maximize $\Pi_A^{LSI}(w_{A1}, w_{A2}, w_{B1})$ and $\Pi_B^{LSI}(w_{A1}, w_{A2}, w_{B1})$ respectively. We apply FOC with respect to w_{A1} and w_{A2} on $\Pi_A^{LSI}(w_{A1}, w_{A2}, w_{B1})$, and with respect to w_{B1} on $\Pi_B^{LSI}(w_{A1}, w_{A2}, w_{B1})$, and solve simultaneously, which yields:

$$w_{A1FOC}^{LSI} = \frac{4a(1-d)(17+d(9+d)) - dh(8+d^2)}{4(34-7d^2)}, \tag{EC.17a}$$

$$w_{A2FOC}^{LSI} = \frac{4a(1-d)(17+d(9+d)) + 15dh(4-d^2)}{4(34-7d^2)}, \text{ and} \tag{EC.17b}$$

$$w_{B1FOC}^{LSI} = \frac{18a(2+d)(1-d) - h(8+d^2)}{2(34-7d^2)}. \tag{EC.17c}$$

Substituting (EC.17a) through (EC.17c) in $s_{i1}^{LSI}(w_{i1}, w_{j1})$, $q_{A2}^{LSI}(w_{A2}, w_{B1})$, and $I_B^{LSI}(w_{A2}, w_{B1})$ from Lemma EC.4, and since $w_{A2} \le w_{A1} + h$, we obtain $s_{A1FOC}^{LSI} =$

$q^{LSI}_{A1FOC} = \frac{4a(1-d)(17+d(9+d))-dh(8+d^2)}{8(1-d)(1+d)(34-7d^2)}$, $s^{LSI}_{B1FOC} = \frac{(8+d^2)(4a(2+d)(1-d))+h(2-d^2))}{8(1-d)(1+d)(34-7d^2)}$, $q^{LSI}_{A2FOC} = s^{LSI}_{A2FOC} = \frac{4a(1-d)(17+d(9+d))+15dh(4-d^2)}{8(1-d)(1+d)(34-7d^2)}$, $I^{LSI}_{AFOC} = 0$, and $I^{LSI}_{BFOC} = \frac{(2+d)(4a(1-d)(5+4d^2)-5h(2-d)(8-5d^2))}{8(1-d)(1+d)(34-7d^2)}$. Substituting these in (12) and (EC.14), respectively, we obtain $w^{LSI}_{B2FOC} = \frac{(2+d)(6a(1-d)+5h(2-d))}{34-7d^2}$, $q^{LSI}_{B2FOC} = \frac{(2+d)(6a(1-d)+5(2-d)h)}{2(34-7d^2)}$. Since $q_{i1} = s_{i1} + I_i$ and $s_{i2} = q_{i2} + I_i$, we obtain q^{LSI}_{B1FOC} and s^{LSI}_{B2FOC}. Substituting w^{LSI}_{A1FOC}, w^{LSI}_{A2FOC}, and w^{LSI}_{B1FOC} in functions $\Pi^{LSI}_{A}(w_{A1}, w_{A2}, w_{B1})$, $\Pi^{LSI}_{B}(w_{A1}, w_{A2}, w_{B1})$, and $\pi^{LSI}(w_{A1}, w_{A2}, w_{B1})$, we obtain the total profits of supplier A, supplier B, and the reseller, given by Π^{LSI}_{AFOC}, Π^{LSI}_{BFOC}, and π^{LSI}_{FOC}, respectively, when inventory is held under LS. These profits are given in Table 3, under column LSI.

We now describe how to derive $h^{LS}_{I}(d)$, and show that when $h < h^{LS}_{I}(d)$, there is an equilibrium in pure strategies where neither the reseller, nor either supplier deviates from above wholesale prices and quantities, which are all positive, $I_A = 0$, and $I_B > 0$. First, by solving $I^{LSI}_{BFOC} = 0$ for h, we obtain the threshold $\overline{h}^{LS}(d) = \frac{4a(1-d)(5+4d^2)}{5(2-d)(8-5d^2)}$, such that if supplier A offers w^{LSI}_{A1FOC} and w^{LSI}_{A2FOC} while supplier B offers w^{LSI}_{B1FOC}, the reseller's best response is to hold $I^{LSI}_{BFOC} > 0$ as long as $h < \overline{h}^{LS}(d)$, since in this region, the condition $w_{B1} \geq \overline{w}^{LS}_{B1}(w_{A2})$ from Lemma EC.4 is satisfied. Therefore, when $h < \overline{h}^{LS}(d)$, the reseller does not unilaterally deviate from the above quantities and inventory.

Suppose supplier B deviates when $h < h^{LS}_{I}(d)$, and offers $w_{B1} > \overline{w}^{LS}_{B1}(w^{LSI}_{A2FOC})$ such that $I_B = 0$, while supplier A offers w^{LSI}_{A1FOC} and w^{LSI}_{A2FOC}. Substituting w^{LSI}_{A1FOC} and w^{LSI}_{A2FOC} we obtain supplier B's profit $\Pi^{LSI}_{B}(w_{B1})$ as a function of w_{B1}. Applying FOC with respect to w_{B1} on $\Pi^{LSI}_{BdevNI_B}(w_{B1})$, we obtain $w^{LSI}_{B1devNI_B} = \frac{4a(1-d)(d+2)(17+d^2)-hd^2(8+d^2)}{8(34-7d^2)}$, which needs to exceed $\overline{w}^{LS}_{B1}(w^{LSI}_{A2FOC})$. Solving $w^{LSI}_{B1devNI_B} - \overline{w}^{LS}_{B1}(w^{LSI}_{A2FOC}) = 0$, we determine the condition to be $h > \underline{h}^{LS}_{B1}(d) = \frac{4a(1-d)(2+d^2)(17+d^2)}{(2-d)(272-154d^2-d^4)}$. Substituting $w^{LSI}_{B1devNI_B}$ in $\Pi^{LSI}_{B}(w_{B1})$, we find supplier B's optimal profit $\Pi^{LSI}_{BdevNI_B}$ from inducing $I_B = 0$. We confirm that $\Pi^{LSI}_{BdevNI_B}$, Π^{LSI}_{BFOC}, and $\Pi^{LSI}_{BFOC} - \Pi^{LSI}_{BdevNI_B}$ are convex in h for $d \in [0, 1)$.

Solving $\Pi^{LSI}_{BFOC} - \Pi^{LSI}_{BdevNI_B} = 0$ for h, we obtain the solution $h = h^{LS}_{I}(d)$, such that (a) $\underline{h}^{LS}_{B1}(d) \leq h^{LS}_{I}(d) \leq \overline{h}^{LS}(d)$, and (b) $\frac{\partial(\Pi^{LSI}_{BFOC} - \Pi^{LSI}_{BdevNI_B})}{\partial h} < 0$ at $h = h^{LS}_{I}(d)$. This implies that it is feasible and beneficial for supplier B to induce the reseller to hold $I_B = 0$ when supplier A offers w^{LSI}_{A1FOC} and w^{LSI}_{A2FOC} if and only if $h > h^{LS}_{I}(d)$. We similarly establish that no other deviations are feasible or beneficial to either supplier when $h < h^{LS}_{I}(d)$, and we omit the details for the sake of brevity.

EC.4.2. Proposition 4, part *(iii)*

We now consider the scenario where the reseller holds $I_A, I_B = 0$ under LS contracts, denoted with superscript $LSNI$. The reseller's profit-to-go $\pi^{LSNI}(s_{A1}, q_{A2}, s_{B1})$ is obtained by substituting $I_B = 0$ in (13). By applying FOC on $\pi^{LSNI}(s_{A1}, q_{A2}, s_{B1})$ with respect to s_{A1}, q_{A2}, and s_{B1}, we obtain: $s^{LSNI}_{i1}(w_{i1}, w_{j1}) = \frac{a(1-d)-w_{i1}+dw_{j1}}{2(1-d)(1+d)}$ $\forall i \in \{A, B\}$, $j \neq i$, and $q^{LSNI}_{A2}(w_{A2}, w_{B1}) = s^{LSNI}_{A2}(w_{A2}) = \frac{a(4-d)-4w_{A2}}{2(2-d)(2+d)}$. Substituting these in (EC.15a) and $\pi^{LSNI}(s_{A1}, q_{A2}, s_{B1})$, we obtain the total profits of supplier A and the reseller, denoted by $\Pi_A^{LSNI}(w_{A1}, w_{A2}, w_{B1})$ and $\pi^{LSNI}(w_{A1}, w_{A2}, w_{B1})$, respectively. Substituting $I_B = 0$ and $q_{A2}^{LSNI}(w_{A2}, w_{B1})$ in (EC.16), we obtain $\Pi_{B2}^{LSNI}(w_{A2}) = \frac{1}{8}\left(a - \frac{2d(a(4-d)-4w_{A2})}{2(2-d)(2+d)}\right)^2$. We substitute $\Pi_{B2}^{LSNI}(w_{A2})$ and $s_{i1}^{LSNI}(w_{i1}, w_{j1})$ in (EC.15b) to obtain supplier B's total profit, $\Pi_B^{LSNI}(w_{A1}, w_{A2}, w_{B1})$.

By simultaneously applying FOC with respect to w_{A1}, w_{A2}, and w_{B1} on $\Pi_A^{LSNI}(w_{A1}, w_{A2}, w_{B1})$ and $\Pi_B^{LSNI}(w_{A1}, w_{A2}, w_{B1})$, we obtain the wholesale price offers under $LSNI$: $w_{A1FOC}^{LSNI} = w_{B1FOC}^{LSNI} = a\left(\frac{1-d}{2-d}\right)$, and $w_{A2FOC}^{LSNI} = \frac{a}{8}(4-d)$. Substituting these in $s_{i1}^{LSNI}(w_{i1}, w_{j1})$ and $q_{A2}^{LSNI}(w_{A2}, w_{B1})$, and since $I_A = 0$, $I_B = 0$, we obtain: $s_{A1FOC}^{LSNI} = q_{A1FOC}^{LSNI} = s_{B1FOC}^{LSNI} = q_{B1FOC}^{LSNI} = \frac{a}{2(1+d)(2-d)}$, $q_{A2FOC}^{LSNI} = s_{A2FOC}^{LSNI} = \frac{a(4-d)}{4(2+d)(2-d)}$, and $I_{AFOC}^{LSNI} = I_{BFOC}^{LSNI} = 0$. Substituting the above in (12) and (EC.14), respectively, we obtain $w_{B2FOC}^{LSNI} = \frac{a(8-d(4+d))}{4(2+d)(2-d)}$, and $q_{B2FOC}^{LSNI} = s_{B2FOC}^{LSNI} = \frac{a(8-d(4+d))}{8(2+d)(2-d)}$. Substituting $w_{A1FOC}^{LSNI} = w_{B1FOC}^{LSNI}$ and w_{A2FOC}^{LSNI} in $\Pi_A^{LSNI}(w_{A1}, w_{A2}, w_{B1})$, $\Pi_B^{LSNI}(w_{A1}, w_{A2}, w_{B1})$, and $\pi^{LSNI}(w_{A1}, w_{A2}, w_{B1})$, we obtain the total profits of supplier A, supplier B, and the reseller, given by Π_{AFOC}^{LSNI}, Π_{BFOC}^{LSNI}, and π_{FOC}^{LSNI}, respectively, when inventory is not held under LS. These profits are given in Table 3, under the column $LSNI$.

We now describe how to derive $h_{NI}^{LS}(d)$, and show that when $h \geq h_{NI}^{LS}(d)$, there is an equilibrium in pure strategies where neither the reseller, nor either supplier deviates from the above wholesale prices and quantities, which are all positive, and $I_A = I_B = 0$. First, note that the reseller's best response, when both suppliers offer the no-inventory wholesale price, is $I_B^{LS}(w_{A2FOC}^{LSNI}, w_{B1FOC}^{LSNI}) = 0$ when $h \geq \overline{h}^{LSNI}(d) = \frac{a(8-d(4-5d))}{8(2+d)(2-d)}$. As we will soon see, $\overline{h}^{LSNI}(d) \leq h_{NI}^{LS}(d)$, therefore the reseller does not unilaterally deviate to hold $I_B > 0$ when $h \geq h_{NI}^{LS}(d)$.

Suppose supplier B deviates from the $LSNI$ equilibrium wholesale prices given by (EC.17a) through (EC.17c), and offers w_{B1} such that $w_{B1} < \overline{w}_{B1}^{LS}(w_{A2FOC}^{LSNI})$ (see Lemma EC.4), resulting in $I_B > 0$. His profit from deviating is given by $\Pi_{Bdev}^{LSNI}(w_{B1})$, substituting (EC.17a) and (EC.17b) in (EC.15b). Applying FOC on $\Pi_{Bdev}^{LSNI}(w_{B1})$ with respect to w_{B1}, we obtain $w_{B1devposI_B}^{LSNI} = \frac{9a(4-d)(8-d(6+d))-8(2-d)\left(4+5d^2\right)h}{16(2-d)\left(17+d^2\right)}$. We find that $\overline{w}_{B1}^{LS}(w_{A2FOC}^{LSNI}) - w_{B1devposI_B}^{LSNI}$ is decreasing in h for all $d \in [0,1)$, and $w_{B1devposI_B}^{LSNI} < \overline{w}_{B1}^{LS}(w_{A2FOC}^{LSNI})$ (i.e., $I_B > 0$ can be induced) as long as $h < \widetilde{h}_{BdevposI_B}^{LSNI}(d) = \frac{a(80-d(40-d(66-5d(4+d))))}{8(2-d)(2+d)\left(10-d^2\right)}$. Substituting $w_{B1devposI_B}^{LSNI}$ in $\Pi_{Bdev}^{LSNI}(w_{B1})$, supplier B's maximum profit from deviation is $\Pi_{BdevposI_B}^{LSNI}$.

We verify that $\Pi_{BdevposI_B}^{LSNI*}$ and $\Pi_{BdevposI_B}^{LSNI*} - \Pi_{BFOC}^{LSNI}$ are convex in h. Solving $\Pi_{BdevposI_B}^{LSNI*} - \Pi_{BFOC}^{LSNI} = 0$ for h, we find that $h = h_{NI}^{LS}(d)$ is the solution that satisfies $h \geq \overline{h}^{LSNI}(d)$. Furthermore, (a) $\frac{\partial}{\partial h}\left(\Pi_{BdevposI_B}^{LSNI*} - \Pi_{BFOC}^{LSNI}\right) < 0$ at $h = h_{NI}^{LS}(d)$, (b) $\widetilde{h}_{BdevposI_B}^{LSNI}(d) - h_{NI}^{LS}(d) \geq 0$ $\forall d \in [0,1)$, and (c) $\Pi_{BdevposI_B}^{LSNI*} - \Pi_{BFOC}^{LSNI} \leq 0$ at $h = \widetilde{h}_{BdevposI_B}^{LSNI}(d)$, $\forall d \in [0,1)$. This implies that it is feasible and beneficial for supplier B to induce the reseller to hold $I_B > 0$ when supplier A offers w_{A1FOC}^{LSNI} and w_{A2FOC}^{LSNI} if and only if $h < h_{NI}^{LS}(d)$. We similarly show that no other deviations are feasible or beneficial for either supplier when $h \geq h_I^{LS}(d)$, and we omit the details for the sake of brevity.

EC.4.3. Proposition 4, part *(ii)*

Now we compare the two thresholds on h, denoted by $h_I^{LS}(d)$ and $h_{NI}^{LS}(d)$, given by the proofs of parts *(i)* and *(iii)* of Proposition 4 above, respectively. It can be verified that $h_{NI}^{LS}(d) = h_I^{LS}(d) = \frac{a}{4}$ only at $d = 0$, and that there exist no $a > 0$ and no $d \in [0,1)$ for which $h_{NI}^{LS}(d) < h_I^{LS}(d)$. Thus, for all $d \in [0,1)$, $h_{NI}^{LS}(d) \geq h_I^{LS}(d)$, and the inequality is strict for all $d > 0$. Since the pure strategy equilibria described in parts *(i)* and *(ii)* of Proposition 4 only hold for $h < h_I^{LS}(d)$ and $h \geq h_{NI}^{LS}(d)$, respectively, when $h \in \left(h_I^{LS}(d), h_{NI}^{LS}(d)\right)$, there exists an equilibrium to the wholesale pricing game under LS contracts only in mixed strategies for supplier B.

If there exists an equilibrium in which supplier B randomizes between two wholesale prices, denoted $w_{B1}^{LS1} < w_{B1}^{LS2}$ with probabilities λ^{LS} and $(1-\lambda^{LS})$ respectively, we need the following to be true: *(i)* The long-term contract prices offered by supplier A, denoted w_{A1}^{LSm} and w_{A2}^{LSm}, must maximize his profits given the randomized strategy of supplier B; *(ii)* Supplier B's lower price, w_{B1}^{LS1}, must maximize his profits among all prices low enough to induce the reseller to hold inventory conditional upon supplier A's long-term contract prices, while supplier B's higher price, w_{B1}^{LS2}, must maximize his profits among all price high enough to discourage inventory from being held conditional upon supplier A's long-term contract prices; and *(iii)* Supplier B must be indifferent between offering w_{B1}^{LS1} and offering w_{B1}^{LS2}. If some set of w_{A1}^{LSm}, w_{A2}^{LSm}, w_{B1}^{LS1}, w_{B1}^{LS2}, and $\lambda^{LS} \in [0,1]$, satisfies these three conditions, then neither firm would have an incentive to deviate, and this would be an equilibrium.

Let $\Pi_A^{LSI}(w_{A1}, w_{A2}, w_{B1})$ be the profit earned by supplier A when $w_{B1} \leq \overline{w}_{B1}^{LS}(w_{A2})$ so that the reseller holds inventory, and let $\Pi_A^{LSNI}(w_{A1}, w_{A2}, w_{B1})$ be the profit earned by supplier A when $w_{B1} > \overline{w}_{B1}^{LS}(w_{A2})$ and the reseller does not hold inventory. Similarly, define $\Pi_B^{LSI}(w_{A1}, w_{A2}, w_{B1})$ and $\Pi_B^{LSNI}(w_{A1}, w_{A2}, w_{B1})$ as the profits earned by supplier B depending on whether w_{B1} is low enough to induce the reseller to hold inventory. We have previously established the concavity of these profit functions. For a given value of λ^{LS}, let $w_{A1}^{LSm}(\lambda^{LS}), w_{A2}^{LSm}(\lambda^{LS}), w_{B1}^{LS1}(\lambda^{LS}), w_{B1}^{LS2}(\lambda^{LS})$ be the solution to the following set of equations:

$$\frac{d}{dw_{A1}}\left(\lambda^{LS}\Pi_A^{LSI}\left(w_{A1}, w_{A2}, w_{B1}^{LS1}\right) + \left(1-\lambda^{LS}\right)\Pi_A^{LSNI}\left(w_{A1}, w_{A2}, w_{B1}^{LS2}\right)\right) = 0, \tag{EC.18a}$$

$$\frac{d}{dw_{A2}}\left(\lambda^{LS}\Pi_A^{LSI}\left(w_{A1}, w_{A2}, w_{B1}^{LS1}\right) + \left(1-\lambda^{LS}\right)\Pi_A^{LSNI}\left(w_{A1}, w_{A2}, w_{B1}^{LS2}\right)\right) = 0, \tag{EC.18b}$$

$$\frac{d}{dw_{B1}^{LS1}}\Pi_B^{LSI}\left(w_{A1}, w_{A2}, w_{B1}^{LS1}\right) = 0, \text{ and } \frac{d}{dw_{B1}^{LS2}}\Pi_B^{LSNI}\left(w_{A1}, w_{A2}, w_{B1}^{LS2}\right) = 0 \tag{EC.18c}$$

It follows that, for any $\lambda^{LS} \in [0,1]$, the first and second-period prices, $w_{A1}^{LSm}(\lambda^{LS})$ and $w_{A2}^{LSm}(\lambda^{LS})$ are supplier A's best response to supplier B's randomization. Similarly, $w_{B1}^{LS1}(\lambda^{LS})$ is supplier B's best response to $w_{A1}^{LSm}(\lambda^{LS})$ and $w_{A2}^{LSm}(\lambda^{LS})$ among all prices low enough to induce inventory to be held, while $w_{B1}^{LS1}(\lambda^{LS})$ is supplier B's best response among all prices high enough to discourage the reseller from holding inventory. As long as there exists a value of $\lambda^{LS*} \in (0,1)$ for which supplier B is indifferent between the two prices, i.e.,

$\Pi_B^{LSI}\left(w_{A1}^{LSm}\left(\lambda^{LS*}\right), w_{A2}^{LSm}\left(\lambda^{LS*}\right), w_{B1}^{LS1}\left(\lambda^{LS*}\right)\right) = \Pi_B^{LSNI}\left(w_{A1}^{LSm}\left(\lambda^{LS*}\right), w_{A2}^{LSm}\left(\lambda^{LS*}\right), w_{B1}^{LS2}\left(\lambda^{LS*}\right)\right)$, neither supplier will have an incentive to deviate, and we will have an equilibrium in mixed strategies. Although finding the expression for λ^{LS*} is challenging, it is possible to numerically confirm that for all $h \in (h_I^{LS}(d), h_{NI}^{LS}(d))$, there exists $\lambda^{LS*} \in (0,1)$ that solves the above equation.

EC.5. Proof of Proposition 5

For this analysis, we consider the scenario in which both suppliers decide their respective wholesale prices after learning the type of contract of the competitor. Supplier $i \in \{A, B\}$ chooses to offer the L contract with probability $\alpha_i \in [0,1]$, and the S contract with probability $1-\alpha_i$. For any probability, α_B, with which supplier B offers a long-term contract, supplier A's expected profits from offering the L and the S contracts are given by: $\Pi_A^{LI}(\alpha_B) = \alpha_B \Pi_A^{LL*} + (1-\alpha_B)\Pi_A^{LSI*}$ and $\Pi_A^{SI}(\alpha_B) = \alpha_B \Pi_A^{SLI*} + (1-\alpha_B)\Pi_A^{SSI*}$, respectively. By substituting Π_A^{LL*}, Π_A^{SSI*}, Π_A^{LSI*}, and $\Pi_A^{SLI*} = \Pi_B^{LSI*}$ from Table 2, we obtain $\Pi_A^{LI}(\alpha_B)$ and $\Pi_A^{SI}(\alpha_B)$. By symmetry, we can also define $\Pi_B^{LI}(\alpha_A)$ and $\Pi_B^{LI}(\alpha_A)$ to be supplier B's expected profits from offering the L and S contracts given that supplier A offers a long-term contract with probability α_A. Of course, $\Pi_A^{SLI*} = \Pi_B^{LSI*}$, and $\Pi_A^{LSI*} = \Pi_B^{SLI*}$. There are three possibilities for supplier A: First, if $\Pi_A^{LI}(\alpha_B) > \Pi_A^{SI}(\alpha_B)$, then supplier A's best response to α_B is to set $\alpha_A = 1$. Second, if $\Pi_A^{LI}(\alpha_B) < \Pi_A^{SI}(\alpha_B)$, then supplier A's best response to α_B is to set $\alpha_A = 0$. Finally, if $\Pi_A^{LI}(\alpha_B) = \Pi_A^{SI}(\alpha_B)$, then any $\alpha_A \in [0,1]$ is the best response to α_B. Supplier B has a symmetric set of three possibilities.

In order to have a pure strategy equilibrium in which both suppliers offer short-term contracts, we must have $\Pi_A^{SI}(\alpha_B) - \Pi_A^{LI}(\alpha_B) \geq 0$ for $\alpha_B = 0$. We illustrate the analysis with the special case of $h = 0$, such that the reseller will hold a positive inventory in any equilibrium in which a short-term contract is offered by at least one supplier. Substituting $\alpha_B = 0$ into $\Pi_A^{LI}(\alpha_B)$ and $\Pi_A^{SI}(\alpha_B)$, and simplifying, we have: $\Pi_A^{SI}(0) - \Pi_A^{LI}(0) = \frac{a^2(1-d)\Gamma}{(1+d)\left(34-7d^2\right)^2\left(102-81d+9d^2+8d^3-2d^4\right)^2}$, where $\Gamma = 176868 - 530604d - 65025d^2 + 327828d^3 - 90070d^4 - 45594d^5 + 34374d^6 - 3100d^7 - 4003d^8 + 896d^9 + 186d^{10} - 40d^{11} - 4d^{12}$. The denominator is positive for $d \in (0,1)$, and the numerator equals $176868a^2 > 0$ when $d = 0$. By continuity, it follows that for sufficiently small values of d, there exists a pure strategy equilibrium in which both suppliers offer short-term contracts, i.e., $\alpha_A = \alpha_B = 0$. We establish that the analysis and the result hold for $h < \frac{a}{4}$, and omit the details for brevity. Of course, as $h \to \frac{a}{4}$, the highest value of d for which there exists a pure strategy equilibrium in which both suppliers offer short-term contracts also approaches zero.

In order to have a mixed strategy equilibrium, we must have some $\alpha_i, \alpha_j \in (0,1)$ such that each supplier is indifferent between offering a long-term and a short-term contract. For supplier i, this means that we need: $\Pi_i^{SI}(\alpha_j) - \Pi_i^{LI}(\alpha_j) = 0$. Denote by $\alpha_i^I(d)$ the solution to the equation $\Pi_i^{LI}(\alpha_j) - \Pi_i^{SI}(\alpha_j) = 0$. It can be confirmed that $\alpha_i^I(d) \leq 1$ and increases in d for all $d \in [0,1)$. By substituting $d = 1$, we can also confirm that $\alpha_i^I(d) = 1$ for $d = 1$. It follows that for sufficiently large values of d within the interval $[0,1)$, there exists an equilibrium in which $\alpha_A = \alpha_B = \alpha_B^I(d)$, and each supplier is indifferent between offering a short-term or a long-term contract.

EC.6. Probability of Offering L-Contract without Strategic Inventory

Consider a scenario where in neither of the SS or LS contracts, the reseller holds inventory in equilibrium (e.g., when $h > \max\{h_I^{SS}(d), h_{NI}^{LS}(d)\}$). When supplier B offers the L contract with probability α_B, supplier A's expected profits from offering the L and the S contract are given by

$$\Pi_A^{LNI}(\alpha_B) = \alpha_B \Pi_A^{LL*} + (1-\alpha_B)\Pi_A^{LSNI*}, \text{ and } \Pi_A^{SNI}(\alpha_B) = \alpha_B \Pi_A^{SLNI*} + (1-\alpha_B)\Pi_A^{SSNI*}, \quad \text{(EC.19)}$$

respectively. Note that due to the symmetric nature of the suppliers, $\Pi_A^{SLNI*} = \Pi_B^{LSNI*}$. Substituting Π_A^{LL*} and Π_A^{SSNI*} from Table 2, and Π_A^{LSNI*} and $\Pi_A^{SLNI*} = \Pi_B^{LSNI*}$ from Table 3, and setting $h = 0$, we obtain $\Pi_A^{LNI}(\alpha_B)$ and $\Pi_A^{SNI}(\alpha_B)$. To have a pure strategy equilibrium in which both suppliers offer long-term contracts, we must have $\Pi_i^{LNI}(\alpha_j) - \Pi_i^{SNI}(\alpha_j) \geq 0$ for $\alpha_j = 1$, where $i, j \in \{A, B\}$ and $j \neq i$. Substituting $\alpha_j = 1$ into (EC.19) and simplifying, we have the following: $\Pi_i^{LNI}(1) - \Pi_A^{SNI}(1) = \frac{d^2(16-24d-9d^2-d^3)}{32(1+d)(4-d^2)^2}$. The denominator of the above is positive for all $d \in [0,1]$. Although the numerator is equal to zero at $d = 0$, because the second term in the numerator decreases in d and is strictly positive at $d = 0$, it follows that $\Pi_i^{LNI}(1) - \Pi_A^{SNI}(1) \geq 0$ for sufficiently small values of $d \in [0,1]$. This implies that there must exist a pure strategy equilibrium in which both suppliers offer long-term contracts, i.e., $\alpha_A = \alpha_B = 1$.

As before, to have a mixed strategy equilibrium, there must exist $\alpha_i, \alpha_j \in (0,1)$ such that each supplier is indifferent between offering a long-term and a short-term contract. Let us denote by $\alpha^{NI}(d)$ the value of α_B that solves $\Pi_A^{LNI}(\alpha_B) - \Pi_A^{SNI}(\alpha_B) = 0$, given by: $\alpha^{NI}(d) = \frac{32+4d+12d^2+7d^3-d^4}{32-12d+36d^2+16d^3}$. It is easy to see that both the numerator and the denominator of $\alpha^{NI}(d)$ are positive, which implies $\alpha^{NI}(d) > 0$, for all $d \in [0,1)$. It can also be confirmed that $\alpha^{NI}(d)$ increases and then decreases in the interval $d \in [0,1]$, and that $\alpha^{NI}(0) = 1$, while $\alpha^{NI}(1) = \frac{3}{4}$. By continuity, it follows that for sufficiently large $d \in [0,1]$, there must be a mixed strategy equilibrium in which each supplier chooses a long-term contract with probability $\alpha_A = \alpha_B = \alpha^{NI}(d)$.

EC.7. Proof of Proposition 6

For this analysis, we consider that a supplier's choice of contract type is hidden from the competitor. Supplier i offers a long-term contract with probability β_i, and cannot learn about the type of contract of supplier j before making the wholesale price(s) corresponding to his own contract type. Note that since suppliers A and B are symmetric, for $k \in \{I, NI\}$, $\Pi_A^{SLk}(w_{B1}, w_{B2}, w_{A1}) = \Pi_B^{LSk}(w_{B1}, w_{B2}, w_{A1})$, and $\Pi_B^{SLk}(w_{B1}, w_{B2}, w_{A1}) = \Pi_A^{LSk}(w_{B1}, w_{B2}, w_{A1})$ where Π_B^{LSI}, Π_A^{LSI}, Π_A^{LSNI}, and Π_B^{LSNI} are obtained from Section EC.4. Because $h_I^{SS}(d) > 0$ and $h_I^{LS}(d) > 0$ for all $d < 1$, we are assured that for $h = 0$, the reseller will maintain a positive inventory in any equilibrium in which a short-term contract is offered by at least one supplier. Substituting $\Pi_i^{SSI}(w_{i1S}, w_{j1S})$, $i \in \{A, B\}$ from (EC.7), $\Pi_A^{SLI}(w_{B1L}, w_{B2L}, w_{A1S})$ and $\Pi_B^{LSI}(w_{A1L}, w_{A2L}, w_{B1S})$ from Section EC.4 in (14a) and (14b), and setting $h = 0$, we obtain the profit of the supplier i under hidden contract types (H), by offering an S-type (short-term) contract, in a low-h scenario such that inventory could be held (I), denoted Π_i^{HSI}. Similarly, substituting $\Pi_i^{LL}(w_{i1L}, w_{i2L}, w_{j1L}, w_{j2L})$, $i \in \{A, B\}$ from (4), $\Pi_B^{SLI}(w_{B1L}, w_{B2L}, w_{A1S})$ and $\Pi_A^{LSI}(w_{A1L}, w_{A2L}, w_{B1S})$ from Section EC.4, in (15a) and (15b), and setting $h = 0$, we obtain supplier i's profit under hidden contract types (H), by offering an L-type (long-term) contract, in a low-h scenario such that inventory could be held (I), denoted Π_i^{HLI}.

By twice differentiating Π_i^{HSI} with respect to w_{i1S}, we can confirm that Π_i^{HSI} is concave in w_{i1S}. Similarly, we can confirm that $\frac{d^2\Pi_i^{HLI}}{dw_{i1L}^2} = \frac{d^2\Pi_i^{HLI}}{dw_{i2L}^2} = \frac{-1}{1-d^2}$, while $\frac{d^2\Pi_i^{HLI}}{dw_{i1L}dw_{i2L}} = 0$, so that Π_i^{HLI} is jointly concave in w_{i1L} and w_{i2L}. We simultaneously apply the FOC on Π_i^{HSI} and Π_i^{HLI}, i.e., solve $\frac{d\Pi_i^{HS}}{dw_{i1S}} = 0$, $\frac{d\Pi_i^{HL}}{\partial dw} = 0$, $\frac{d\Pi_i^{HL}}{dw_{i2L}} = 0$, $i = 1, 2$, and obtain, when $h = 0$, $w_{i1S}^{H*}(\beta_i, \beta_j) = w_{i1S}^{HI}(\beta_i, \beta_j)$, and $w_{itL}^{H*}(\beta_i, \beta_j) = w_{itL}^{HI}(\beta_i, \beta_j)$, $\forall t \in \{1, 2\}$, $\forall i, j \in \{A, B\}$, $j \neq i$. Here w_{i1S}^{H*} denotes supplier i's first-period wholesale price offer if he chooses a short-term contract, and w_{itL}^{H*} is his t-th period wholesale price offer if he chooses a long-term contract, where the probability of him choosing a long-term contract is β_i, and β_j is his belief about the probability of his rival choosing a long-term contract. Substituting w_{i1S}^{H*} and w_{itL}^{H*} into Π_i^{HSI} and Π_i^{HLI}, we obtain supplier i's profit from offering the wholesale prices given by w_{i1S}^{H*} and w_{itL}^{H*}, where $\forall i, j \in \{A, B\}$, $j \neq i$: $\Pi_i^{HSI*}(\beta_i, \beta_j) = \Pi_i^{HS}\left(w_{i1S}^{HI}(\beta_i, \beta_j), w_{j1S}^{HI}(\beta_j, \beta_i), w_{j1L}^{HI}(\beta_j, \beta_i), w_{j2L}^{HI}(\beta_j, \beta_i), \beta_j\right)$, and $\Pi_i^{HLI*}(\beta_i, \beta_j) = \Pi_i^{HL}\left(w_{i1L}^{HI}(\beta_i, \beta_j), w_{i2L}^{HI}(\beta_i, \beta_j), w_{j1S}^{HI}(\beta_j, \beta_i), w_{j1L}^{HI}(\beta_j, \beta_i), w_{j2L}^{HI}(\beta_j, \beta_i), \beta_j\right)$.

Substituting the FOC wholesale prices, for $\beta_i = \beta_j = 0$, we have: $\Pi_i^{HSI*}(0,0) - \Pi_i^{HLI*}(0,0) = \frac{612-2448d+2295d^2-105d^3-463d^4+97d^5+16d^6-4d^7}{4(1+d)(102+d(-81+d(9+2(4-d)d)))^2}$. The denominator of this expression is clearly positive, and it is easy to see that the numerator is positive at $d = 0$. By continuity, it follows that for sufficiently small d, we have $\Pi_i^{HSI*}(0,0) - \Pi_i^{HLI*}(0,0) \geq 0$, which ensures that there is a pure strategy equilibrium in which both suppliers offer short-term contracts, i.e., $\beta^I(d) = 0$. In order to have a mixed strategy equilibrium, there must exist $\beta_i, \beta_j \in (0,1)$ such that each supplier is indifferent between offering a long-term and a short-term contract. For supplier i, this means that we need: $\Pi_i^{HSI*}(\beta_i, \beta_j) - \Pi_i^{HLI*}(\beta_i, \beta_j) = 0$. It can be confirmed that $\Pi_i^{HSI*}(\beta, \beta) - \Pi_i^{HLI*}(\beta, \beta) = 0$ for $\beta = \beta^I(d)$, such that $\beta^I(d)$ increases in $d \in [0,1]$ and $\lim_{d \to 1} \beta^I(d) = 1$. It follows that when d is sufficiently large, there exists a mixed strategy equilibrium in which each supplier offers a long-term contract with probability $\beta^I(d) \in (0,1)$ and a short-term contract with probability $1 - \beta^I(d)$.

EC.8. Prisoner's Dilemma in One-way Commitment Contracts

In the one-way wholesale-price-only *commitment* contracts described in Anand et al. (2008), Desai et al. (2010), and Li et al. (2022), the suppliers announce their wholesale prices for both periods at the beginning of the first period (stage $1a$ of our model in Table 1), while there is no commitment of future purchase quantity from the reseller, unlike the L contract. Let us refer to such a *commitment* contract as a C-contract. Note that when both suppliers offer C contracts, the reseller holds inventory from neither supplier, and the wholesale prices, purchase quantities, and selling prices are indistinguishable from those in the LL contracts (see, e.g., Desai et al. 2010). The other type of contract in the strategic inventory literature is the *dynamic* (D) contract, which is identical to the short-term (S) contract in our model. We show that the trade-offs that arise when suppliers choose between long-term and short-term contracts are different from when they choose between commitment and dynamic contracts. Now we focus on when one supplier (e.g., A) offers the C contract, while the other supplier (B) offers the D contract.

In stage $2b$, the reseller determines q_{A2} and q_{B2} in response to wholesale price offers w_{A2} (announced in stage $1a$) and w_{B2} (announced in stage $2a$) from suppliers A and B, respectively. These quantities are functionally identical to (EC.2), with $I_A = 0$, since supplier A does not set $w_{A1} + h < w_{A2}$ while offering a long-term contract. In stage $2a$, supplier B determines w_{B2} to maximize Π_{B2}, which is given by setting $I_A = 0$ in (EC.3). Since supplier B does not set w_{B1} so low that the reseller would hold very high I_B precluding $q_{B2} > 0$, supplier B's second-period wholesale price offer is given by: $w_{B2}^{CD}(w_{A2}, I_B) = \frac{1}{2}\left(a(1-d) - 2(1-d^2) I_B + dw_{A2}\right)$. In response, the reseller buys $q_{A2}^{CD}(w_{A2}, I_B) = \frac{a(1-d)(2+d)-\left(2-d^2\right)w_{A2}}{4\left(1-d^2\right)} - \frac{dI_B}{2}$ and $q_{B2}^{CD}(w_{A2}, I_B) = \frac{a(1-d)+dw_{A2}}{4\left(1-d^2\right)} - \frac{I_B}{2}$, obtained by substituting w_{B2}^{CD} into (EC.2). It can be verified that, in equilibrium, neither w_{B2} nor I_B is so high that it results in $q_{A2} = 0$ or $q_{B2} = 0$. Substituting $q_{A2}^{CD}(w_{A2}, I_B)$, $q_{B2}^{CD}(w_{A2}, I_B)$, and $w_{B2}^{CD}(w_{A2}, I_B)$ into (EC.1) and (EC.3), we obtain the *SPE* profits of the reseller and suppliers, indicated by $\pi_2^{CD*}(w_{A2}, I_B)$, $\Pi_{A2}^{CD*}(w_{A2}, I_B)$, and $\Pi_{B2}^{CD*}(w_{A2}, I_B)$, respectively.

In stage $1b$, in response to the suppliers' wholesale price offers w_{A1}, w_{A2}, and w_{B1}, the reseller determines s_{A1}, s_{B1}, $I_A = 0$, and I_B, which maximize her total profit π^{CD}, given by substituting $\pi_2^{CD}(w_{A2}, I_B)$ in (EC.5). Applying FOC on π^{CD}, we observe that $s_{A1}^{CD}(w_{A1}, w_{B1}, w_{A2})$ and $s_{B1}^{CD}(w_{A1}, w_{B1}, w_{A2})$ are functionally identical to $s_{i1}^{SS}(w_{i1}, w_{j1})$, $i \in \{A, B\}$, $j \neq i$, from (EC.6); while $I_B^{CD}(w_{A1}, w_{B1}, w_{A2}) = \frac{3a(1-d)+3dw_{A2}+4(w_{B1}+h)}{6\left(1-d^2\right)}$. We substitute I_B^{CD} in $\Pi_{A2}^{CD*}(w_{A2}, I_B)$ and $\Pi_{B2}^{CD*}(w_{A2}, I_B)$ to obtain the suppliers' *SPE* profits as functions of w_{A1}, w_{B1}, and w_{A2}. Substituting Π_{A2}^{CD*}, Π_{B2}^{CD*}, s_{A1}^{CD}, and s_{B1}^{CD} into (EC.7), we obtain the total profits of the two suppliers, given by $\Pi_A^{CD}(w_{A1}, w_{B1}, w_{A2})$ and $\Pi_B^{CD}(w_{A1}, w_{B1}, w_{A2})$. Simultaneously applying FOC on these profit functions with respect to w_{A1}, w_{B1}, and w_{A2}, we obtain the equilibrium wholesale prices w_{A1}^{CD*}, w_{B1}^{CD*}, and w_{A2}^{CD*}. At these wholesale prices, the reseller holds $I_B^{CD*} > 0$ as long as $h < h_I^{CD}(d) = \frac{a(1-d)}{2(2-d)}$.

Note that $h_I^{CD}(d) < h_I^{SS}(d)$ $\forall d \in [0, 1)$, where $h_I^{SS}(d)$ is the highest holding cost at which inventory is held in SS contracts (see Proposition 1). We limit our attention to $h < h_I^{CD}(d)$ under CD contracts to focus on the effect of strategic inventory. Following the method in Section EC.4, we establish that neither the two suppliers nor the reseller deviate from the FOC wholesale prices, quantities, and inventory, when $h < h_I^{CD}(d)$. Substituting w_{A1}^{CD*}, w_{B1}^{CD*}, and w_{A2}^{CD*} into $\Pi_A^{CD}(w_{A1}, w_{B1}, w_{A2})$ and $\Pi_B^{CD}(w_{A1}, w_{B1}, w_{A2})$, we obtain the suppliers' equilibrium profits under (C, D) contracts, denoted by Π_A^{CD*} and Π_B^{CD*}, respectively. The equilibrium decisions and profits are provided in Table EC.1.

Recall from Proposition 3 that Π_i^{SS*} is convex while Π_i^{LL*} is constant with respect to h, and $\Pi_i^{SS*} > \Pi_i^{LL*}$ only if $h < \widehat{h}(d)$, where $\widehat{h}(d) < h_I^{SS}(d)$ for all $d \in [0, 1)$. From Table EC.1, it can be

Table EC.1 Equilibrium Decisions and Profits under CD

	$h < h_I^{CD}(d)$
w_{A1}^*, w_{A2}^*	$\frac{a(1-d)(68+3d(12+d))-2dh(4-3d^2)}{2(68-15d^2)}, \frac{a(68-d(44+3d(9-d)))+2dh(20-3d^2)}{2(68-15d^2)}$
w_{B1}^*, w_{B2}^*	$\frac{2(9a(1-d)(2+d)-h(4-3d^2))}{68-15d^2}, \frac{2(6a(1-d)(2+d)+h(20-3d^2))}{68-15d^2}$
q_{A1}^*, q_{A2}^*	$\frac{a(1-d)(68+3d(12+d))-2dh(4-3d^2)}{4(1-d)(1+d)(68-15d^2)}, \frac{a(1-d)(68+3d(8-d))+2dh(20-3d^2)}{4(1-d)(1+d)(68-15d^2)}$
q_{B1}^*, q_{B2}^*	$\frac{13a(1-d)(2+d)-h(36-11d^2)}{(1-d)(1+d)(68-15d^2)}, \frac{6a(1-d)(2+d)+h(20-3d^2)}{(1-d)(1+d)(68-15d^2)}$
s_{A1}^*, s_{A2}^*	$\frac{a(1-d)(68+3d(12+d))-2dh(4-3d^2)}{4(1-d)(1+d)(68-15d^2)}, \frac{a(1-d)(68+3d(8-d))+2dh(20-3d^2)}{4(1-d)(1+d)(68-15d^2)}$
s_{B1}^*, s_{B2}^*	$\frac{a(1-d)(2+d)(32+3d^2)+2h(2-d^2)(4-3d^2)}{4(1-d)(1+d)(68-15d^2)}, \frac{a(1-d)(2+d)(44-3d^2)-2h(2-d^2)(20-3d^2)}{4(1-d)(1+d)(68-15d^2)}$
I_A^*, I_B^*	$0, \frac{(2+d)(20-3d^2)(a(1-d)-2h(2-d))}{4(1-d)(1+d)(68-15d^2)}$
π^*, Π_A^*, Π_B^*	$\frac{E}{8(1-d)(1+d)(68-15d^2)^2}, \frac{F}{4(1-d)(1+d)(68-15d^2)^2}, \frac{G}{(1-d)(1+d)(68-15d^2)^2}$
E	$a^2(1-d)(9584+3d(3264+d(976+d(68-3d(3+d))))) - 2a(1-d)(1888+d(2032+3d(144-d(8-9d(2+d)))))h + 16(608-448d^2+102d^4-9d^6)h^2$
F	$a^2(1-d)^2(4624+3d(1360+3d(104+d(4+d)))) - 4a(-1+d)d(544+3d(56+3d(-4+d(2+d))))h + 4d^2(208+9d^2(-8+d^2))h^2$
G	$306a^2(-2+d+d^2)^2 - 68a(-1+d)(2+d)(-4+3d^2)h + 4(272-136d^2+21d^4)h^2$

verified similarly that Π_A^{CD*} increases, while Π_B^{CD*} is convex in h for all $d \in [0,1)$. We compare the suppliers' profits under CD (from Table EC.1) with each other and with those under LL and SS contracts (from Table 2). This comparison allows us to establish the following two thresholds on the holding cost h: $\widehat{h}_A(d)$ and $\widehat{h}_B(d)$. These thresholds are ordered with respect to the previously established h-thresholds as follows: $\widehat{h}(d) < \widehat{h}_A(d) < \widehat{h}_B(d) < h_I^{CD}(d)$ for all $d \in (0,1)$, and they all converge to $\frac{a}{4}$ when $d = 0$. We make the following observations.

REMARK EC.1. (a) For all $h < h_I^{CD}(d)$, $\Pi_A^{CD*} < \Pi_B^{CD*}$, $\Pi_A^{CD*} < \Pi_A^{LL*}$, and $\Pi_B^{CD*} < \Pi_B^{SS*}$.

(b) $\Pi_A^{CD*} < \Pi_A^{SS*}$ when $h < \widehat{h}_A(d)$, and $\Pi_A^{CD*} > \Pi_A^{SS*}$ when $\widehat{h}_A(d) < h < h_I^{CD}(d)$.

(c) $\Pi_B^{CD*} > \Pi_B^{LL*}$ when $h < \widehat{h}_B(d)$, and $\Pi_B^{CD*} < \Pi_B^{LL*}$ when $\widehat{h}_B(d) < h < h_I^{CD}(d)$.

Based on the observations above, we can conclude that when wholesale-price-only commitment contracts (C) are used, CC contracts arise in equilibrium only when $h \geq \widehat{h}_B(d)$, and CD contracts only when $\widehat{h}_A(d) \leq h < \widehat{h}_B(d)$. However, when $\widehat{h}(d) < h < \widehat{h}_A(d)$, there exists a prisoner's dilemma. Although both suppliers would have been better off if the reseller could not hold strategic inventory (i.e., under CC contracts), supplier B would deviate and offer the D contract and make supplier A worse-off. Thus, in equilibrium, the suppliers offer the *inferior* DD contracts for $\widehat{h}(d) < h < \widehat{h}_A(d)$, even though they are worse off than if both offered the C contract, which by definition is the prisoner's dilemma.

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