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Online Supplement for: Reining in Onion Prices by Introducing a Vertically Differentiated Substitute: Models, Analysis, and Insights

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Problem Definition: Onion is an indispensable ingredient of the Indian diet, and plays a vital role in Indian economy, society, and politics. The ever-lasting volatility in its prices leads to significant social unrest. In this paper, we are interested in helping decision-makers to rigorously evaluate a recent policy proposal to make dehydrated onion widely available to remedy the situation.

Methodology/ Results: Using a stylized analytical model, we look for conditions under which it is optimal to introduce a processed substitute and whether it should be managed by nonprofit or for-profit firms. We find that the solution is identified by threshold-based policies and outcomes are far better under the non-profit management. We also find that a non-profit processing firm may purposefully choose a strategy where consumers do not purchase its offering for a certain medium range of raw onion deterioration levels. In addition, we find that a for-profit firm would always choose to be the lower-quality substitute in the market unless the raw onion deterioration is high. We also find that when supply capacity is constrained, sales of the processed substitute might decrease with increased supply availability.

Managerial Implications: This is the first paper that takes perishability and consumer welfare into account in a two-period vertically differentiated market model and compares various scenarios of competition when there is consumer prejudice for the processed substitute. For India's policymakers, we find ample evidence to work towards implementing the processed substitute policy. We go deep and discuss tailored insights for certain regions in India. We find that while improved consumer perception is favorable in general, policymakers should be careful about some unintended consequences such as increased prices and lower availability.

Key words: Vertical differentiation; perishability; consumer prejudice; non-profit competition; India

Appendix A: Thresholds

We define the following threshold levels for describing the optimal policies.

$$\hat{\delta}_1 = \frac{q_{f2}}{q_d}\ ,\ \hat{\delta}_2 = \frac{3q_{f1}q_{f2}}{2q_{f1}q_d+q_{f2}q_d}$$

$$\begin{aligned}
\hat{c}_{f1} &= \frac{q_d\delta(-q_{f2}+q_d\delta)(-4q_{f1}q_{f2}+3q_{f1}q_d\delta+q_{f2}q_d\delta)}{2(-2q_{f2}+q_d\delta)(-2q_{f1}q_{f2}+q_{f1}q_d\delta+q_{f2}q_d\delta)} & \hat{c}_{n1} &= \frac{q_d\delta(q_{f2}-q_d\delta)}{2q_{f2}-q_d\delta} \\
\hat{c}_{f2} &= \frac{q_d\delta(-q_{f1}+q_d\delta)}{-2q_{f1}+q_d\delta} & \hat{c}_{n2} &= \hat{c}_{f2} \\
\hat{c}_{f3} &= \frac{q_d\delta}{2} & \hat{c}_{n3} &= \hat{c}_{f3} \\
\hat{c}_{f4} &= \frac{q_d\delta(q_{f2}-q_d\delta)(q_{f1}(7q_{f2}-10q_d\delta)+q_d\delta(-q_{f2}+4q_d\delta))}{2(q_{f2}-2q_d\delta)(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))} & \hat{c}_{n4} &= \frac{2q_d\delta(q_d\delta-q_{f2})}{2q_d\delta-q_{f2}} \\
\hat{c}_{f5} &= -\frac{q_{f2}}{2}+q_d\delta & \hat{c}_{n5} &= \hat{c}_{f5} \\
\hat{c}_{f6} &= \frac{q_d\delta(q_{f1}-q_d\delta)^2(q_{f2}+2q_d\delta)}{2(2q_{f1}-q_d\delta)(-q_{f1}q_{f2}+2q_{f1}q_d\delta-q_d^2\delta^2)} & \hat{c}_{n6} &= \frac{q_d\delta(-q_{f1}+q_d\delta)}{-2q_{f1}+q_d\delta} \\
\hat{c}_{f7} &= \frac{2q_d\delta(q_{f2}-q_d\delta)}{q_{f2}-2q_d\delta} & \hat{c}_{n7} &= \hat{c}_{f7} \\
\hat{c}_{X1} &= \frac{q_d\delta(-q_{f2}+q_d\delta)(q_{f2}(-4+q_d\delta)+q_d\delta(2+q_d\delta))}{{q_d}^3\delta^3+{q_{f2}}^2(8-4q_d\delta)+q_{f2}q_d\delta(-6+q_d\delta)} \\
\hat{c}_{X2} &= \frac{2(q_{f2}-q_d\delta)(3q_{f2}+q_d\delta(-5+2q_d\delta))}{(-6+q_{f2})q_{f2}+(8+q_{f2})q_d\delta-4{q_d}^2\delta^2} \\
\hat{c}_{X3} &= \frac{q_d\delta(-1+q_d\delta)(-q_{f2}+(-2+3q_{f2})q_d\delta)}{-4q_{f2}+(8+q_{f2})q_d\delta+(-6+q_{f2}){q_d}^2\delta^2}
\end{aligned}$$

Appendix B: Trader-Only (TO) Model

As detailed before, the current market situation in India is dysfunctional where hoarding, price manipulation, and collusive behavior are commonly observed (CCI 2012). Trying to model this situation by a verifiable, insightful, and generalizable economic analysis has proven to be impossible (Sharma et al. 2011). Since our main goal is to investigate the prospects of dehydrated onion markets, we focus our attention to modeling the competition as opposed to modeling the current practice. Given that collusion among the handful of traders is common, we model a conventional monopolist fresh onion trader as our benchmark.

We admit that this conventional monopoly model will result in a higher social welfare, lower and stable prices, and greater availability in the market as compared to the current dysfunctional situation in India. Hence, *any improvement reported in this paper should be regarded as a lower bound to the potential improvements* that can be achieved when dehydrated onion is introduced in Indian markets.

Following the classical vertically differentiated demand model (Moorthy 1988, Tirole 1988), consumer types θ are uniformly distributed in the unit interval [0,1] with unit total mass. We denote the price of onion as p_{fi} in period i $(i=1,2)$. The utility derived by a θ-type consumer from buying onion in period i is $U_{fi}=\theta\cdot q_{fi}-p_{fi}$. A θ-type consumer buys onion in period i only when $U_{fi}\geq 0$.

In the monopolist-trader-only (TO) model, the fraction of consumers buying onion in period i $(i=1,2)$ is given by $x_{fi}^{TO}=1-p_{fi}/q_{fi}$. Given this demand model, the monopolist trader maximizes its profit across the two periods as follows:

$$\pi^{TO}=\max_{p_{fi}\geq 0}\{p_{f1}\cdot x_{f1}+p_{f2}\cdot x_{f2}\} \text{ subject to } x_{fi}\geq 0 \quad i=1,2 \tag{A1}$$

Lemma $A1$ summarizes the outcomes for the TO model.

Lemma A1. *The optimal prices and quantities for the trader-only (TO) model are as follows:*

$$p_{fi}^{TO}=q_{fi}/2;\ x_{fi}^{TO}=1/2;\ \pi^{TO}=(q_{f1}+q_{f2})/4;\ i=1,2$$

Lemma A1 adapts the well-established monopoly results from the economic theory to our model. It sets the benchmark that will be useful in the main paper where we investigate how things change when there exists a competitive force in this two-period vertically-differentiated perishable produce market.

Appendix C: Numerical Experiment Design

We construct parametric sets where we fix fresh onion harvest quality $q_{f1} = 1$ (without loss of generality) with various cost coefficients ($\gamma = 0.001, 0.01$, and 0.1); second period fresh onion quality levels ($q_{f2} = 0.5, 0.6, ..., 0.9$, and 1); and prejudice parameters ($\delta = 0.5, 0.525, 0.55, ..., 0.975$, and 1). For each parametric set, we list all available strategy alternatives, confirm the concavity of the objective function and convexity of the constraints within the limits of piecewise function in which it belongs, compare the results of each strategy alternative, and pick the best strategy for that set of parameters.

Appendix D: Proofs

Proof of Lemma A1: Objective function of the monopolist's problem is jointly concave on a convex set defined by linear constraints. Then, the optimal solution can be obtained by solving the first order conditions.

$$\partial_{p_{fi}} \pi^{TO} = 1 - 2p_{fi}/q_{fi} = 0$$

Hence, $p_{fi}^{FO} = q_{fi}/2$, $i = 1,2 \Rightarrow x_{fi}^{TO} = 1 - p_{fi}/q_{fi} = 1/2$, $i = 1,2$, $\Rightarrow \pi^{TO} = (q_{f1} + q_{f2})/4$

Proof of Proposition 1: Results listed in the proposition directly follow from the equilibrium solutions presented in Appendix Section E.1.

Proof of Proposition 2: Results listed in the proposition directly follow from the equilibrium solutions presented in Appendix Section E.1.

Proof of Proposition 3: For each parametric case, we compare the key variable outcomes of NP versus FP using the equilibrium solutions presented in Appendix Sections E.1 and E.2.

Region 1: $0 < \delta < \hat{\delta}_1$

Ω_{1A}. In this region FP solution is FP-1A and NP solution is NP-1A. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > q_{f2} > \delta q_d, \delta < \hat{\delta}_1$, and $c_d < \hat{c}_{f1} < \hat{c}_{n1}$

Using these conditions it is straightforward to show the outcomes listed in Table 2.

Ω_{1X}. In this region FP solution is FP-1B and NP solution is NP-1A. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > q_{f2} > \delta q_d, \delta < \hat{\delta}_1$, and $\hat{c}_{f1} < c_d < \hat{c}_{n1}$

Using these conditions it is straightforward to show the outcomes listed in Table 2. Algebra of x_{f2} requires additional condition on c_d:

$x_{f2}^{NP1A} < x_{f2}^{FP1B}$ when $c_d < \hat{c}_{X1}$ and $x_{f2}^{NP1A} > x_{f2}^{FP1B}$ when $c_d > \hat{c}_{X1}$.

Ω_{1B}. In this region FP solution is FP-1B and NP solution is NP-1B. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > q_{f2} > \delta q_d, \delta < \hat{\delta}_1$, and $\hat{c}_{n1} < c_d < \hat{c}_{f2}$

Using these conditions it is straightforward to show the outcomes listed in Table 2.

Region 2: $\hat{\delta}_1 < \delta < \hat{\delta}_2$

Ω_{2A}. In this region FP solution is FP-2A and NP solution is NP-2A. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > \delta q_d > q_{f2}, \hat{\delta}_1 < \delta < \hat{\delta}_2$, and $c_d < \hat{c}_{f4} < \hat{c}_{n4}$

Using these conditions it is straightforward to show the outcomes listed in Table 2.

Ω_{2X}. In this region FP solution is FP-2B and NP solution is NP-2A. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > \delta q_d > q_{f2}, \hat{\delta}_1 < \delta < \hat{\delta}_2$, and $\hat{c}_{f4} < c_d < \hat{c}_{n4}$

Using these conditions it is straightforward to show the outcomes listed in Table 2. Algebra of x_{f2} requires additional condition on c_d:

$x_{f2}^{NP2A} < x_{f2}^{FP2B}$ when $c_d < \hat{c}_{X2}$ and $x_{f2}^{NP2A} > x_{f2}^{FP2B}$ when $c_d > \hat{c}_{X2}$.

Ω_{2B}. In this region FP solution is FP-2B and NP solution is NP-2B. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > \delta q_d > q_{f2}, \hat{\delta}_1 < \delta < \hat{\delta}_2$, and $\hat{c}_{n4} < c_d < \hat{c}_{f2}$

Using these conditions it is straightforward to show the outcomes listed in Table 2.

Region 3: $\hat{\delta}_2 < \delta < 1$

Ω_{3A}. In this region FP solution is FP-3A and NP solution is NP-3A. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > \delta q_d > q_{f2}, \hat{\delta}_2 < \delta < 1$, and $c_d < \hat{c}_{f6} < \hat{c}_{n6}$

Using these conditions it is straightforward to show the outcomes listed in Table 2.

Ω_{3X}. In this region FP solution is FP-3B and NP solution is NP-3A. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > \delta q_d > q_{f2}, \hat{\delta}_2 < \delta < 1$, and $\hat{c}_{f6} < c_d < \hat{c}_{n6}$

Using these conditions it is straightforward to show the outcomes listed in Table 2. Algebra of x_{f1} requires additional condition on c_d:

$x_{f1}^{NP3A} < x_{f1}^{FP3B}$ when $c_d < \hat{c}_{X3}$ and $x_{f1}^{NP3A} > x_{f1}^{FP3B}$ when $c_d > \hat{c}_{X3}$.

Ω_{3B}. In this region FP solution is FP-3B and NP solution is NP-3B. Parametric conditions that are satisfied in this region are as follows:

$q_d > 0, \delta > 0, c_d > 0, q_{f1} > \delta q_d > q_{f2}, \hat{\delta}_2 < \delta < 1$, and $\hat{c}_{n6} < c_d < \hat{c}_{f7}$

Using these conditions it is straightforward to show the outcomes listed in Table 2.

Proof of Corollary 1: $\partial_{\delta,\delta}(\hat{c}_{f2}) = (4q_{f1}^2 q_d^2)/(-2q_{f1} + q_d\delta)^3 < 0$ then $\hat{c}_{f2}$ is a strictly concave function of δ.

$\partial_\delta(\hat{c}_{f2}) = q_d - \frac{2q_{f1}^2 q_d}{(-2q_{f1}+q_d\delta)^2} = q_d(1 - 2(q_{f1}/(2q_{f1} - q_d\delta))^2 = 0 \Rightarrow \{\delta = -(((-2+\sqrt{2})q_{f1})/q_d)\}, \{\delta = ((2+\sqrt{2})q_{f1})/q_d\}$. Since $\delta \leq 1$, maximum is achieved at $\delta = ((2-\sqrt{2})q_{f1})/q_d$.

We can also show that the first derivative is negative when $2 > 2(q_{f1}/(2q_{f1} - q_d\delta))^2 > 1 \Rightarrow \delta > ((2-\sqrt{2})q_{f1})/q_d$. Since $\delta \leq 1 \Rightarrow q_d/q_{f1} > (2-\sqrt{2})$.

Proof of Corollary 2: $\partial_{q_{f2}}(\hat{c}_{f1}) = (q_d^2\delta^2(12q_{f1}^2q_{f2}^2 - 2q_{f1}q_{f2}(8q_{f1} + 5q_{f2})q_d\delta + (5q_{f1}^2 + 10q_{f1}q_{f2} + q_{f2}^2)q_d^2\delta^2 - 2q_{f1}q_d^3\delta^3))/(2(-2q_{f2} + q_d\delta)^2(-2q_{f1}q_{f2} + (q_{f1} + q_{f2})q_d\delta)^2)$

It is straightforward to show that $\partial_{q_{f2}}(\hat{c}_{f1}) > 0$ when $\delta > 0, q_d > 0, q_{f1} > q_{f2} > 0, q_{f2} > \delta q_d, q_{f1} > \delta q_d$.

Proof of Corollary 3: The relevant sales quantities are listed in Appendix E.2. NP Equilibrium solutions. NP Regions 2A and 3A.

Using these values for x_{d1} and x_{d2}, it is straightforward to show that $\partial_{q_{f2}}(x_{d1}) < 0$ and $\partial_{q_{f2}}(x_{d2}) < 0$ in both Regions 2A and 3A when $\delta > 0, q_d > 0, c_d > 0, q_{f1} > q_{f2} > 0, q_{f2} < \delta q_d, q_{f1} > \delta q_d$.

Proof of Corollary 4: The relevant sales quantities are listed in Appendix E.2. NP Equilibrium solutions. NP Regions 2A and 3A.

Using these values for x_{d1} and x_{d2}, it is straightforward to show that $\partial_\delta(x_{d2}) > 0$ in both Regions 2A and 3A when $\delta > 0, q_d > 0, c_d > 0, q_{f1} > q_{f2} > 0, q_{f2} < \delta q_d, q_{f1} > \delta q_d$.

We also find that $\partial_\delta(x_{d1})$ becomes negative using the Western India parametric estimates $q_{f1} = 1, q_d = 0.9, q_{f2} = 0.72$, and $\delta = 0.9$ in both Regions 2A and 3A.

Proof of Proposition 4: We focus on Case (i) where $q_{f2} > \delta * q_d$ and supply S is binding where it is smaller than the unconstrained problem solution quantity: $S < \hat{S}_1$ where $\hat{S}_1 = x^*_{f1} + x^*_{f2} + x^*_{d1} + x^*_{d2}$.

β percentage is allocated to the raw onion trader and the rest is allocated to the processor and both firms are strictly capacity constrained:

$1 - ((p_{f1} - p_d)/(q_{f1} - \delta q_d)) + 1 - ((p_{f2} - p_d)/(q_{f2} - \delta q_d)) = \beta S$

$((p_{f1} - p_d)/(q_{f1} - \delta q_d)) - (p_d/(\delta q_d)) + ((p_{f2} - p_d)/(q_{f2} - \delta q_d)) - (p_d/(\delta q_d)) = (1 - \beta)S$

Solving these price-demand equations together, we get:

$p_{f2} = (2p_{f1}(q_{f2} - q_d\delta) + q_d\delta(q_{f2}(2 + S - 2S\beta) + 2q_dS(-1 + \beta)\delta) + q_{f1}(2q_{f2}(-2 + S\beta) + q_d(2 + S - 2S\beta)\delta))/(-2q_{f1} + 2q_d\delta)$

$p_d = (1/2)q_d(2 - S)\delta$

Next we solve the trader's problem for p_{f1}: $\partial_{p_{f1}} \pi_f^{FP}(p^*_d) = 0$ to find:

$p_{f1} = -((-q_{f1}^2 + q_d\delta(q_{f2}(1 + S - 2S\beta) + 2q_dS(-1 + \beta)\delta) + q_{f1}(q_{f2}(-3 + 2S\beta) + q_d(3 + S - 2S\beta)\delta))/(2(q_{f1} + q_{f2} - 2q_d\delta)))$

Plugging this solution back in the equations as presented above, we find the values for $p_{f2}, x_{f1}, x_{f2}, x_{d1}$ and x_{d2} when the problem is supply capacity constrained. Checking the feasibility conditions for each, we find the second threshold $S \geq \hat{S}_2$ where $\hat{S}_2 = (q_{f1} - q_{f2})/(q_{f1} + q_{f2} - 2q_{f2}\beta + 2q_d(-1 + \beta)\delta)$.

Proof of Corollary 5: From the solutions presented in Proposition 4, we find that
$x_{d2} = \frac{(q_{f2}(-1+S)+q_{f1}(1+S-2S\beta)+2q_dS(-1+\beta)\delta)}{(2(q_{f1}+q_{f2}-2q_d\delta))}$
Then, simple algebra shows that $\partial_S x_{d2} < 0$ when $\beta > \hat{\beta}_1 = \frac{(q_{f1}+q_{f2}-2q_d\delta)}{(2q_{f1}-2q_d\delta)}$

Appendix E: Equilibrium Solutions

In summary, we enumerate and solve all unconstrained equilibriums that may arise in this game. Once we achieve closed form solutions including interior and boundary solutions, we check the feasibility constraints and eliminate the infeasible ones. If there are multiple feasible solutions, we compare the objective functions and choose the best one as the optimal solution for that parametric set. Limits on δ and unit processing cost c_d are determined by the feasibility conditions.

For all problems, demand equations listed in Equation set 1 are used and equilibrium is obtained by simultaneously solving the first-order conditions using the backward induction based solution techniques for problems 3 and 4 in the FP model, and for problems 3 and 5 in the NP model.

Below are the list of feasibility conditions that need to be satisfied in this game:

$$x_{fi} \geq 0, x_{di} \geq 0, p_{fi} \geq 0, p_d \geq 0, x_{fi} + x_{di} \leq 1 \text{ for } i = 1, 2$$

Below are the list of possible Nash equilibriums that may arise in this game:

1. ***Both periods: Both types of produce are sold at competitive prices:***

Unconstrained problems are solved using the equations as listed.

2. ***First period: Both types of produce are sold at competitive prices; Second period: Only raw-onion is sold at entry-deterring low prices:***

The raw onion trader fixes p_{f2} at a certain low value so FP cannot have an option but $x_{d2}=0$. All other equations remain the same.

3. ***First period: Only raw-onion is sold at entry-deterring low prices; Second period: Both types of produce are sold at competitive prices:***

The raw onion trader fixes p_{f1} at a certain low value so FP cannot have an option but $x_{d1}=0$. All other equations remain the same.

4. ***Both periods: Only raw-onion is sold at entry-deterring low prices:***

The raw onion trader fixes both p_{f1} and p_{f2} at a certain low value so FP cannot have an option but $x_{d1}=0$ and $x_{d2}=0$. All other equations remain the same.

5. ***Both periods: Only raw-onion is sold; First period: at monopoly prices; Second period: at entry-deterring low prices:***

The raw onion trader is able to charge monopoly price in the first period $p_{f1}=q_{f1}/2$. It fixes p_{f2} at a certain low value so FP cannot have an option but $x_{d2}=0$. All other equations remain the same.

6. ***Both periods: Only raw-onion is sold; First period: at entry-deterring low prices; Second period: at monopoly prices:***

The raw onion trader is able to charge monopoly price in the second period $p_{f2}=q_{f2}/2$. It fixes p_{f1} at a certain low value so FP cannot have an option but $x_{d1}=0$. All other equations remain the same.

7. ***Both periods: Only raw-onion is sold at monopoly prices:***

The raw onion trader is able to charge monopoly prices in both periods $p_{f1}=q_{f1}/2$ and $p_{f2}=q_{f2}/2$.

E.1. FP Equilibrium Solutions

Below is the summary of results for each parametric combination in FP's problem:

FP-Region 1. $0<\delta<\hat{\delta}_1$

FP-1A. $0<c_d<\hat{c}_{f1}$

Nash equilibrium 1 is optimal:

$$p_d=\frac{2(q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+c_d(2q_{f1}q_{f2}-q_{f1}q_d\delta q_{f2}q_d\delta))}{q_{f1}(8q_{f2}-5q_d\delta)+q_d\delta(-5q_{f2}+2q_d\delta)}$$

$$p_{f1}=\frac{(q_{f1}-q_d\delta)(8q_{f1}q_{f2}-5q_{f1}q_d\delta-3q_{f2}q_d\delta)+c_d(4q_{f1}q_{f2}-2q_{f1}q_d\delta-2q_{f2}q_d\delta)}{2(q_{f1}(8q_{f2}-5q_d\delta)+q_d\delta(-5q_{f2}+2q_d\delta))}$$

$$p_{f2}=\frac{(q_{f2}-q_d\delta)(8q_{f1}q_{f2}-3q_{f1}q_d\delta-5q_{f2}q_d\delta)+c_d(4q_{f1}q_{f2}-2q_{f1}q_d\delta-2q_{f2}q_d\delta)}{2(q_{f1}(8q_{f2}-5q_d\delta)+q_d\delta(-5q_{f2}+2q_d\delta))}$$

$$x_{f1}=\frac{c_d(-4q_{f1}q_{f2}+2q_{f1}q_d\delta+2q_{f2}q_d\delta)+(q_{f1}-q_d\delta)(-8q_{f1}q_{f2}+5q_{f1}q_d\delta+3q_{f2}q_d\delta)}{2(q_{f1}-q_d\delta)(q_d\delta(5q_{f2}-2q_d\delta)+q_{f1}(-8q_{f2}+5q_d\delta))}$$

$$x_{f2}=\frac{c_d(-4q_{f1}q_{f2}+2q_{f1}q_d\delta+2q_{f2}q_d\delta)+(q_{f2}-q_d\delta)(-8q_{f1}q_{f2}+3q_{f1}q_d\delta+5q_{f2}q_d\delta)}{2(q_{f2}-q_d\delta)(q_d\delta(5q_{f2}-2q_d\delta)+q_{f1}(-8q_{f2}+5q_d\delta))}$$

$$x_{d1}=\frac{-2c_d(2q_{f1}-q_d\delta)(-2q_{f1}q_{f2}+q_{f1}q_d\delta+q_{f2}q_d\delta)+q_d\delta(q_{f1}-q_d\delta)(-4q_{f1}q_{f2}+q_{f1}q_d\delta+3q_{f2}q_d\delta)}{2q_d\delta(-q_{f1}+q_d\delta)(q_{f1}(8q_{f2}-5q_d\delta)+q_d\delta(-5q_{f2}+2q_d\delta))}$$

$$x_{d2}=\frac{1}{6}(-\frac{3c_d}{q_d\delta}-\frac{2c_d}{q_{f2}-q_d\delta}+\frac{12q_{f1}q_{f2}-3(3q_{f1}+q_{f2})q_d\delta+c_d(q_{f1}-3q_{f2}+2q_d\delta)}{8q_{f1}q_{f2}-5(q_{f1}+q_{f2})q_d\delta+2q_d^2\delta^2}$$

FP-1B. $\hat{c}_{f1}<c_d<\hat{c}_{f2}$

Nash equilibrium 2 is optimal:

$$p_d=\frac{q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+c_d(4q_{f1}q_{f2}-2q_{f1}q_d\delta-2q_{f2}q_d\delta)}{q_{f1}(6q_{f2}-4q_d\delta)+q_d\delta(-3q_{f2}+q_d\delta)}$$

$p_{f1} = \frac{c_d(2q_{f1}q_{f2}-q_{f1}q_d\delta-q_{f2}q_d\delta)-(q_{f1}-q_d\delta)(-3q_{f1}q_{f2}+2q_{f1}q_d\delta+q_{f2}q_d\delta)}{q_{f1}(6q_{f2}-4q_d\delta)+q_d\delta(-3q_{f2}+q_d\delta)}$
$p_{f2} = \frac{q_{f2}(q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+c_d(4q_{f1}q_{f2}-2q_{f1}q_d\delta-2q_{f2}q_d\delta))}{q_d\delta(q_{f1}(6q_{f2}-4q_d\delta)+q_d\delta(-3q_{f2}+q_d\delta))}$
$x_{f1} = \frac{c_d(-2q_{f1}q_{f2}+q_{f1}q_d\delta+q_{f2}q_d\delta)+(q_{f1}-q_d\delta)(-3q_{f1}q_{f2}+2q_{f1}q_d\delta+q_{f2}q_d\delta)}{(q_{f1}-q_d\delta)(q_d\delta(3q_{f2}-q_d\delta)+q_{f1}(-6q_{f2}+4q_d\delta))}$
$x_{f2} = \frac{q_d\delta(5q_{f1}q_{f2}-3q_{f1}q_d\delta-2q_{f2}q_d\delta)+c_d(-4q_{f1}q_{f2}+2q_{f1}q_d\delta+2q_{f2}q_d\delta)}{q_d\delta(q_{f1}(6q_{f2}-4q_d\delta)+q_d\delta(-3q_{f2}+q_d\delta))}$
$x_{d1} = -\frac{(-2q_{f1}q_{f2}+q_{f1}q_d\delta+q_{f2}q_d\delta)(c_d(2q_{f1}-q_d\delta)+q_d\delta(-q_{f1}+q_d\delta))}{q_d\delta(-q_{f1}+q_d\delta)(q_{f1}(6q_{f2}-4q_d\delta)+q_d\delta(-3q_{f2}+q_d\delta))}$ $x_{d2} = 0$

FP-1C. $\hat{c}_{f2} < c_d < \hat{c}_{f3}$

Nash equilibrium 4 is optimal:

$p_d = c_d; p_{f1} = \frac{c_d q_{f1}}{q_d\delta}; p_{f2} = \frac{c_d q_{f2}}{q_d\delta}$ $x_{f1} = 1 - \frac{c_d}{q_d\delta}; x_{f2} = 1 - \frac{c_d}{q_d\delta}; x_{d1} = 0; x_{d2} = 0$

FP-1D. $c_d > \hat{c}_3$

Nash equilibrium 7 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = \frac{q_{f2}}{2}; x_{f1} = \frac{1}{2}; x_{f2} = \frac{1}{2}$

FP-Region 2. $\hat{\delta}_1 < \delta < \hat{\delta}_2$

FP-2A. $0 < c_d < \hat{c}_{f4}$

Nash equilibrium 1 is optimal:

$p_d = \frac{3q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta)}$
$p_{f1} = \frac{2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))+(q_{f1}-q_d\delta)(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(q_{f2}+2q_d\delta))}{2(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta))}$
$p_{f2} = \frac{q_{f2}(3q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta)))}{2q_d\delta(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta))}$
$x_{f1} = -\frac{2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))+(q_{f1}-q_d\delta)(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(q_{f2}+2q_d\delta))}{2(q_{f1}-q_d\delta)(q_d\delta(2q_{f2}-5q_d\delta)+q_{f1}(-5q_{f2}+8q_d\delta))}$
$x_{f2} = \frac{3q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{2(q_{f2}-q_d\delta)(q_d\delta(2q_{f2}-5q_d\delta)+q_{f1}(-5q_{f2}+8q_d\delta))}$
$x_{d1} = \frac{q_d\delta(q_{f1}-q_d\delta)^2(q_{f2}+2q_d\delta)+2c_d(2q_{f1}-q_d\delta)(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{2q_d\delta(-q_{f1}+q_d\delta)(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta))}$
$x_{d2} = \frac{1}{30}(12 - \frac{6c_d}{q_d\delta} + \frac{10c_d}{q_{f2}-q_d\delta} + \frac{-8c_dq_{f1}-12c_dq_{f2}+45q_{f1}q_{f2}+(20c_d+9(-6q_{f1}+q_{f2}))q_d\delta}{5q_{f1}q_{f2}-2(4q_{f1}+q_{f2})q_d\delta+5q_d^2\delta^2})$

FP-2B. $\hat{c}_{f4} < c_d < \hat{c}_{f2}$

Nash equilibrium 2 is optimal:

$p_{f1} = \frac{c_dq_d^2\delta^2+c_dq_{f1}(q_{f2}-2q_d\delta)+(q_{f1}-q_d\delta)(2q_{f1}q_{f2}-3q_{f1}q_d\delta+q_d^2\delta^2)}{q_{f1}(4q_{f2}-6q_d\delta)+q_d\delta(-q_{f2}+3q_d\delta)}$
$p_{f2} = \frac{2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))+(q_{f2}-q_d\delta)(q_{f1}(4q_{f2}-5q_d\delta)+q_d\delta(-q_{f2}+2q_d\delta))}{q_{f1}(4q_{f2}-6q_d\delta)+q_d\delta(-q_{f2}+3q_d\delta)}$
$p_d = \frac{q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{q_{f1}(4q_{f2}-6q_d\delta)+q_d\delta(-q_{f2}+3q_d\delta)}$
$x_{f1} = \frac{(q_{f1}-q_d\delta)(-2q_{f1}q_{f2}+3q_{f1}q_d\delta-q_d^2\delta^2)-c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{(q_{f1}-q_d\delta)(q_d\delta(q_{f2}-3q_d\delta)+q_{f1}(-4q_{f2}+6q_d\delta))}$
$x_{f2} = \frac{q_d\delta(3q_{f1}q_{f2}-5q_{f1}q_d\delta+2q_d^2\delta^2)-2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{q_{f2}(q_{f1}(4q_{f2}-6q_d\delta)+q_d\delta(-q_{f2}+3q_d\delta))}$
$x_{d1} = \frac{(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))(c_d(2q_{f1}-q_d\delta)+q_d\delta(-q_{f1}+q_d\delta))}{q_d\delta(-q_{f1}+q_d\delta)(q_{f1}(4q_{f2}-6q_d\delta)+q_d\delta(-q_{f2}+3q_d\delta))}$ $x_{d2} = 0$

FP-2C. $\hat{c}_{f2} < c_d < \hat{c}_{f3}$

Nash equilibrium 4 is optimal:

$p_{f1} = \frac{c_dq_{f1}}{q_d\delta}; p_{f2} = c_d + q_{f2} - q_d\delta; p_d = c_d;$ $x_{f1} = 1 - \frac{c_d}{q_d\delta}; x_{f2} = \frac{-c_d+q_d\delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$

FP-2D. $\hat{c}_{f3} < c_d < \hat{c}_{f5}$

Nash equilibrium 5 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = c_d + q_{f2} - q_d\delta; p_d = c_d;$ $x_{f1} = \frac{1}{2}; x_{f2} = \frac{-c_d+q_d\delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$

FP-2E. $c_d > \hat{c}_{f5}$

Nash equilibrium 7 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = \frac{q_{f2}}{2}; x_{f1} = \frac{1}{2}; x_{f2} = \frac{1}{2}$

FP-Region 3. $\hat{\delta}_2 < \delta < 1$

FP-3A. $0 < c_d < \hat{c}_{f6}$

Nash equilibrium 1 is optimal:

$$p_d = \frac{3q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta)}$$
$$p_{f1} = \frac{2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))+(q_{f1}-q_d\delta)(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(q_{f2}+2q_d\delta))}{2(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta))}$$
$$p_{f2} = \frac{q_{f2}(3q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta)))}{2q_d\delta(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta))}$$
$$x_{f1} = -\frac{2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))+(q_{f1}-q_d\delta)(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(q_{f2}+2q_d\delta))}{2(q_{f1}-q_d\delta)(q_d\delta(2q_{f2}-5q_d\delta)+q_{f1}(-5q_{f2}+8q_d\delta))}$$
$$x_{f2} = \frac{3q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)+2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{2(q_{f2}-q_d\delta)(q_d\delta(2q_{f2}-5q_d\delta)+q_{f1}(-5q_{f2}+8q_d\delta))}$$
$$x_{d1} = \frac{q_d\delta(q_{f1}-q_d\delta)^2(q_{f2}+2q_d\delta)+2c_d(2q_{f1}-q_d\delta)(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{2q_d\delta(-q_{f1}+q_d\delta)(q_{f1}(5q_{f2}-8q_d\delta)+q_d\delta(-2q_{f2}+5q_d\delta))}$$
$$x_{d2} = \frac{1}{30}(12 - \frac{6c_d}{q_d\delta} + \frac{10c_d}{q_{f2}-q_d\delta} + \frac{-8c_dq_{f1}-12c_dq_{f2}+45q_{f1}q_{f2}+(20c_d+9(-6q_{f1}+q_{f2}))q_d\delta}{5q_{f1}q_{f2}-2(4q_{f1}+q_{f2})q_d\delta+5q_d^2\delta^2})$$

FP-3B. $\hat{c}_{f6} < c_d < \hat{c}_{f7}$

Nash equilibrium 3 is optimal:

$$p_d = \frac{2(c_dq_d^2\delta^2+c_dq_{f1}(q_{f2}-2q_d\delta)+q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta))}{3q_{f1}(q_{f2}-2q_d\delta)+q_d\delta(-q_{f2}+4q_d\delta)}$$
$$p_{f1} = \frac{2q_{f1}(c_dq_d^2\delta^2+c_dq_{f1}(q_{f2}-2q_d\delta)+q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta))}{q_d\delta(3q_{f1}(q_{f2}-2q_d\delta)+q_d\delta(-q_{f2}+4q_d\delta))}$$
$$p_{f2} = \frac{q_{f2}(c_dq_d^2\delta^2+c_dq_{f1}(q_{f2}-2q_d\delta)+q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta))}{q_d\delta(3q_{f1}(q_{f2}-2q_d\delta)+q_d\delta(-q_{f2}+4q_d\delta))}$$
$$x_{f1} = \frac{-2c_d(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))+q_d\delta(q_{f1}(q_{f2}-4q_d\delta)+q_d\delta(q_{f2}+2q_d\delta))}{q_d\delta(3q_{f1}(q_{f2}-2q_d\delta)+q_d\delta(-q_{f2}+4q_d\delta))}$$
$$x_{f2} = \frac{c_dq_d^2\delta^2+c_dq_{f1}(q_{f2}-2q_d\delta)+q_d\delta(q_{f1}-q_d\delta)(q_{f2}-q_d\delta)}{(q_{f2}-q_d\delta)(q_d\delta(q_{f2}-4q_d\delta)-3q_{f1}(q_{f2}-2q_d\delta))}$$
$$x_{d1} = 0;\ x_{d2} = \frac{(c_dq_{f2}-2(c_d+q_{f2})q_d\delta+2q_d^2\delta^2)(q_d^2\delta^2+q_{f1}(q_{f2}-2q_d\delta))}{q_d\delta(q_{f2}-q_d\delta)(q_d\delta(q_{f2}-4q_d\delta)-3q_{f1}(q_{f2}-2q_d\delta))}$$

FP-3C. $\hat{c}_{f7} < c_d < \hat{c}_{f3}$

Nash equilibrium 4 is optimal:

$$p_{f1} = \frac{c_dq_{f1}}{q_d\delta}; p_{f2} = c_d + q_{f2} - q_d\delta; p_d = c_d;\ x_{f1} = 1 - \frac{c_d}{q_d\delta}; x_{f2} = \frac{-c_d+q_d\delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$$

FP-3D. $\hat{c}_{f3} < c_d < \hat{c}_{f5}$

Nash equilibrium 5 is optimal:

$$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = c_d + q_{f2} - q_d\delta; p_d = c_d;\ x_{f1} = \frac{1}{2}; x_{f2} = \frac{-c_d+q_d\delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$$

FP-3E $c_d > \hat{c}_{f5}$

Nash equilibrium 7 is optimal:

$$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = \frac{q_{f2}}{2}; x_{f1} = \frac{1}{2}; x_{f2} = \frac{1}{2}$$

E.2. NP Equilibrium solutions

Below is the summary of results for each parametric combination in NP's problem. These solutions are compared to FP solutions in Figure 4.

NP-Region 1. $(0 < \delta < \hat{\delta}_1$

NP-1A. $0 < c_d < \hat{c}_{n1}$

Nash equilibrium 1 is optimal:

$$p_d = c_d; p_{f1} = \frac{1}{2}(c_d + q_{f1} - q_d\delta); p_{f2} = \frac{1}{2}(c_d + q_{f2} - q_d\delta)$$
$$x_{f1} = \frac{c_d+q_{f1}-q_d\delta}{2q_{f1}-2q_d\delta}; x_{f2} = \frac{c_d+q_{f2}-q_d\delta}{2q_{f2}-2q_d\delta}; x_{d1} = \frac{1}{2} - \frac{c_d}{q_d\delta} - \frac{c_d}{2q_{f1}-2q_d\delta}; x_{d2} = \frac{1}{2} - \frac{c_d}{q_d\delta} - \frac{c_d}{2q_{f2}-2q_d\delta}$$

NP-1B. $\hat{c}_{n1} < c_d < \hat{c}_{n2}$

Nash equilibrium 2 is optimal:

$$p_d = c_d; p_{f1} = \frac{1}{2}(c_d + q_{f1} - q_d\delta); p_{f2} = \frac{c_dq_{f2}}{q_d\delta};\ x_{f1} = \frac{c_d+q_{f1}-q_d\delta}{2q_{f1}-2q_d\delta}; x_{f2} = 1 - \frac{c_d}{q_d\delta}; x_{d1} = \frac{1}{2} - \frac{c_d}{q_d\delta} - \frac{c_d}{2q_{f1}-2q_d\delta}; x_{d2} = 0$$

NP-1C. $\hat{c}_{n2} < c_d < \hat{c}_{n3}$

Nash equilibrium 4 is optimal:

$p_d = c_d; p_{f1} = \frac{c_d q_{f1}}{q_d \delta}; p_{f2} = \frac{c_d q_{f2}}{q_d \delta};\ x_{f1} = 1 - \frac{c_d}{q_d \delta}; x_{f2} = 1 - \frac{c_d}{q_d \delta}; x_{d1} = 0; x_{d2} = 0$

NP-1D $c_d > \hat{c}_{n3}$

Nash equilibrium 7 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = \frac{q_{f2}}{2}; x_{f1} = \frac{1}{2}; x_{f2} = \frac{1}{2}$

NP-Region 2. $\hat{\delta}_1 < \delta < \hat{\delta}_2$

NP-2A. $0 < c_d < \hat{c}_{n4}$

Nash equilibrium 1 is optimal:

$p_d = c_d; p_{f1} = \frac{1}{2}(c_d + q_{f1} - q_d \delta); p_{f2} = \frac{c_d q_{f2}}{2 q_d \delta}$

$x_{f1} = \frac{c_d + q_{f1} - q_d \delta}{2 q_{f1} - 2 q_d \delta}; x_{f2} = -\frac{c_d}{2 q_{f2} - 2 q_d \delta}; x_{d1} = \frac{1}{2} - \frac{c_d}{q_d \delta} - \frac{c_d}{2 q_{f1} - 2 q_d \delta}; x_{d2} = 1 - \frac{c_d q_{f2} - 2 c_d q_d \delta}{2 q_{f2} q_d \delta - 2 {q_d}^2 \delta^2}$

NP-2B $\hat{c}_{n4} < c_d < \hat{c}_{n2}$

Nash equilibrium 2 is optimal:

$p_{f1} = \frac{1}{2}(c_d + q_{f1} - q_d \delta); p_{f2} = c_d + q_{f2} - q_d \delta; p_d = c_d$

$x_{f1} = \frac{c_d + q_{f1} - q_d \delta}{2 q_{f1} - 2 q_d \delta}; x_{f2} = \frac{-c_d + q_d \delta}{q_{f2}}; x_{d1} = \frac{1}{2} - \frac{c_d}{q_d \delta} - \frac{c_d}{2 q_{f1} - 2 q_d \delta}; x_{d2} = 0$

NP-2C. $\hat{c}_{n2} < c_d < \hat{c}_{n3}$

Nash equilibrium 4 is optimal:

$p_{f1} = \frac{c_d q_{f1}}{q_d \delta}; p_{f2} = c_d + q_{f2} - q_d \delta; p_d = c_d;\ x_{f1} = 1 - \frac{c_d}{q_d \delta}; x_{f2} = \frac{-c_d + q_d \delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$

NP-2D. $\hat{c}_{n3} < c_d < \hat{c}_{n5}$

Nash equilibrium 5 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = c_d + q_{f2} - q_d \delta; p_d = c_d;\ x_{f1} = \frac{1}{2}; x_{f2} = \frac{-c_d + q_d \delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$

NP-2E. $c_d > \hat{c}_{n5}$

Nash equilibrium 7 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = \frac{q_{f2}}{2}; x_{f1} = \frac{1}{2}; x_{f2} = \frac{1}{2}$

NP-Region 3. $\hat{\delta}_2 < \delta < 1$

NP-3A. $0 < c_d < \hat{c}_{n6}$

Nash equilibrium 1 is optimal:

$p_d = c_d; p_{f1} = \frac{1}{2}(c_d + q_{f1} - q_d \delta); p_{f2} = \frac{c_d q_{f2}}{2 q_d \delta}$

$x_{f1} = \frac{c_d + q_{f1} - q_d \delta}{2 q_{f1} - 2 q_d \delta}; x_{f2} = -\frac{c_d}{2 q_{f2} - 2 q_d \delta}; x_{d1} = \frac{1}{2} - \frac{c_d}{q_d \delta} - \frac{c_d}{2 q_{f1} - 2 q_d \delta}; x_{d2} = 1 - \frac{c_d q_{f2} - 2 c_d q_d \delta}{2 q_{f2} q_d \delta - 2 {q_d}^2 \delta^2}$

NP-3B. $\hat{c}_{n6} < c_d < \hat{c}_{n7}$

Nash equilibrium 3 is optimal:

$p_d = c_d; p_{f1} = \frac{c_d q_{f1}}{q_d \delta}; p_{f2} = \frac{c_d q_{f2}}{2 q_d \delta};\ x_{f1} = 1 - \frac{c_d}{q_d \delta}; x_{f2} = -\frac{c_d}{2 q_{f2} - 2 q_d \delta}; x_{d1} = 0; x_{d2} = 1 - \frac{c_d q_{f2} - 2 c_d q_d \delta}{2 q_{f2} q_d \delta - 2 {q_d}^2 \delta^2}$

NP-3C. $\hat{c}_{n7} < c_d < \hat{c}_{n3}$

Nash equilibrium 4 is optimal:

$p_{f1} = \frac{c_d q_{f1}}{q_d \delta}; p_{f2} = c_d + q_{f2} - q_d \delta; p_d = c_d;\ x_{f1} = 1 - \frac{c_d}{q_d \delta}; x_{f2} = \frac{-c_d + q_d \delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$

NP-3D. $\hat{c}_{n3} < c_d < \hat{c}_{n5}$

Nash equilibrium 5 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = c_d + q_{f2} - q_d \delta; p_d = c_d;\ x_{f1} = \frac{1}{2}; x_{f2} = \frac{-c_d + q_d \delta}{q_{f2}}; x_{d1} = 0; x_{d2} = 0$

NP-3E. $c_d > \hat{c}_{n5}$

Nash equilibrium 7 is optimal:

$p_{f1} = \frac{q_{f1}}{2}; p_{f2} = \frac{q_{f2}}{2}; x_{f1} = \frac{1}{2}; x_{f2} = \frac{1}{2}$

Appendix F: Figures and Tables

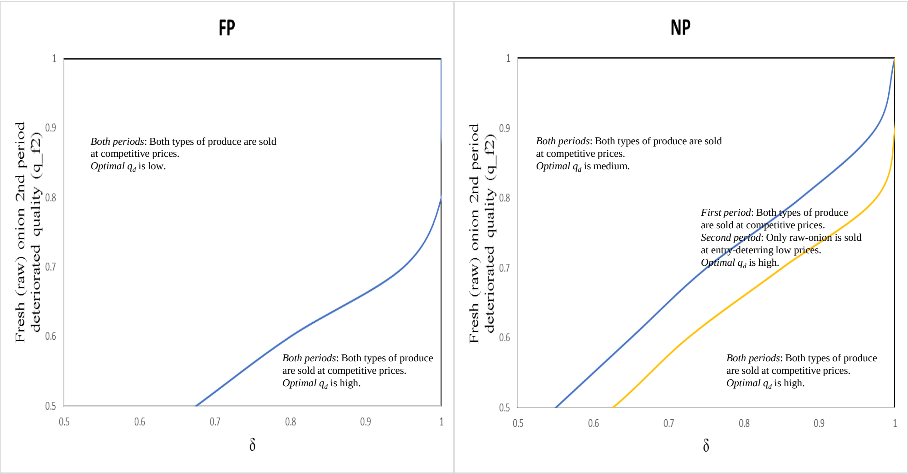
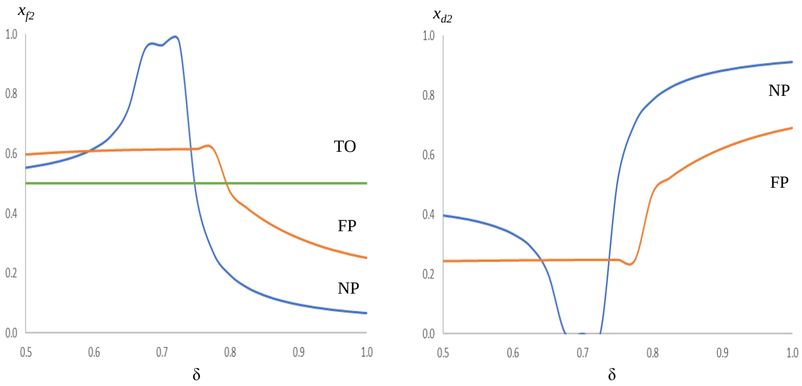


Figure F1 **Strategy choices of FP vs. NP wrt δ and q_{f2} when processing costs are low $\gamma = 0.001$**

Comparison	Low Cost	Medium Cost	High Cost	Overall Average
Consumer Surplus				
NP/TO	302.0%	248.7%	178.7%	243.1%
FP/TO	228.5%	188.4%	143.6%	186.8%
NP/FP	133.1%	132.1%	123.9%	129.7%
Optimal Quality				
NP/FP	131.9%	143.2%	160.6%	145.2%
Raw Onion Price in the First Period				
NP/TO	39.7%	53.2%	72.9%	55.3%
FP/TO	58.0%	69.9%	84.2%	70.7%
NP/FP	68.0%	75.7%	86.4%	76.7%
Raw Onion Price in the Second Period				
NP/TO	22.3%	36.3%	61.9%	40.2%
FP/TO	44.8%	58.8%	77.9%	60.5%
NP/FP	45.1%	58.9%	78.4%	60.8%
Processed-Onion Price				
NP/FP	41.5%	78.0%	130.0%	83.2%

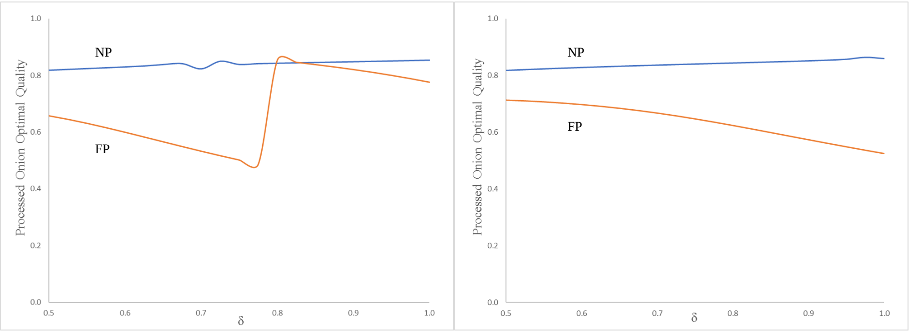
Table F1 **Summary of numerical results**



(a) Fresh (Raw) Onion Sales
in the Second period

(b) Processed Onion Sales
in the Second period

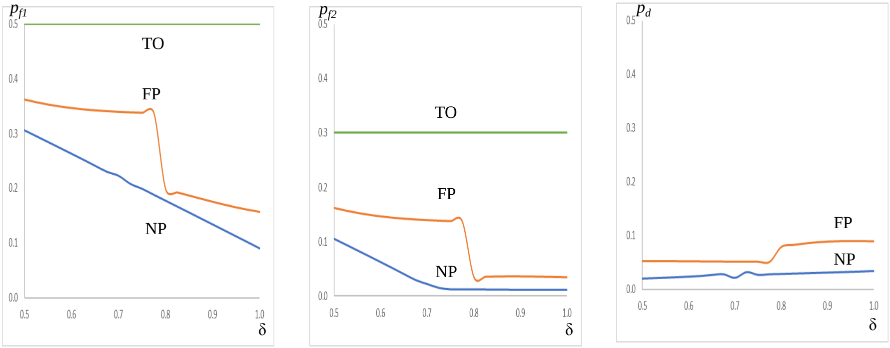
Figure F2 Second period sales quantities with optimal quality when $q_{f2} = 0.6$ and $\gamma = 0.001$



(a) Optimal Quality of Processed Onion
when γ = 0.001 and q_{f2}=0.6

(b) Optimal Quality of Processed Onion
when γ = 0.001 and q_{f2}=0.9

Figure F3 Optimal quality of processed produce



(a) Fresh (Raw) Onion Prices in the 1st period

(b) Fresh (Raw) Onion Prices in the 2nd period

(c) Processed Onion Prices

Figure F4 **Prices with optimal quality when** $q_{f2} = 0.6$ **and** $\gamma = 0.001$