Online Appendix for "Lemons, Trade-Ins and Certified Pre-Owned Programs"

SA1. Proofs

Proof of Lemma 1. First, we summarize the single-period net utilities for a type θ consumer in Table SA1 below, given the consumer actions in the current and previous periods (denoted as a and a', respectively). For example, when a consumer buys a new product after having purchased a new one in the previous period, (s)he gains p_u^C from selling the used product (that was purchased new in the previous period) through the trade-in program, (s)he also pays p_T^C and enjoys the utility θ from the new product. Therefore, the corresponding net utility in the current period is $\theta - p_T^C + p_u^C$. All other single-period net utilities can be constructed in a similar manner. All infeasible cases (e.g., a consumer cannot keep a peach or lemon unless a new one was purchased in the previous period) are represented by a dash.

Table SA1 Single-Period Net Utilities for a Type θ Consumer Under a CPO Program

$a' \backslash a$	Buy new	Keep a peach	Keep a lemon	Buy a peach	Buy a lemon	Remain inactive
Buy new	$\theta - p_T^C + p_u^C$	$\delta\theta$	$\delta\theta(1-s)$	$\delta\theta - p_p^C + p_u^C$	$\delta(1-s)\theta - p_l^C + p_u^C$	p_u^C
Keep a peach	$\theta - p^C$	–	–	$\delta\theta - p_p^C$	$\delta(1-s)\theta - p_l^C$	0
Keep a lemon	$\theta - p^C$	–	–	$\delta\theta - p_p^C$	$\delta(1-s)\theta - p_l^C$	0
Buy a peach	$\theta - p^C$	–	–	$\delta\theta - p_p^C$	$\delta(1-s)\theta - p_l^C$	0
Buy a lemon	$\theta - p^C$	–	–	$\delta\theta - p_p^C$	$\delta(1-s)\theta - p_l^C$	0
Remain inactive	$\theta - p^C$	–	–	$\delta\theta - p_p^C$	$\delta(1-s)\theta - p_l^C$	0

We next eliminate the dominated consumer strategies based on the consumer utilities. Buying a lemon in one period and remaining inactive in the next period is dominated: A type θ consumer obtains a utility of $(1-s)\delta\theta - p_l^C$ in the period (s)he buys a lemon from the secondary market, and a utility of 0 in the inactive period. If $(1-s)\delta\theta - p_l^C \geq 0$, then the consumer will gain a higher utility from always buying a lemon; otherwise (if $(1-s)\delta\theta - p_l^C < 0$), the consumer will obtain a higher utility from remaining inactive in all periods. Following a similar argument, we rule out all dominated consumer strategies and identify all the non-dominated strategies to be NN, NP, NH, PP, LL and II, as described in Lemma 1.

We now construct the expected per-period utilities from all the non-dominated strategies for a type θ consumer.

(i) NN segment consumers trade in the used product from the previous period to get p_u^C, pay p_T^C for the new product and enjoy utility θ from it. Therefore, $V_{NN}^C(\theta) = \theta - p_T^C + p_u^C$.

(ii) To calculate the expected per-period utility for the NP consumers, we need to determine the fraction of this segment that uses a new product purchased without a trade-in (this happens after using a peach), the fraction that uses a new product purchased with a trade-in (this happens after using a new product that becomes a lemon) and the fraction that uses a peach (this happens after using a new product that becomes a peach). Let z denote the fraction that uses a peach and $1-z$ denote the fraction that uses a new product in a given period. In the next period, all the peach owners in the previous period have to replace the peaches (as they reach their end-of-life) by buying new products without trade-ins. On the other hand, α fraction of the new product owners in the previous period now will be using the used products that turn out to be peaches, while the other $1-\alpha$ fraction have to trade in their used products that turn out to be lemons and buy new replacements. Therefore, in this period, the total peach owners will be $\alpha(1-z)$ and the total new product owners will be $z+(1-\alpha)(1-z)$. In the stationary equilibrium, these fractions remain constant over time; i.e., $z=\alpha(1-z)$ (for peach owners) and $1-z=z+(1-\alpha)(1-z)$ (for new product owners). Solving these equations gives $z=\frac{\alpha}{1+\alpha}$. Therefore, the fractions of consumers that buy new products with and without trade-ins are $(1-\alpha)(1-z)=\frac{1-\alpha}{1+\alpha}$ and $\frac{\alpha}{1+\alpha}$, respectively. As the per-period utilities of buying a new product with and without trade-ins are $\theta-p_T^C+p_u^C$ and $\theta-p^C$, respectively, and that of keeping a peach is $\theta\delta$, the expected per-period utility for NP consumers is $V_{NP}^C(\theta)=\frac{1-\alpha}{1+\alpha}(\theta-p_T^C+p_u^C)+\frac{\alpha}{1+\alpha}(\theta-p^C)+\frac{\alpha}{1+\alpha}(\delta\theta)=\frac{1+\alpha\delta}{1+\alpha}\theta-\frac{1}{1+\alpha}(\alpha p^C+(1-\alpha)(p_T^C-p_u^C))$.

(iii) NH segment consumers buy a new product in one period and then use it for a second period regardless of the quality realization. Therefore, the expected per-period utility is $V_{NH}^C(\theta)=\frac{1}{2}(\theta-p^C+\alpha(\theta\delta)+(1-\alpha)(1-s)\delta\theta)=\frac{1+\delta(1-(1-\alpha)s)}{2}\theta-\frac{p^C}{2}$.

(iv) For the PP/LL consumers, they always buy peaches/lemons in every period, and the corresponding per-period utilities are therefore $V_{PP}^C(\theta)=\delta\theta-p_p^C$ and $V_{LL}^C(\theta)=\delta(1-s)\theta-p_l^C$, respectively.

Proof of Lemma 2. As consumers self-select into one of the six non-dominated consumer strategies (as presented in Lemma 1) based on their type θ, there are six segments in the market. It is easy to check that $V_{NN}^C(\theta)-V_{NP}^C(\theta)$ is an increasing function of θ; therefore, consumers that optimally choose the NP strategy are of lower type θ than those choosing the NN strategy. Following a similar argument, consumers that optimally choose the NN, NP, NH, PP, LL and II strategies are of descending type θ. As such, there exist $\theta_1^C,\theta_2^C,\theta_3^C,\theta_4^C,\theta_5^C$ with $0\le\theta_5^C\le\theta_4^C\le\theta_3^C\le\theta_2^C\le\theta_1^C\le 1$ such that consumers of type $\theta\in(\theta_1^C,1]$, $\theta\in(\theta_2^C,\theta_1^C]$, $\theta\in(\theta_3^C,\theta_2^C]$, $\theta\in(\theta_4^C,\theta_3^C]$, $\theta\in(\theta_5^C,\theta_4^C]$, and $\theta\in[0,\theta_5^C]$ choose the NN, NP, NH, PP, LL, and II strategies, respectively.

To derive these thresholds, note that consumers of type θ_1^C are indifferent between choosing the NN and NP strategies; i.e., $V_{NN}^C(\theta_1^C)=V_{NP}^C(\theta_1^C)$. Solving this yields $\theta_1^C=\frac{-2p_T^C+2p_u^C+p^C}{\delta-1}$; and all other thresholds can be derived in a similar manner.

Demand Characterization. Based on Lemma 2, the size of the NN segment can be calculated as $1-\theta_1^C=\frac{p^C-2p_T^C+2p_T^u}{1-\delta}+1$. We solve for all the other segment sizes in a similar manner. They are $\theta_1^C-\theta_2^C=\frac{(\alpha+1)\delta s(p^C-2p_T^C+2p_u^C)}{(\delta-1)(\delta(\alpha s+s-1)+1)}$, $\theta_2^C-\theta_3^C=-\frac{p^C-2p_p^C}{\delta((\alpha-1)s-1)+1}-\frac{p^C-2p_T^C+2p_u^C}{\delta(\alpha s+s-1)+1}$, $\theta_3^C-\theta_4^C=\frac{p^C-2p_p^C}{\delta((\alpha-1)s-1)+1}+\frac{p_l^C-p_p^C}{\delta s}$, $\theta_4^C-\theta_5^C=\frac{p_l^C+p_p^C(s-1)}{\delta(s-1)s}$ and $\theta_5^C=\frac{p_l^C}{\delta-\delta s}$, for the NP, NH, PP, LL and II segments, respectively.

We can now construct the demand functions based on these segment sizes. First, the total demand for new products purchased with trade-ins comes from both a fraction $\frac{1-\alpha}{1+\alpha}$ of NP consumers (as discussed in the derivation of $V_{NP}^C(\theta)$ in the proof of Lemma 1 above), as well as the NN consumers; i.e., $D_{wTI}^C = (1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C) = 1 - \frac{(p^C - 2p_T^C + 2p_u^C)(\delta(2\alpha s-1)+1)}{(\delta-1)(\delta(\alpha s+s-1)+1)}$. Second, the total demand for new products purchased without trade-ins comes from both a fraction $\frac{\alpha}{1+\alpha}$ of NP consumers and a fraction $\frac{1}{2}$ of NH consumers; i.e., $D_{woTI}^C = \frac{\alpha}{1+\alpha}(\theta_1^C - \theta_2^C) + \frac{1}{2}(\theta_2^C - \theta_3^C) = \frac{2p_p^C - p^C}{2\delta((\alpha-1)s-1)+2} - \frac{p^C - 2p_T^C + 2p_u^C}{2(\delta(\alpha s+s-1)+1)} + \frac{\alpha\delta s(p^C - 2p_T^C + 2p_u^C)}{(\delta-1)(\delta(\alpha s+s-1)+1)}$. Third, the total peach and lemon demands are $D_p^C = \theta_3^C - \theta_4^C = \frac{p_l^C - p_p^C}{\delta s} + \frac{p^C - 2p_p^C}{\delta((\alpha-1)s-1)+1}$ and $D_l^C = \theta_4^C - \theta_5^C = \frac{p_l^C + p_p^C(s-1)}{\delta(s-1)s}$, respectively.

Next, we derive the market-clearing prices p_u^C and p_l^C based on the following market-clearing conditions: $(1-\alpha)(1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C) = (\theta_4^C - \theta_5^C)$ for lemons and $\alpha(1-\theta_1^C) = (\theta_3^C - \theta_4^C)$ for peaches. Solving them yields $p_u^C = \frac{(\delta-1)(s-1)(\delta(\alpha s+s-1)+1)\left(\frac{2p_p^C - p^C}{(s-1)(\delta((\alpha-1)s-1)+1)} - \frac{\alpha(p^C-2p_T^C)((\alpha-1)s+2)}{(\alpha+1)(\delta-1)(s-1)} + \frac{(\alpha-1)(p^C-2p_T^C)}{\alpha+(\alpha+1)\delta(\alpha s+s-1)+1} + \frac{p_p^C}{\delta(s-1)} + \frac{\alpha s}{s-1} - 1\right)}{2(-\delta+s(\alpha+\delta(\alpha+(\alpha-1)\alpha s+1)-1)+1)}$ and $p_l^C = -\frac{(s-1)((\alpha-1)\delta s(\delta p^C(\alpha s-1)+p^C+\alpha\delta s(\delta((\alpha-1)s-1)+1)) - (\delta-1)p_p^C(\delta(\alpha s+s-1)+1))}{(\delta((\alpha-1)s-1)+1)(-\delta+s(\alpha+\delta(\alpha+(\alpha-1)\alpha s+1)-1)+1)}$. Substituting them back, we can rewrite the demand functions as: $D_{wTI}^C = \frac{\delta(\delta p^C(2\alpha s-1)+p^C-(\alpha-1)\alpha\delta s^2(\delta((\alpha-1)s-1)+1)) - p_p^C(\delta+(\alpha-1)\delta s+1)(\delta(2\alpha s-1)+1)}{\delta(\delta((\alpha-1)s-1)+1)(-\delta+s(\alpha+\delta(\alpha+(\alpha-1)\alpha s+1)-1)+1)}$, $D_{woTI}^C = \frac{\delta(((\alpha-1)s+1)(\delta((\alpha-1)s-1)+1)(-\delta+2\alpha\delta s+1) - p^C(-2\delta+s(\alpha+\delta+\alpha\delta((\alpha-1)s+3)-1)+2)) + \delta^2 p_p^C(s(\alpha(4(\alpha-1)s+3)+3)-3) + \delta p_p^C((5\alpha-3)s+2) + p_p^C}{2\delta(\delta((\alpha-1)s-1)+1)(-\delta+s(\alpha+\delta(\alpha+(\alpha-1)\alpha s+1)-1)+1)}$, $D_p^C = \frac{\delta(p^C(-4\delta+6\delta s+4) - \delta(s-1)s(\delta(s+2)-2)) + p_p^C(\delta(s-2)-2)(\delta(3s-2)+2)}{\delta(\delta(s+2)-2)(\delta((s-6)s+4)+2(s-2))}$, and $D_l^C = \frac{4\delta p^C + \delta^2(2p^C(s-2) + p_p^C(s-2)^2 + 2s) - 4p_p^C + \delta^3(-s)(s+2)}{\delta(\delta(s+2)-2)(\delta((s-6)s+4)+2(s-2))}$.

Formulation of the Firm's Problem. The firm generates profit from selling new products with and without trade-ins, and selling peaches and lemons. Therefore $\Pi^C(p^C, p_T^C, p_p^C) = D_{wTI}^C(p_T^C - p_u^C - c - e) + D_{woTI}^C(p^C - c) + D_p^C(p_p^C) + D_l^C(p_l^C)$. The firm then solves $\max_{p^C, p_T^C, p_p^C} \Pi^C$, subject to $1-\theta_1^C, \theta_1^C - \theta_2^C, \theta_2^C - \theta_3^C, \theta_3^C - \theta_4^C, \theta_4^C - \theta_5^C, \theta_5^C \geq 0$. The constraints ensure non-negative segment sizes. They also ensure that all prices are non-negative: For example, since $\theta_5 = \frac{p_l^C}{\delta - \delta s}$, $p_l^C \geq 0$ follows from $\theta_5 \geq 0$ (note that $0 \leq \delta, s \leq 1$), and similarly for other prices.

Proof of Proposition 1. Based on the profit function, we can solve the maximization problem with the constraints for the firm. To this end, we first identify all the candidate optimal solutions by constructing the Lagrangian and applying the Karush–Kuhn–Tucker (KKT) conditions. Then we rule out the dominated ones by comparing the profits from candidate solutions that have overlapping ranges to finalize the optimal solution.

When $e = 0$ (i.e., Case(O) in Proposition 1), $p^{C*} = \frac{2c(\delta(s-3)-1)+\delta(-6\delta+s(\delta(s+2)+4)-8)-2}{\delta(7s-12)-4}$, $p_T^{C*} = \frac{-4(c+4)\delta+2(c+5)\delta s-4(c+1)+\delta^2(s-2)^2}{2\delta(7s-12)-8}$ and $p_p^{C*} = \frac{\delta(2c(s-4)+\delta s(s+8)-2(8\delta+s))}{2\delta(7s-12)-8}$, and the resulting profit is $\Pi^{C*} = \frac{(2c+\delta(s-2)-2)^2}{4(\delta(12-7s)+4)}$. In this case, the market segmentation outcome is [NN PP LL II] (where the list shows the strategy choices by consumers of descending type θ); i.e., the firm optimally chooses the prices such that the strategic peach-holding behavior is dominated for all consumers and the NP segment no longer exists in the market. Moreover, there exist $\overline{e_1}(c,s,\delta), \overline{e_2}(c,s,\delta), \overline{e_3}(c,s,\delta) > 0$, with $\overline{e_2}(c,s,\delta) \leq \overline{e_3}(c,s,\delta)$ (the expressions of these thresholds are available but complicated and

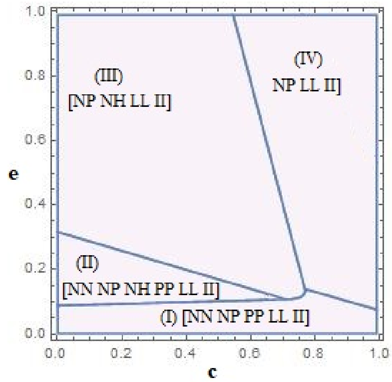


Figure SA1 **The equilibrium market segmentation outcome under a CPO program (figure constructed with $s=0.3$ and $\delta=0.3$).**

omitted for brevity), such that we have the other cases as presented in the proposition. Figure SA1 presents a graphical example of this market segmentation outcome.

Proof of Proposition 2. We replicate and extend the analysis in Rao et al. (2009), by using slightly different notation adapted to our CPO program notation as described in the paper, and allowing $c \geq 0$ (Rao et al. 2009 assume $c=0$). In this case, consumers can choose from the following actions in every period: buy a new product (and trade-in the used unit if a new one was purchased in the previous period), buy a used product from the secondary market, keep the peach/lemon for a second period of use (if a new unit was purchased in the previous period), or remain inactive. Given the two-period lifespan of products, it will suffice to focus on two-period consumer strategies. We first summarize the single-period net utilities for a type θ consumer in Table SA2 below, given the consumer actions in the current and previous periods (a and a'). Here w denotes the average quality of used products (with $(1-s)\delta \leq w \leq \delta$ since used products in the secondary market consist of both lemons and peaches); other details are similar to the case under the CPO program.

Table SA2 **Single-Period Net Utilities for a Type θ Consumer Under a Pure Trade-In Program**

$a' \backslash a$	Buy a new	Keep a peach	Keep a lemon	Buy a used	Remain inactive
Buy new	$\theta - p_T^{TI} + p_u^{TI}$	$\delta\theta$	$\delta\theta(1-s)$	$w\theta$	p_u^{TI}
Keep a peach	$\theta - p^{TI}$	–	–	$w\theta - p_u^{TI}$	0
Keep a lemon	$\theta - p^{TI}$	–	–	$w\theta - p_u^{TI}$	0
Buy a used	$\theta - p^{TI}$	–	–	$w\theta - p_u^{TI}$	0
Remain inactive	$\theta - p^{TI}$	–	–	$w\theta - p_u^{TI}$	0

Next, we eliminate all dominated consumer strategies in a way similar to the CPO case and find that the non-dominated strategies are NN, NP, NH, UU and II; here, UU refers to the strategy of buying a used product from the secondary market in every period.

The expected per-period consumer utility for the UU segment consumers is $V_{UU}^{TI}(\theta) = w\theta - p_u^{TI}$. All other utilities are similar to those in the CPO case (with the corresponding prices under the pure

TI program). Based on the utilities, we next characterize the demand functions. It is easy to check that consumers who optimally choose the strategies NN, NP, NH, UU and II are of descending type θ. Therefore, there exist $\theta_1^{TI}, \theta_2^{TI}, \theta_3^{TI}, \theta_4^{TI}$ with $0 \le \theta_4^{TI} \le \theta_3^{TI} \le \theta_2^{TI} \le \theta_1^{TI} \le 1$ such that consumers of type $\theta \in (\theta_1^{TI}, 1]$, $\theta \in (\theta_2^{TI}, \theta_1^{TI}]$, $\theta \in (\theta_3^{TI}, \theta_2^{TI}]$, $\theta \in (\theta_4^{TI}, \theta_3^{TI}]$ and $\theta \in [0, \theta_4^{TI}]$ will choose the NN, NP, NH, UU and II strategies, respectively. Similar to the solution approach in the CPO case, we identify the thresholds θ_i^{TI} (for $i \in \{1,2,3,4\}$) based on the utilities of the indifferent consumers. Then the NN, NP, NH, UU and II segment sizes can be calculated by $1-\theta_1^{TI}$, $\theta_1^{TI}-\theta_2^{TI}$, $\theta_2^{TI}-\theta_3^{TI}$, $\theta_3^{TI}-\theta_4^{TI}$, and θ_4^{TI}, respectively. Then the consumer demands for used products, and new products purchased with and without trade-ins can be written as $D_u^{TI} = \theta_3^{TI} - \theta_4^{TI}$, $D_{wTI}^{TI} = (1-\theta_1^{TI}) + \frac{1-\alpha}{1+\alpha}(\theta_1^{TI} - \theta_2^{TI})$ and $D_{woTI}^{TI} = \frac{\alpha}{1+\alpha}(\theta_1^{TI} - \theta_2^{TI}) + \frac{1}{2}(\theta_2^{TI} - \theta_3^{TI})$, respectively; and the average quality in the secondary market will be $w = \frac{\delta(\alpha(1-\theta_1^{TI})) + (1-s)\delta((1-\alpha)(1-\theta_1^{TI}) + \frac{1-\alpha}{1+\alpha}(\theta_1^{TI}-\theta_2^{TI}))}{(1-\theta_1^{TI}) + \frac{1-\alpha}{1+\alpha}(\theta_1^{TI}-\theta_2^{TI})}$. The used product price p_u^{TI} is endogenously determined by the market-clearing condition that balances all the supply and demand of used products; i.e., we can derive p_u^{TI} from $(1-\theta_1^{TI}) + \frac{1}{1+\alpha}(\theta_1^{TI} - \theta_2^{TI}) = \theta_3^{TI} - \theta_4^{TI}$. Substituting the market-clearing price p_u^{TI} into the demand and the secondary market average quality functions, we can then write the firm profit as a function of the firm decisions p^{TI}, p_T^{TI}. In particular, $\Pi^{TI}(p^{TI}, p_T^{TI}) = D_{wTI}^{TI}(p_T^{TI} - c) + D_{woTI}^{TI}(p^{TI} - c)$. The firm solves $\max_{p^{TI}, p_T^{TI}} \Pi^{TI}$, subject to $1-\theta_1^{TI}, \theta_1^{TI}-\theta_2^{TI}, \theta_2^{TI}-\theta_3^{TI}, \theta_3^{TI}-\theta_4^{TI}, \theta_4^{TI} \ge 0$. The results are summarized below.

Case (TI-I) When $0 \le \delta \le \frac{4}{4+s}$, the market segmentation outcome is [NN NP UU II] (i.e., $\theta_3^{TI} = \theta_2^{TI}$ at optimality and the NH segment disappears). In this case, $p_T^{TI*} = \frac{-8c(\delta-1)(\delta(-48\delta+s(\delta(s+18)-26)+32)+16)+\delta(\delta^3(-(s-4))(s(7s-60)+96)+\delta^2(s(s(9s-208)+528)-256)+8\delta(5s(3s+2)-64)-272s+256)+128}{(\delta(s+8)-8)(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}$, $p^{TI*} = \frac{-4c(\delta-1)(3\delta(s-4)-4)+\delta(16(3\delta^2+\delta-3)+\delta(7-5\delta)s^2-4(\delta-1)(3\delta+7)s)-16}{\delta(96\delta+s(\delta(s-48)+48)-64)-32}$; as a result, $\Pi^{TI*} = \frac{2(\delta-1)(2c+\delta(s-2)-2)^2}{\delta(96\delta+s(\delta(s-48)+48)-64)-32}$.

Case(TI-II) When $\frac{4}{4+s} < \delta \le 1$, the market segmentation outcome is [NP UU II] (i.e., $\theta_1^{TI} = 1$ at optimality and the NN segment also disappears). In this case, $p_T^{TI*} = \frac{2c(\delta(5s-6)-2)-8\delta(\delta+2)+\delta s(14-9\delta(s-2))-8}{2(\delta(7s-10)-6)}$, $p^{TI*} = \frac{2c(\delta(s-2)-2)+\delta(\delta(s(s+3)-6)+3s-8)-2}{\delta(7s-10)-6}$; as a result, $\Pi^{TI*} = -\frac{(2c+\delta(s-2)-2)^2}{4(\delta(7s-10)-6)}$.

Proof of Corollary 1. Under the CPO program, based on the Proposition 1 results, $p^C - p_T^C = \frac{\delta(2c(s-4)+\delta s(s+8)-2(8\delta+s))}{2\delta(7s-12)-8}$, $p_p^C = \frac{\delta(2c(s-4)+\delta s(s+8)-2(8\delta+s))}{2\delta(7s-12)-8}$ and $p_l^C = \frac{\delta(s-1)(4c+\delta(8-5s))}{\delta(7s-12)-4}$ at $e = 0$ in equilibrium. On the other hand, under the pure TI program, $p^{TI} - p_T^{TI} = \frac{2(\delta-1)\delta s(-2c(\delta(s-24)+24)+16\delta(9\delta-8)+\delta s(\delta(s-70)+74)-16)}{(\delta(s+8)-8)(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}$ and $p_u^{TI} = -\frac{\delta(-8\delta+(3\delta-4)s+8)(32\delta(c+s-2)-32c+\delta^2((s-32)s+64))}{(\delta(s+8)-8)(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}$ when $0 \le \delta \le \frac{4}{4+s}$; and $p^{TI} - p_T^{TI} = \frac{c(\delta(4-6s)-4)+\delta^2(s(11s-12)-4)-8\delta s+4}{2(\delta(7s-10)-6)}$, $p_u^{TI} = \frac{\delta(s-1)(4c+\delta(6-5s)+2)}{\delta(7s-10)-6}$ otherwise. The conclusions $p^C - p_T^C > p^{TI} - p_T^{TI}$, $p_p^C < p_T^{TI}$ and $p_l^C < p_u^{TI}$ follow by comparing the corresponding expressions.

Proof of Lemma 3. We derive this result by comparing the optimal firm profits in the cases under the CPO program (as presented in Proposition 1) and the pure TI program (as presented in Proposition 2). Specifically, $\bar{e} = -\frac{4(2c\delta-2c-2\delta^2+\delta^2 s-\delta s+2)}{-4\delta+7\delta s-12} - \sqrt{\frac{2(1-\delta)(\delta(7s-12)-4)(\delta^2(s(s+8)+64)-8\delta(s+16)+64)(2c+\delta(s-2)-2)^2}{(\delta(4-7s)+12)^2(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}}$ when $\delta \le \frac{4}{s+4}$, and $\bar{e} = -\frac{4(2c\delta-2c-2\delta^2+\delta^2 s-\delta s+2)}{-4\delta+7\delta s-12} - 2\sqrt{\frac{3(\delta-1)^2(\delta(7s-12)-4)(2c+\delta(s-2)-2)^2}{(\delta(4-7s)+12)^2(\delta(7s-10)-6)}}$ otherwise; note that the threshold depends on product characteristics captured by c, s, δ. Then we can derive $\frac{\partial \bar{e}}{\partial c} = 2(\delta -$

$1)(\frac{4}{\delta(4-7s)+12} - \sqrt{2}\sqrt{-\frac{(\delta-1)(\delta(7s-12)-4)\left(\delta^2(s(s+8)+64)-8\delta(s+16)+64\right)(2c+\delta(s-2)-2)^2}{(\delta(4-7s)+12)^2(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}}/(\delta-1)(2c+\delta(s-2)-2))$ when $\delta \leq \frac{4}{s+4}$, and $\frac{\partial \overline{e}}{\partial c} = -4\sqrt{3}\sqrt{\frac{(\delta-1)^2(\delta(7s-12)-4)(2c+\delta(s-2)-2)^2}{(\delta(4-7s)+12)^2(\delta(7s-10)-6)}}/2c+\delta(s-2)-2-\frac{8(\delta-1)}{\delta(7s-4)-12}$ otherwise. Based on these results, we can show that $\frac{\partial \overline{e}}{\partial c} < 0$ always holds.

Proof of Proposition 3. First, by comparing the optimal firm profits under the pure TI and CPO programs, we can show that $\overline{e} < \overline{e_1}(c,s,\delta)$ ($\overline{e_1}(c,s,\delta)$ is specified in Proposition 1). Therefore, the equilibrium market segmentation outcome under the CPO program for $e > 0$ is always [NN NP PP LL II] when the CPO program leads to a higher firm profit (i.e., $e < \overline{e}$) and is adopted. Moreover, in this case (which is Case (I) in Proposition 1), the NP segment size is $\frac{3e}{4-4\delta}$, which tends to 0 as $e \to 0$. This proves Part (i) of the proposition.

For Part (ii), we first study the VOT. Based on our previous analysis, we have $VOT^C = \frac{D_p^C + D_l^C}{D_{wTI}^C + D_{woTI}^C} = \frac{8c(\delta-1)+\delta(-8\delta+e(7s-4)+4(\delta-1)s)-12e+8}{4(\delta-1)(2(c+e-1)+\delta(s-2))}$ and $VOT^{TI} = \frac{D_u^{TI}}{D_{wTI}^{TI} + D_{woTI}^{TI}} = \frac{\delta(s+8)-8}{8(\delta-1)}$ when $\delta \leq \frac{4}{s+4}$; and $VOT^{TI} = \frac{1}{2}$ otherwise. Therefore, when $\delta \leq \frac{4}{s+4}$, $VOT^C - VOT^{TI} = \frac{\delta s(-2c+2\delta-\delta s+2)+4e(3\delta(s-2)-2)}{8(\delta-1)(2(c+e-1)+\delta(s-2))}$. Given $e < \overline{e}$, we can prove that $8(\delta-1)(2(c+e-1)+\delta(s-2)) > 0$ for the denominator (as we also have $0 < \delta < 1$), and $\delta s(-2c+2\delta-\delta s+2)+4e(3\delta(s-2)-2) > 0$ for the numerator; therefore, $VOT^C - VOT^{TI} > 0$. The case when $\delta > \frac{4}{s+4}$ can be proved similarly. Next, we explore the effect of the CPO inspection cost (e) on VOT^C (since VOT^{TI} is independent of e). We can calculate the derivative to have $\frac{\partial VOT^C}{\partial e} = \frac{(\delta(7s-12)-4)(2c+\delta(s-2)-2)}{4(\delta-1)(2(c+e-1)+\delta(s-2))^2} < 0$ (as $0 < s,c,\delta < 1$). Moreover, it is straightforward to check that $\lim_{e\to 0} VOT^C = 1$. Below, We also study the trade-in discount. Based on previous analysis, when $e < \overline{e}$, we have $p^C - p_T^C = \frac{\delta(2c(s-4)+\delta s(s+8)-2(8\delta+s))+e(\delta(4-5s)+4)}{2\delta(7s-12)-8}$ under the CPO program; and $p^{TI} - p_T^{TI} = \frac{2(\delta-1)\delta s(-2c(\delta(s-24)+24)+16\delta(9\delta-8)+\delta s(\delta(s-70)+74)-16)}{(\delta(s+8)-8)(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}$ when $\delta \leq \frac{4}{s+4}$, and $p^{TI} - p_T^{TI} = \frac{2(\delta-1)\delta s(-2c(\delta(s-24)+24)+16\delta(9\delta-8)+\delta s(\delta(s-70)+74)-16)}{(\delta(s+8)-8)(\delta(96\delta+s(\delta(s-48)+48)-64)-32)}$ otherwise under the pure TI program. Given $e < \overline{e}$, $(p^C - p_T^C) - (p^{TI} - p_T^{TI}) = \frac{4\delta(c+e+4)-2\delta s(c+e+5)+4(c+e+1)+\delta^2\left(-(s-2)^2\right)}{2\delta(7s-12)-8} + \frac{4c(\delta(s-3)-1)+\delta(-3es+2\delta(s(s+2)-6)+8s-16)-4}{2\delta(7s-12)-8} - \frac{2(\delta-1)\delta s(-2c(\delta(s-24)+24)+16\delta(9\delta-8)+\delta s(\delta(s-70)+74)-16)}{(\delta(s+8)-8)(\delta(96\delta+s(\delta(s-48)+48)-64)-32)} > 0$ when $\delta \leq \frac{4}{s+4}$, and $(p^C - p_T^C) - (p^{TI} - p_T^{TI}) = \frac{8c(\delta(s-2)(\delta(7s-8)-1)-2)-e(-4\delta+5\delta s-4)(-10\delta+7\delta s-6)+2\delta^3(s((131-35s)s-154)+56)+4\delta^2(s(20s-43)+20)-48\delta(s-1)+16}{2(\delta(7s-12)-4)(\delta(7s-10)-6)} > 0$ otherwise. Moreover, while $p^{TI} - p_T^{TI}$ is independent of e, $\frac{\partial p^C - p_T^C}{\partial e} = \frac{\delta(4-5s)+4}{2\delta(7s-12)-8} < 0$ (as $0 < s,c,\delta < 1$).

For Part (iii), the fraction of consumers served refers to the fraction of all active consumers that buy either new or used products. Under a CPO program, it can be calculated as $MC^C = 1 - \theta_5^C$ (based on the Lemma 2 results), evaluated at the optimal prices chosen by the firm in the stationary equilibrium. The fraction of consumers served under a pure TI program (MC^{TI}) can be derived in a similar way. Specifically, $MC^C = \frac{4(c+e-1)+2\delta(s-2)}{\delta(7s-12)-4}$; and $MC^{TI} = -\frac{16(\delta-1)(2c+\delta(s-2)-2)}{\delta(96\delta+s(\delta(s-48)+48)-64)-32}$ when $\delta \leq \frac{4}{s+4}$, and $MC^{TI} = \frac{-4c+4\delta-2\delta s+4}{10\delta-7\delta s+6}$ otherwise. We can then show that $MC^C > MC^{TI}$ holds when $e < \overline{e}$. Consumer surplus under a CPO program can be calculated as $CS^C = \int_{\theta_1^C}^1 V_{NN}^C(\theta)d\theta + \int_{\theta_2^C}^{\theta_1^C} V_{NP}^C(\theta)d\theta + \int_{\theta_3^C}^{\theta_2^C} V_{NH}^C(\theta)d\theta + \int_{\theta_4^C}^{\theta_3^C} V_{PP}^C(\theta)d\theta + \int_{\theta_5^C}^{\theta_4^C} V_{LL}^C(\theta)d\theta$ (based on the Lemma 2 results), evaluated at the optimal equilibrium optimum. Consumer surplus under a pure TI program (CS^{TI}) can be derived in a similar way. Specifically, $CS^C = -\frac{8c^2(\delta-1)+8c(\delta-1)(2e+\delta(s-2)-2)+e^2(-4\delta+7\delta s-12)+8(\delta-1)e(\delta(s-2)-2)+2(\delta-1)(\delta(s-2)-2)^2}{16(\delta-1)(\delta(7s-12)-4)}$; and $CS^{TI} = \frac{(\delta-1)(2c+\delta(s-2)-2)^2}{\delta(96\delta+s(\delta(s-48)+48)-64)-32}$ when $\delta \leq \frac{4}{s+4}$, and $CS^{TI} = -\frac{(2c+\delta(s-2)-2)^2}{8(\delta(7s-10)-6)}$ otherwise. $CS^C > CS^{TI}$ when $e < \bar{e}$ follows from direct comparisons.

SA2. Details of the Partial CPO Coverage Models in §4.2

Under partial CPO coverage where the CPO program only inspects a β $(0 < \beta \leq 1)$ fraction of all used products traded-in, the new product sales and trade-in model features remain the same as in the main model. Below, we explain the key differences of the partial coverage models.

First, we focus on the Partial CPO Coverage Model 1. Recall that in our main model with the full-coverage CPO program, the total volume of used products traded-in is $(1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C)$ (where $(1-\theta_1^C)$ and $(\theta_1^C - \theta_2^C)$ respectively represent the NN and NP segment sizes). Among them, the peaches come from the trade-ins by the NN consumers, with volume $\alpha(1-\theta_1^C)$ (since α fraction of the NN consumer trade-ins are peaches). Meanwhile, all the rest of the trade-ins are non-certified, with volume $\left((1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C)\right) - \alpha(1-\theta_1^C) = (1-\alpha)(1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C)$. With partial CPO coverage, as we assume the proportion of peaches from the NN trade-ins in the inspected fraction to be the same as that in the entire used product population (i.e., α), the expected total amount of identified peaches (that go through inspection and are identified to be of high quality) will be $\beta(\alpha(1-\theta_1^C))$. Meanwhile, all the others, of total volume $\left((1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C)\right) - \beta(\alpha(1-\theta_1^C))$, will be sold at price $p_l^{C,\beta}$. As such, the market-clearing conditions now become $\beta(\alpha(1-\theta_1^C)) = \theta_3^C - \theta_4^C$ for the identified peaches and $\left((1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C)\right) - \beta(\alpha(1-\theta_1^C)) = \theta_4^C - \theta_5^C$ for all the other trade-ins. Similar to the main model, we can now solve for the prices $p_l^{C,\beta}$ and $p_u^{C,\beta}$ based on the market clearing-conditions (expressions omitted here for brevity). Then we can formulate the firm profit as a function of the firm's price decisions; in particular, $\Pi^{C,\beta}(p^{C,\beta}, p_T^{C,\beta}, p_p^{C,\beta}) = D_{wTI}^{C,\beta}(p_T^{C,\beta} - p_u^{C,\beta} - c - \beta e) + D_{woTI}^{C,\beta}(p^{C,\beta} - c) + D_p^{C,\beta}(p_p^{C,\beta}) + D_l^{C,\beta}(p_l^C)$. Following the same solution approach as described in the main model, we can derive the full optimal solution characterization under the CPO program, and compare the resulting profit to that under the pure TI program. Next, we resort to an extensive numerical analysis to study the effects of the CPO program size β on the secondary market efficiency (captured by VOT) and the trade-in discount, as the corresponding analytical analysis becomes intractable due to the added complexity introduced by the partial CPO coverage. In particular, our numerical analysis covers feasible combinations of the parameters c, δ and s, over their valid ranges $(0,1)$ using increments of 0.1. As β increases, the effectiveness of the CPO program in helping increase the trade-in discount and the VOT is expected to increase; i.e., an increase in β should drive up both the trade-in discount and the VOT. This is indeed consistently reflected in our numerical study, and Figure 2 in the paper presents an example.

In Partial CPO Coverage Model 2, we extend the analysis to also capture the possibility that as the program size β decreases (e.g., only used cars with lower age and mileage are inspected in the

CPO program), the proportion of peaches among the inspected fraction will be higher. To this end, we assume the peach proportion to be $\phi(\beta) = \frac{\alpha}{\alpha+\beta(1-\alpha)}$, which decreases in β. As such, all the model and analysis details remain unchanged, except that the market-clearing conditions now become $\beta(\phi(\beta)(1-\theta_1^C)) = \theta_3^C - \theta_4^C$ for the identified peaches, and $\left((1-\theta_1^C) + \frac{1-\alpha}{1+\alpha}(\theta_1^C - \theta_2^C)\right) - \beta(\phi(\beta)(1-\theta_1^C)) = \theta_4^C - \theta_5^C$ for all the other trade-ins. We next follow a similar solution approach as in the main model to solve this problem. We also conduct an extensive numerical study to identify the effects of β in this case. We observe that the increase in β drives up the VOT and trade-in discount, and Figure 3 in the paper presents an example.

SA3. Extensions to Competitive Markets

While we focus on a monopolistic model in the main analysis in the paper to highlight the unexplored mechanism underlying CPO programs and their market impacts, we now generalize our analysis to consider two types of market competitions.

Market Competition in the Secondary Market (CP1). For durable products such as automotives, peer-to-peer (P2P) trade of used products outside of the producer-managed channels (i.e., through the CPO program) commonly exists. We therefore incorporate this resale market in our analysis below. Specially, we consider that among all used products sold by consumers, only a y portion of them are traded in the producer-managed channel through the CPO program (as in the main model), while the rest are traded in the P2P resale market. The producer sells the new products, peaches, and lemons similarly as in the main model: In addition to the regular new product sales, the producer also offers the CPO program that acquires used product through trade-ins from consumers and then inspects and resells the peaches with (lemons without) the CPO certification. While the producer sells peaches and lemons separately under the CPO program, the P2P market sells a mix of both due to the absence of CPO inspection and certification. In the analysis, we include "$CP1$" in the superscript to denote quantities in this competitive case.

First, we analyze the consumer choices. The consumer utility of buying a used product from the P2P resale market may be influenced by a myriad of factors, such as reputation of the sales platform, individual consumer preferences towards the different sales channels, transaction risks, or perceived average quality. These factors may result in different market segmentation outcomes: For example, used products from the P2P market may be considered a higher- or lower-value substitute purchase option to buying lemons from the firm. We capture the overall effect of these factors in the parameter w_P, so the per-period utility of such a purchase of used product from the P2P resale market will be $w_P\delta\theta$ with $0 < w_P < 1$ for a θ type consumer. Next, we list all the possible single-period consumer actions and the associated per-period utilities (similar to the analysis in Table SA1), based on which we can then identify and compare all the possible consumer strategies. As such, we derive the non-dominated consumer strategies, which include NN, NP, NH, PP, LL and II (as defined in the main model), and the additional option of always buying a used product from the P2P market (denoted as UU2). In particular, the expected per-period utility of the consumer strategy UU2 is $V_{UU2}^{CP1}(\theta) = w_P\delta\theta - p_{u2}^{CP1}$ (with the used product price p_{u2}^{CP1} in the P2P market). Moreover, as used product owners can either sell to the P2P market or trade in through the CPO program, the NN and NP consumer utilities become $V_{NN}^{CP1}(\theta) = \theta - p^{CP1} + ((y)(p_{u1}^{CP1} + (p^{CP1} - p_T^{CP1})) + (1-y)(p_{u2}^{CP1}))$ and $V_{NP}^{CP1}(\theta) = \frac{\alpha}{1+\alpha}(\theta - p^{CP1} + \delta\theta) + \frac{1-\alpha}{1+\alpha}(\theta - p^{CP1} + ((y)(p_{u1}^{CP1} + (p^{CP1} - p_T^{CP1})) + (1-y)(p_{u2}^{CP1})))$. We compare the per-period utilities of all the non-dominated consumer strategies (similar to the proof of Lemma 2 in §SA1). We find that consumers who choose NN NP NH PP are of descending types. To see this, we solve $V_{NN}^{CP1}(\theta_1^{CP1}) - V_{NP}^{CP1}(\theta_1^{CP1}) = 0$ for θ_1^{CP1}, such that when $\theta > \theta_1^{CP1}$ $(\theta \leq \theta_1^{CP1})$, $V_{NN}^{CP1}(\theta) - V_{NP}^{CP1}(\theta) > 0 (\leq 0)$; in other words, consumers who optimally choose NN (with $V_{NN}^{CP1}(\theta) - V_{NP}^{CP1}(\theta) > 0$) will be of higher type θ than the NP consumers (with $V_{NN}^{CP1}(\theta) - V_{NP}^{CP1}(\theta) \leq 0$). In particular, consumers of type θ_1^{CP1} are indifferent between choosing NN or NP. Other strategy comparisons can be conducted similarly. We further find that the ordering of consumers (in descending types) who choose PP, LL, and UU2 depends on a condition on w_P and s. When $1 - s \leq w_P$ (hereafter referred to as Case (CP1-I)), the consumer segments are ordered as PP, UU2, LL; and otherwise when $w_P < 1 - s$ (hereafter referred to as Case (CP1-II)), they are ordered as PP, LL, UU2. Both cases are possible; e.g., when the P2P sales platform is more (less) credible, w_P may be higher (lower) as in Case (CP1-I) (Case (CP1-II)). Below, we present the solution details of Case (CP1-I) for illustration. Case (CP1-II) can be solved similarly (details omitted) to confirm that, although with differences in the specific expressions, all main insights are directionally the same.

For Case (CP1-I), we first characterize the consumer demands. As shown above, consumers of type $\theta \in (\theta_1^{CP1}, 1]$ will choose NN, we can then calculate the size of the NN segment, with $sgNN^{CP1} = (1 - \theta_1^{CP1})$; and other segment sizes (sgZ^{CP1} with $Z \in \{NP, NH, PP, UU2, LL\}$) can be derived similarly. For the firm, the total demands of new products purchased with and without trade-ins can be calculated as in the main model: $D_{wTI}^{CP1} = sgNN^{CP1} + \frac{1-\alpha}{1+\alpha} sgNP^{CP1}$ and $D_{woTI}^{CP1} = \frac{\alpha}{1+\alpha} sgNP^{CP1} + \frac{1}{2} sgNH^{CP1}$. In this case with two resale markets, the market clearing conditions for used products traded in the producer-managed channel (through CPO) become $sgPP^{CP1} = (y)((\alpha)sgNN^{CP1})$ and $sgLL^{CP1} = (y)((1-\alpha)sgNN^{CP1} + (\frac{1-\alpha}{1+\alpha})sgNP^{CP1})$ (to respectively balance the supply and demand of peaches and lemons). For the used products traded in the P2P market, the market clearing condition will be $sgUU2^{CP1} = (1-y)(sgNN^{CP1} + (\frac{1-\alpha}{1+\alpha})sgNP^{CP1})$. We can then solve for the market clearing prices p_u^{CP1} and p_l^{CP1} for the firm, and p_{u2}^{CP1} for the P2P market. The firm profit can then be calculated by $\Pi^{CP1} = (p_T^{CP1} - p_u^{CP1} - c - e)D_{wTI}^{CP1} + (p^{CP1} - c)D_{woTI}^{CP1} + (p_p^{CP1})D_p^{CP1} + (p_l^{CP1})D_l^{CP1}$.

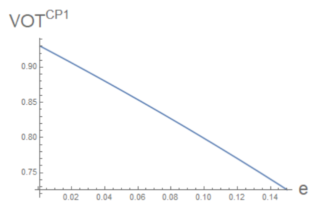
We next maximize the firm profit by solving $\max_{p^{CP1}, p_T^{CP1}, p_p^{CP1}} \Pi^{CP1}$, subject to $sgNN^{CP1}$, $sgNP^{CP1}$, $sgNH^{CP1}$, $sgPP^{CP1}$, $sgUU2^{CP1}$, $sgLL^{CP1} \geq 0$. To this end, we construct the Lagrangian and solve the KKT conditions to derive the full characterization of the optimal solution, similarly to the main model (details available from the authors but omitted here for brevity). Specifically, $\exists \overline{e_1}^{CP1} > 0$ such that when $0 \leq e \leq \overline{e_1}^{CP1}$, the equilibrium market segmentation outcome is [NN NP PP UU2 LL II]. Based on these results, we can then compare the respective optimal firm profits under the CPO and pure TI programs. The comparison shows that CPO will lead to a higher firm profit than pure TI programs if and only if $e < \overline{e}^{CP1}$ (with $\overline{e}^{CP1} > 0$). We focus on this case (in which CPO will be adopted by the firm) in the following discussion to highlight the effects of CPO programs. Comparing the thresholds of inspection cost yields $\overline{e}^{CP1} < \overline{e_1}^{CP1}$. Therefore, the market segmentation outcome will always be [NN NP PP UU2 LL II] (in descending consumer type θ) when $e < \overline{e}^{CP1}$. This result suggests that the secondary market competition reduces the CPO effect in mitigating the lemons problem, as the NP segment (and hence the strategic peach-holding) persists at $e = 0$ rather than completely eliminated. This result is consistent with our discussion on the CPO working mechanism: CPO programs take effect by increasing the resale revenue (by selling peaches and lemons separately with the certification), which will in turn enable the firm to increase the trade-in discount to suppress strategic peach-holding. Yet, when the firm is faced with competition (on both the supply and demand of used products) from the P2P market, the firm's pricing power in the secondary market is reduced. Meanwhile, we show below that our key findings regarding the working mechanism and market impacts of CPO programs (in restoring market efficiency in the lemons context) remain robust.

In the main model, we show that while CPO helps increase the trade-in discount with the inspection and subsequent certification, its effect is constrained by the unit inspection cost. We show below that this result continues to hold.

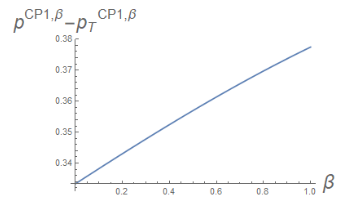
PROPOSITION SA1. *Let $1 - s \leq w_P$, then in the presence of competition in the secondary market, the trade-in discount of the firm's CPO program (i.e., $p^{CP1} - p_T^{CP1}$) weakly decreases in e.*

Proof of Proposition SA1: Based on the optimal solution of the firm's new product prices under the CPO program in this case, we derive $\frac{\partial p^{CP1} - p_T^{CP1}}{\partial e} = (\delta(-(y-1)w_p(\delta(s((y-5)y+8)y - 2(y-4)(y-3)y-4) - 8y+20) + \delta(-(y^2-5y+4)^2)w_p^2 + \delta(y(s(y((y-6)y+7)+8) - (y-5)(y-1)y-4)-4) + 2y(s(3y-8)-4y+6)-4)+8)/(2(\delta((y-1)w_p(\delta(s(y-8)(y-1)y - 2(y-6)(y-3)y-4) + 4(y+5)) + \delta(y^2-5y+4)^2 w_p^2 + \delta(y(-s(y-8)((y-2)y-1) + (y-7)(y-3)y+8)+4) - 2y(s(y-8)+2(y+4))+4)-8)) \leq 0$, given $0 < s, \delta, w_p, y < 1$. □

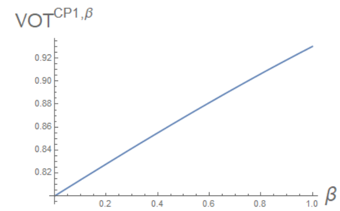
To study the market efficiency (reflected by VOT), we resort to extensive numerical analysis due to the additional complexity in the extension. As demonstrated in Figure SA2(a), while CPO helps



(a) Effect of e on VOT.



(b) Effect of β on the trade-in discount with partial CPO coverage.



(c) Effect of β on VOT with partial CPO coverage.

Figure SA2 Effect of the CPO program with market competition in the secondary market (figures constructed with $c = 0.5$, $s = 0.5$, $\delta = 0.5$, $x = 0.75$, $w_P = 0.75$, $y = 0.5$).

increase the market efficiency, its effectiveness is constrained by the inspection cost, and therefore the VOT under CPO programs decreases in e.

These results together affirm the robustness of our findings in Proposition 3 in the main model.

Next, we further extend our analysis on this competitive case to consider the partial CPO coverage model as in §4.2. To keep the analysis concise, we focus on the Partial CPO Coverage Model 1 here, with the assumption that within the β fraction of all the trade-ins covered by the firm's CPO program, the peach proportion among the NN trade-ins is the same as that in the entire used product population, i.e., α. With a similar analysis approach, we present the results in Figures SA2(b,c) (where we include "$CP1, \beta$" in the superscript for this case). Specifically, Figure SA2(b) demonstrates that the CPO program acts through increasing the trade-in discount, as is evidenced by the increase in the trade-in discount when the CPO program size (β) increases. Figure SA2(c) shows that the CPO program is effective in increasing the market efficiency for the firm when faced with the lemons problem, as the VOT increases in the program size. These results are similar to those in the main model.

Competition in the Primary Market (CP2). We also study the case where the focal firm competes with another durable product firm. While competition primarily emerges from the primary market of new products in this case, it also exists in the secondary market as used products from both firms are simultaneously available. To highlight the effects of CPO programs while keeping the model concise, we assume that the uncertainty and asymmetric information of used product quality only exist for the focal firm (i.e., the used products by the competing firm are of the same quality), and we focus on the CPO program offered by the focal firm.

Specifically, as in the main model in the paper, the focal firm sells new products and offers the CPO program. Its new product provides a per-period utility of θ to a type θ consumer. Meanwhile, quality uncertainty exists for its used products as they can be peaches or lemons. For the competing firm, we assume it sells new products that provide a per-period utility of $x\theta$, with x capturing the product differentiation between the two firms. Without loss of generality, we assume $x \in (0, 1)$.

Used products from the competing firm (that are of the same quality) provide a per-period utility of $x\delta\theta$. Neither pure TI nor CPO programs will be needed and these used units will be traded in their secondary market at the market clearing price. Both firms first set their new product prices (and the trade-in discount for the focal firm), after which consumers make their trade-in and purchase decisions. We include "1" and "2" in the subscript of the notation for quantities related to the focal and competing firms, respectively; and "$CP2$" in the superscript to denote quantities of this competitive case. We solve the problem again, following a similar analysis as in the main model (e.g., in §SA1). We outline the main steps below.

We start by analyzing the consumer choices. First, we list all the single-period consumer actions and the associated utilities (similar to the analysis in Table SA1), based on which we can then compare all the possible consumer strategies. Next, we identify the non-dominated consumer strategies, which include NN1, NP1, NH1, PP1, LL1 and II that are similar to the ones in the main model, and two additional strategies: sell the used competing product in the secondary market and replace it with a new competing product in every period (NN2), and always buy a used competing product from the secondary market (UU2). In particular, the expected utilities for a type θ consumer from the two additional strategies are $V_{NN2}^{CP2} = x\theta - p_2^{CP2} + p_{u2}^{CP2}$ and $V_{UU2}^{CP2} = x\delta\theta - p_{u2}^{CP2}$. By comparing the utilities from different consumer strategies, we conclude that there are six mutually exclusive and collectively exhaustive cases. Here, we present the analysis for one such case; the main insights for all other cases are directionally the same (details are available from the authors). Let $\delta < x < \min[1-s, \frac{1}{2}(\delta + \alpha\delta s + \delta(-s) + 1)]$. Then we can show (similar to the proof of Lemma 2 in §SA1) that consumers who choose NN1, NP1, NH1, NN2, PP1, LL1, UU2, and II are of decreasing types. Next, we characterize the profit functions for each firm. For the focal firm, the resale under CPO program is similar as in the main model. For the competing firm, the used products are traded at the price p_{u2}^{CP2}, which is endogenously determined by the market clearing condition $sgNN2^{CP2} = sgUU2^{CP2}$ (which balances the supply and demand of the used competing products). Based on these results, we can then formulate the profits. For the focal firm, $\Pi_1^{CP2} = (p_{T1}^{CP2} - p_{u1}^{CP2} - c - e)D_{wTI}^{CP2} + (p_1^{CP2} - c)D_{woTI}^{CP2} + (p_{p1}^{CP2})D_p^{CP2} + (p_{l1}^{CP2})D_l^{CP2}$; for the competing firm, $\Pi_2^{CP2} = (p_2^{CP2} - xc)D_2^{CP2}$ (note that the product differentiation captured by x is also reflected in the competing firm's production cost).

We next solve $\max_{p_1^{CP2}, p_{T1}^{CP2}, p_p^{CP2}} \Pi_1^{CP2}$ and $\max_{p_2^{CP2}} \Pi_2^{CP2}$ simultaneously to fully characterize the optimal solution, as in the main model (details omitted for brevity). Specifically, the equilibrium market segmentation outcome is [NN1 NP1 NN2 PP1 LL1 UU2 II] when $0 \le e \le \overline{e_1}^{CP2}$. To compare the CPO and pure TI programs in this case, we also model and analyze the TI program in the presence of market competition with another durable product firm. In this case, the focal firm acquires trade-ins from consumers but sells all of them as is (without inspection nor certification) in the secondary market. Following a similar analysis of the consumer strategies as discussed above, the non-dominated consumer strategies now also include the option UU1 (i.e., to always buy used products from the focal firm's secondary market through the pure TI program). The associated per-period utility will be $V_{UU1}^{CP2} = w^{CP2}\theta - p_{u1}$, with w^{CP2} denoting the expected average used product quality where $w^{CP2} = \frac{((1-\alpha)sgNN^{CP2}+\frac{1-\alpha}{1+\alpha}sgNP^{CP2})(\delta)(1-s)+((\alpha)sgNN^{CP2})(\delta)}{sgNN^{CP2}+\frac{1-\alpha}{1+\alpha}sgNP^{CP2}}$. The rest of the solution process is similar to the main model. Maximizing the focal and competing firms' profits by choosing their respective price decisions will yield the optimal solution, based on which we can then compare the focal firm's profit under the CPO and pure TI programs. The analysis shows that when $e < \overline{e}^{CP2}$ (with $\overline{e}^{CP2} > 0$), the CPO program will lead to higher focal firm profit and hence will be adopted; we focus on this case in the following discussion. Comparing the inspection cost thresholds yields that $\overline{e}^{CP2} < \overline{e_1}^{CP2}$. Therefore, the market segmentation outcome will be [NN1 NP1 NN2 PP1 LL1 UU2 II] (in descending consumer type θ) in equilibrium. We observe that the market competition reduces the CPO program effectiveness in alleviating the lemons problem: The NP1 segment persists at $e = 0$, as opposed to completely disappearing as in the monopolistic case (e.g., Proposition 1). This is intuitive and driven by the competition, which also aligns with our discussion on how CPO functions: Recall that CPO takes effect through increasing the resale revenue for the focal firm, which then enables the focal firm to increase the trade-in discount in the primary market to alter the strategic peach-holding behavior of the used product sellers. The competition pressure, however, limits how the focal firm can set the prices both in the secondary market (which affects its resale revenue) and the primary market (which affects the trade-in discount). Despite this weaker effect, however, below we show that our key findings regarding the working mechanism and market impacts of CPO programs continue to hold.

Proposition SA2. *Let $\delta < x < \min[1-s, \frac{1}{2}(\delta + \alpha\delta s + \delta(-s) + 1)]$, then in the presence of a competing durable product firm, the trade-in discount of the focal firm's CPO program (i.e., $p_1^{CP2} - p_{T1}^{CP2}$) weakly decreases in e.*

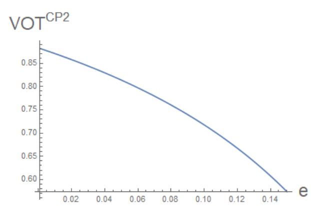
Proof of Proposition SA2.: Based on the optimal decisions of the focal firm under the CPO program, we can check that (expressions omitted here for brevity) $\frac{\partial p_1^{CP2} - p_{T1}^{CP2}}{\partial e} \leq 0$, given $0 < s, \delta, x < 1$. □

To study the CPO impacts on the market efficiency (captured by VOT), we resort to extensive numerical analysis due to the additional complexity of the extension. As demonstrated in Figure SA3(a), while CPO helps increase the market efficiency, its effectiveness is constrained by the inspection cost, and therefore the VOT under CPO programs decreases in e.

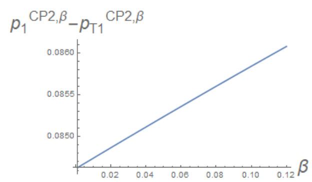
These results together affirm the robustness of our findings in Proposition 3 in the main model.

Next, we further extend our analysis on this competitive case to consider the partial CPO coverage model as in §4.2. To keep the analysis concise, we focus on the Partial CPO Coverage

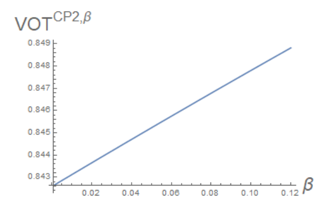
Model 1 here. With a similar analysis approach, we present the results in Figures SA3(b,c) (where we include "$CP2, \beta$" in the superscript for this case). Specifically, Figure SA3(b) illustrates that the CPO program acts through increasing the trade-in discount, as is evidenced by the increase in the trade-in discount when the CPO program size (β) increases. Figure SA3(c) also illustrates that the CPO program is effective in increasing the market efficiency for the focal firm when faced with the lemons problem, as the VOT increases in the CPO program size. These results are similar to those in the main model.



(a) Effect of e on VOT.



(b) Effect of β on the trade-in discount with partial CPO coverage.



(c) Effect of β on VOT with partial CPO coverage.

Figure SA3 Effect of the CPO program with market competition in the primary market (figures constructed with $c = 0.5$, $s = 0.1$, $\delta = 0.5$, $x = 0.7$).

SA4. Impact of Product Characteristics

As the cost-effectiveness of inspection critically influences the overall CPO effects, Lemma 3 explicitly characterizes the inspection cost threshold $\overline{e}$, such that the CPO program guarantees a higher firm profit than the pure TI program when $e < \overline{e}$. We further investigate the impacts of the product durability and reliability below.

Corollary SA1. *$\overline{e}$ first increases and then decreases in both s and δ.*

Proof of Corollary SA1: Based on the characterization of $\overline{e}$ (in the proof of Lemma 3), we can show that for $\delta \leq \frac{4}{4+\delta}$, $\frac{\partial \overline{e}}{\partial \delta} > 0$ when $\delta < \overline{\delta_e} \doteq \sqrt{2}(-(49cs^2 - 196cs + 192c + 49es^2 - 196es + 192e - 35s^2 + 136s - 128)/((s-2)(7s-12)^2))^{\frac{1}{2}} + \frac{4}{7s-12}$, and $\frac{\partial \overline{e}}{\partial \delta} < 0$ when $\delta > \overline{\delta_e}$; similarly for the case with $\delta < \frac{4}{4+\delta}$. Therefore, $\overline{e}$ first increases and then decreases in δ. The conclusion for s can be proved in a similar way. □

SA5. Summary of Data and Descriptive Statistics

In Table SA3, we summarize details of the dataset we assemble from various sources for the empirical study for easy reference. We then present the descriptive statistics of the variables we utilize in our empirical analyses in Table SA4.

Table SA3 Summary of Data

Source	Administered by	Data	Related Variables
Vehicle Dependability Study	J.D. Power	Reliability ($Reliability_{bt}$)	Reliability ($Reliability_{bt}$)
National Automobile Dealers Association Official Used Car Guide	J.D. Power	Prices of used cars ($p_{b1t}, p_{b4t}, p_{b7t}$)	Durability ($Durability_{bt} = \frac{p_{b4t}-p_{b7t}}{p_{b1t}-p_{b4t}}$)
Consumer Expenditure Survey (CES) data	U.S. Bureau of Labor Statistics	Total number of used car purchase transactions and total number of car ownership ($UsedPurchase_{bt}$, $UsedStock_{bt}$)	VOT ($VOT_{bt} = \frac{UsedPurchase_{bt}}{UsedStock_{bt}}$)
		Total number of vehicles sold to another individual ($UsedSales_{bt}$) and total number of traded-in vehicles ($UsedTradeIns_{bt}$)	VOT ($VOT_{Rao} = (UsedPurchase_{bt} + UsedSales_{bt} + UsedTradeIns_{bt}) / (UsedStock_{bt})$)
		Net purchase price paid ($FinalPricePaid_{ibt}$)	Net purchase price paid ($FinalPricePaid_{ibt}$)
		Household characteristics	
The Guide to Certified Pre-Owned Vehicle Programs	Automotive News	Maximum allowable length of ownership and maximum allowable use after purchase in the CPO qualification criteria	CPO program size (CPO_{bt})
		Number of CPO inspection points	CPO inspection cost ($Insp\_Cost_{bt}$)

Table SA4 Descriptive Statistics of Key Variables for the VOT Analysis (at the brand-year level) and the Final Price Paid Analysis (at the household-brand-year level).

Variable	Mean	S.D.	Min.	Max.
VOT Analysis				
VOT	0.13	0.10	0	1
CPO	0	0.82	-1.25	2.50
$Insp\_Cost$	156.83	38.67	111.00	300.00
$Durability$	0.77	0.69	0.02	7.79
$Reliability$	157.82	37.60	71.00	278.00
Final Price Paid Analysis				
$FinalPricePaid$	23,087.26	9,064.53	2.00	90,000.00
$TradeIn$	0.41	0.49	0	1
CPO	0.07	0.69	-1.25	2.50
$Insp\_Cost$	153.22	20.57	111.00	300.00
$Durability$	0.76	0.63	0.24	7.79
$Reliability$	162.31	34.69	71.00	255.00
$q_{P,1}$	0.03	0.17	0	1
$q_{C,2}$	0.24	0.43	0	1
$q_{P,2}$	0.02	0.12	0	1
$q_{C,3}$	0.20	0.40	0	1
$q_{P,3}$	0.01	0.09	0	1
$q_{C,4}$	0.09	0.29	0	1
$q_{P,4}$	0.01	0.06	0	1
$q_{N,4}$	0.11	0.31	0	1
$hhIncome$	85,561.81	80,922.08	-64,762.00	1,096,545.00
$hhFemale$	0.49	0.50	0	1
$hhMinority$	0.17	0.37	0	1
$hhAge$	46.91	14.41	17.00	87.00

SA6. Impacts of CPO Program Size on VOT and Trade-In Discount

In this preliminary exploratory analysis, we investigate the effect of CPO program size, defined through the qualification criteria (with thresholds on maximum allowable length of ownership and use after purchase), on the VOT and the trade-in discount. Utilizing the coefficient estimates from Model (1) in §5, we first calculate the corresponding increase in the CPO measure, since we use factor analysis on maximum allowable length of ownership and maximum allowable use after purchase to create the CPO measure (see §5.3). The average estimated factor loading across 2008-2013 is $\lambda_{CPO} = 0.775$. In the data, the standard deviation of the maximum allowable length of ownership is $\sigma_{year} = 0.883$. Hence, an increase by a year in maximum allowable length of ownership corresponds to an increase in CPO by $\delta_{Year} = \frac{1}{\sigma_{year} \times \lambda_{CPO}} = 1.461$. The standard deviation of the maximum allowable use after purchase is $\sigma_{mileage} = 11032.13$. Then, an increase by 10,000 miles in maximum allowable use after purchase corresponds to an increase in CPO by $\delta_{10000\_Miles} = \frac{10000}{\sigma_{mileage} \times \lambda_{CPO}} = 1.17$. Note that the coefficient estimate for CPO in Model (1) is 0.048. Hence, we find that an increase by a year in maximum allowable length of ownership corresponds to an increase of VOT by $\bar{\Delta}_{VOT,Year} = 7\%$, and an increase of 10,000 miles in maximum allowable use after purchase corresponds to an increase of VOT by $\bar{\Delta}_{VOT,10000\_Miles} = 5.62\%$ on average.

We next utilize the coefficient estimates for the final price paid specification to carry out a similar analysis for the trade-in discount. Since we focus on a brand-level trade-in discount provided to an individual at every transaction, we also need to account for the variations across brands due to their inspection and product characteristics. Hence, we use the coefficient estimates from Model (8) for this analysis. Using δ_{Year} and δ_{10000_Miles}, we can calculate the corresponding average changes, $\bar{\Delta}_{TradeIn,Year}$ and $\bar{\Delta}_{TradeIn,10000\_Miles}$:

$$\bar{\Delta}_{TradeIn,Month} = \frac{\delta_{Year}}{5|\mathbf{B}|} \sum_{i \in \mathbf{B}} \sum_{t=2008}^{2013} (13,050.83 - 82.13 Insp\_Cost_{i,t}),$$

$$\bar{\Delta}_{TradeIn,10000\_Miles} = \frac{\delta_{10000\_Miles}}{5|\mathbf{B}|} \sum_{i \in \mathbf{B}} \sum_{t=2008}^{2013} (13,050.83 - 82.13 Insp\_Cost_{i,t}).$$

We find that an increase by a year in maximum allowable length of ownership corresponds to an increase of the trade-in discounts by $\bar{\Delta}_{TradeIn,Month} = \$682.50$ on average, and that an increase of 10,000 miles in maximum allowable use after purchase corresponds to an increase of the trade-in discount by $\bar{\Delta}_{TradeIn,10000\_Miles} = \$546.56$ on average.

References

Rao RS, Narasimhan O, John G (2009) Understanding the role of trade-ins in durable goods markets: Theory and evidence. *Marketing Science* 28(5):950–967.