Online Appendices

“Disclosing Delivery Performance Information when Consumers are Sensitive to Promised Delivery Time, Delivery Reliability, and Price”

Appendix A Average Delivery-Time Ratings and Conformance

We used T-mall/Cainiao data to explore the relationship between average delivery-time ratings and delivery reliability. First, we compute the reliability of each merchant in the data set and the average of the individual-level ratings provided by buyers, across all orders with complete data. This yields 3,665 observations. Then we perform an OLS regression of average delivery-time ratings on reliability. The estimated regression equation is

$$\text{average delivery-time rating} = \underset{(0.060)}{4.396} + \underset{(0.062)}{0.438} \times \text{reliability} + \epsilon,$$

where the standard errors of the estimates appear in parenthesis. The coefficient of reliability is highly significant (p-value$<$ 0.0001), indicating that there is a positive and statistically significant relationship between reliability and ratings. The intercept is positive, large, and statistically significant, so that ratings are positive even when reliability is zero. This implies that ratings exhibit a significant upward bias. We then inspect the residuals to verify that the conditions for inference are satisfied (residuals should be normally distributed, homoskedastic, independent, and the relationship linear). We note, however, that the distribution of reliability is highly skewed, with 50% of the observations having a value above 0.98 and fewer than 5% of the observations having values below 0.90. This implies that residuals could appear heteroskedastic simply because dispersion is normally proportional to sample size and therefore we could expect higher dispersion for high levels of reliability. For this reason, we randomly sample 500 observations from the data with probability weights inversely proportional to the square of their reliability values. We then replicate the regression analysis and obtain the estimated regression equation

$$\text{average delivery-time rating} = \underset{(0.113)}{4.248} + \underset{(0.116)}{0.593} \times \text{reliability} + \epsilon.$$

Once again, we find evidence of a positive and statistically-significant correlation (p-value$<$ 0.0001) between reliability and average delivery-time ratings. The standardized residuals do not exhibit any clear sign of homoskedasticity, non-linearities, or non-normality. Data are not ordered and the possibility of autocorrelation is ruled out. The conditions for inference are satisfied.

Appendix B Information Quality and Inference Distribution

The propositions that $E(X)$ and $Var(X)$ are both positive and decreasing on information quality are backed by several studies in the literature. For example, field experiments in the context of consumer expectations of prices show that making high-quality information available to consumers reduces the heterogeneity and bias of their expectations (Fuster et al. 2002). In the context of online retailing, consumer expectations exhibit a positive bias because online reviews influence consumer expectations and online reviews are biased upward (see Park et al. 2018 and references therein).

Similarly, research has shown that both the mean and dispersion of consumer attitudes towards offerings decline with the quality of reviews. This occurs because high-quality negative reviews affect attitude (expectations) more than low-quality reviews do, whereas positive reviews are less effective regardless of their quality (Lee et al. 2008). Because attitudes informed by ratings are normally biased upwards, the negative effect of increased information quality on attitudes should reduce such positive bias. Conversely, lowering the quality of online reviews can increase the bias and variance of subsequent evaluations (Muchnik et al. 2013). In summary, the literature indicates that expectations exhibit a positive bias, which decreases as information quality increases. The dispersion (heterogeneity) of these expectations also decreases as information quality increases.

One may note that the firm may benefit from increasing information quality even if the optimal degree of delivery trust decreases when both $E(X)$ and $Var(X)$ decrease. Specifically, information error inflating the degree of trust can harm the firm because it implies that consumers "overestimate" delivery reliability q. When consumers overestimate the reliability, they are less satisfied with the actual delivery service and provide lower delivery ratings. It follows that a higher degree of trust resulting from a larger informational error makes the firm worse off. That is, a "lower degree of trust" is better for the firm than an "inflated degree of trust." Therefore, the firm has an incentive to improve information quality (i.e., increase the value of ϕ), reducing $E(X)$ and $Var(X)$ and deflating the degree of trust. Nonetheless, the firm can benefit from a high degree of trust that is made by a large delivery reliability q.

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Appendix C The Mean and Variance of the $C(\phi)$ Distribution

Cardell (1997) derived the characteristic function of the $C(\phi)$ distribution as

$$\psi_\phi(\tau) = \Gamma(1 - \iota\tau)/\Gamma(1 - \iota\phi\tau), \tag{16}$$

where $\iota \equiv \sqrt{-1}$ is the imaginary unit, $\Gamma(z) \equiv \int_0^\infty x^{z-1}\exp(-x)dx$ is the standard gamma function, and $\tau \in R$ is the argument of the characteristic function. However, Cardell (1997) did not derive closed-form expressions for the mean and variance and these are not readily available in the literature. (Cardell did show that, although there is no closed-form cumulative distribution function for $C(\phi)$, there is a closed-form probability density function.)

Next, we derive closed-form expressions of the mean and variance of Z. From (16), the first and second moments of the $C(\phi)$ distribution are given by

$$\begin{cases} E(Z) & = \iota^{-1} \times \left.\dfrac{\partial \psi_\phi(\tau)}{\partial \tau}\right|_{\tau=0} & = \gamma(1-\phi), \\ E(Z^2) & = \iota^{-2} \times \left.\dfrac{\partial^2 \psi_\phi(\tau)}{\partial \tau^2}\right|_{\tau=0} & = \dfrac{(1-\phi)[\pi^2(1+\phi)+6\gamma^2(1-\phi)]}{6}, \end{cases}$$

where $\gamma \equiv 0.57721$ is the Euler-Mascheroni constant. The mean of Z is given by the first moment $E(Z)$ and, letting σ_Z denote the standard deviation of Z, we have the variance as

$$Var(Z) = \sigma_Z^2 = \pi^2(1-\phi^2)/6. \tag{17}$$

It follows that $\lim_{\phi\to 1} E(Z) = 0$ and $\lim_{\phi\to 1} Var(Z) = 0$, which is consistent with Cardell's finding, i.e., $\phi = 1$ implies that $\Pr(Z = 0) = 1$. In addition, we find that $\lim_{\phi\to 0} E(Z) = \gamma$ and $\lim_{\phi\to 0} Var(Z) = \pi^2/6$. This also is consistent with Cardell's finding: the $C(\phi = 0)$ distribution is the standard Gumbel distribution.

Appendix D Proofs

Proof of Lemma 1. Given the value of p, we compute the first and second-order derivatives of $\Pi(p,t)$ w.r.t. t as

$$\frac{\partial \Pi(p,t)}{\partial t} = \frac{\partial \lambda(p,t)}{\partial t}(p - b\phi - c) \text{ and } \frac{\partial^2 \Pi(p,t)}{\partial t^2} = \frac{\partial^2 \lambda(p,t)}{\partial t^2}(p - b\phi - c),$$

where $\lambda(p,t) \equiv \lambda(p,t|\phi)$. It follows from expression (4) that $\lambda(p,t,\phi) = \Lambda/[1+\exp(-(V-V_e)/\omega_\theta)]$, from which we obtain $V = V_e + \omega_\theta[\ln(\lambda(p,t)) - \ln(\Lambda - \lambda(p,t))]$. For simplicity, we hereafter define $K \equiv V_e + \omega_\theta[\ln(\lambda(p,t)) - \ln(\Lambda - \lambda(p,t))]$. We differentiate K once w.r.t. t to obtain

$$\frac{\partial K}{\partial t} \equiv \frac{\omega_\theta \Lambda}{\lambda(p,t)(\Lambda - \lambda(p,t))} \frac{\partial \lambda(p,t)}{\partial t}.$$

Note that $V = \alpha - p + \beta_r(\rho q + \delta_2) - \beta_t t$, where $q = 1 - \exp(-(\mu - \lambda)t)$ as given in (1). We then differentiate V once w.r.t. t to obtain

$$\frac{\partial V}{\partial t} = \beta_r \rho \exp(-(\mu - \lambda(p,t))t) \left[\mu - \lambda(p,t) - \frac{\partial \lambda(p,t)}{\partial t} t\right] - \beta_t.$$

Solving $\partial V/\partial t = \partial K/\partial t$ for $\partial \lambda(p,t)/\partial t$, we obtain

$$\frac{\partial \lambda(p,t)}{\partial t} = \frac{\lambda(p,t)(\Lambda - \lambda(p,t))\xi}{\beta_r \rho t \lambda(p,t)(\Lambda - \lambda(p,t)) \exp(-(\mu - \lambda(p,t))t) + \omega_\theta \Lambda},$$

where $\xi \equiv \beta_r \rho(\mu - \lambda(p,t)) \exp(-(\mu - \lambda(p,t))t) - \beta_t$. One can easily note that the sign of $\partial \lambda(p,t)/\partial t$ only depends on the sign of ξ. Because $S\Lambda \le \bar{S}\Lambda$, we have $\mu - \lambda(p,t) \ge \mu - \bar{S}\Lambda$. Noting that, for $a > 0$ and $ax \ge 1$, the function $x\exp(-ax)$ is decreasing in x, we find that, because $t \ge 1/(\mu - \lambda(p,t))$, then $(\mu - \lambda(p,t)) \exp(-(\mu - \lambda(p,t))t) \le (\mu - \bar{S}\Lambda) \exp(-(\mu - \bar{S}\Lambda)t)$. In addition, because $\exp(-(\mu-\bar{S}\Lambda)t)\le 1$, we have $(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t)\le(\mu-\bar{S}\Lambda)$. Multiplying both side of the inequality by $\beta_r\rho$, we obtain

$$\xi \le \beta_r\rho(\mu-\bar{S}\Lambda)-\beta_t. \tag{18}$$

Thus, for a given value of p, if $\beta_r\rho(\mu-\bar{S}\Lambda)-\beta_t<0$, or $\mu<\bar{S}\Lambda+\beta_t/(\beta_r\rho)$, then $\xi<0$ or $\partial\lambda(p,t)/\partial t<0$; and t should be $\underline{t}$, i.e., $t^*(p)=\underline{t}$.

Otherwise, if $\mu\ge\bar{S}\Lambda+\beta_t/(\beta_r\rho)$, then the RHS of the inequality in (18) is non-negative and therefore ξ is no larger than a non-negative number. It remains to show that there exists at least one solution for $\xi=0$. As argued above, $(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t)$ is decreasing in t; that is, ξ decreases with t. Because $0\le\lambda(p,t)\le\bar{S}\Lambda$,

$$\lim_{t\to+\infty}\xi=\lim_{t\to+\infty}[\beta_r\rho(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t)-\beta_t]=-\beta_t<0.$$

Noting that $t\in[\underline{t},+\infty)$ and $\underline{t}\ge 0$, we should also examine if $\xi|_{t\to0}\ge0$. Since $\mu\ge\bar{S}\Lambda+\beta_t/(\beta_r\rho)$ and $0\le\lambda(p,t)\le\bar{S}\Lambda$, $\mu-\lambda(p,t)\ge\beta_t/(\beta_r\rho)$. In addition, when $t\to0$, $\exp(-(\mu-\lambda(p,t))t)\to1$ and thus $(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t)\ge\beta_t/(\beta_r\rho)$, which means $\xi|_{t\to0}\ge0$.

These results imply that we can find at least one solution satisfying $\xi=0$. Hence, there is at least one solution for the first order condition $d\lambda(p,t)/dt=0$. For any such solution t and $\mu\ge\bar{S}\Lambda+\beta_t/(\beta_r\rho)$, we compute the second-order derivatives of $\lambda(p,t)$ w.r.t. t as

$$\left.\frac{\partial^2\lambda(p,t)}{\partial t^2}\right|_{\frac{\partial\lambda(p,t)}{\partial t}=0}=-\frac{\beta_t\lambda(p,t)(\Lambda-\lambda(p,t))(\mu-\lambda(p,t))}{\beta_r\rho t\lambda(p,t)(\Lambda-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t)+\omega_\theta\Lambda}<0.$$

The above implies that, $\forall p$, $\Pi(p,t)$ is a unimodal function of t with a unique optimal solution $t^*(p)$ that can be obtained by solving the following equation for t: $\beta_r\rho(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t)=\beta_t$. That is, there is a unique solution for the first order condition if $\mu\ge\bar{S}\Lambda+\beta_t/(\beta_r\rho)$.

Next, we find the optimal price for the two scenarios: (1) $\mu<\bar{S}\Lambda+\beta_t/(\beta_r\rho)$ and (2) $\mu\ge\bar{S}\Lambda+\beta_t/(\beta_r\rho)$. For scenario (1), $t^*(p)=\underline{t}$, $\lambda(p,t^*(p))=\lambda(p,\underline{t})$, $q=1-\exp(-(\mu-\lambda(p,\underline{t}))\underline{t})$, and $V=\alpha+\beta_r\rho[1-\exp(-(\mu-\lambda(p,\underline{t}))\underline{t})]+\beta_r\delta_2-p-\beta_t\underline{t}$. We use $\underline{t}$ to replace t in $\Pi(p,t)=\lambda(p,t,\phi)(p-b\phi-c)$, and compute the first- and second-order derivatives of $\Pi(p)$ w.r.t. p as

$$\frac{\partial\Pi(p)}{\partial p}=\frac{\partial\lambda(p,\underline{t})}{\partial p}(p-b\phi-c)+\lambda(p,\underline{t})\text{ and }\frac{\partial^2\Pi(p)}{\partial p^2}=\frac{\partial^2\lambda(p,\underline{t})}{\partial p^2}(p-b\phi-c)+2\frac{\partial\lambda(p,\underline{t})}{\partial p}.$$

Partially differentiating V once w.r.t. p, we find

$$\begin{aligned}\frac{\partial V}{\partial p}&=-1-\underline{t}\beta_r\rho\exp(-(\mu-\lambda(p,\underline{t}))\underline{t})\frac{\partial\lambda(p,\underline{t})}{\partial p},\\ \frac{\partial^2 V}{\partial p^2}&=-\underline{t}\beta_r\rho\exp(-(\mu-\lambda(p,\underline{t}))\underline{t})\frac{\partial^2\lambda(p,\underline{t})}{\partial p^2}-\underline{t}^2\beta_r\rho\exp(-(\mu-\lambda(p,\underline{t}))\underline{t})\left(\frac{\partial\lambda(p)}{\partial p}\right)^2.\end{aligned}$$

Similarly, we compute the first- and second-order derivatives of K (replacing t by $\underline{t}$) w.r.t. p as

$$\begin{cases}\dfrac{\partial K}{\partial p}= & \dfrac{\omega_\theta\Lambda}{\lambda(p,\underline{t})(\Lambda-\lambda(p,\underline{t}))}\dfrac{\partial\lambda(p,\underline{t})}{\partial p},\\ \dfrac{\partial^2 K}{\partial p^2}= & \dfrac{\omega_\theta\Lambda}{\lambda(p,\underline{t})(\Lambda-\lambda(p,\underline{t}))}\dfrac{\partial^2\lambda(p,\underline{t})}{\partial p^2}+\omega_\theta\left[-\dfrac{1}{(\lambda(p,\underline{t}))^2}+\dfrac{1}{(\Lambda-\lambda(p,\underline{t}))^2}\right]\left(\dfrac{\partial\lambda(p,\underline{t})}{\partial p}\right)^2.\end{cases}$$

Solving $\partial V/\partial p = \partial K/\partial p$ gives

$$\frac{\partial \lambda(p,\underline{t})}{\partial p} = -\frac{\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))}{\beta_r \underline{t}\rho\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))\exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta \Lambda}.$$

We also solve $\partial^2 V/\partial p^2 = \partial^2 K/\partial p^2$ to find

$$\frac{\partial^2 \lambda(p,\underline{t})}{\partial p^2} = \left(\frac{\partial \lambda(p,\underline{t})}{\partial p}\right)^3 \left\{\underline{t}^2 \beta_r \rho \exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta \left[\frac{1}{(\Lambda - \lambda(p,\underline{t}))^2} - \frac{1}{(\lambda(p,\underline{t}))^2}\right]\right\}.$$

Using $\partial\lambda(p)/\partial p$ and $\partial^2\lambda(p)/\partial p^2$, we can rewrite $\partial\Pi(p)/\partial p$ and $\partial^2\Pi(p)/\partial p^2$ as

$$\frac{\partial \Pi(p)}{\partial p} = -\lambda(p,\underline{t})\frac{(\Lambda - \lambda(p,\underline{t}))(p - b\phi - c) - \beta_r \underline{t}\rho\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))\exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) - \omega_\theta\Lambda}{\beta_r \underline{t}\rho\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))\exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta\Lambda}.$$

The first order condition (i.e., $\partial\Pi(p)/\partial p = 0$) is $(\Lambda - \lambda(p,\underline{t}))(p - b\phi - c) = \beta_r\underline{t}\rho\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))\exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta\Lambda$ and, at any point where $\partial\Pi(p)/\partial p = 0$, we have that

$$\begin{aligned}
\left.\frac{\partial^2 \Pi(p)}{\partial p^2}\right|_{\frac{\partial\Pi(p)}{\partial p}=0} &= \frac{\partial^2 \lambda(p,\underline{t})}{\partial p^2}(p - b\phi - c) + 2\frac{\partial\lambda(p,\underline{t})}{\partial p} \\
&= \frac{\partial\lambda(p,\underline{t})}{\partial p}\left\{\frac{(\lambda(p,\underline{t}))^2\underline{t}^2\beta_r\rho\exp(-(\mu - \lambda(p,\underline{t}))\underline{t})}{p-c}\right. \\
&\quad \left. + \frac{\omega_\theta(\lambda(p,\underline{t}))^2}{(p - b\phi - c)(\Lambda - \lambda(p,\underline{t}))^2} + \frac{2(p - b\phi - c) - \omega_\theta}{p - b\phi - c}\right\} \\
&< 0.
\end{aligned}$$

Therefore, the optimal price p^* is found as in (5).

Next, we find the optimal price for scenario (ii). As $t^*(p)$ is the unique solution of $\beta_r\rho(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) = \beta_t$, we let $\lambda(p) \equiv \lambda(p, t^*(p))$ and rewrite the delivery reliability as $q = 1 - \exp(-(\mu - \lambda(p))t^*(p)) = 1 - \beta_t/[\beta_r\rho(\mu - \lambda(p))]$. Then, we can rewrite V as $V = \alpha + \beta_r(\rho + \delta_2) - p - \beta_t/[\mu - \lambda(p)] - \beta_t t^*(p)$. Substituting $t^*(p)$ into the firm's profit function gives $\Pi(p) \equiv \Pi(t^*(p), p) = \lambda(p)(p - b\phi - c)$. Partially differentiating V once w.r.t. p, we have

$$\frac{\partial V}{\partial p} = -1 - \frac{\beta_t}{(\mu - \lambda(p))^2}\frac{\partial\lambda(p)}{\partial p} - \beta_t\frac{\partial t^*(p)}{\partial p}.$$

To find $\partial t^*(p)/\partial p$, we take the first-order partial derivative of two sides in (6) w.r.t. p, and solve the resulting equation for $\partial t^*(p)/\partial p$ as

$$\frac{\partial t^*(p)}{\partial p} = \frac{t^*(p)(\mu - \lambda(p)) - 1}{(\mu - \lambda(p))^2}\frac{\partial\lambda(p)}{\partial p}.$$

We then rewrite $\partial V/\partial p$ as

$$\frac{\partial V}{\partial p} = -1 - \frac{\beta_t t^*(p)}{\mu - \lambda(p)}\frac{\partial\lambda(p)}{\partial p},$$

and also compute

$$\frac{\partial^2 V}{\partial p^2} = -\frac{\beta_t t^*(p)}{\mu - \lambda(p)}\frac{\partial^2\lambda(p)}{\partial p^2} - \frac{2\beta_t t^*(p)(\mu - \lambda(p)) - \beta_t}{(\mu - \lambda(p))^3}\left(\frac{\partial\lambda(p)}{\partial p}\right)^2.$$

Similarly, we compute the first- and second-order derivatives of $K = V_e + \omega_\theta[\ln(\lambda(p,t)) - \ln(\Lambda - \lambda(p,t))]$ w.r.t. p as

$$\begin{cases} \dfrac{\partial K}{\partial p} = \dfrac{\omega_\theta \Lambda}{\lambda(p)(\Lambda - \lambda(p))} \dfrac{\partial \lambda(p)}{\partial p}, \\ \dfrac{\partial^2 K}{\partial p^2} = \dfrac{\omega_\theta \Lambda}{\lambda(p)(\Lambda - \lambda(p))} \dfrac{\partial^2 \lambda(p)}{\partial p^2} + \omega_\theta \left[-\dfrac{1}{(\lambda(p))^2} + \dfrac{1}{(\Lambda - \lambda(p))^2} \right] \left(\dfrac{\partial \lambda(p)}{\partial p} \right)^2. \end{cases}$$

Solving $\partial V/\partial p = \partial K/\partial p$ gives

$$\frac{\partial \lambda(p)}{\partial p} = -\frac{\lambda(p)(\Lambda - \lambda(p))(\mu - \lambda(p))}{\omega_\theta \Lambda(\mu - \lambda(p)) + \beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p))}.$$

We also solve $\partial^2 V/\partial p^2 = \partial^2 K/\partial p^2$ to find

$$\frac{\partial^2 \lambda(p)}{\partial p^2} = \left(\frac{\partial \lambda(p)}{\partial p} \right)^3 \left[\frac{\beta_t}{(\mu - \lambda(p))^2} \left(2t^*(p) - \frac{1}{\mu - \lambda(p)} \right) - \frac{\omega_\theta}{(\lambda(p))^2} + \frac{\omega_\theta}{(\Lambda - \lambda(p))^2} \right].$$

Using $\partial \lambda(p)/\partial p$ and $\partial^2 \lambda(p)/\partial p^2$, we can rewrite $\partial \Pi(p)/\partial p$ and $\partial^2 \Pi(p)/\partial p^2$ as

$$\frac{\partial \Pi(p)}{\partial p} = \lambda(p) \frac{\beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p)) + \omega_\theta \Lambda(\mu - \lambda(p)) - (\mu - \lambda(p))(\Lambda - \lambda(p))(p - b\phi - c)}{\omega_\theta \Lambda(\mu - \lambda(p)) + \beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p))}.$$

The first order condition (i.e., $\partial \Pi(p)/\partial p = 0$) is $\beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p)) + \omega_\theta \Lambda(\mu - \lambda(p)) = (\mu - \lambda(p))(\Lambda - \lambda(p))(p - b\phi - c)$ and, at any point where $\partial \Pi(p)/\partial p = 0$,

$$\begin{aligned} \left. \frac{\partial^2 \Pi(p)}{\partial p^2} \right|_{\frac{\partial \Pi(p)}{\partial p} = 0} &= \frac{\partial^2 \lambda(p)}{\partial p^2}(p - b\phi - c) + 2\frac{\partial \lambda(p)}{\partial p} \\ &= (p - b\phi - c) \left(\frac{\partial \lambda(p)}{\partial p} \right)^3 \left[\frac{\beta_t}{(\mu - \lambda(p))^2} \left(2t^*(p) - \frac{1}{\mu - \lambda(p)} \right) + \frac{\omega_\theta}{(\Lambda - \lambda(p))^2} \right] \\ &\quad + \frac{2(p - b\phi - c) - \omega_\theta}{p - b\phi - c} \frac{\partial \lambda(p)}{\partial p}. \end{aligned}$$

Noting that (7) implies that $\partial \lambda(p)/\partial p < 0$ and that

$$p - b\phi - c = \frac{\beta_t t^*(p)\lambda(p)}{\mu - \lambda(p)} + \frac{\omega_\theta \Lambda}{\Lambda - \lambda(p)} > \frac{\omega_\theta \Lambda}{\Lambda - \lambda(p)} > \omega_\theta,$$

we find that, if $t^*(p) > \mu - \lambda(p)$, then

$$\left. \frac{\partial^2 \Pi(p)}{\partial p^2} \right|_{\frac{\partial \Pi(p)}{\partial p} = 0} < 0,$$

which means that $\Pi(p)$ is strictly concave function of p. We thus prove this lemma. ■

Proof of Lemma 2. The first-order derivative of $\lambda^*(\phi) = \lambda(p^*(\phi), t^*(\phi), \phi)$ in (4) w.r.t. ω_θ is

$$\begin{aligned} \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta} &= -\frac{\Lambda \exp[-(V - V_e)/\omega_\theta]}{\{1 + \exp[-(V - V_e)/\omega_\theta]\}^2} \left[-\frac{1}{\omega_\theta} \frac{\partial V}{\partial \omega_\theta} + \frac{V - V_e}{\omega_\theta^2} \right] \\ &= \frac{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))}{\Lambda} \left(\frac{1}{\omega_\theta} \frac{\partial V}{\partial \omega_\theta} - \frac{V - V_e}{\omega_\theta^2} \right). \end{aligned} \quad (19)$$

To find $\partial\lambda^*(\phi)/\partial\omega_\theta$, we differentiate $V = \alpha - p^*(\phi) + \beta_r(\rho q^*(\phi) + \delta_2) - \beta_t t^*(\phi)$ once w.r.t. ω_θ, as

$$\frac{\partial V}{\partial\omega_\theta} = -\frac{\partial p^*(\phi)}{\partial\omega_\theta} + \beta_r\rho\frac{\partial q^*(\phi)}{\partial\omega_\theta} - \beta_t\frac{\partial t^*(\phi)}{\partial\omega_\theta},$$

where $\partial q^*(\phi)/\partial\omega_\theta$ is given as in (8). We differentiate both sides of the equation $\beta_r\rho(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t) = \beta_t$ in (6) once w.r.t. ω_θ, and obtain

$$\beta_r\rho\exp(-(\mu-\lambda^*(\phi))t^*(\phi))\left[\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}[t^*(\phi)(\mu-\lambda^*(\phi))-1] - (\mu-\lambda^*(\phi))^2\frac{\partial t^*(\phi)}{\partial\omega_\theta}\right] = 0.$$

Because $\beta_r\rho\exp(-(\mu-\lambda^*(\phi))t^*(\phi)) > 0$, we have

$$(\mu-\lambda^*(\phi))^2\frac{\partial t^*(\phi)}{\partial\omega_\theta} = [t^*(\phi)(\mu-\lambda^*(\phi))-1]\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta},$$

or,

$$\frac{\partial t^*(\phi)}{\partial\omega_\theta} = \frac{t^*(\phi)(\mu-\lambda^*(\phi))-1}{(\mu-\lambda^*(\phi))^2}\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta},$$

which implies that $\partial t^*(\phi)/\partial\omega_\theta$ and $\partial\lambda^*(\phi)/\partial\omega_\theta$ have the same sign.

We compute the first-order derivative of $p^*(\phi)$ in (7) w.r.t. ω_θ as

$$\frac{\partial p^*(\phi)}{\partial\omega_\theta} = b\frac{\partial\phi}{\partial\omega_\theta} + \frac{\beta_t\lambda^*(\phi)}{\mu-\lambda^*(\phi)}\frac{\partial t^*(\phi)}{\partial\omega_\theta} + \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\frac{\beta_t t^*(\phi)\mu}{(\mu-\lambda^*(\phi))^2} + \frac{\omega_\theta\Lambda}{(\Lambda-\lambda^*(\phi))^2}\right].$$

Together with the definition of ω_θ this yields $\partial\phi/\partial\omega_\theta = -\omega_\theta/[(\beta_r\delta_1)^2\phi]$ and allows us to rewrite $\partial p^*(\phi)/\partial\omega_\theta$ as

$$\frac{\partial p^*(\phi)}{\partial\omega_\theta} = -\frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} + \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\frac{\beta_t\lambda^*(\phi)[t^*(\phi)(\mu-\lambda^*(\phi))-1]}{(\mu-\lambda^*(\phi))^3} + \frac{\beta_t t^*(\phi)\mu}{(\mu-\lambda^*(\phi))^2} + \frac{\omega_\theta\Lambda}{(\Lambda-\lambda^*(\phi))^2}\right],$$

which indicates that if $\partial\lambda^*(\phi)/\partial\omega_\theta \le 0$, then $\partial p^*(\phi)/\partial\omega_\theta < 0$; but, if $\partial\lambda^*(\phi)/\partial\omega_\theta > 0$, then $\partial p^*(\phi)/\partial\omega_\theta$ may be positive or may be negative.

Using the expressions of $\partial p^*(\phi)/\partial\omega_\theta$, $\partial q^*(\phi)/\partial\omega_\theta$, and $\partial t^*(\phi)/\partial\omega_\theta$, we find

$$\frac{\partial V}{\partial\omega_\theta} = \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\frac{2\beta_t t^*(\phi)\mu(\mu-\lambda^*(\phi)) - \beta_t\lambda^*(\phi)}{(\mu-\lambda^*(\phi))^3} + \frac{\omega_\theta\Lambda}{(\Lambda-\lambda^*(\phi))^2}\right]. \tag{20}$$

We rewrite (19) to obtain

$$\frac{\partial V}{\partial\omega_\theta} = \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\frac{\omega_\theta\Lambda}{\lambda^*(\phi)(\Lambda-\lambda^*(\phi))} + \frac{V-V_e}{\omega_\theta}, \tag{21}$$

and using the RHS of the above equation to replace $\partial V/\partial\omega_\theta$ in (20) yields

$$\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\beta_t\frac{2t^*(\phi)\mu(\mu-\lambda^*(\phi))-\lambda^*(\phi)}{(\mu-\lambda^*(\phi))^3} + \frac{\omega_\theta\Lambda^2}{\lambda^*(\phi)(\Lambda-\lambda^*(\phi))^2}\right] = \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V-V_e}{\omega_\theta}, \tag{22}$$

where $2t^*(\phi)\mu(\mu-\lambda^*(\phi)) - \lambda^*(\phi) > 0$, because $t(\mu-\lambda^*(\phi)) > 1$. It thus follows that the sign of

$\partial\lambda^*(\phi)/\partial\omega_\theta$ depends on the sign of

$$\frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi}-\frac{V-V_e}{\omega_\theta}.$$

We also differentiate the firm's expected profit $\Pi^*(\phi)=\Pi(p^*(\phi),t^*(\phi),\phi)$ once w.r.t. ω_θ:

$$\begin{aligned}\frac{\partial\Pi^*(\phi)}{\partial\omega_\theta} &= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}(p^*(\phi)-b\phi-c)+\lambda^*(\phi)\left(\frac{\partial p^*(\phi)}{\partial\omega_\theta}-b\frac{\partial\phi}{\partial\omega_\theta}\right)\\ &= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}(p^*(\phi)-b\phi-c)+\lambda^*(\phi)\frac{\partial p^*(\phi)}{\partial\omega_\theta}+\lambda^*(\phi)\frac{b\omega_\theta}{\beta_r\phi}\\ &= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\times\left\{p^*(\phi)-b\phi-c+\frac{\beta_t(\lambda^*(\phi))^2[t^*(\phi)(\mu-\lambda^*(\phi))-1]}{(\mu-\lambda^*(\phi))^3}\right.\\ &\left.+\frac{\beta_t t^*(\phi)\mu\lambda^*(\phi)}{(\mu-\lambda^*(\phi))^2}+\frac{\omega_\theta\Lambda\lambda^*(\phi)}{(\Lambda-\lambda^*(\phi))^2}\right\}\\ &= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\times\left\{\frac{\beta_t(\lambda^*(\phi))^2[t^*(\phi)(\mu-\lambda^*(\phi))-1]}{(\mu-\lambda^*(\phi))^3}\right.\\ &\left.+\frac{\beta_t t^*(\phi)\mu\lambda^*(\phi)}{(\mu-\lambda^*(\phi))^2}+\frac{\beta_t t^*(\phi)\lambda^*(\phi)}{\mu-\lambda^*(\phi)}+\frac{\omega_\theta\Lambda^2}{(\Lambda-\lambda^*(\phi))^2}\right\},\end{aligned}$$

which shows that, as the terms in the curly bracket are all positive, $\partial\Pi^*(\phi)/\partial\omega_\theta$ and $\partial\lambda^*(\phi)/\partial\omega_\theta$ have an identical sign. Because, as shown above, the sign of $\partial\lambda^*(\phi)/\partial\omega_\theta$ is the same as the sign of $b\omega_\theta/[(\beta_r\delta_1)^2\phi]-(V-V_e)/\omega_\theta$, we find that, when the inequality in (9) is satisfied, $\partial\lambda^*(\phi)/\partial\omega_\theta\geq 0$ and thus, $\partial\Pi^*(\phi)/\partial\omega_\theta\geq 0$ and $\partial t^*(\phi)/\partial\omega_\theta\geq 0$. As $\partial\phi/\partial\omega_\theta=-\omega_\theta/[(\beta_r\delta_1)^2\phi]<0$, we find that $\partial\lambda^*(\phi)/\partial\phi\leq 0$, $\partial\Pi^*(\phi)/\partial\phi\leq 0$, and $\partial t^*(\phi)/\partial\phi\leq 0$, as indicated in the first item of this lemma.

Next, we analyze the impact of ϕ on the expected ratings. Using (2), we find $E(R^*)=\rho q^*(\phi)+E(X)=\rho q^*(\phi)+\delta_1\gamma(1-\phi)+\delta_2$, and compute

$$\frac{\partial E(R^*)}{\partial\omega_\theta}=-\frac{\beta_t}{\beta_r\rho(\mu-\lambda^*(\phi))^2}\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}+\frac{\gamma\omega_\theta}{\beta_r^2\delta_1\phi}.$$

Thus, if the inequality in (9) is not satisfied, then $\partial\lambda^*(\phi)/\partial\omega_\theta<0$ and as $\partial\phi/\partial\omega_\theta<0$, $\partial\lambda^*(\phi)/\partial\phi=\partial\lambda^*(\phi)/\partial\omega_\theta\times\partial\omega_\theta/\partial\phi>0$ and $\partial E(R^*)/\partial\phi=\partial E(R^*)/\partial\omega_\theta\times\partial\omega_\theta/\partial\phi<0$. Otherwise, $\partial\lambda^*(\phi)/\partial\phi<0$ and $\partial E(R^*)/\partial\phi$ may be non-negative, which depends on the comparison between $-\partial\lambda^*(\phi)/\partial\omega_\theta\times\beta_t/[\rho(\mu-\lambda^*(\phi))^2]$ and $\gamma\omega_\theta/(\beta_r\delta_1\phi)$.

Moreover, according to (20), we find that if $\partial\lambda^*(\phi)/\partial\phi\geq 0$, then we must have $\partial V/\partial\phi<0$. Then, using (21), we have $V>V_e$. We thus prove this lemma. ■

Proof of Theorem 1. We denote the difference between the LHS and RHS of (9) by

$$h(\phi)\equiv\frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi}-\frac{V-V_e}{\omega_\theta}.$$

Because $\lim_{\phi\to 0}h(\phi)=+\infty$, we consider the following two cases only:

1. **Case 1**: $\forall\phi\in[0,1]$, $h(\phi)\geq 0$ or the condition in (9) is always satisfied. In this case, the firm's expected profit is decreasing in ϕ and thus, $\phi^*=0$.
2. **Case 2**: there exists one point $\phi_0\in[0,1)$ such that $h(\phi_0)<0$ but $h(\phi)\geq 0$ for $\phi\in[0,\phi_0)$. That is, there exists a sufficiently small, positive number ϕ_0 such that $h(\phi)\geq 0$ for $\phi\in[0,\phi_0)$.

As we already have the optimal solution for Case 1, we next conduct our analysis for Case 2.

Note that, for Case 2, $\phi_0 \neq 1$ because, otherwise, if $\phi_0 = 1$, then $h(\phi) \geq 0$, $\forall \phi \in [0,1]$ and thus Case 2 is identical to Case 1. Hence, for Case 2, $h(\phi) < 0$ at point $\phi = \phi_0 \neq 1$. This implies that $V > V_e$, because $b\omega_\theta/[(\beta_r\delta_1)^2\phi] > 0$. We differentiate $h(\phi)$ once w.r.t. ω_θ and find

$$\frac{\partial h(\phi)}{\partial \omega_\theta} = \frac{b\omega_\theta^2}{(\beta_r\delta_1)^4\phi^3} + \frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))}, \tag{23}$$

where an expression for $\partial\lambda^*(\phi)/\partial\omega_\theta$ is given as in (19) in the proof of Lemma 2. We consider the following two possibilities.

1. If, at point $\phi = \phi_0$, $\partial\lambda^*(\phi)/\partial\omega_\theta \leq 0$, then $\partial h(\phi)/\partial\omega_\theta > 0$ when $\phi = \phi_0$.
2. If, at point $\phi = \phi_0$, $\partial\lambda^*(\phi)\partial\omega_\theta > 0$, we need to analyze the RHS of (23). We find from equation (22) in the proof of Lemma 2 that

$$\frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\left[\beta_t \frac{2t^*(\phi)\mu(\mu - \lambda^*(\phi)) - \lambda^*(\phi)}{(\mu - \lambda^*(\phi))^3} + \frac{\omega_\theta \Lambda^2}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))^2}\right] = \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V - V_e}{\omega_\theta},$$

which can be rewritten as

$$\frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda^2}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))^2} = \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\beta_t}{\omega_\theta}\frac{2t^*(\phi)\mu(\mu - \lambda^*(\phi)) - \lambda^*(\phi)}{(\mu - \lambda^*(\phi))^3} + \frac{V - V_e}{\omega_\theta^2} > 0,$$

where the last inequality follows the conditions: (i) $V \geq V_e$, (ii) $\partial\lambda^*(\phi)/\partial\omega_\theta > 0$, and (iii) $2t^*(\phi)\mu(\mu - \lambda^*(\phi)) - \lambda^*(\phi) > 0$ because $t(\mu - \lambda^*(\phi)) > 1$ and $2\mu > \lambda$.

In addition, as $\partial\lambda^*(\phi)/\partial\omega_\theta > 0$ and $\Lambda > \Lambda - \lambda^*(\phi)$, we have

$$\frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))} > \frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda^2}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))^2} > 0.$$

Thus, we find from (23) that

$$\frac{\partial h(\phi)}{\partial \omega_\theta} = \frac{b\omega_\theta^2}{(\beta_r\delta_1)^4\phi^3} + \frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))} > 0.$$

The above results indicate that, at point $\phi = \phi_0$, $h(\phi)$ is increasing in ω_θ, or, $h(\phi)$ is decreasing in ϕ because $\partial\phi/\partial\omega_\theta < 0$, i.e.,

$$\left.\frac{\partial h(\phi)}{\partial \phi}\right|_{\phi=\phi_0} < 0. \tag{24}$$

We claim that $h(\phi) < 0$ for all values of ϕ in $[\phi_0, 1]$. *Suppose* this is not the case. Then, let $\hat{\phi} = \min\{\phi : \phi \in (\phi_0, 1), h(\phi) \geq 0\}$. Then, because of continuity of $h(\phi)$, there must exist a $\tilde{\phi} \in (\phi_0, \hat{\phi})$ such that

$$\left.\frac{\partial h(\phi)}{\partial \phi}\right|_{\phi=\tilde{\phi}} \geq 0.$$

However, this *cannot* happen because, by definition, $h(\tilde{\phi}) < 0$, from which we can conclude that

$$\left.\frac{\partial h(\phi)}{\partial \phi}\right|_{\phi=\tilde{\phi}} < 0,$$

along similar lines of proof that we derived (24). This proves the claim.

It follows that, if the firm's expected profit $\Pi^*(\phi)$ is increasing in ϕ when $\phi = \phi_0$, then $\Pi^*(\phi)$

is increasing in ϕ for all values of ϕ in $[\phi_0, 1]$. Furthermore, if $\Pi^*(\phi)$ increases with ϕ at point $\phi' \in [0,1]$, then $\Pi^*(\phi)$ is increasing in any value of $\phi \in (\phi', 1]$. We can thus conclude that it is impossible for $\Pi^*(\phi)$ to be a unimodal function with a single maximum point. That is, $\Pi^*(\phi)$ cannot first monotonically increase until $\phi = \phi' \in [0,1)$ and then monotonically decrease when $\phi > \phi'$.

As $\lim_{\phi \to 0} h(\phi) > 0$, $\Pi^*(\phi)$ is a monotonically decreasing when ϕ is sufficiently small in the range $(0,1)$. Hence, $\Pi^*(\phi)$ (for $\phi \in [0,1]$) has two possible shapes: (1) $\Pi^*(\phi)$ is monotonically decreasing in ϕ, and (2) $\Pi^*(\phi)$ is a reverse unimodal function with a single minimum point that first monotonically decreases until $\phi = \phi' \in [0,1]$ and then monotonically increases when $\phi > \phi'$. Thus, the optimal decision ϕ^* must be one of end points (0 and 1). Specifically, since $h(\phi)$ is always decreasing in ϕ, we find that, if $h(1) \geq 0$, then $h(\phi)|_{\phi \in [0,1]} \geq 0$ and thus, $\Pi^*(\phi)$ is decreasing in $\phi \in [0,1]$. That is, if $h(1) \geq 0$, then the firm's optimal decision is $\phi^* = 0$. Let $V_1 \equiv V|_{\phi=1}$. Because

$$h(1) = \lim_{\phi \to 1} \left(\frac{b\omega_\theta}{(\beta_r \delta_1)^2 \phi} - \frac{V - V_e}{\omega_\theta} \right) = \frac{b\omega_Y}{(\beta_r \delta_1)^2} - \frac{V_1 - V_e}{\omega_Y},$$

we conclude that if the condition in (12) is satisfied, then $h(1) \geq 0$ and $\phi^* = 0$.

Otherwise, if the condition in (12) does not hold, i.e., $h(1) < 0$, then there exists one point $\phi_0 \in [0,1]$ such that $h(\phi) \geq 0$ for $\phi \in (0, \phi_0]$ but $h(\phi) < 0$ for $\phi \in (\phi_0, 1]$. For this case, $\Pi^*(\phi)$ decreases with ϕ over the range $(0, \phi_0]$ but $\Pi^*(\phi)$ increases with ϕ over the range $(\phi_0, 1]$. Hence, the optimal decision is $\phi^* = 1$ if $\Pi^*(1) \geq \Pi^*(0)$ and $\phi^* = 0$ otherwise. This proves the theorem. ■

Proof of Corollary 1. We analyze the impact of β_t for $\phi^* = 1$ and $\phi^* = 0$. For the case $\phi^* = 1$, we differentiate the two sides of the expression (6) at $t_1^* \equiv t^*|_{\phi^*=1}$ once w.r.t. β_t, and obtain

$$\frac{\partial t_1^*}{\partial \beta_t} = \frac{\partial \lambda_1^*}{\partial \beta_t} \left(\frac{t_1^*}{\mu - \lambda_1^*} - \frac{1}{(\mu - \lambda_1^*)^2} \right) - \frac{1}{\beta_t (\mu - \lambda_1^*)}, \tag{25}$$

where $\lambda_1^* \equiv \lambda^*|_{\phi^*=1}$. Then, differentiating λ_1^* once w.r.t. β_t yields

$$\frac{\partial \lambda_1^*}{\partial \beta_t} = \frac{\lambda_1^*}{\omega_Y} \frac{\Lambda - \lambda_1^*}{\Lambda} \left[-\frac{\partial p_1^*}{\partial \beta_t} + \beta_r \rho \frac{\partial q_1^*}{\partial \beta_t} - t_1^* - \beta_t \frac{\partial t_1^*}{\partial \beta_t} \right], \tag{26}$$

where $p_1^* \equiv p^*|_{\phi^*=1}$ and $q_1^* \equiv q^*|_{\phi^*=1}$. Taking the first-order derivative of q_1^* w.r.t. β_t gives

$$\frac{\partial q_1^*}{\partial \beta_t} = -(1 - q_1^*) \left[-(\mu - \lambda_1^*) \frac{\partial t_1^*}{\partial \beta_t} + t_1^* \frac{\partial \lambda_1^*}{\partial \beta_t} \right].$$

Using (25), we reduce the above expression of $\partial q_1^* / \partial \beta_t$ to

$$\frac{\partial q_1^*}{\partial \beta_t} = -(1 - q_1^*) \left[\frac{\partial \lambda_1^*}{\partial \beta_t} \frac{1}{\mu - \lambda_1^*} + \frac{1}{\beta_t} \right]. \tag{27}$$

Differentiating p_1^* w.r.t. β_t gives

$$\begin{aligned} \frac{\partial p_1^*}{\partial \beta_t} &= \frac{\beta_t \lambda_1^* t_1^*}{\mu - \lambda_1^*} \left(\frac{1}{\beta_t} + \frac{1}{\mu - \lambda_1^*} \frac{\partial \lambda_1^*}{\partial \beta_t} \right) + \frac{\beta_t \lambda_1^*}{\mu - \lambda_1^*} \frac{\partial t_1^*}{\partial \beta_t} + \frac{\beta_t t_1^*}{\mu - \lambda_1^*} \frac{\partial \lambda_1^*}{\partial \beta_t} + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2} \frac{\partial \lambda_1^*}{\partial \beta_t} \\ &= \frac{\lambda_1^* t_1^*}{\mu - \lambda_1^*} + \frac{\partial t_1^*}{\partial \beta_t} \frac{\beta_t \lambda_1^*}{\mu - \lambda_1^*} + \frac{\partial \lambda_1^*}{\partial \beta_t} \left[\frac{\beta_t t_1^* \mu}{(\mu - \lambda_1^*)^2} + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2} \right]. \end{aligned} \tag{28}$$

Using (25), (27), and (28), we rewrite (26) as

$$\begin{aligned}\frac{\partial\lambda_1^*}{\partial\beta_t} &= -\frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\Lambda\omega_Y}\left\{\frac{\partial\lambda_1^*}{\partial\beta_t}\left[\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}+\frac{\beta_r\rho(1-q_1^*)+\beta_t t_1^*}{\mu-\lambda_1^*}\right]\right.\\ &\left.\quad+\frac{\beta_r}{\beta_t}\rho(1-q_1^*)+\left(1+\frac{\partial\lambda_1^*}{\partial\beta_t}\frac{\beta_t}{\mu-\lambda_1^*}\right)\left(1+\frac{\lambda_1^*}{\mu-\lambda_1^*}\right)\left(t_1^*-\frac{1}{\mu-\lambda_1^*}\right)\right\}.\end{aligned} \tag{29}$$

If $\partial\lambda_1^*/\partial\beta_t \geq 0$, then the RHS of (29) is negative because $1 > q_1^*$ and $t_1^* \geq 1/(\mu-\lambda_1^*)$. As a result, the equation in (29) does not hold. That is, $\partial\lambda_1^*/\partial\beta_t \geq 0$ is *impossible*. It thus follows that $\partial\lambda_1^*/\partial\beta_t < 0$. According to (25), we also have $\partial t_1^*/\partial\beta_t < 0$. However, we find from (27) that $\partial q_1^*/\partial\beta_t \geq 0$ if

$$\frac{\partial\lambda_1^*}{\partial\beta_t}\frac{1}{\mu-\lambda_1^*}+\frac{1}{\beta_t}\leq 0, \text{ or, } -\frac{\partial\lambda_1^*/\lambda_1^*}{\partial\beta_t/\beta_t}\geq\frac{\mu-\lambda_1^*}{\lambda_1^*},$$

and $\partial q_1^*/\partial\beta_t < 0$ otherwise. Moreover, $\partial p_1^*/\partial\beta_t \geq 0$ if

$$\frac{\lambda_1^* t_1^*}{\mu-\lambda_1^*}\geq-\frac{\partial t_1^*}{\partial\beta_t}\frac{\beta_t\lambda_1^*}{\mu-\lambda_1^*}-\frac{\partial\lambda_1^*}{\partial\beta_t}\left[\frac{\beta_t t_1^*\mu}{(\mu-\lambda_1^*)^2}+\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right].$$

and $\partial p_1^*/\partial\beta_t < 0$ otherwise.

We take the first-order derivative of $\Pi_1^* \equiv \Pi^*|_{\phi^*=1}$ w.r.t. β_t and find

$$\frac{\partial\Pi_1^*}{\partial\beta_t}=\frac{(\lambda_1^*)^2 t_1^*}{\mu-\lambda_1^*}+\frac{\partial\lambda_1^*}{\partial\beta_t}\left[p_1^*-b-c+\frac{\beta_t\lambda_1^* t_1^*\mu}{(\mu-\lambda_1^*)^2}+\frac{\Lambda\lambda_1^*\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]+\frac{\partial t_1^*}{\partial\beta_t}\frac{\beta_t(\lambda_1^*)^2}{\mu-\lambda_1^*}.$$

Hence, $\partial\Pi_1^*/\partial\beta_t \geq 0$ if

$$\frac{(\lambda_1^*)^2 t_1^*}{\mu-\lambda_1^*}\geq-\frac{\partial\lambda_1^*}{\partial\beta_t}\left[p_1^*-b-c+\frac{\beta_t\lambda_1^* t_1^*\mu}{(\mu-\lambda_1^*)^2}+\frac{\Lambda\lambda_1^*\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]-\frac{\partial t_1^*}{\partial\beta_t}\frac{\beta_t(\lambda_1^*)^2}{\mu-\lambda_1^*},$$

and $\partial\Pi_1^*/\partial\beta_t < 0$ otherwise. To derive meaning implications from the above results, we simplify them to find that $\partial p_1^*/\partial\beta_t < 0$ and $\partial\Pi_1^*/\partial\beta_t < 0$ if

$$\frac{\lambda_1^* t_1^*}{\mu-\lambda_1^*}\leq-\frac{\partial t_1^*}{\partial\beta_t}\frac{\beta_t\lambda_1^*}{\mu-\lambda_1^*}, \text{ or, } -\frac{\partial t_1^*/t_1^*}{\partial\beta_t/\beta_t}\geq 1.$$

Following a similar process, we obtain the impacts of β_t for $\phi^* = 0$. The proof is complete. ■

Proof of Corollary 2. For $\phi^* = 1$, we differentiate both sides of expression (6) w.r.t. β_r and obtain

$$\frac{\partial t_1^*}{\partial\beta_r}(\mu-\lambda_1^*)=\frac{1}{\beta_r}+\frac{\partial\lambda_1^*}{\partial\beta_r}\left(t_1^*-\frac{1}{\mu-\lambda_1^*}\right). \tag{30}$$

We then compute the first-order derivative of λ_1^* w.r.t. β_r and find

$$\frac{\partial\lambda_1^*}{\partial\beta_r}=\frac{\lambda_1^*}{\omega_Y}\frac{\Lambda-\lambda_1^*}{\Lambda}\left(-\frac{\partial p_1^*}{\partial\beta_r}+\rho q_1^*+\delta_2+\beta_r\rho\frac{\partial q_1^*}{\partial\beta_r}-\beta_t\frac{\partial t_1^*}{\partial\beta_r}\right). \tag{31}$$

Taking the first-order derivative of q_1^* in (1) w.r.t. β_r and using (6) and (30) to simplify it yields

$$\frac{\partial q_1^*}{\partial\beta_r}=\frac{\beta_t}{\beta_r\rho}\left(\frac{\partial t_1^*}{\partial\beta_r}-\frac{t_1^*}{\mu-\lambda_1^*}\frac{\partial\lambda_1^*}{\partial\beta_r}\right)=\frac{\beta_t}{\beta_r\rho(\mu-\lambda_1^*)}\left(\frac{1}{\beta_r}-\frac{\partial\lambda_1^*}{\partial\beta_r}\frac{1}{\mu-\lambda_1^*}\right),$$

which is non-negative when

$$\frac{1}{\beta_r}-\frac{\partial\lambda_1^*}{\partial\beta_r}\frac{1}{\mu-\lambda_1^*}\geq 0,\text{ or, }\frac{\partial\lambda_1^*/\lambda_1^*}{\partial\beta_r/\beta_r}\leq\frac{\mu-\lambda_1^*}{\lambda_1^*}.$$

Also, we differentiate p_1^* once w.r.t. β_r and obtain

$$\frac{\partial p_1^*}{\partial\beta_r}=\frac{\beta_t\lambda_1^*}{\beta_r(\mu-\lambda_1^*)^2}+\frac{\partial\lambda_1^*}{\partial\beta_r}\left[\frac{\beta_t\lambda_1^*t_1^*}{\mu-\lambda_1^*}+\frac{2\beta_t\lambda_1^*t_1^*}{(\mu-\lambda_1^*)^2}-\frac{\beta_t\lambda_1^*}{(\mu-\lambda_1^*)^3}+\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right].$$

We then use the above results to rewrite $\partial\lambda_1^*/\partial\beta_r$ in (31) as

$$\begin{aligned}\frac{\partial\lambda_1^*}{\partial\beta_r} &= \frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\omega_Y\Lambda}\left\{\rho+\delta_2-\frac{\beta_t\mu}{\beta_r(\mu-\lambda_1^*)^2}-\frac{\partial\lambda_1^*}{\partial\beta_r}\left[\frac{\beta_t\lambda_1^*t_1^*}{\mu-\lambda_1^*}+\frac{\beta_t t_1^*}{\mu-\lambda_1^*}\right.\right.\\ &\qquad\left.\left.+\frac{2\beta_t\lambda_1^*t_1^*}{(\mu-\lambda_1^*)^2}-\frac{\beta_t\lambda_1^*}{(\mu-\lambda_1^*)^3}+\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]\right\}.\end{aligned}\tag{32}$$

If $\rho+\delta_2\leq\beta_t\mu/[\beta_r(\mu-\lambda_1^*)^2]$, or,

$$q_1^*\leq\frac{\beta_t\lambda_1^*-\beta_r\delta_2(\mu-\lambda_1^*)^2}{\beta_t\mu-\beta_r\delta_2(\mu-\lambda_1^*)^2},$$

then $\partial\lambda_1^*/\partial\beta_r\leq 0$. Otherwise, $\partial\lambda_1^*/\partial\beta_r$ may be positive. In particular, when the expected delivery time $1/(\mu-\lambda^*)$ is negligibly short, i.e., $1/(\mu-\lambda^*)\to 0$, in (32) we can remove all terms involving $1/(\mu-\lambda^*)$. As a result, (32) can be reduced to

$$\frac{\partial\lambda_1^*}{\partial\beta_r}=\frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\omega_Y\Lambda}\left[\rho+\delta_2-\frac{\partial\lambda_1^*}{\partial\beta_r}\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]=\frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\omega_Y\Lambda}(\rho+\delta_2)-\frac{\partial\lambda_1^*}{\partial\beta_r}\frac{\lambda_1^*}{\Lambda-\lambda_1^*},$$

which can be rewritten as

$$\frac{\partial\lambda_1^*}{\partial\beta_r}=\frac{\lambda_1^*(\Lambda-\lambda_1^*)^2}{\omega_Y\Lambda^2}(\rho+\delta_2)>0.$$

We take the first-order derivative of Π_1^* w.r.t. β_r and find

$$\frac{\partial\Pi_1^*}{\partial\beta_r}=\frac{\beta_t(\lambda_1^*)^2}{\beta_r(\mu-\lambda_1^*)^2}+\frac{\partial\lambda_1^*}{\partial\beta_r}\left[p_1^*-b-c+\frac{\beta_t(\lambda_1^*)^2t_1^*}{\mu-\lambda_1^*}+\frac{2\beta_t(\lambda_1^*)^2t_1^*}{(\mu-\lambda_1^*)^2}-\frac{\beta_t(\lambda_1^*)^2}{(\mu-\lambda_1^*)^3}+\frac{\lambda_1^*\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right],$$

which means $\partial\Pi_1^*/\partial\beta_r\geq 0$ if $\partial\lambda_1^*/\partial\beta_r\geq 0$.

The analyses for $\phi^*=0$ are similar to those for $\phi^*=1$. We differentiate $\lambda_0^*\equiv\lambda^*|_{\phi^*=0}$ once w.r.t. β_r and get

$$\begin{aligned}\frac{\partial\lambda_0^*}{\partial\beta_r} &= \frac{\lambda_0^*(\Lambda-\lambda_0^*)}{\Lambda\sqrt{(\beta_r\delta_1)^2+\omega_Y^2}}\left\{\rho+\delta_2-\frac{\beta_t\mu}{\beta_r(\mu-\lambda_0^*)^2}-\frac{\partial\lambda_0^*}{\partial\beta_r}\left[\frac{\beta_t\lambda_0^*t_0^*}{\mu-\lambda_0^*}+\frac{\beta_t t_0^*}{\mu-\lambda_0^*}+\frac{2\beta_t\lambda_0^*t_0^*}{(\mu-\lambda_0^*)^2}\right.\right.\\ &\qquad\left.\left.-\frac{\beta_t\lambda_0^*}{(\mu-\lambda_0^*)^3}+\frac{\Lambda\sqrt{(\beta_r\delta_1)^2+\omega_Y^2}}{(\Lambda-\lambda_0^*)^2}+\frac{\beta_r\delta_1(V_0-V_e)}{(\beta_r\delta_1)^2+\omega_Y^2}\right]\right\}-\frac{\lambda_0^*\beta_r\delta_1}{(\beta_r\delta_1)^2+\omega_Y^2},\end{aligned}$$

where $t_0^* \equiv t^*|_{\phi^*=0}$. Taking the first-order derivative of $p_0^* \equiv p^*|_{\phi^*=0}$ w.r.t. β_r gives

$$\frac{\partial p_0^*}{\partial \beta_r} = \frac{\beta_t \lambda_0^*}{\mu - \lambda_0^*}\frac{\partial t_0^*}{\partial \beta_r} + \frac{\partial \lambda_0^*}{\partial \beta_r}\left[\frac{\beta_t t_0^*}{\mu - \lambda_0^*} + \frac{\beta_t \lambda_0^* t_0^*}{(\mu - \lambda_0^*)^2} + \frac{\Lambda\sqrt{(\beta_r\delta_1)^2 + \omega_Y^2}}{(\Lambda - \lambda_0^*)^2}\right] + \frac{\Lambda \beta_r \delta_1}{(\Lambda - \lambda_0^*)\sqrt{(\beta_r\delta_1)^2 + \omega_Y^2}}.$$

Other derivatives are similar to those for $\phi^* = 1$. Using the arguments for $\phi^* = 1$, we produce the results for $\phi^* = 0$ as shown in this corollary. ■

Proof of Corollary 3. For $\phi^* = 1$, we differentiate the two sides of the expression for t_1^* once w.r.t. μ, and obtain

$$\frac{\partial t_1^*}{\partial \mu}(\mu - \lambda_1^*)^2 = [1 - t_1^*(\mu - \lambda_1^*)]\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right).$$

Then, differentiating λ_1^* once w.r.t. μ yields

$$\frac{\partial \lambda_1^*}{\partial \mu} = \frac{\lambda_1^*}{\omega_Y}\frac{\Lambda - \lambda_1^*}{\Lambda}\left(-\frac{\partial p_1^*}{\partial \mu} + \beta_r \rho \frac{\partial q_1^*}{\partial \mu} - \beta_t \frac{\partial t_1^*}{\partial \mu}\right), \tag{33}$$

Taking the first-order derivative of q_1^* w.r.t. μ gives

$$\frac{\partial q_1^*}{\partial \mu} = \frac{\beta_t}{\beta_r \rho}\left[\frac{\partial t_1^*}{\partial \mu} + \frac{t_1^*}{\mu - \lambda_1^*}\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right)\right] = \frac{\beta_t}{\beta_r \rho (\mu - \lambda_1^*)^2}\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right).$$

Differentiating p_1^* w.r.t. μ gives

$$\frac{\partial p_1^*}{\partial \mu} = \frac{\beta_t t_1^*}{\mu - \lambda_1^*}\frac{\partial \lambda_1^*}{\partial \mu} + \frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left(\frac{1}{\mu - \lambda_1^*} - 2t_1^*\right)\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right) + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\frac{\partial \lambda_1^*}{\partial \mu}.$$

We then rewrite $\partial \lambda_1^* / \partial \mu$ in (33) as

$$\frac{\partial \lambda_1^*}{\partial \mu} = \frac{\lambda_1^*(\Lambda - \lambda_1^*)}{\omega_Y \Lambda} - \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\frac{\partial \lambda_1^*}{\partial \mu} + \frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left[\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right)\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right) + \frac{\beta_t t_1^*}{\mu - \lambda_1^*}\left(1 - 2\frac{\partial \lambda_1^*}{\partial \mu}\right)\right],$$

where the RHS is positive if the LHS (i.e., $\partial \lambda_1^* / \partial \mu \leq 0$). That is, $\partial \lambda_1^* / \partial \mu > 0$. Moreover, if $\partial \lambda_1^* / \partial \mu \geq 1$, the RHS of the above equation is negative, which breaks the equation up. Therefore, we have $0 < \partial \lambda_1^* / \partial \mu < 1$. As a result, $\partial t_1^* / \partial \mu < 0$ and $\partial q_1^* / \partial \mu > 0$.

We take the first-order derivative of Π_1^* w.r.t. μ and find

$$\frac{\partial \Pi_1^*}{\partial \mu} = \left[p_1^* - b - c + \frac{\beta_t \lambda_1^* t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \lambda_1^* \omega_Y}{(\Lambda - \lambda_1^*)^2}\right]\frac{\partial \lambda_1^*}{\partial \mu} + \frac{\beta_t (\lambda_1^*)^2}{(\mu - \lambda_1^*)^2}\left(\frac{1}{\mu - \lambda_1^*} - 2t_1^*\right)\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right).$$

We can conclude that for $\phi^* = 1$, if the expected delivery time is sufficiently short, then both $\partial p_1^* / \partial \mu$ and $\partial \Pi_1^* / \partial \mu$ are positive.

The analysis for $\phi^* = 0$ is analogous to that for $\phi^* = 1$. The corollary is thus proved. ■

Proof of Corollary 4. For $\phi^* = 1$, we differentiate the two sides of the expression for t_1^* once w.r.t. ω_Y, and have

$$\frac{\partial t_1^*}{\partial \omega_Y}(\mu - \lambda_1^*)^2 = [t_1^*(\mu - \lambda_1^*) - 1]\frac{\partial \lambda_1^*}{\partial \omega_Y}.$$

Then, differentiating λ_1^* once w.r.t. ω_Y yields

$$\frac{\partial \lambda_1^*}{\partial \omega_Y} = \frac{\lambda_1^*}{\omega_Y}\frac{(\Lambda - \lambda_1^*)}{\Lambda}\left(-\frac{\partial p_1^*}{\partial \omega_Y} + \beta_r \rho \frac{\partial q_1^*}{\partial \omega_Y} - \beta_t \frac{\partial t_1^*}{\partial \omega_Y} + \frac{V_1 - V_e}{\omega_Y}\right). \tag{34}$$

Taking the first-order derivative of q_1^* w.r.t. ω_Y gives

$$\frac{\partial q_1^*}{\partial \omega_Y} = -\frac{\beta_t}{\beta_r \rho (\mu - \lambda_1^*)}\left[-(\mu - \lambda_1^*)\frac{\partial t_1^*}{\partial \omega_Y} + \frac{\partial \lambda_1^*}{\partial \omega_Y} t_1^*\right] = -\frac{\beta_t}{\beta_r \rho (\mu - \lambda_1^*)^2}\frac{\partial \lambda_1^*}{\partial \omega_Y}.$$

Differentiating p_1^* w.r.t. ω_Y gives

$$\frac{\partial p_1^*}{\partial \omega_Y} = \frac{\Lambda}{\Lambda - \lambda_1^*} + \frac{\partial \lambda_1^*}{\partial \omega_Y}\left[\frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right) + \frac{\beta_t t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\right].$$

Using the above to simplify $\partial \lambda_1^* / \partial \omega_Y$ gives

$$\begin{aligned}\frac{\partial \lambda_1^*}{\partial \omega_Y} \quad &= \quad \frac{\lambda_1^*}{\omega_Y}\frac{(\Lambda - \lambda_1^*)}{\Lambda}\left\{\beta_r \delta_2 + \frac{V_1 - V_e}{\omega_Y} - \frac{\Lambda}{\Lambda - \lambda_1^*}\right. \\ &\left. -\frac{\partial \lambda_1^*}{\partial \omega_Y}\left[\frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right) + \frac{2\beta_t t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\right]\right\}. \end{aligned} \tag{35}$$

We can then use the expression of λ_1^* to find that

$$\frac{\Lambda}{\Lambda - \lambda_1^*} = \frac{\Lambda}{\lambda_1^*}\exp\left(\frac{V_1 - V_e}{\omega_Y}\right) > \exp\left(\frac{V_1 - V_e}{\omega_Y}\right),$$

which implies

$$\frac{V_1 - V_e}{\omega_Y} > \frac{\Lambda}{\Lambda - \lambda_1^*}.$$

Therefore, $\partial \lambda_1^* / \partial \omega_Y \geq 0$, because the equation in (35) does not hold if $\partial \lambda_1^* / \partial \omega_Y < 0$. Hence, $\partial t_1^* / \partial \omega_Y \geq 0$, $\partial p_1^* / \partial \omega_Y \geq 0$, and $\partial q_1^* / \partial \omega_Y \leq 0$. We also differentiate Π_1^* once w.r.t. ω_Y and have

$$\frac{\partial \Pi_1^*}{\partial \omega_Y} = \frac{\Lambda \lambda_1^*}{\Lambda - \lambda_1^*} + \frac{\partial \lambda_1^*}{\partial \omega_Y}\left[p_1^* - b - c + \frac{\beta_t (\lambda_1^*)^2}{(\mu - \lambda_1^*)^2}\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right) + \frac{\beta_t \lambda_1^* t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \lambda_1^* \omega_Y}{(\Lambda - \lambda_1^*)^2}\right],$$

which is non-negative because $\partial \lambda_1^* / \partial \omega_Y \geq 0$.

The results for $\phi^* = 0$ are the same as those for $\phi^* = 1$, because $\partial\sqrt{(\beta_r \delta_1)^2 + \omega_Y^2}/\partial \omega_Y = \omega_Y / \sqrt{(\beta_r \delta_1)^2 + \omega_Y^2} > 0$. This corollary is thus proved. ■

Appendix E Additional Numerical Results

We extend our numerical analyses of the *baseline* scenario to consider a broader set of model parameters and to test the robustness of the results to the assumption that $Z \sim C(\phi)$ (which implies that $X \sim C(\phi)$ because $\delta_1 = 1$ and $\delta_2 = 0$). We generate data to characterize the relationship between ϕ and Π^* for different values of the parameters b, c, d, α, ρ, V_e, and the parameters already considered in the main text (β_r, β_t, μ, and ω_Y). This time, we evaluate the parameters at the values $\{0, 5, 9\}$ because of computational singularities with the value 10 when considering alternative distributions.

The results for $Z \sim C(\phi)$ appear in Figure A and replicate those presented in the main text for parameters d, β_r, β_t, μ, and ω_Y. The numerical results for the additional parameters indicate that profits are higher when costs (b and c) are lower, when the perceived quality of the firm's offering (α) is high, when the perceived value of balking (V_e) is low, and when the effect of delivery reliability on delivery trust (ρ) is strong. The intuition behind the results for b, c, V_e, and α is self-evident. The intuition behind the result for ρ is similar to the intuition behind the result for β_r because both parameters determine the weight of delivery reliability information in the utility function in similar ways. In addition, we note that variation in the values of β_r and ρ results in the largest variation in profits among all parameters. This suggests that β_r and ρ are the most important determinants of profitability. This result is confirmed by an elasticity analyses (available upon request).

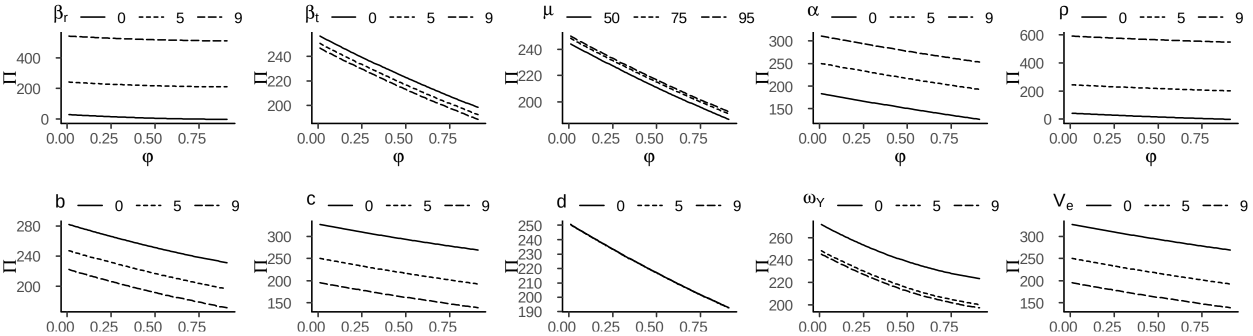

Figure A: Sensitivity analyses under the assumption that $Z \sim C(\phi)$.

The $C(\phi)$ distribution allows us to obtain closed form solutions and analytical results but it is of interest to determine whether results hold for other popular distributions. Because closed-form solutions do not exist for distributions other than the $C(\phi)$ distribution, we perform these analyses numerically. We consider the cases in which $Z \sim Gumbel$, $Z \sim \log normal$, and $Z \sim Normal$ because these distributions are widely used in individual-level models of demand. We use these alternative distributions to replicate the sensitivity analyses so as to draw easily interpretable results while considering a wide range of parameter values. To ensure comparability to the base results obtained with the $C(\phi)$ distribution, we restrict the mean and variance of these alternative distributions to be equal to those of the $C(\phi)$ distribution for each value of ϕ considered.

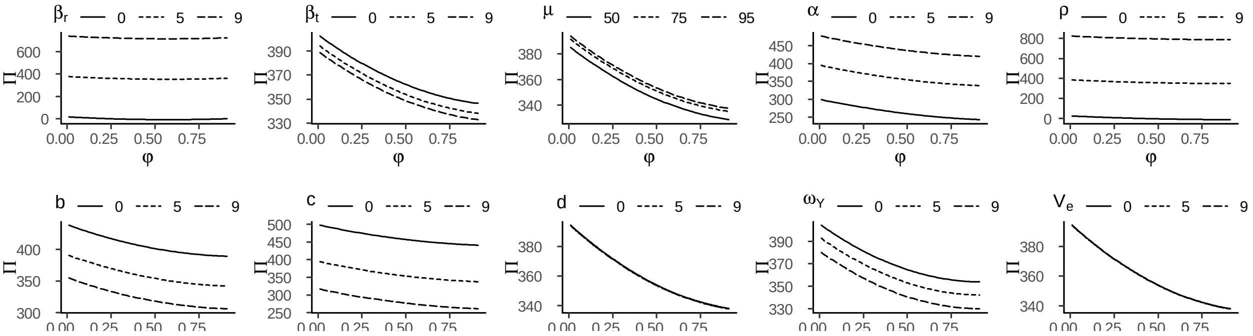

Figure B: Sensitivity analyses under the assumption that $Z \sim Gumbel$.

Results for the Gumbel distribution appear in Figure B and for the lognormal and normal distributions in Figures C and D, respectively. All three figures indicate that the shapes of the profit curves are very similar across distributions. Most importantly, the main result that only $\phi = 0$ or $\phi = 1$ can be optimal holds regardless of the distributional assumption.

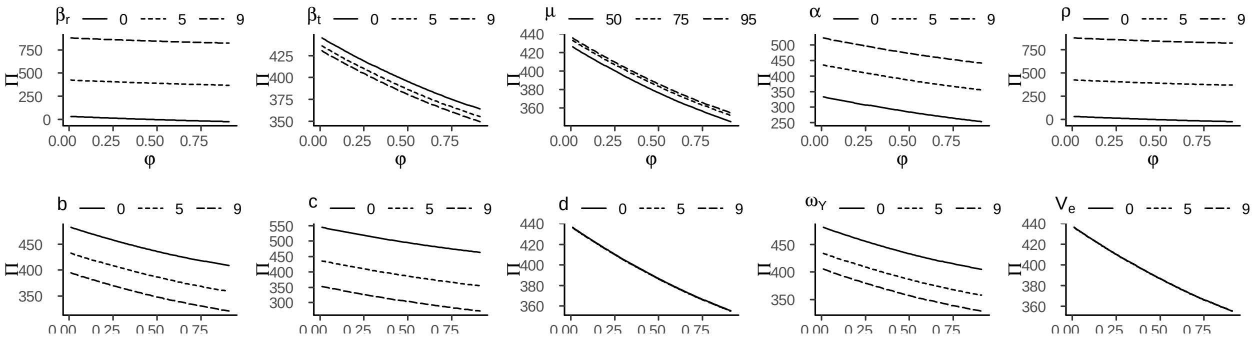

Figure C: Sensitivity analyses under the assumption that $Z \sim \log normal$.

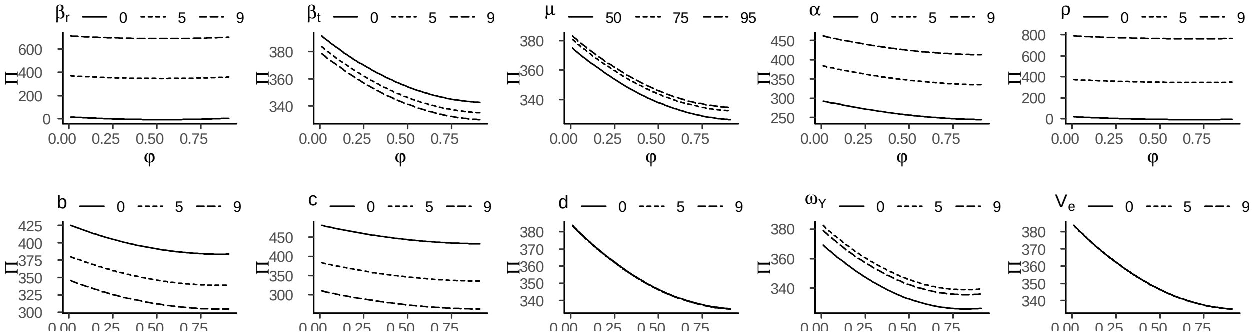

Figure D: Sensitivity analyses under the assumption that $Z \sim Normal$.