Online Appendix for "Reducing Traffic Incidents in Meal Deliveries: Penalize the Platform or its Independent Drivers?".

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- In Appendix A, we provide comprehensive anecdotal evidence on the causes of delivery-related incidents.
- In Appendix B, we examine the robustness of our findings by analyzing various model extensions.
- In Appendix C, we provide proofs for the results in §4.
- In Appendix D, we provide proofs for the results in §5, which shows government's optimal penalty scheme.
- In Appendix E, we provide proofs for the results in §B.
- In Appendix F, we include auxiliary results, as well as their proofs, to support the reasoning presented.

Throughout the document, we use the following notation to ease the exposition of our derivations and results:

Definition 1. $\underline{\mathcal{F}}(A) = \left\{(x,v) : v \geq \frac{\alpha(A+w)}{s} x\right\} \cap (0,\bar{x}] \times (0,\bar{v}]$, *and* $\bar{\mathcal{F}} = \left\{(x,v) : v \geq \frac{\beta+\alpha w}{s} x\right\} \cap (0,\bar{x}] \times (0,\bar{v}]$.

Definition 2.

$$T_\star := \frac{\gamma + R + \left(1 - \frac{1}{\alpha}\right) v}{2\left(1 - \frac{1}{\alpha}\right)\beta}.$$

Definition 3. *Special social surplus:*

1. *Pro-platform:* $S_p(x,v;A,R) := \pi^*(x,v;A,R) + (1 - G(T^*(x,v;A,R)))P$.
2. *Pro-driver:* $S_d(x,v;A,R) := u^*(x,v;A,R)T^*(x,v;A,R) + (1 - G(T^*(x,v;A,R)))P$.

Appendix A: Anecdotal Evidence on the Causes of Delivery-Related Incidents

First, according to the NYC report (Figueroa et al. 2021), drivers face serious consequences if they complete a delivery later than the time promised to the customer—even when delays stem from factors beyond their control, such as restaurant preparation times or traffic congestion. Platform algorithms closely monitor delivery timeliness and customer complaints, and drivers frequently report being penalized through lower ratings, restricted access to shifts, or even deactivation. Many drivers noted that customers can easily file complaints or retract tips following a late delivery, directly reducing their earnings. In most cases, platforms tend to side with the customer. As a result, drivers often feel compelled to speed—not just to maximize their income but to protect themselves from punitive measures that jeopardize their ability to continue working.

To provide more evidence, we share some of the quotes from drivers in the NYC report (Figueroa et al. 2021), which make it clear that by penalizing lateness and rewarding on-time performance regardless of circumstances, platforms effectively force drivers to complete deliveries no later than the quoted time as a defensive strategy to protect their earnings and continue working.

"They don't care for our safety. I had an accident once and instead of asking for my well-being they asked if I had delivered the food. They care for the food, not the [worker]. We could get shot, assaulted, and they wouldn't care."

"If something happens while we travel [the apps] aren't liable"

". . . we tell customers that there is a 30-minute wait time, and they respond with 'that's your problem, if you want to cancel the order do so,' which will bring down my rating."

". . . you risk getting a deactivation for a customer reporting that they didn't get the food (they call it 'fraud'), and unless you're wearing a camera that shows that you left the food, you're out."

They closed [my] account . . . . due to a discussion with the restaurant because I asked [the restaurant employee] why [the food] wasn't ready, and she wouldn't let me wait inside and she cursed at me. I defended myself.

In food delivery markets outside the U.S., we observe fairly similar patterns: platforms prioritize speed over safety. Below, we cite several quotes from food delivery drivers (Banerji et al. 2021, Jung and Lee 2021):

"Food delivery drivers have to race against the clock. When companies want money, they come up with delivery targets. And since we want money, we try to hit those targets."

— Maneewan Maneesecom, a driver, injuried during a delivery for Line Man

"(When we do deliveries) we keep getting calls saying 'Dear partner, you're running late. Deliver the order immediately or else you might have to face a penalty'. Your focus is on your phone and it's buzzing and buzzing and buzzing and buzzing. Every single order feels manic. We speed and put our lives in danger."

"I got hit from behind by a guy who was speeding on a scooter. Thank God he wasn't driving a car because I wouldn't have survived. My motorcycle skidded about 20 metres, flinging me off as cars and trucks zipped past. The first thing I did was to go and check if the cake was fine."

— An anonymous driver, injuried during a delivery for Zomato

"...the reasons behind industrial accidents by food delivery drivers are not individual matters, but are structural parts of the industry and how it works."

"What happened to those delivery drivers could have happened to any one of us. We know that our driving can be dangerous, but at the same time, it's what we feel we have to do in order to survive in this industry."

— Members in Korean Federation of Service Workers' Union

"If the delivery drivers reject any calls or cannot deliver food on time, they receive penalties such as bad ratings."

— Park Jeong-hoon, Chairman of the Driver Union (via JoongAng Ilbo)

The following driver testimonies are drawn from a widely discussed investigative article "Food Delivery Drivers, Trapped in the System" in China (Lai 2020), which resonated deeply with the public.[15] Almost the same pattern emerges.

"*Jin Zhuangzhuang, who worked as a Meituan delivery station manager for three years, recalled that from 2016 to 2019 he received three notifications from Meituan about accelerating delivery times: in 2016, the maximum allowed time for a 3-kilometer delivery was one hour; in 2017, it was reduced to 45 minutes; in 2018, shortened again by 7 minutes, to 38 minutes. Data shows that by 2019, the average delivery time in China had decreased by 10 minutes compared with three years earlier.*"

"*In the system's settings, delivery time is the most important indicator, and lateness is not allowed. Once it happens, it means bad reviews, reduced income, or even elimination. As one driver wrote in an online forum: delivering food is like racing against death, competing with traffic police, and befriending red lights.*"

"To remind himself constantly, a driver in Jiangsu changed his social media nickname to: 'Lateness means losing your head.'"

"A driver in Shanghai's Songjiang district said he rides against traffic on almost every order, which saves him about five minutes each time."

"Zhu Dahe recalled sweating with fear of being late. He once crashed while swerving to avoid a bicycle, his scooter skidding across the road and the spicy hotpot he was delivering flying into the air. What struck him first was not the pain, but the thought: 'Oh no, I'll be late.' In another case, he paid out of pocket to buy the meal himself to avoid a late order and a bad review. As he put it: 'It cost me more than 80 yuan, but I still remember it clearly — the delivery fee was only 6.5 yuan."'

[15] Upon release, the article received more than 100,000 reads within just two hours, and it quickly climbed to over 3 million views in total. Its influence extended well beyond readership, sparking commentary from at least 11 state-level media outlets and hundreds of social media influencers with millions of followers, making it one of the most impactful reports on the risks faced by food delivery drivers in China.

> *"Driver Wei Lai described witnessing a colleague run a red light and be hit by a speeding car, dying instantly at the intersection. He admitted that he did not stop, because his own order was about to be late, and at that very moment, his phone buzzed with a new order assignment."*

Lastly, New York City's passage of multiple local laws in 2021 targeting the working conditions of app-based delivery drivers provides strong evidence that platform system designs have been creating unsafe pressures—even in the absence of explicit instructions. In particular, algorithms that control work pace amplify stress by rewarding speed and penalizing delays, making the speed of work a major health and safety risk factor. To meet delivery demands and avoid negative consequences such as poor reviews, lower tips, or platform-imposed penalties, drivers often resort to unsafe practices—speeding, using phones while driving, running red lights, skipping breaks, or working through inclement weather. These laws, such as mandating per-trip minimum payments, implicitly acknowledge that the design of the platforms themselves creates incentives for unsafe behaviors. Without a guaranteed base payment, drivers are compelled to increase delivery volume to secure a living wage, which in turn pressures them to rush, cut corners, and accept riskier assignments. By legislating minimum per-trip pay, the City effectively recognized that the absence of such protections was not a neutral design choice but one that structurally encouraged unsafe practices as drivers sought to maximize earnings under the algorithm's pace-driven incentives (Plasencia et al. 2022).

Based on the overwhelming evidence we collected from public sources and our private discussions with meal delivery drivers, we are confident that the platforms' penalty mechanisms and algorithmic controls (particularly delivery time targets) play a greater role than the drivers' own incentives to earn more in causing them to exceed safe speeds. Our model builds on this notion and assumes that drivers do not drive slower than the platform's promised delivery time, given the severe consequences of lateness. Instead, drivers may choose either to deliver faster than the promised time or to decline the request.

Appendix B: Robustness Checks

We examine the robustness of our primary results by systematically relaxing key assumptions and exploring alternative formulations. Specifically, we extend our analysis to the following scenarios: (1) the platform has imperfect information about customer valuation; (2) the delivery rate and the driver's commission rate are independent; (3) the incident probability follows an exponential distribution; (4) the government imposes a fixed penalty per incident on drivers; and (5) drivers have positive outside earning options. Our findings consistently hold across all these extensions, demonstrating the reliability and broad applicability of our original conclusions.

B.1. Imperfect Information about Customer Valuation: from v to $v+\epsilon$

Our main model in §3 is based on the assumption that the platform knows each customer's valuation v. This assumption relies on the capabilities of large meal-delivery platforms leveraging extensive data analytics and advanced machine-learning algorithms. These technologies, while not perfect, significantly enhance the platform's ability to predict customer behavior and valuation more accurately than ever before (Ping et al. 2021). In practice, however, predictions of customer valuation are inherently subject to error. To address this, as a robustness check, we consider the case when the platform does not fully observe the customer valuation. In this analysis, for a given customer, we assume that the customer's valuation is $v+\epsilon$, where v is observable by the platform whereas ϵ is not. We assume that $\epsilon \sim U[-\epsilon_0, \epsilon_0]$, where $\epsilon_0 > 0$ captures the platform's prediction error on the customer's valuation.

First, a customer will place an order if:

$$\sigma(x, v, \epsilon, k, t) = v + \epsilon - kx - \beta t \geq 0.$$

Then, from the platform's perspective, the customer's purchase probability is:

$$P(\epsilon \geq kx + \beta t - v) = \frac{1}{2\epsilon_0}\left(\epsilon_0 - \min\{\epsilon_0, \max\{-\epsilon_0, kx + \beta t - v\}\}\right).$$

Second, the driver's expected earning rate becomes:

$$\begin{aligned} u(t; b, x, v, A) &= \frac{1}{2\epsilon_0}\left(\epsilon_0 - \min\{\epsilon_0, \max\{-\epsilon_0, kx + \beta t - v\}\}\right)\left(sb - \max\left\{1 - \frac{s}{x}t, 0\right\}A\right) \\ &= \begin{cases} sb - \max\left\{1 - \frac{s}{x}t, 0\right\}A, & \text{if } kx + \beta t - v \leq -\epsilon_0 \\ \frac{1}{2\epsilon_0}(v + \epsilon_0 - kx - \beta t)\left(sb - \max\left\{1 - \frac{s}{x}t, 0\right\}A\right), & \text{if } -\epsilon_0 < kx + \beta t - v \leq \epsilon_0 \\ 0, & \text{if } kx + \beta t - v > \epsilon_0 \end{cases}, \end{aligned}$$

where $k = \alpha b$. It is straightforward to verify that it is suboptimal for the driver to choose $t > \frac{x}{s}$. Hence, the driver's optimization problem follows:

$$\max_{0 \leq t \leq \frac{x}{s}} \frac{1}{2\epsilon_0}\left(\epsilon_0 - \min\{\epsilon_0, \max\{-\epsilon_0, \alpha bx + \beta t - v\}\}\right)\left(sb - A + \frac{s}{x}At\right),$$

and the driver's optimal delivery time can be characterized by:

$$\hat{t}^* = \max\left\{0, \min\left\{\hat{t}, \frac{x}{s}\right\}\right\},$$

where

$$\hat{t} = \begin{cases} \max\left\{\frac{1}{\beta}(v-\epsilon_0-\alpha bx), \min\left\{\frac{1}{\beta}(v+\epsilon_0-\alpha bx), \frac{\frac{s}{x}A(v+\epsilon_0-\alpha bx)-\beta(sb-A)}{2\beta\frac{s}{x}A}\right\}\right\}, & \text{if } A>0 \\ \frac{1}{\beta}(v-\epsilon_0-\alpha bx), & \text{if } A=0 \end{cases}.$$

Thus, a driver will participate if and only if $u(\hat{t}^*; b, x, A) \geq w$.

Considering the decision of customers and drivers, the platform's optimization problem for customer (x,v) can be formulated as:

$$\begin{aligned} \max_{b\geq 0} \quad & \frac{1}{2\epsilon_0}\left(\epsilon_0 - \min\left\{\epsilon_0, \max\left\{-\epsilon_0, \alpha bx + \beta\hat{t}^* - v\right\}\right\}\right)\left(\frac{s}{x}\hat{t}^*(\gamma+(\alpha-1)bx) - \left(1-\frac{s}{x}\hat{t}^*\right)R\right) \\ \text{s.t.} \quad & u(\hat{t}^*; b, x, A) \geq w. \end{aligned} \tag{18}$$

We then use b^* to represent the platform's optimal policy and t^* to denote the driver's optimal delivery time under b^*.

Subsequently, we consider the government's objective of maximizing the total social surplus under the conditions set by the platform's optimal policy. The utility of customer (x,v) in this scenario is as follows:

$$\sigma^*(x,v,b^*,t^*) = \mathbb{E}\left[\max\{v+\epsilon-\alpha b^*x-\beta t^*, 0\}\right] = \frac{\epsilon_0-\underline{\epsilon}}{2\epsilon_0}\left(v-\alpha b^*x-\beta t^*+\frac{\epsilon_0+\underline{\epsilon}}{2}\right),$$

where $\underline{\epsilon} = \min\{\epsilon_0, \max\{-\epsilon_0, \alpha b^*x+\beta t^*-v\}\}$. Then, the driver's expected earning rate is:

$$u^*(t^*; b^*, x, A) = \frac{\epsilon_0-\underline{\epsilon}}{2\epsilon}\left(sb^*-A+\frac{s}{x}At^*\right),$$

and the platform's expected profit follows:

$$\pi^*(x,v,A,R) = \frac{\epsilon_0-\underline{\epsilon}}{2\epsilon_0}\left(\frac{s}{x}t^*(\gamma+(\alpha-1)b^*x)-\left(1-\frac{s}{x}t^*\right)R\right).$$

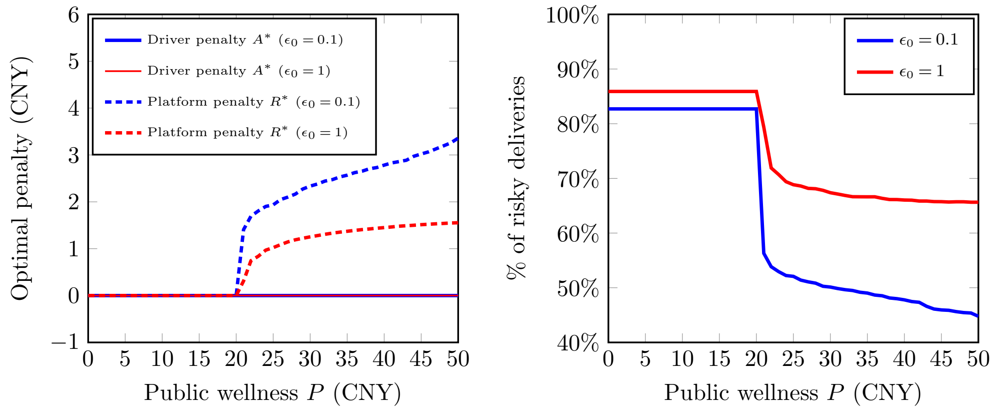
Therefore, the expected surplus from serving customer (x,v) is given by $S(x,v;A,R) = \sigma^* + u^*t^* + \pi^* + \frac{\epsilon_0-\underline{\epsilon}}{2\epsilon_0}\cdot\frac{s}{x}t^*P$. Building on this, the government's problem of maximizing the total social surplus can be expressed as:

$$\max_{A\geq 0, R\geq 0}\int_{(x,v)\in\mathcal{D}(A,R)} S(x,v;A,R)\mathbb{1}\{\pi^*(x,v;A,R)\geq 0\}F_{\mathcal{C}}(dx,dv),$$

where $\mathcal{D}(A,R) = \{(x,v)|$ Program (18) is feasible under $(A,R)\}$. Given the complexity and intractability of the problem in this extension, we focus on numerically examining the two market-level mechanisms—risky-delivery reduction and market expansion—and the optimal penalty scheme (A^*, R^*). In these numerical analyses, we use the customer distribution and model primitives calibrated in §6

Figure 8 presents the government's optimal penalty scheme (A^*, R^*) across different levels of P (left panel) and the relationship between the percentage of risky deliveries and P (right panel). As shown in the left plot, the optimal driver penalty remains $A^* = 0$ when the prediction error is set to $\epsilon_0 \in \{0.1, 1\}$ (note that the setting reduces to the main model when $\epsilon_0 = 0$). This finding indicates that even when the platform cannot perfectly observe each customer's valuation, it is still optimal for the government to impose penalties on the platform rather than on drivers. This result is consistent with the main findings in §5.3, thereby reinforcing the robustness and generalizability of our conclusions.

Figure 8. Optimal government penalty scheme (left) and the percentage of deliveries with a positive incident probability under the optimal penalty scheme (right) under imperfect information cases (i.e., $\epsilon_0 = 0.1$ and $\epsilon_0 = 1$) .



B.2. Independent Delivery Fee k and Driver Commission Rate b

In our main model (§3), we assume that the delivery rate k and driver commission rate b are linked by an exogenous factor $\alpha > 1$, such that $b = k/\alpha$. This allows us to determine b^* via T^* in Propositions 1 and 2, and then recover $k^* = \alpha b^*$. This simplification, adopted by Uber in its early operations, is empirically supported (Bai et al. 2019). To assess robustness, we now relax the constraint $b = k/\alpha$, enabling the platform to independently optimize both k and b. This generalization decouples the delivery fee kx charged to customers from the driver payment bx, aligning with current practices in meal delivery platforms (Mao et al. 2025).

To begin, recall from Inequality (1) that customer (x, v) would place an order if and only if $\sigma(x, v, k, t) = v - kx - \beta t \geq 0$ (which is equivalent to $t \leq \frac{v-kx}{\beta}$). Then, suppose a driver participates in the meal delivery for customer (x, v); their optimal delivery time t^* satisfies:

$$t^*(b, x, v, A) = \underset{0 < t \leq \frac{1}{\beta}(v - kx)}{\arg\max} \; u(t; b, x, A),$$

where the driver's earning $u(\cdot)$ is given in Expression (3). By considering the first-order condition, we obtain the following lemma that is analogous to Lemma 1.

Lemma 7. *For any delivery rate k, the driver's optimal delivery time t^* for customer (x, v) satisfies:*

$$t^*(k) = \max\left\{\min\left\{\frac{x}{s}, \frac{v - kx}{\beta}\right\}, 0\right\}.$$

By considering $t^*(k)$ along with the driver's outside option value w as defined in §4.2, the driver will accept the order if and only if $u(t^*(k); b, x, A) \geq w$ and the customer (x, v) will place the order if $v - kx - \beta t^*(k) \geq 0$. Also, it follows from (5) that the platform's profit is:

$$\pi(k, b; x, v, t^*, R) = \min\left\{\frac{s}{x}t^*, 1\right\} \cdot (\gamma + (k - b)x) - \max\left\{1 - \frac{s}{x}t^*, 0\right\} \cdot R.$$

Hence, by considering both the customer's and the driver's participation constraints, the platform's profit maximization problem over k and b can be formulated as:

$$\begin{aligned} &\max_{k,b} \frac{s}{x}t^*(k)(\gamma+(k-b)x)-\left(1-\frac{s}{x}t^*(k)\right)R, \text{ s.t.} \\ \text{(Customer's purchase constraint) } & v-kx-\beta t^*(k)\geq 0, \\ \text{(Driver's participation constraint) } & \frac{s}{x}bx-\left(1-\frac{s}{x}t^*(k)\right)A\geq w. \end{aligned} \tag{19}$$

Before we determine the platform's optimal decisions regarding the delivery rate k and the driver commission rate b, let us characterize the optimal solution to the platform's problem in the following lemma:

Lemma 8. *Under the optimal solution to Problem (19), both the customer's purchase and the driver's participation constraints are binding.*

Unlike the main model based on the assumption $b=k/\alpha$, which has only one free variable, our extension enables the platform to use two free variables k and b to extract the surplus from both the driver and the customer, making both the customer's participation constraint and the driver's participation constraint binding. By applying Lemma 8, the platform's optimal policy (k^*, b^*) satisfies:

$$k^*=\frac{v-\beta t^*(k^*)}{x} \text{ and } b^*=\frac{w}{s}+\left(\frac{1}{s}-\frac{t^*(k^*)}{x}\right)A.$$

Different from our main model based on the assumption $b=k/\alpha$ with $\alpha>1$, so that $b<k$, the above equation reveals that it is possible to have $b^*>k^*$. For instance, when the driver penalty A is sufficiently large, the platform might pay the driver more than the amount the platform collects from the customer, resulting in $b^*>k^*$.

Next, by noting that the optimal delivery time $t^*(k^*)$ must satisfy $0<t^*(k^*)\leq \frac{x}{s}$, we can utilize the above lemma to determine the optimal delivery time as follows:

Proposition 5. *For any given penalty policy (A,R), the platform's optimal delivery time for customer (x,v) satisfies:*

1. **Low-A regime.** *Suppose $A<\beta$. Then,*

$$t^*(k^*)=\begin{cases} 0, & \text{if } v<(w+A)\frac{x}{s}-\gamma-R \\ \frac{\gamma+v+R-(w+A)\frac{x}{s}}{2(\beta-A)}\left(<\frac{x}{s}\right), & \text{if } (w+A)\frac{x}{s}-\gamma-R\leq v<(2\beta-A+w)\frac{x}{s}-\gamma-R \\ \frac{x}{s}, & \text{if } v\geq(2\beta-A+w)\frac{x}{s}-\gamma-R \end{cases}.$$

2. **High-A regime.** *Suppose $A\geq\beta$. Then,*

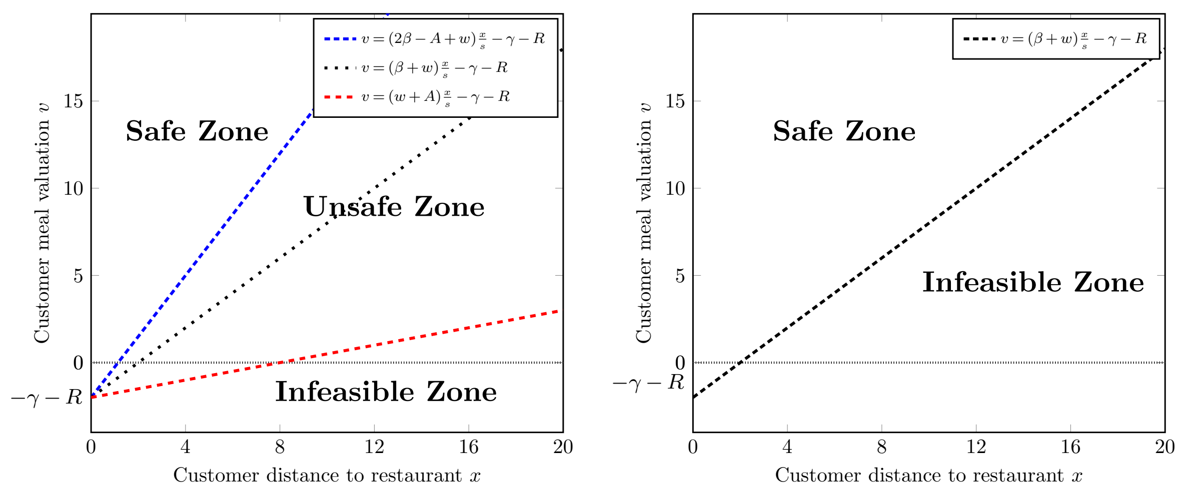
$$t^*(k^*)=\begin{cases} 0, & \text{if } v<(\beta+w)\frac{x}{s}-\gamma-R \\ \frac{x}{s}, & \text{if } v\geq(\beta+w)\frac{x}{s}-\gamma-R \end{cases}.$$

Using the above proposition, we partition the customer space into three zones: a "safe zone", where no delivery incidents occur; an "unsafe zone", where the probability of delivery incidents is strictly positive; and an "infeasible zone", where $t^*=0$ and no deliveries are made. Also, analogous to (10), the feasible zone $\mathcal{D}(A,R)$ is:

$$\mathcal{D}(A,R)=\left\{(x,v)\,\middle|\, \begin{matrix} v\geq(w+A)\frac{x}{s}-\gamma-R, & \text{if } A<\beta \\ v\geq(\beta+w)\frac{x}{s}-\gamma-R, & \text{if } A\geq\beta \end{matrix}, (x,v)\in(0,\bar{x}]\times(0,\bar{v}]\right\}.$$

Figure 9 illustrates the customer zones under $A<\beta$ and $A\geq\beta$. Notably, Proposition 5 and Figure 9 parallel Propositions 1 and 2, as well as Figures 2 and 3. These similarities confirm that the structural results remain valid even when we relax the the assumption $b=k/\alpha$ with $\alpha>1$.

Figure 9. Customer zones when $A<\beta$ (left) and when $A\geq\beta$ (right).



Parameter value: $A=0.125,\ R=1,\ \alpha=2, \beta=0.5, \gamma=1,\ s=0.5,$ *and* $w=0$.

Next, in the following proposition, we show that increasing the platform penalty R is more effective in reducing delivery-related risks, whereas decreasing driver penalty A is more effective in expanding market coverage, as discussed in §5.2.

Proposition 6. *Increasing the platform penalty R always reduces the incident probability $G(t^*)$, whereas increasing the driver penalty A may increase the incident probability. Also, decreasing A can expand the platform's feasible zone for delivery, $\mathcal{D}(A,R)$, whereas decreasing R cannot.*

Proposition 6 suggests that the government should increase R to curb risky deliveries while reducing A to expand the platform's market coverage.

Finally, it remains to examine the government's problem of designing a penalty policy (A,R) to maximize total social surplus. For customer (x,v), the platform will participate in serving the customer if $\pi^*(x,v,A,R)\geq 0$, where $\pi^*(x,v,A,R)$ is the maximum profit solved from Problem (19). Then, given the platform's participation, the social surplus created by customer (x,v)'s order is:

$$S(x,v;A,R):=\pi^*+\sigma^*+u^*t^*+(1-G(t^*))\,P=\pi^*+wt^*+\frac{s}{x}t^*P.$$

As in the main model, we set the social surplus to zero when the platform chooses not to serve customer (x,v). Therefore, the government's objective of maximizing the total social surplus across the entire market can be formulated as:

$$\max_{A\geq 0,R\geq 0}\bar{S}(A,R):=\int_{(x,v)\in\mathcal{D}(A,R)}S(x,v;A,R)\mathbb{1}\{\pi^*(x,v;A,R)\geq 0\}F_{\mathcal{C}}(dx,dv). \tag{20}$$

A close examination of Program (20) reveals the following:

Theorem 2. *Suppose the customer distribution satisfies:*

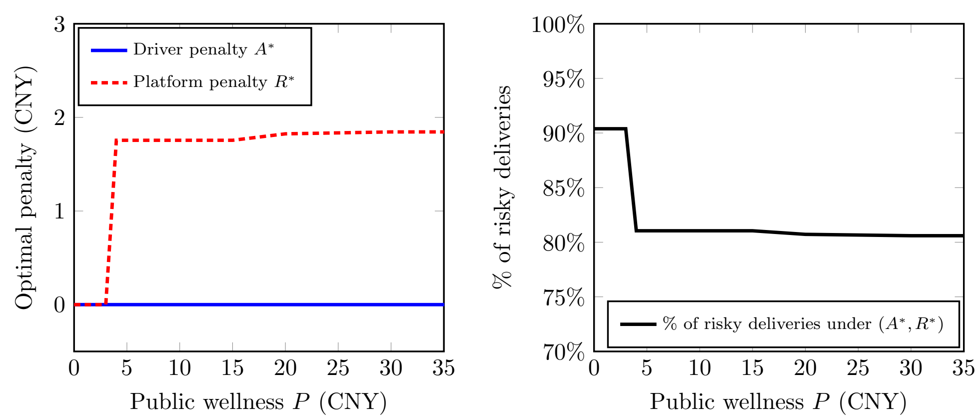
$$\mathcal{C}=\left\{(x,v)\;\middle|\;\begin{array}{l}\underline{\ell}<(2\beta+w)\frac{x}{s}-\gamma-v-2\sqrt{\beta\frac{x}{s}\left((\beta+w)\frac{x}{s}-\gamma-v\right)}<\bar{\ell}\\ \bar{\ell}<\min\left\{(\beta+w)\frac{x}{s}-\gamma-v,(2\beta-w)\frac{x}{s}-2P-\gamma-v\right\}\end{array}\right\}$$

for given $\underline{\ell}$ *and* $\bar{\ell}$*, which satisfy* $0<\underline{\ell}<\bar{\ell}$*. Then,* $A^*=0$ *is the optimal solution to Program* (19)*.*

The above theorem establishes that when customers are relatively impatient—i.e., their valuation v is low relative to their distance x—the government's optimal policy for maximizing total social surplus is to impose no penalty on drivers for delivery-related incidents. This result also reinforces our earlier finding in Theorem 1.

Figure 10 presents the numerically derived optimal penalty scheme (A^*,R^*), based on the customer distribution and model primitives calibrated in §6. By evaluating a broad range of values for public wellness P, the left panel reveals that it is optimal to penalize the platform rather than the driver for delivery incidents—i.e., $A^*=0$ and $R^*\geq 0$. Moreover, the optimal platform penalty R^* increases with P. These findings are consistent with the insights discussed in §5 and §6, confirming that our main structural results remain robust even when the assumption $b=k/\alpha$ is relaxed.

Figure 10. Optimal government penalty scheme (left) and the percentage of deliveries with a positive incident probability under the optimal penalty scheme (right).



Additionally, the right panel of Figure 10 illustrates the percentage of deliveries with a positive incident probability (i.e., $G(t^*)>0$) under the optimal policy (A^*,R^*). The solid curve represents this percentage. As the optimal platform penalty R^* increases with public wellness, the percentage of risky deliveries declines from 90% to slightly above 80%, suggesting the effectiveness of the platform penalty in mitigating delivery-related incidents.

B.3. Exponential Incident Probability Distribution: $H(\xi)=1-e^{-\lambda\xi}$

Recall from Equation (2) in §3 that, for analytical tractability, we transform the incident probability function such that $G(t;x)=\max\left\{1-\frac{s}{x}t,0\right\}$, where $t=\frac{x}{s+\xi}$ denotes the delivery time, and $\xi\geq 0$ represents the excess travel speed beyond the safe speed limit s. This transformation reflects the simplifying assumption that the incident probability decreases piecewise linearly with delivery time t, consistent with empirical findings in the transportation safety literature (Garber and Gadirau 1988, Taylor et al. 2002). Notably, when drivers adhere to the safe speed limit (so that the excess speed $\xi = 0$), the delivery time satisfies $t \geq \frac{x}{s}$, and the incident probability becomes zero. To examine the robustness of our results associated with the incident probability $G(t;x)$, we now consider an "alternative" incident probability $H(\xi)$ that depends directly on the excessive speed ξ (instead of the delivery time t).

Specifically, we assume that the alternative incident probability $H(\xi)$ follows an exponential probability distribution such that $H(\xi) = 1 - e^{-\lambda\xi}$ with the "rate parameter" $\lambda > 0$, $H(0) = 0$, and $H(\infty) = 1$ (Hussain et al. 2019). Hence, the incident probability $H(\xi) = 1 - e^{-\lambda\xi}$ is concave and increasing in the excessive speed ξ. By focusing on ξ (instead of t), the driver's expected earning rate given in (3) can be rewritten as:

$$u(\xi; x, b, A) = (1 - H(\xi))\frac{bx}{t} - H(\xi)A.$$

Because $t = \frac{x}{s+\xi}$, the driver's earning maximization problem can be formulated as follows:

$$\begin{aligned} \max_{\xi \geq 0} \quad & e^{-\lambda\xi} b(s+\xi) - \left(1 - e^{-\lambda\xi}\right) A \\ \text{s.t.} \quad & \frac{x}{s+\xi} \leq \frac{1}{\beta}(v - \alpha b x). \end{aligned}$$

It is straightforward to verify that the driver's optimal excessive speeding decision ξ^* satisfies:

$$\xi^* = \max\left\{\frac{b - \lambda A}{\lambda b} - s, \frac{\beta x}{v - \alpha b x} - s, 0\right\}.$$

Thus, the driver will participate if and only if $u(\xi^*; x, b, A) \geq w$.

Using the same approach presented in §4, the platform's profit function can be rewritten as:

$$\pi(b; x, v, \xi^*, R) = (1 - H(\xi^*))(\gamma + (\alpha - 1)bx) - H(\xi^*)R.$$

Then, the corresponding profit maximization problem can be written as:

$$\begin{aligned} \max_{b \geq 0} \quad & e^{-\lambda\xi^*}(\gamma + (\alpha - 1)bx) - \left(1 - e^{-\lambda\xi^*}\right) R \\ \text{s.t.} \quad & e^{-\lambda\xi^*} b(s+\xi^*) - \left(1 - e^{-\lambda\xi^*}\right) A \geq w. \end{aligned} \tag{21}$$

Let b^* denote the optimal solution to the above program. Then, the expected surplus from serving customer (x, v) can be given by:

$$S(x, v; A, R) = \sigma\left(x, v, b^*, \frac{x}{s+\xi^*}\right) + u(\xi^*; x, b^*, A)\frac{x}{s+\xi^*} + \pi(b^*; x, v, \xi^*, A) + (1 - H(\xi^*))P.$$

Because customer (x, v) is distributed according to $F_{\mathcal{C}}$, the government's problem that maximizes the total expected social surplus can be expressed as:

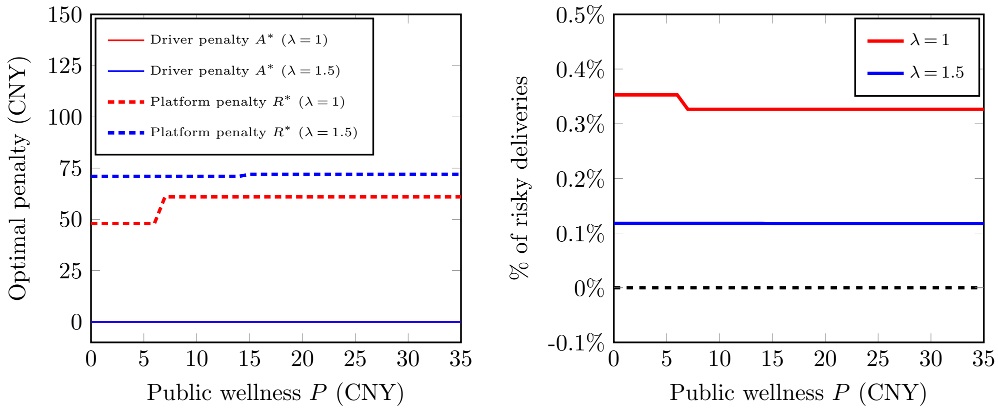
$$\max_{A \geq 0, R \geq 0} \int_{(x,v) \in \mathcal{D}_\xi(A,R)} S(x, v; A, R) \mathbb{1}\{\pi^*(x, v; A, R) \geq 0\} F_{\mathcal{C}}(dx, dv).$$

where $\mathcal{D}_\xi(A, R) = \{(x, v) |$ Program (21) is feasible under $(A, R)\}$.

Due to the analytical intractability of the government's optimization problem, we employ numerical methods to derive the optimal policy. Specifically, we examine three values of the incident probability rate parameter, setting $\lambda \in \{1, 1.5\}$. The customer distribution and model primitives are calibrated as described in §6. The resulting optimal penalty scheme, denoted by (A^*, R^*), is depicted in Figure 11. As shown, we find that it is optimal to set the driver penalty $A^* = 0$ across all cases, as evidenced by the "overlapping" solid lines that collectively form the solid purple line in the left panel. This observation suggests that our main results remain robust under the alternative incident probability specification $H(\xi)$.[16]

[16] When λ is sufficiently small (e.g., $\lambda = 0.1$), we numerically find that $A^* > 0$. This is because a small λ implies that drivers place little weight on the penalty relative to their earning rate, making driver-initiated speeding the dominant

Figure 11. Optimal government penalty scheme (left) and the percentage of deliveries with a positive incident probability under the optimal penalty scheme (right) under exponential incident probability distributions.



B.4. Driver Penalty Per Incident B

Recall from §3 that the driver penalty A is conceptualized not merely as a utility penalty per delivery-related incident but, more substantively, as an adjustment to the driver's hourly wage contingent upon the occurrence of such incidents. To facilitate a meaningful comparison with the platform's unit penalty per incident R, we now consider an "alternative" specification: a flat driver penalty per incident B.

First, the customer utility stated in (1) remains the same as $\sigma(x,v,k,t) = v - kx - \beta t$ so that customer will place the order if and only if $t \le \frac{1}{\beta}(v - \alpha bx)$. Under the flat per-incident driver penalty B, the driver's earning rate given in (3) can be rewritten as:

$$u(t,b,x,B) = \min\left\{\frac{s}{x}t, 1\right\}\frac{bx}{t} - \max\left\{1 - \frac{s}{x}t, 0\right\}\frac{B}{t},$$

By using the same arguments as in Lemma 1, the driver's optimal delivery time is:

$$t^* = \max\left\{\min\left\{\frac{x}{s}, \frac{1}{\beta}(v - \alpha bx)\right\}, 0\right\}.$$

Second, by considering that $t^* = \frac{1}{\beta}(v - \alpha bx)$, the platform's profit stated in (5) is: $\pi(b;x,v) = \frac{s}{x}t^*(\gamma + (\alpha - 1)bx) - \left(1 - \frac{s}{x}t^*\right)R$. By letting $T := \frac{1}{\beta}(v - \alpha bx)$, we can express $b = \frac{1}{\alpha x}(v - \beta T)$, and the platform's optimization problem stated in (8) under the alternative penalty B can be rewritten as:

$$\begin{aligned} \max_{T \in \left(0, \frac{x}{s}\right]} \quad & \frac{s}{x}T\left(\gamma + \left(1 - \frac{1}{\alpha}\right)(v - \beta T)\right) - \left(1 - \frac{s}{x}T\right)R \\ \text{s.t.} \quad & \frac{s}{\alpha x}(v - \beta T) - \frac{B}{T} + \frac{s}{x}B \ge w. \end{aligned}$$

It is easy to check that the above problem can be simplified as:

$$\begin{aligned} \max_{T \in \left(0, \frac{x}{s}\right]} \quad & \frac{s}{x}T\left(\gamma + \left(1 - \frac{1}{\alpha}\right)(v - \beta T)\right) - \left(1 - \frac{s}{x}T\right)R \\ \text{s.t.} \quad & T_1 \le T \le T_2, \end{aligned} \tag{22}$$

source of risky behavior. In this case, increasing A becomes an effective tool to deter such behavior, leading to a positive optimal driver penalty.

where

$$T_1 = \frac{v + \alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta}}{2\beta} \geq 0, T_2 = \frac{v + \alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta}}{2\beta} \geq 0,$$

and $\Delta := \left(v + \alpha B - \alpha w \frac{x}{s}\right)^2 - 4\alpha\beta B \frac{x}{s}$. Upon examination, we can identify the feasibility conditions associated with Problem (22) in the following lemma:

Lemma 9. *For a given customer (x,v), the platform Problem (22) is feasible if and only if one of the following two scenarios holds: (1) $\alpha B \leq \beta \frac{x}{s}$ and $v \geq 2\sqrt{\alpha\beta B \frac{x}{s}} - \alpha B + \alpha w \frac{x}{s}$; or (2) $\alpha B > \beta \frac{x}{s}$ and $v \geq (\beta + \alpha w)\frac{x}{s}$.*

Then, analogous to Expression (10), the feasible zone $\mathcal{D}(A)$ in this extension is:

$$\mathcal{D}(B) = \left\{ (x,v) \middle| \begin{array}{ll} v \geq 2\sqrt{\alpha\beta B \frac{x}{s}} - \alpha B + \alpha w \frac{x}{s}, & \text{if } \alpha B \leq \beta \frac{x}{s} \\ v \geq (\beta + \alpha w)\frac{x}{s}, & \text{if } \alpha B > \beta \frac{x}{s} \end{array}, (x,v) \in (0,\bar{x}] \times (0,\bar{v}] \right\}.$$

Additionally, the platform's optimal solution can be denoted by:

$$T^* = \max\left\{T_1, \min\left\{T_\star, T_2, \frac{x}{s}\right\}\right\},$$

where $T_\star$ maximizes the objective function of Problem (22) without considering the boundary constraints. It follows from Definition 2 at the beginning of Appendix, $T_\star := \frac{\gamma + R + \left(1 - \frac{1}{\alpha}\right)v}{2\left(1 - \frac{1}{\alpha}\right)\beta}$. By considering $T_\star$, we characterize T^* as follows:

Proposition 7. (Low-B regime) *Suppose $\alpha B \leq \beta \frac{x}{s}$ and $v \geq 2\sqrt{\alpha\beta B \frac{x}{s}} - \alpha B + \alpha w \frac{x}{s}$. Then, the optimal T^* that solves the platform's Problem (22) satisfies:*

$$T^* = \begin{cases} T_1 \ (< \frac{x}{s}), & \textit{if } \frac{\gamma+R}{1-\frac{1}{\alpha}} \leq \min\left\{\alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta}, 2\beta \frac{x}{s} - v\right\} \\ T_\star \ (< \frac{x}{s}), & \textit{if } \alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta} \leq \frac{\gamma+R}{1-\frac{1}{\alpha}} \leq \min\left\{\alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta}, 2\beta \frac{x}{s} - v\right\} \\ T_2 \ (< \frac{x}{s}), & \textit{if } \alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta} \leq \min\left\{\frac{\gamma+R}{1-\frac{1}{\alpha}}, 2\beta \frac{x}{s} - v\right\} \\ \frac{x}{s}, & \textit{if } 2\beta \frac{x}{s} - v \leq \min\left\{\frac{\gamma+R}{1-\frac{1}{\alpha}}, \alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta}\right\} \end{cases}.$$

Proposition 8. (High-B regime) *Suppose $\alpha B \geq \beta \frac{x}{s}$ and $v \geq (\beta + \alpha w)\frac{x}{s}$. Then, the optimal T^* that solves platform's Problem (22) satisfies:*

$$T^* = \begin{cases} T_1 \ (< \frac{x}{s}), & \textit{if } \frac{\gamma+R}{1-\frac{1}{\alpha}} \leq \min\left\{\alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta}, 2\beta \frac{x}{s} - v\right\} \\ T_\star \ (< \frac{x}{s}), & \textit{if } \alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta} \leq \frac{\gamma+R}{1-\frac{1}{\alpha}} \leq 2\beta \frac{x}{s} - v \\ \frac{x}{s}, & \textit{if } 2\beta \frac{x}{s} - v \leq \frac{\gamma+R}{1-\frac{1}{\alpha}} \end{cases}.$$

By using the above propositions, we can show in the following proposition that the two market mechanisms generated by A and R established in §5.2 continue to hold in this extension. Specifically, we show that increasing the platform penalty R is more effective for reducing risky deliveries than increasing the driver penalty B. Also, decreasing B is more effective in expanding market coverage than decreasing R.

Proposition 9. *Increasing the platform penalty R (weakly) reduces the incident probability $G(T^*)$, whereas increasing the driver penalty B may increase the incident probability. Also, decreasing B can expand the platform's feasible zone for delivery, $\mathcal{D}(B)$, whereas decreasing R cannot.*

Utilizing the platform's optimal decisions T^* and b^*, the social surplus for serving customer (x,v) is:

$$S(x,v;B,R) := \pi^* + \sigma^* + u^*T^* + (1-G(T^*))\,P = \pi^* + u^*T^* + (1-G(T^*))\,P,$$

where $\pi^* = \frac{s}{x}T^*\left(\gamma + \left(1-\frac{1}{\alpha}\right)(v-\beta T^*)\right) - \left(1-\frac{s}{x}T^*\right)R$ and $u^* = \frac{s}{\alpha x}(v-\beta T^*) - \frac{B}{T^*} + \frac{s}{x}B$. By simplifying $S(x,v;B,R)$ as:

$$S(x,v;B,R) = \frac{s}{x}T^*\left(\gamma + v + P - \beta T^*\right) - \left(1-\frac{s}{x}T^*\right)(R+B),$$

we can rewrite the government optimization problem defined in §5.3 as:

$$\max_{B\geq 0, R\geq 0} \int_{(x,v)\in\mathcal{D}(B)} S(x,v;B,R)\mathbb{1}\{\pi^*(x,v;B,R)\geq 0\}F_{\mathcal{C}}(dx,dv). \tag{23}$$

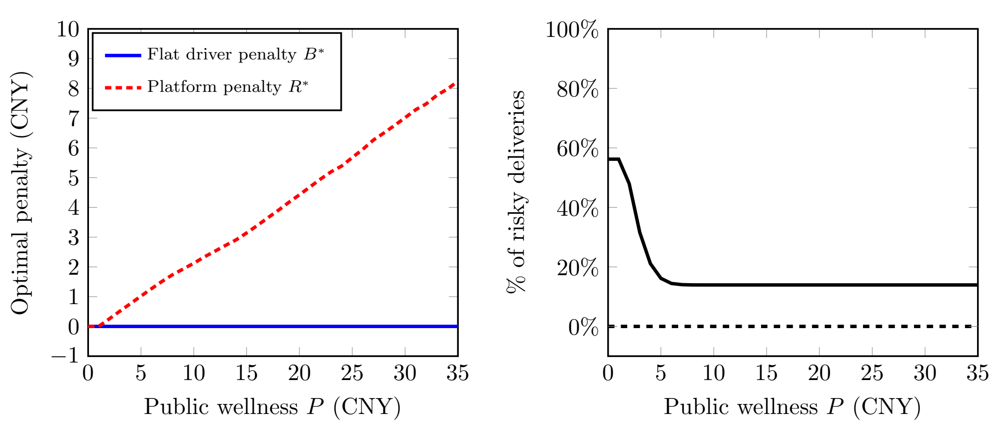
We observe the following regarding the optimal solution to Program (23):

Theorem 3. *Suppose $\beta < \alpha w$ and the customer distribution satisfies $\mathcal{C} = \left\{(x,v) | v \leq (\beta+\alpha w)\frac{x}{s}\right\}$. Then, $B^* = 0$ is the optimal solution to Program (23).*

The above theorem establishes that when customers are relatively impatient—i.e., their valuation v is low relative to their distance x—the government's optimal policy for maximizing total social surplus is to impose no penalty on drivers for delivery-related incidents. This result also reinforces our earlier finding in Theorem 1.

We further numerically determine the optimal penalty scheme (B^*, R^*) using the calibrated customer distribution and model primitives described in §6. Figure 12 illustrates the resulting optimal policy. The analysis reveals that the government's optimal strategy involves imposing penalties exclusively on the platform, while refraining from penalizing drivers (i.e., $B^* = 0$). Therefore, even when a per-incident penalty B is levied on drivers, the structural insights derived from our main model remain robust.

Figure 12. Optimal government penalty scheme (left) and the percentage of deliveries with a positive incident probability under the optimal penalty scheme (right).



B.5. Heterogeneous Weights Across Stakeholders' Payoffs

Recall from §5.3 that Theorem 1 is predicated on the assumption that the social surplus function $S(x,v;A,R)$, as defined in Equation (14), assigns equal weights to the payoffs of all stakeholders. We now generalize this framework by allowing heterogeneous weights across stakeholders when computing the social surplus associated with an order placed by customer (x,v):

$$\widehat{S}(x,v,A,R) = a_1\pi^* + a_2\sigma^* + a_3u^*T^* + (1-G(T^*))P, \tag{24}$$

where each weight $a_i \geq 0$ and $\sum_{i=1}^{3} a_i = 1$.

Theorem 4. *Suppose* $\left(1-\frac{1}{\alpha}\right)(\beta-\alpha w)\frac{x}{s} \leq P+\gamma$ *holds for any* $x \in (0,\bar{x}]$. *Then, even when the government assigns different weights to the various stakeholders' payoffs, the optimal penalty policy* (A^*,R^*) *satisfies* $A^*=0$ *and* $R^*<P$.

The above theorem reveals that our results and main insights continue to hold under this generalization: (1) the optimal driver penalty $A^*=0$ to maximize market expansion; and (2) the optimal platform penalty R^* should be set at a moderate level to balance risky-delivery reduction with value creation through market coverage.

Appendix C: Proofs for §4

Proof of Lemma 1

We prove the lemma by considering cases of $0 \le t \le \min\left\{\frac{x}{s}, \frac{1}{\beta}(v-\alpha bx)\right\}$ and $\frac{x}{s} < t \le \frac{1}{\beta}(v-\alpha bx)$, separately. If $0 \le t \le \min\left\{\frac{x}{s}, \frac{1}{\beta}(v-\alpha bx)\right\}$, the driver's utility, Equation (3), can be simplified as $u(t) = sb + \frac{s}{x}At - A$, which increases in t. Hence, the utility maximum, in this case, is obtained at $t^* = \min\left\{\frac{x}{s}, \frac{1}{\beta}(v-\alpha bx)\right\}$. If $\frac{x}{s} < t \le \frac{1}{\beta}(v-\alpha bx)$, the driver's utility, Equation (3), can be simplified as $u(t) = \frac{bx}{t}$, which decreases in t. Then, the driver's optimal delivery time, in this case, is $t^* = \frac{x}{s}$. Combine the two cases above, we conclude that $t^*(b) = \max\left\{\min\left\{\frac{x}{s}, \frac{1}{\beta}(v-\alpha bx)\right\}, 0\right\}$. □

Proof of Lemma 2

We prove this by contradiction. Suppose Equation (7) does not hold. Then Lemma 1 implies that the optimal commission b^* satisfies $t^*(b^*) = \frac{x}{s} < \frac{1}{\beta}(v-\alpha b^* x)$. When $t^*(b) = \frac{x}{s}$, the two constraints of Problem (6) can be simplified as:

$$\frac{\left(1-\frac{s}{x}t^*(b)\right)A}{s} = 0 \le b \le \frac{v-\beta\frac{x}{s}}{\alpha x} = \frac{v-\beta t^*(b)}{\alpha x}.$$

Also, by noting that Problem (6)'s objective function is increasing in b when $t^*(b) = \frac{x}{s}$, the optimal value $b^* = \frac{v-\beta\frac{x}{s}}{\alpha x}$ so that Equation (7) holds. This contradicts the supposition. □

Proof of Lemma 3

For Problem (8), we examine its feasibility in three cases, $\beta-\alpha A > 0$, $\beta-\alpha A = 0$, and $\beta-\alpha A < 0$, separately. If $\beta-\alpha A > 0$, then Problem (8)'s constraint becomes $T \le \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$; the feasible region is non-empty if and only if $0 \le \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$, which is equivalent to $\frac{v}{x} \ge \frac{\alpha(A+w)}{s}$. If $\beta-\alpha A = 0$, then the constraint becomes $v-\alpha(A+w)\frac{x}{s} \ge 0$; the feasible region is non-empty if and only if $\frac{v}{x} \ge \frac{\alpha(A+w)}{s}$. If $\beta-\alpha A < 0$, then the constraint becomes $T \ge \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$; the feasible region is non-empty if and only if $\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A} \le \frac{x}{s}$, which is equivalent to $\frac{v}{x} \ge \frac{1}{s}(\alpha w+\beta)$. □

Proof of Proposition 1

We prove the proposition under $A < \frac{\beta}{\alpha}$ and $A = \frac{\beta}{\alpha}$, separately.

Under $A < \frac{\beta}{\alpha}$, Problem (8) can be further simplified as

$$\max_{T\ge 0} \; -\left(1-\tfrac{1}{\alpha}\right)\beta\tfrac{s}{x}T^2 + \left(\gamma + R + \left(1-\tfrac{1}{\alpha}\right)v\right)\tfrac{s}{x}T - R, \quad \text{s.t.} \tag{25}$$

$$\text{(Driver participation constraint)} \qquad T \le \min\left\{\tfrac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}, \tfrac{x}{s}\right\}.$$

First, we note that Problem (25) is feasible if and only if $\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A} \ge 0$, or equivalently, $v \ge \alpha(A+w)\frac{x}{s}$. Second, it is straightforward to show that the objective function, which is quadratic and concave in T, is maximized at $T_\star > 0$ (Definition 2). Hence, the objective function is maximized at

$$T^* = \min\left\{T_\star, \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}, \frac{x}{s}\right\}. \tag{26}$$

In addition, we note that $\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A} \le \frac{x}{s}$ is equivalent to $v \le (\beta+\alpha w)\frac{x}{s}$. Hence, the characterization of T^* above can be rewritten as

$$T^* = \begin{cases} \min\left\{T_\star, \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}\right\}, & \text{if } v \le (\beta+\alpha w)\frac{x}{s} \\ \min\left\{T_\star, \frac{x}{s}\right\}, & \text{if } v > (\beta+\alpha w)\frac{x}{s} \end{cases}. \tag{27}$$

Third, following simple algebra, we can further show that $T_\star \geq \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ is equivalent to

$$v \leq \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta}, \tag{28}$$

and $T_\star < \frac{x}{s}$ is equivalent to

$$v < 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}}. \tag{29}$$

Fourth, we proceed by specifying the characterization of T^* in Statements (A) (i.e., $(\beta-\alpha w)\frac{x}{s} \leq \frac{\gamma+R}{1-\frac{1}{\alpha}}$) and (B) (i.e., $(\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}}$), separately, based on Equation (27), as well as Inequalities (28) and (29).

(A) By Equation (27)'s first case and Inequality (28), we can show that $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ when $\alpha(A+w)\frac{x}{s} \leq v < \frac{\beta+\alpha w}{s}x$. In particular, Inequality (28) holds because

$$v < (\beta+\alpha w)\frac{x}{s} \leq \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta},$$

where the second inequality above follows since it is equivalent to $(\beta-\alpha w)\frac{x}{s} \leq \frac{\gamma+R}{1-\frac{1}{\alpha}}$, which is given in Statement (A). Then, Inequality (28) implies that $T_\star \geq \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$. In addition, $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A} < \frac{x}{s}$ holds because it is equivalent to $v < (\beta+\alpha w)\frac{x}{s}$.

Second, we show that when $v \geq (\beta+\alpha w)\frac{x}{s}$, we have $T^* = \frac{x}{s}$ by Equation (27)'s second case. It is equivalent to show that $T_\star \geq \frac{x}{s}$, which is equivalent to $v \geq 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}}$ by inverting Inequality (29). Then, we point out that the last inequality holds because

$$2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}} \leq (\beta+\alpha w)\frac{x}{s} \leq v,$$

where the first inequality above is equivalent to $(\beta-\alpha w)\frac{x}{s} \leq \frac{\gamma+R}{1-\frac{1}{\alpha}}$, which is given. Therefore, we have characterized T^* in Statement (A) and proved $T^* \leq \frac{x}{s}$.

(B) By Equation (27)'s first case and Inequality (28), we can claim that $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ when

$$\alpha(A+w)\frac{x}{s} \leq v < \min\left\{(\beta+\alpha w)\frac{x}{s}, \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta}\right\}.$$

Notice that the above condition can be simplified as

$$\alpha(A+w)\frac{x}{s} \leq v < \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta} \tag{30}$$

because it is straightforward to verify that

$$\frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta} < (\beta+\alpha w)\frac{x}{s},$$

which is equivalent to $(\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}}$, and it is given in Statement (B). Also, $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A} < \frac{x}{s}$ holds because it is equivalent to $v < (\beta+\alpha w)\frac{x}{s}$.

Second, it follows that $T^* = T_\star$ when

$$\frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta} \leq v < (\beta+\alpha w)\frac{x}{s} \tag{31}$$

by Equation (27)'s first case and by inverting Inequality (28). In addition, $T^* = T_\star$ continues to hold when

$$(\beta + \alpha w)\frac{x}{s} \le v < 2\frac{\beta}{s}x - \frac{\gamma + R}{1 - \frac{1}{\alpha}} \tag{32}$$

because of Equation (27)'s second case and Inequality (29). It is worth mentioning that the last interval for v is not empty because $(\beta+\alpha w)\frac{x}{s} < 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}}$, which is equivalent to $(\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}}$. Combining Inequalities (31) and (32), we then obtain that $T^* = T_\star$ when

$$\frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta - \alpha A)}{\alpha A + \beta} \le v < 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}}.$$

In addition, Inequality (29) implies $T^* = T_\star < \frac{x}{s}$.

Third, using Equation (27)'s second case and the inversion of Inequality (29), we obtain that $T^* = \frac{x}{s}$ when $v \ge 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}}$. Note that $2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}} > (\beta+\alpha w)\frac{x}{s}$ holds because it is equivalent to $(\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}}$ which is given. Therefore, we have completed T^*'s characterization in Statement (B) and proved $T^* \le \frac{x}{s}$.

Lastly, it is straightforward to verify that $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A}$ is equivalent to $u(T^*; b^*, x, A) = w$, where b^* satisfies Equation (7). That is, the driver's optimal hourly earning is w.

Under $A = \frac{\beta}{\alpha}$, Problem (8) can be further simplified as

$$\max_{0 \le T \le \frac{x}{s}} \; -\left(1 - \tfrac{1}{\alpha}\right)\beta\tfrac{s}{x}T^2 + \left(\gamma + R + \left(1 - \tfrac{1}{\alpha}\right)v\right)\tfrac{s}{x}T - R, \quad \text{s.t.}$$

$$\text{(Driver participation constraint)} \qquad v \ge (\beta + \alpha w)\tfrac{x}{s}.$$

That is, the above problem is feasible if and only if $v \ge (\beta+\alpha w)\frac{x}{s}$. By Equation (27), we can show $T^* = \min\{T_\star, \frac{x}{s}\}$. Then we note that $T^* = T_\star$ if and only if Inequality (29) holds. By reasoning similarly to that in case $A < \frac{\beta}{\alpha}$, we obtain the following characterizations for T^* under $A = \frac{\beta}{\alpha}$

$$T^* = \begin{cases} \frac{x}{s}, & \text{if } v \ge (\beta+\alpha w)\frac{x}{s} \text{ and } (\beta-\alpha w)\frac{x}{s} \le \frac{\gamma+R}{1-\frac{1}{\alpha}} \\ T_\star, & \text{if } (\beta+\alpha w)\frac{x}{s} \le v < 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}} \text{ and } (\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}} \\ \frac{x}{s}, & \text{if } v \ge 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}} \text{ and } (\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}} \end{cases}. \tag{33}$$

We claim that the T^*'s characterization above is equivalent to what is provided in the proposition. To see this, firstly, it is easy to verify that:

$$\frac{\alpha(A+w)}{s}x = \frac{\beta+\alpha w}{s}x = \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A + \beta}.$$

Statement (A)'s characterization then reduces to $T^* = \frac{x}{s}$ if $v \ge \frac{(\beta+\alpha w)}{s}x = \frac{\alpha(A+w)}{s}x$; Statement (B)'s characterization then reduces to:

$$T^* = \begin{cases} T_\star, & \text{if } \frac{\alpha(A+w)}{s}x = \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta} \le v < 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}} \\ \frac{x}{s}, & \text{if } v \ge 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}} \end{cases},$$

which are equivalent to Expression (33). Lastly, $T^* < \frac{x}{s}$ when $T^* = T_\star$ is guaranteed by Inequality (29). Combining the derivations for cases $A < \frac{\beta}{\alpha}$ and $A = \frac{\beta}{\alpha}$ above, we complete the proof of the proposition. ☐

Proof of Proposition 2

The proof follows a similar argument as that in Proposition 1. We hence omit it for brevity. ☐

Appendix D: Proof for §5

Proof of Corollary 1

We characterize the effects of A and R on T^*, $G(T^*)$, b^*, π^*, u^*, and u^*T^* when $\beta \geq \alpha A$ and $\frac{v}{x} \geq \frac{\alpha(A+w)}{s}$.

First, we characterize A and R's effects on the optimal delivery time T^*. By Proposition 1, we note that A has an effect on T^* only when $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$. Then, $\frac{\partial T^*}{\partial A} = \frac{1}{(\beta-\alpha A)^2}\left(-\alpha\frac{x}{s}(\beta-\alpha A)+\alpha\left(v-\alpha(A+w)\frac{x}{s}\right)\right)$, which has the same sign as $v-(\beta+\alpha w)\frac{x}{s}$. On the other hand, we know that $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ only occurs when one of the following conditions holds:

$$\text{Condition (i): } (\beta-\alpha w)\frac{x}{s} \leq \frac{\gamma+R}{1-\frac{1}{\alpha}} \text{ and } \frac{\alpha(A+w)}{s}x \leq v < \frac{\beta+\alpha w}{s}x,$$

$$\text{Condition (ii): } (\beta-\alpha w)\frac{x}{s} > \frac{\gamma+R}{1-\frac{1}{\alpha}} \text{ and } \frac{\alpha(A+w)}{s}x \leq v < \frac{2\alpha(A+w)\frac{\beta}{s}x+\frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\beta+\alpha A}.$$

It immediately follows that under Condition (i), $\frac{\partial T^*}{\partial A} < 0$. Under condition (ii), we can show that

$$\frac{2\alpha(A+w)\frac{\beta}{s}x+\frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\beta+\alpha A} \leq \frac{\beta+\alpha w}{s}x$$

is equivalent to $\frac{\gamma+R}{1-\frac{1}{\alpha}} \leq (\beta-\alpha w)\frac{x}{s}$. Note that the last inequality is given by Condition (ii). Therefore, in this case, we also have $\frac{\partial T^*}{\partial A} < 0$. Regarding R's effect on T^*, Proposition 1 suggests that the effect may only exist when $T^* = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$. In this case, it follows that $\frac{\partial T^*}{\partial R} = \frac{1}{2\left(1-\frac{1}{\alpha}\right)\beta} > 0$.

Second, for A and R's effects on incident probability $G(T^*)$, we immediately observe that $\frac{\partial G(T^*)}{\partial A} \geq 0$ and $\frac{\partial G(T^*)}{\partial R} \leq 0$ because $G(T^*) = 1-\frac{s}{x}T^*$.

Third, for A and R's effects on commission rate b^*, we note that $T^* = \frac{1}{\beta}(v-\alpha b^* x)$, which is equivalent to $b^* = \frac{v-\beta T^*}{\alpha x}$. Therefore, $\frac{\partial b^*}{\partial A} = -\frac{\beta}{\alpha x}\frac{\partial T^*}{\partial A} > 0$ and $\frac{\partial b^*}{\partial R} = -\frac{\beta}{\alpha x}\frac{\partial T^*}{\partial R} < 0$.

Fourth, we characterize A and R's effects on platform profit π^*. We discuss cases when $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ and $T^* = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, separately. When $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$, by Expression (26), we have $T^* < T_\star = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, which is the maxima of π. Therefore, at T^*, π increases with T. Then, since π is independent of A given T and $\frac{\partial T^*}{\partial A} \leq 0$, we have $\frac{\partial \pi}{\partial A} \leq 0$. Besides, T^* is independent of R in this case. Then, $\frac{\partial \pi^*}{\partial R} = -\left(1-\frac{s}{x}T\right) \leq 0$. When $T^* = T_\star = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, which is independent of A. We have $\frac{\partial \pi^*}{\partial A} = 0$. In addition, at $T = T_\star$, the platform profit can be simplified as

$$\pi^* = \frac{s}{x}\frac{\left(\gamma+R+\left(1-\frac{1}{\alpha}\right)v\right)^2}{4\left(1-\frac{1}{\alpha}\right)\beta} - R.$$

Therefore, $\frac{\partial \pi^*}{\partial R} = \frac{s}{x}\frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta} - 1 = \frac{s}{x}T_\star - 1 \leq 0$.

Fifth, we characterize A and R's effects on the driver earning rate u^*. Note that $u^* = w$, which is a constant, when $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$. Hence, we focus on $T^* = T_\star = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, which is independent of A. In this case, $u^* = sb^* - \left(1-\frac{s}{x}T^*\right)A$. Note that $b^* = \frac{v-\beta T^*}{\alpha x}$. Then, the driver earning rate can be rewritten as $u^* = \frac{s}{x}\frac{v}{\alpha} - A - \frac{s}{\alpha x}(\beta-\alpha A)T^*$. Therefore, $\frac{\partial u^*}{\partial A} = -1+\frac{s}{x}T^* \leq 0$ and $\frac{\partial u^*}{\partial R} = -\frac{s}{\alpha x}(\beta-\alpha A)\frac{\partial T^*}{\partial R} \leq 0$.

Sixth, we characterize A and R's effects on the driver earning u^*T^*. In particular, we consider $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ and $T^* = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, separately. When $T^* = \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$, we have $u^* = w$, which yields $u^*T^* = wT^*$. Since we have shown that $\frac{\partial T^*}{\partial A} \leq 0$, it follows that $\frac{\partial}{\partial A}(u^*T^*) \leq 0$ in this case. In addition, since T^* is independent of R, we have $\frac{\partial(u^*T^*)}{\partial R} = 0$. When $T^* = T_\star = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, the driver earning rate can be expressed as $u = \frac{s}{\alpha x}\left(v - \frac{\alpha x}{s}A - (\beta - \alpha A)T\right)$. Subsequently, we have $u^*T^* = \frac{s}{\alpha x}\left(v - \frac{\alpha x}{s}A - (\beta - \alpha A)T^*\right)T^*$, and $\frac{\partial(u^*T^*)}{\partial A} = \left(-1 + \frac{s}{x}T^*\right)T^* \leq 0$. In addition, we note that $\frac{\partial}{\partial R}u^*T^* = \frac{s}{\alpha x}\left[\left(v - \frac{\alpha x}{s}A\right) - 2(\beta - \alpha A)T^*\right]\frac{\partial T^*}{\partial R}$, where $\frac{\partial T^*}{\partial R} \geq 0$. Then, $\frac{\partial}{\partial R}u^*T^* > 0$ holds if and only if $T^* > \frac{v - \frac{\alpha x}{s}A}{2(\beta - \alpha A)}$, which is equivalent to $\frac{\gamma + R}{1 - \frac{1}{\alpha}} \leq \frac{\alpha A}{\beta - \alpha A}\left(v - \beta\frac{x}{s}\right)$. □

Proof of Corollary 2

The proof follows a similar argument as that in Corollary 1. We hence omit it for brevity. □

Proof of Lemma 4

By Proposition 2 and suggested by Figure 3, the entire unsafe zone can be characterized by:

$$\mathcal{U}^{(2)} = \left\{(x,v)\middle|\frac{\beta + \alpha w}{s}x \leq v < 2\frac{\beta}{s}x - \frac{\gamma + R}{1 - \frac{1}{\alpha}}, x \in (0, \bar{x}],\ \text{and}\ v \in (0, \bar{v}]\right\},$$

which is independent of A. Then, if $\mathcal{U}^{(2)} \neq \emptyset$ and there exists $(x,v) \in \mathcal{U}^{(2)}$, we know that the T^* for (x,v) satisfies $T^* < \frac{x}{s}$ for any $A \in [\beta/\alpha, +\infty)$ from Proposition 2. Therefore, the incident probability remains positive regardless of the value of A. □

Proof of Proposition 3

Regarding the claim that penalizing drivers by setting A sufficiently high may not eliminate incidents during deliveries, this has been established in Lemma 4. In this proof, we instead focus on how a sufficiently high R can eliminate delivery-related incidents for any customer distribution $\mathcal{C}$.

It suffices to show that for any customer $(x,v) \in (0,\bar{x}] \times (0,\bar{v}]$, the platform either serves the order at the safe speed or declines it. First, note that when R is sufficiently large, only Statement (A) in Propositions 1 and 2 applies. In the low-A regime (Proposition 1), incidents can occur only when serving customers in zone $\mathcal{U}_0^{(1)}$, because the optimal delivery time satisfies $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A} < \frac{x}{s}$. Moreover, T^* is independent of R. From the objective function of Program (8), it follows that the platform's profit decreases in R and approaches $-\infty$ as $R \to +\infty$. Therefore, when R is sufficiently large, the platform will decline orders placed by customers in $\mathcal{U}_0^{(1)}$. In the high-A regime, the platform serves customers only at the safe speed (Statement (A) of Proposition 2). Therefore, no incidents occur during deliveries. □

Proof of Proposition 4

When A is relatively low, the results immediately follow as the boundary line of the feasible zone $\mathcal{D}(A)$, which is $v = \alpha(A+w)\frac{x}{s}$, decreases with A. □

Proof of Lemma 5

We show that for any policy (A,R) with $A > 0$, which is referred to as the original policy, it is dominated by $(0,R)$, which is referred to as the alternative policy. First, note that the social surplus for serving customer (x,v) can be written as $S(x,v;A,R) = \pi^* + u^*T^* + (1 - G(T^*))P$. By Corollary 1, we have π^* and u^*T^* (weakly) decrease in A and $G(T^*)$ (weakly) increase in A. Hence, for any customer $(x,v) \in \left\{(x,v)\middle|\frac{v}{x} > \frac{\alpha(A+w)}{s}\right\} \cap ([0,\overline{x}] \times [0,\overline{v}])$, it follows that $S(x,v;A,R) \leq S(x,v;0,R)$. Second, by Proposition 4, we know that the platform serves more customers under $(0,R)$ than it does under (A,R). Therefore, $(0,R)$ dominates (A,R) by (1) increasing the social surplus for any customer (x,v) who is served under (A,R) and (2) serving new customers (x,v) who are not served under (A,R). As a result, it is optimal to set $A_1^* = 0$. □

Proof of Lemma 6

We first point out that the lemma apparently holds when $\beta-\alpha w\le 0$. In this case, Proposition 2 states that $T^*\equiv\frac{x}{s}$; both A and R impose no effect because of the zero incident rate. Besides, the proposition implies that the platform's feasible zone remains independent of (A,R). Therefore, the policy with $A^\dagger=\frac{\beta}{\alpha}$ is optimal.

In what follows, we focus on the case of $\beta-\alpha w>0$. First of all, we can characterize $S(x,v,A,R)$ as follows:

$$\begin{aligned}S(x,v,A,R)&=\pi^*+u^*T^*+(1-G(T^*))P\\&=\frac{s}{x}T^*(\gamma+(\alpha-1)b^*x)-\left(1-\frac{s}{x}T^*\right)R+\left(sb^*-A+\frac{s}{x}AT^*\right)T^*+\frac{s}{x}T^*P.\end{aligned}$$

Note that $b^*=\frac{v-\beta T^*}{\alpha x}$ by Lemma 2. Then, $S(x,v,A,R)$ can be simplified as:

$$S(x,v,A,R)=(A-\beta)\frac{s}{x}(T^*)^2+\left((\gamma+v+R+P)\frac{s}{x}-A\right)T^*-R.$$

Under a given policy (A,R) with $A>\frac{\beta}{\alpha}$ (referred to as the original policy), we can partition the feasible customer space, $\bar{\mathcal{F}}=\left\{\frac{v}{x}>\frac{\beta+\alpha w}{s}\right\}\cap([0,\bar{x}]\times[0,\bar{v}])$, into $\mathcal{U}_1^{(2)}$, $\mathcal{U}_0^{(2)}$, and $\bar{\mathcal{F}}\setminus(\mathcal{U}_1^{(2)}\cup\mathcal{U}_0^{(2)})$ such that

$$T^*=\begin{cases}\frac{x}{s}, & \text{if } (x,v)\in\bar{\mathcal{F}}\setminus(\mathcal{U}_1^{(2)}\cup\mathcal{U}_0^{(2)})\\ \frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}, & \text{if } (x,v)\in\mathcal{U}_0^{(2)}\\ \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}, & \text{if } (x,v)\in\mathcal{U}_1^{(2)}\end{cases},$$

as illustrated by the right panel of Figure 3. Then, we propose the following policy:

$$A^\dagger=\frac{\beta}{\alpha}\text{ and }R^\dagger=\max\left\{(\beta-\alpha w)\left(1-\frac{1}{\alpha}\right)\frac{\bar{x}}{s}-\gamma,R\right\},\tag{34}$$

and we refer to it as the alternate policy. Note that, under $(A^\dagger,R^\dagger)$, $T^*=\frac{x}{s}$ for any $(x,v)\in\bar{\mathcal{F}}$. This is because $(\beta-\alpha w)\frac{x}{s}\le\frac{\gamma+R^\dagger}{1-\frac{1}{\alpha}}$ holds true for any $x\le\bar{x}$, under which Proposition 2 renders $T^*=\frac{x}{s}$. In what follows, we show that $(A^\dagger,R^\dagger)$ achieves (weakly) higher social surplus than (A,R) for customers in $\bar{\mathcal{F}}\setminus(\mathcal{U}_1^{(2)}\cup\mathcal{U}_0^{(2)})$, $\mathcal{U}_0^{(2)}$, and $\mathcal{U}_1^{(2)}$, separately.

First, we show that when serving customer $(x,v)\in\bar{\mathcal{F}}\setminus(\mathcal{U}_1^{(2)}\cup\mathcal{U}_0^{(2)})$, the original policy and the alternative policy achieve the same social surplus. We begin by noting that if customer (x,v) is feasible to serve by the platform under the original policy, she remains feasible to be served by the platform under the alternative policy. This is because $T^*=\frac{x}{s}$ for this customer under both policies; constraint (9) becomes independent of A and identical under both policies. Likewise, their social surpluses for serving the customer are the same, which is $S(x,v;A,R)=\gamma+v+P-\beta\frac{x}{s}$.

Second, we show that when serving customer $(x,v)\in\mathcal{U}_0^{(2)}$, the alternative policy weakly dominates the original policy. We begin by showing that if customer (x,v) is feasible to serve under the original policy, she is also feasible to serve under the alternative policy. On one hand, the premise suggests that constraint (9) holds, which is equivalent to $T\beta+\alpha A\left(\frac{x}{s}-T\right)\le v-\alpha w\frac{x}{s}$ with $T<\frac{x}{s}$. On the other hand, for (x,v) to be feasible to serve under the alternative policy, we need $\frac{x}{s}\beta\le v-\alpha w\frac{x}{s}$ (note that $T^*=\frac{x}{s}$ under the alternative policy). The last inequality holds true because we can show that $\frac{x}{s}\beta\le T\beta+\alpha A\left(\frac{x}{s}-T\right)$, which is equivalent to $\beta\left(\frac{x}{s}-T\right)\le\alpha A\left(\frac{x}{s}-T\right)$, holds. Next, note that under the alternative policy, $T^*=\frac{x}{s}$ and $\pi^*=\gamma+\left(v-\beta\frac{x}{s}\right)\left(1-\frac{1}{\alpha}\right)\ge 0$. So, the platform will participate. If $\pi^*(x,v,A,R)\le 0$ for a given $(x,v)\in\mathcal{U}_0^{(2)}$

under the original policy, the platform will not serve the customer, rendering zero social surplus; the original policy is dominated by the alternative policy. Then, we show that the claim still holds when $\pi^*(x,v;A,R)\geq 0$ under the original policy. In other words, we show that

$$S(x,v;A,R)\leq S(x,v;A^\dagger,R^\dagger)=\gamma+v-\beta\frac{x}{s}+P.$$

In this case, we have $S(x,v;A,R)=\pi^*+wT^*+(1-G(T^*))P$ because the driver's earning is w. Next, we point out that $\frac{\partial S}{\partial A}\geq 0$. Specifically,

$$S=-\frac{s}{x}\left(1-\frac{1}{\alpha}\right)\beta(T^*)^2+\frac{s}{x}\left(\gamma+R+\frac{x}{s}w+P+\left(1-\frac{1}{\alpha}\right)v\right)T^*-R,$$

and

$$\frac{\partial S}{\partial A}=\frac{\partial S}{\partial T^*}\cdot\frac{\partial T^*}{\partial A}=\left[-2\frac{s}{x}\left(1-\frac{1}{\alpha}\right)\beta T^*+\frac{s}{x}\left(\gamma+R+\frac{x}{s}w+P+\left(1-\frac{1}{\alpha}\right)v\right)\right]\frac{\partial T^*}{\partial A}$$

where it has been shown that $\frac{\partial T^*}{\partial A}\geq 0$ in Corollary 2. Then, we focus on showing

$$-2\frac{s}{x}\left(1-\frac{1}{\alpha}\right)\beta T^*+\frac{s}{x}\left(\gamma+R+\frac{x}{s}w+P+\left(1-\frac{1}{\alpha}\right)v\right)\geq 0.$$

Following basic algebra, we show that the last inequality is equivalent to:

$$v\geq\frac{2\alpha\beta(A+w)\frac{x}{s}-\frac{(\alpha A-\beta)(\gamma+R+\frac{x}{s}w+P)}{1-\frac{1}{\alpha}}}{\alpha A+\beta}.$$

Since $v\geq(\beta+\alpha w)\frac{x}{s}$, the inequality above holds if

$$(\beta+\alpha w)\frac{x}{s}\geq\frac{2\alpha\beta(A+w)\frac{x}{s}-\frac{\gamma+R+\frac{x}{s}w+P}{1-\frac{1}{\alpha}}(\alpha A-\beta)}{\alpha A+\beta},$$

which is equivalent to (note $\alpha A>\beta$):

$$\left(\left(1-\frac{1}{\alpha}\right)\beta-\alpha w\right)\frac{x}{s}\leq\gamma+R+P.$$

The inequality above immediately holds given the condition and $R\geq 0$.

Therefore, $\frac{\partial S}{\partial T^*}\geq 0$, which yields $\frac{\partial S}{\partial A}\geq 0$. Then, we have

$$\begin{aligned}S(x,v;A,R)&\leq\lim_{A\to+\infty}S(x,v;A,R)\\&=\lim_{A\to+\infty}-\frac{s}{x}\left(1-\frac{1}{\alpha}\right)\beta(T^*)^2+\frac{s}{x}\left(\gamma+R+\frac{x}{s}w+P+\left(1-\frac{1}{\alpha}\right)v\right)T^*-R\\&=\gamma+\frac{x}{s}w+P+\left(1-\frac{1}{\alpha}\right)\left(v-\beta\frac{x}{s}\right)\\&\leq\gamma+P+v-\beta\frac{x}{s}\\&=S(x,v;A^\dagger,R^\dagger).\end{aligned}$$

The last inequality holds because $v\geq(\beta+\alpha w)\frac{x}{s}$. Thus, in $\mathcal{U}_0^{(2)}$, the alternative policy (weakly) dominates the original policy.

Third, we show that when serving customer $(x,v)\in\mathcal{U}_1^{(2)}$, the alternative policy weakly dominates the original policy. Following a similar argument as in case $\mathcal{U}_0^{(2)}$, we can show that the customer is feasible to serve under the alternative policy if she is feasible to serve under the original policy. In addition, if $\pi^*(x,v;A,R)\leq 0$

for a given customer $(x,v)\in\mathcal{U}_1^{(2)}$, then $S(x,v;A,R)=0$, and the claim naturally holds. In what follows, we consider the more general case where customer $(x,v)\in\mathcal{U}_1^{(2)}$, such that $\pi^*(x,v;A,R)>0$. Note that in this case $T^*=\frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$, which is independent of A. Also, the social surplus can be characterized by $S=(A-\beta)\frac{s}{x}(T^*)^2+\left((\gamma+v+R+P)\frac{s}{x}-A\right)T^*-R$ (note that the driver utility is greater than or equal to w). Hence, we have $\frac{\partial S}{\partial A}=-T^*\left(1-\frac{s}{x}T^*\right)\le 0$. Since $A>\frac{\beta}{\alpha}$,

$$S(x,v;A,R)\le S\left(x,v;\frac{\beta}{\alpha},R\right)=-\beta\left(1-\frac{1}{\alpha}\right)\frac{s}{x}(T^*)^2+\left((\gamma+v+R+P)\frac{s}{x}-\frac{\beta}{\alpha}\right)T^*-R.$$

Next, we show that

$$-\beta\left(1-\frac{1}{\alpha}\right)\frac{s}{x}(T^*)^2+\left((\gamma+v+R+P)\frac{s}{x}-\frac{\beta}{\alpha}\right)T^*-R\le\gamma+v+P-\beta\frac{x}{s}=S\left(x,v;A^\dagger,R^\dagger\right). \tag{35}$$

We claim that the left-hand side of the inequality above increases with R under the premise of the lemma. In particular, it is straightforward to verify that its derivative with respect to R follows:

$$\frac{(\gamma+v+R+P)\frac{s}{x}-\frac{\beta}{\alpha}}{2\left(1-\frac{1}{\alpha}\right)\beta}-1,$$

which has the same sign as $v+\gamma+P+R-\beta\left(2-\frac{1}{\alpha}\right)\frac{x}{s}$. Next, we claim that the last quantity is greater than or equal to zero. Note that $v\ge(\beta+\alpha w)\frac{x}{s}$ and $R\ge 0$. To prove the claim, it suffices to show that $(\beta+\alpha w)\frac{x}{s}+\gamma+P-\beta\left(2-\frac{1}{\alpha}\right)\frac{x}{s}=\gamma+P-\left(\beta\left(1-\frac{1}{\alpha}\right)-\alpha w\right)\frac{x}{s}\ge 0$, which holds true under the lemma's premise. As a result, the left-hand side of Inequality (35) increases with R. Then, as R increases such that $T^*\uparrow\frac{x}{s}$, the left-hand side of Inequality (35) converges to $\gamma+v+P-\beta\frac{x}{s}$. Therefore, the inequality holds for any R such that the customer $(x,v)\in\mathcal{U}_1^{(2)}$.

In sum, we have proved that policy $(A^\dagger,R^\dagger)$ dominates the original policy. That is, the optimal policy for Subproblem (17) satisfies $A^\dagger=\frac{\beta}{\alpha}$. □

Proof of Theorem 1

By Lemmas 5 and 6, it immediately follows that the optimal solution for Problem (15) satisfies $A^*=0$.

In what follows, we focus on proving $R^*\le\bar{R}:=\max\left\{0,\frac{\alpha-1}{2-\alpha}P-\frac{1}{2-\alpha}\gamma\right\}$ under the optimal policy. In particular, we show that for any policy $(0,R)$ with $R>\bar{R}$, we can find an alternative policy with $\tilde{R}=R-\epsilon$ (where $\epsilon\downarrow 0$) that can improve social surplus.

First, we note that by Proposition 1 and $A^*=0$, T^* can be characterized by:

(A) If $(\beta-\alpha w)\frac{x}{s}\le\frac{\gamma+R}{1-\frac{1}{\alpha}}$, then

$$T^*=\begin{cases}\frac{v-\alpha w\frac{x}{s}}{\beta}, & \text{if } \frac{\alpha w}{s}x\le v<\frac{\beta+\alpha w}{s}x\\ \frac{x}{s}, & \text{if } v>\frac{\beta+\alpha w}{s}x\end{cases}.$$

(B) If $(\beta-\alpha w)\frac{x}{s}>\frac{\gamma+R}{1-\frac{1}{\alpha}}$, then

$$T^*=\begin{cases}\frac{v-\alpha w\frac{x}{s}}{\beta}, & \text{if } \frac{\alpha w}{s}x\le v<2\alpha w\frac{x}{s}+\frac{\gamma+R}{1-\frac{1}{\alpha}}\\ \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}, & \text{if } 2\alpha w\frac{x}{s}+\frac{\gamma+R}{1-\frac{1}{\alpha}}\le v<2\frac{\beta}{s}x-\frac{\gamma+R}{1-\frac{1}{\alpha}}\\ \frac{x}{s}, & \text{if } v\ge 2\frac{\beta}{s}x-\frac{\gamma+R}{1-\frac{1}{\alpha}}\end{cases}.$$

Besides, the social surplus, Equation (14), is defined as follows:

$$S(x,v;0,R) = -\beta\frac{s}{x}T^{*2} + (\gamma+R+v+P)\frac{s}{x}T^* - R.$$

Second, we claim that for any customer (x,v) within $\underline{\mathcal{F}}(0) = (0,\bar{x}]\times(0,\bar{v}]$ (where $\underline{\mathcal{F}}(0)$ is given by Definition 1), the social surplus generated from the customer's order (weakly) decreases in R when $R > \bar{R}$. Specifically, if (x,v) is within the safe zone (i.e., $(x,v)\in\underline{\mathcal{F}}\setminus\left(\mathcal{U}_0^{(1)}\cup\mathcal{U}_1^{(1)}\right)$, then the social surplus is independent of R (since the incident rate is zero), and the claim holds automatically. If $(x,v)\in\mathcal{U}_0^{(1)}$, then $T^* < \frac{x}{s}$ is independent of R. Then, $\frac{d}{dR}S(x,v;0,R) = \frac{\partial}{\partial R}S(x,v;0,R) = -\left(1-\frac{s}{x}T^*\right) < 0$. Hence, the claim holds in this case. Lastly, when $(x,v)\in\mathcal{U}_1^{(1)}$, we have $T^* = \frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$ and $\frac{dT^*}{dR} = \frac{1}{2\beta\left(1-\frac{1}{\alpha}\right)}$. Then, following basic algebra, we obtain:

$$\begin{aligned}\frac{d}{dR}S(x,v;0,R) =& \frac{\partial}{\partial T^*}S(x,v;0,R)\cdot\frac{dT^*}{dR} + \frac{\partial}{\partial R}S(x,v;0,R)\\ =& -\frac{s}{x}\cdot\frac{\alpha(2-\alpha)}{2\beta(\alpha-1)^2}R - \frac{s}{x}\cdot\frac{1}{2\beta(\alpha-1)}\left(\frac{\alpha(2-\alpha)}{\alpha-1}\gamma-(\alpha-1)v-\alpha P\right)-1,\end{aligned}$$

which is a decreasing function of R (owing to $\alpha\in(1,2)$). We also note that $\frac{d}{dR}S(x,v;0,R)=0$ when $R = R^\diamond := \frac{\alpha-1}{2-\alpha}P - \gamma - \frac{(\alpha-1)^2}{\alpha(2-\alpha)}\left(2\beta\frac{x}{s}-v\right)$. That is, $\frac{d}{dR}S(x,v;0,R)<0$ if and only if $R > R^\diamond$. Moreover, we note that $R^\diamond \le \frac{\alpha-1}{2-\alpha}P - \frac{1}{2-\alpha}\gamma$. This is because $v < 2\frac{\beta}{s}x - \frac{\gamma+R}{1-\frac{1}{\alpha}}$ when $(x,v)\in\mathcal{U}_1^{(1)}$; the last inequality implies that $2\beta\frac{x}{s} - v > \frac{\gamma+R}{1-\frac{1}{\alpha}} \ge \frac{\gamma}{1-\frac{1}{\alpha}}$. Therefore, $R^\diamond < \frac{\alpha-1}{2-\alpha}P - \gamma - \frac{(\alpha-1)^2}{\alpha(2-\alpha)}\cdot\frac{\gamma}{1-\frac{1}{\alpha}} = \frac{\alpha-1}{2-\alpha}P - \frac{\gamma}{2-\alpha}$. In other words, $S(x,v;0,R)$ decreases with R, and the claim holds in this case.

Third, the claim implies that for a given original policy $(0,R)$, the alternative policy $(0,\tilde{R})$ weakly dominates for any $(x,v)\in\underline{\mathcal{F}}(0)$. Besides, since $\tilde{R}<R$, it is straightforward to verify that under $(0,\tilde{R})$, the platform will continue to serve customers who are served under $(0,R)$. In sum, the total social surplus across the entire market will be (weakly) higher under the alternative policy; $R > \max\left\{0, \frac{\alpha-1}{2-\alpha}P - \frac{1}{2-\alpha}\gamma\right\}$ cannot be optimal. □

Appendix E: Proofs for Appendix B

Proof of Lemma 7

It follows the same proof of Lemma 1, and we omit it for brevity. $\square$

Proof of Lemma 8

First, the platform's profit maximization problem follows:

$$\begin{aligned}
\max_{k,b} \quad & \frac{s}{x}t^*(\gamma+(k-b)x) - \left(1-\frac{s}{x}t^*\right)R, \quad \text{s.t.} \\
\text{(Customer's purchase constraint)} \quad & v-kx-\beta t^* \geq 0, \\
\text{(Driver's participation constraint)} \quad & \frac{s}{x}bx - \left(1-\frac{s}{x}t^*\right)A \geq w, \\
\text{(Driver's optimal delivery time)} \quad & t^* = \max\left\{\min\left\{\frac{x}{s}, \frac{v-kx}{\beta}\right\}, 0\right\}.
\end{aligned}$$

We note that t^* is independent of the commission rate b, and the objective function is decreasing in b. Hence, the optimal b^* is found when the driver's participation constraint is binding. That is,

$$b^* = \frac{w}{s} + \frac{1}{s}\left(1-\frac{s}{x}t^*\right)A = \frac{w}{s} + \left(\frac{1}{s}-\frac{t^*}{x}\right)A.$$

With b^*, the platform's problem can be simplified as:

$$\begin{aligned}
\max_{k} \quad & \frac{s}{x}t^*\left(\gamma+kx-\frac{w}{s}x-\left(\frac{x}{s}-t^*\right)A\right) - \left(1-\frac{s}{x}t^*\right)R, \quad \text{s.t.} \\
\text{(Customer's purchase constraint)} \quad & v-kx-\beta t^* \geq 0, \\
\text{(Driver's optimal delivery time)} \quad & t^* = \max\left\{\min\left\{\frac{x}{s}, \frac{v-kx}{\beta}\right\}, 0\right\}.
\end{aligned}$$

We then claim that $\frac{x}{s} < \frac{v-kx}{\beta}$ does not hold at optimum. To see this, we note that the condition is equivalent to $k < \frac{v}{x} - \frac{\beta}{s}$, and under this condition, it follows $t^* = \frac{x}{s}$, which is independent of k. In addition, the objective function increases with k, holding t^* fixed. Therefore, any $k < \frac{v}{x} - \frac{\beta}{s}$ is suboptimal; the optimal delivery fee k^* satisfies $k^* \geq \frac{v}{x} - \frac{\beta}{s}$. Under k^*, we can show that $t^* = \frac{v-k^*x}{\beta}$, which yields zero surplus for the customer. $\square$

Proof of Proposition 5

By Lemmas 7 and 8, the platform's profit maximization problem can be simplified as:

$$\begin{aligned}
\max_{k} \quad & \frac{s}{x}t^*\left(\gamma+kx-w\frac{x}{s}-\left(\frac{x}{s}-t^*\right)A\right) - \left(1-\frac{s}{x}t^*\right)R, \quad \text{s.t.} \\
\text{(Driver's optimal delivery time)} \quad & t^* = \frac{v-kx}{\beta} \text{ and } 0 \leq t^* \leq \frac{x}{s}.
\end{aligned}$$

We let $T := \frac{v-kx}{\beta}$. Then, substituting k in the program above with T, we obtain:

$$\max_{0\leq T\leq \frac{x}{s}} \quad -\frac{s}{x}(\beta-A)T^2 + \frac{s}{x}T\left(\gamma+v-\frac{x}{s}(w+A)+R\right) - R. \tag{36}$$

Next, we solve for Problem (36) by considering the following three cases separately: (1) $A<\beta$; (2) $A=\beta$; (3) $A>\beta$.

In case (1), the objective of Problem (36), platform profit π, is concave in T. By $\frac{d\pi}{dT} = -2\frac{s}{x}(\beta-A)T + \frac{s}{x}\left(\gamma+v-\frac{x}{s}(w+A)+R\right) = 0$, we solve for $T_\star = \frac{\gamma+v-\frac{x}{s}(w+A)+R}{2(\beta-A)}$. Therefore, the optimal solution to Problem (36) follows $T^* = \max\left\{\min\left\{T_\star, \frac{x}{s}\right\}, 0\right\}$. Then, we have $T^*=0$ if $T_\star<0$, which is equivalent to $v<(w+A)\frac{x}{s} - \gamma - R$; we have $T^* = T_\star$ if $0\leq T_\star \leq \frac{x}{s}$, which is equivalent to $(w+A)\frac{x}{s}-\gamma-R \leq v \leq (2\beta-A+w)\frac{x}{s}-\gamma-R$; we have $T^* = \frac{x}{s}$ if $T_\star > \frac{x}{s}$, which is equivalent to $v > (2\beta-A+w)\frac{x}{s}-\gamma-R$.

Cases (2) and (3)'s derivations follow similar arguments, and we omit the details for brevity. $\square$

Proof of Proposition 6

First, we compare A and R's effectiveness in risky-delivery reduction. By Proposition 5, we note that it suffices to examine A and R's effects on $T^* = \frac{\gamma+v+R-(w+A)\frac{x}{s}}{2(\beta-A)} \left(< \frac{x}{s}\right)$. It is straightforward to show that increasing the platform penalty R always prolongs the delivery time (i.e., $\frac{\partial t^*}{\partial R} > 0$). However, increasing the driver penalty A prolongs the delivery time (i.e., $\frac{\partial t^*}{\partial A} > 0$) if and only if $v \geq (\beta+w)\frac{x}{s} - \gamma - R$.

Second, we examine A and R's role in expanding the platform's feasible zone $\mathcal{D}(A,R)$. Since the lower boundary of the feasible zone (weakly) increases with A but decreases with R. Therefore, reducing A expands $\mathcal{D}(A,R)$ whereas lowering R shrinks $\mathcal{D}(A,R)$. □

Proof of Theorem 2

We prove the theorem by showing that any policy (A,R) with $A>0$ is not optimal. That is, we can always find a policy $(\tilde{A},\tilde{R})$ with $\tilde{A}=0$ such that

$$\bar{S}(\tilde{A},\tilde{R}) \geq \bar{S}(A,R) \tag{37}$$

where $\bar{S}(A,R)$ is defined in Program (19). It suffices to show that Inequality (37) holds in the following two cases: (i) $R \in [0,\bar{\ell}]$ and (ii) $R \in (\bar{\ell},+\infty)$.

For case (i) with $R \in [0,\bar{\ell}]$, we let $\tilde{A}=0$ and $\tilde{R}=R$. By proposition 5, we can partition the platform's feasible zone, $\mathcal{D}(A,R)$, under policy (A,R) into the following zones:

$$\text{Safe zone}\quad \mathcal{S}(A,R) := \left\{ (x,v) \,\middle|\, \begin{array}{ll} v \geq (2\beta - A + w)\frac{x}{s} - \gamma - R, & \text{if } A < \beta \\ v \geq (\beta+w)\frac{x}{s} - \gamma - R, & \text{if } A \geq \beta \end{array} \right\},$$

$$\text{Unsafe zone 1}\quad \mathcal{U}_1(A,R) := \left\{ (x,v) \,\middle|\, \begin{array}{ll} (\beta+w)\frac{x}{s} - \gamma - R < v \leq (2\beta - A + w)\frac{x}{s} - \gamma - R, & \text{if } A < \beta \\ \emptyset, & \text{if } A \geq \beta \end{array} \right\},$$

$$\text{Unsafe zone 2}\quad \mathcal{U}_2(A,R) := \left\{ (x,v) \,\middle|\, \begin{array}{ll} (w+A)\frac{x}{s} - \gamma - R < v \leq (\beta+w)\frac{x}{s} - \gamma - R, & \text{if } A < \beta \\ \emptyset, & \text{if } A \geq \beta \end{array} \right\}.$$

First, we claim that $\mathcal{C} \cap \mathcal{S}(A,R) = \mathcal{C} \cap \mathcal{U}_1(A,R) = \emptyset$. Note that $R < \bar{\ell} < (\beta+w)\frac{x}{s} - \gamma - v$ for any $(x,v) \in \mathcal{C}$. Therefore, $v < (\beta+w)\frac{x}{s} - \gamma - R$ for any $(x,v) \in \mathcal{C}$, which has no overlap with $\mathcal{S}(A,R)$ and $\mathcal{U}_1(A,R)$.

Next, we show that policy $(0,R)$ dominates policy (A,R) in $\mathcal{U}_2(A,R)$. We claim that $\frac{d}{dA}S(x,v;A,R) < 0$ for $(x,v) \in \mathcal{U}_2(A,R)$. To see this, we first note that (we omit arguments in functions for brevity):

$$S = \pi^* + wt^* + \frac{s}{x}t^*P = \pi^* + \left(w + \frac{s}{x}P\right)t^*,$$

where π^* is given by the objective function of Program (36) and $t^* = \frac{\gamma+v+R-(w+A)\frac{x}{s}}{2(\beta-A)}$. Then,

$$\begin{aligned} \frac{dS}{dA} &= \frac{\partial \pi}{\partial t^*} \cdot \frac{dt^*}{dA} + \frac{\partial \pi}{\partial A} + \left(w + \frac{s}{x}P\right) \cdot \frac{dt^*}{dA} \\ &= 0 - \left(1 - \frac{s}{x}t^*\right)t^* + \left(w + \frac{s}{x}P\right) \cdot \frac{1}{2(\beta - A)} \left(\gamma + v + R - (w+\beta)\frac{x}{s}\right). \end{aligned}$$

Since $t^* < \frac{x}{s}$ (Proposition 5) and $\gamma + v + R - (w+\beta)\frac{x}{s} < 0$ for any $(x,v) \in \mathcal{C}$ (this is because $R < \bar{\ell} < (\beta+w)\frac{x}{s} - \gamma - v$), we obtain $\frac{dS}{dA} < 0$. Therefore, policy $(0,R)$ dominates policy (A,R) in $\mathcal{U}_2(A,R)$. Besides, it is straightforward to show that $\mathcal{U}_2(A,R) \subseteq \mathcal{U}_2(0,R)$. That is, policy $(0,R)$ serves more customers than policy (A,R). In summary, Inequality (37) holds in case (i).

For case (ii) with $R \in (\bar{\ell}, +\infty)$, we first claim that under policy (A,R), customers in $\mathcal{C}$, if served by the platform, can only be served at the safe speed (i.e., the delivery time is always $t^* = \frac{x}{s}$). We prove the claim by showing that for any customer $(x,v) \in (\mathcal{U}_1(A,R) \cup \mathcal{U}_2(A,R)) \cap \mathcal{C}$, the platform profit π is negative. Specifically, for $(x,v) \in (\mathcal{U}_1(A,R) \cup \mathcal{U}_2(A,R)) \cap \mathcal{C}$, we have $t^* = \frac{\gamma+v+R-(w+A)\frac{x}{s}}{2(\beta-A)} < \frac{x}{s}$ and

$$\frac{d\pi^*}{dA} = \frac{\partial \pi^*}{\partial t^*} \cdot \frac{dt^*}{dA} + \frac{\partial \pi^*}{\partial A} = -\left(1 - \frac{s}{x} t^*\right) t^* < 0.$$

Then, to show $\pi^*(x,v;A,R) < 0$, it suffices to show $\pi^*(x,v;0,R) < 0$ since $\pi^*(x,v;A,R) \leq \pi^*(x,v;0,R)$. For $\pi^*(x,v;0,R)$, it can be simplified as:

$$\pi^*(x,v;0,R) = \frac{s}{x} \cdot \frac{\left(\gamma + v + R - \frac{x}{s} w\right)^2}{4\beta} - R.$$

Then, $\pi^*(x,v;0,R) < 0$ is equivalent to:

$$r^2 - 2\sqrt{\beta \frac{x}{s}} r + \gamma + v - \frac{x}{s} w < 0,$$

where $r = \sqrt{R}$. Then, the above inequality holds if

$$\sqrt{\beta \frac{x}{s}} - \sqrt{(\beta + w)\frac{x}{s} - \gamma - v} < r < \sqrt{\beta \frac{x}{s}} + \sqrt{(\beta + w)\frac{x}{s} - \gamma - v}, \text{or equivalently}$$

$$(2\beta + w)\frac{x}{s} - \gamma - v - 2\sqrt{\beta \frac{x}{s}\left((\beta + w)\frac{x}{s} - \gamma - v\right)} < R < (2\beta + w)\frac{x}{s} - \gamma - v + 2\sqrt{\beta \frac{x}{s}\left((\beta + w)\frac{x}{s} - \gamma - v\right)}. \tag{38}$$

Since $R \geq \bar{\ell} > (2\beta + w)\frac{x}{s} - \gamma - v - 2\sqrt{\beta \frac{x}{s}\left((\beta + w)\frac{x}{s} - \gamma - v\right)}$, the first inequality in Expression (38) holds. To show the second inequality of Expression (38), we note that for $(x,v) \in \mathcal{U}_1(A,R) \cup \mathcal{U}_2(A,R)$, we have $t^* = \frac{\gamma+v+R-\frac{x}{s}(w+A)}{2(\beta-A)} < \frac{x}{s}$, which is equivalent to $R < (2\beta + w - A)\frac{x}{s} - \gamma - v$ (note $A < \beta$). Hence, $R < (2\beta + w)\frac{x}{s} - \gamma - v < (2\beta + w)\frac{x}{s} - \gamma - v + 2\sqrt{\beta \frac{x}{s}\left((\beta + w)\frac{x}{s} - \gamma - v\right)}$. Therefore, we have shown that $\pi^*(x,v;A,R) < 0$ holds for any $(x,v) \in (\mathcal{U}_1(A,R) \cup \mathcal{U}_2(A,R)) \cap \mathcal{C}$.

Next, we show that policy $(0,\underline{\ell})$ dominates (A,R) in total social surplus. First, we show that for customer $(x,v) \in \mathcal{C}$, if she is served under policy (A,R), then she is also served under policy $(0,\underline{\ell})$ and $S(x,v;0,\underline{\ell}) \geq S(x,v;A,R)$. As established above, $t^* = \frac{x}{s}$ under policy (A,R). Hence,

$$S(x,v;A,R) = \pi + w\frac{x}{s} + P = \gamma + v + P - \beta \frac{x}{s}.$$

Then, $S(x,v;0,\underline{\ell}) \geq S(x,v;A,R)$ naturally holds if the customer is served at the safe speed under policy $(0,\underline{\ell})$. If the customer is served at $t^* = \frac{\gamma+v+\underline{\ell}-w\frac{x}{s}}{2\beta} < \frac{x}{s}$ (i.e., she is in the unsafe zone under policy $(0,\underline{\ell})$), then

$$S(x,v;0,\underline{\ell}) = -\frac{s}{x}\beta \cdot (t^*)^2 + \frac{s}{x} t^* \left(\gamma + v + \underline{\ell} - \frac{x}{s} w\right) - \underline{\ell} + \left(w + \frac{s}{x} P\right) t^*.$$

Then, $S(x,v;0,\underline{\ell}) \geq S(x,v;A,R)$ is equivalent to:

$$-\frac{s}{x}\beta \cdot (t^*)^2 + \frac{s}{x} t^* \left(\gamma + v + \underline{\ell} + P\right) - \underline{\ell} \geq \gamma + v + P - \beta \frac{x}{s}. \tag{39}$$

The inequality above can be further simplified as:

$$\underline{\ell} \le (2\beta - w)\frac{x}{s} - \gamma - v - 2P,$$

which holds given the definition of $\mathcal{C}$.

Lastly, we show that the customer is also served by the platform under policy $(0, \underline{\ell})$. From $S(x,v;0,\underline{\ell}) \ge S(x,v;A,R)$, we can show that $t^* > 0$, which implies that $(x,v) \in \mathcal{D}(0,\underline{\ell})$. This is because if $t^* = 0$, Inequality (39) is violated. Besides, it is straightforward to show that $\pi^*(x,v;0,\underline{\ell}) > \pi^*(x,v;A,R)$ from $S(x,v;0,\underline{\ell}) \ge S(x,v;A,R)$ and $\pi^* = S - wt^* - \frac{s}{x}t^*P$.

In summary, we have shown that Inequality (37) holds in cases (i) and (ii). Therefore, any policy with $A > 0$ is dominated by a policy with $A = 0$, which shows that the optimal policy satisfies $A = 0$. □

Proof of Lemma 9

For Problem (22) to be feasible, we need $\left(v + \alpha B - \alpha w\frac{x}{s}\right)^2 - 4\alpha\beta B\frac{x}{s} \ge 0$ and $\left(0, \frac{x}{s}\right] \cap [T_1, T_2] \ne \emptyset$. The first condition is equivalent to:

$$v \ge 2\sqrt{\alpha\beta B\frac{x}{s}} - \alpha B + \alpha w\frac{x}{s},$$

and the second condition is equivalent to $T_1 \le \frac{x}{s}$ (since $T_1 > 0$ under the first condition). We note that $T_1 \le \frac{x}{s}$ is equivalent to:

$$v + \alpha B - \alpha w\frac{x}{s} - 2\beta\frac{x}{s} \le \sqrt{\left(v + \alpha B - \alpha w\frac{x}{s}\right)^2 - 4\alpha\beta B\frac{x}{s}}, \tag{40}$$

which holds automatically if $v < (\alpha w + 2\beta)\frac{x}{s} - \alpha B$; if $v \ge (\alpha w + 2\beta)\frac{x}{s} - \alpha B$, then Inequality (40) can be rewritten as $v \ge (\alpha w + \beta)\frac{x}{s}$. Besides, it always holds that $\beta\frac{x}{s} \ge 2\sqrt{\alpha\beta B\frac{x}{s}} - \alpha B$.

With the observations above, we note that Problem (22) is feasible if and only if:

$$v \ge \max\left\{(\alpha w + 2\beta)\frac{x}{s} - \alpha B, (\alpha w + \beta)\frac{x}{s}\right\} \text{ or } 2\sqrt{\alpha\beta B\frac{x}{s}} - \alpha B + \alpha w\frac{x}{s} \le v \le (\alpha w + 2\beta)\frac{x}{s} - \alpha B. \tag{41}$$

If $\alpha B \le \beta\frac{x}{s}$, which is equivalent to $2\sqrt{\alpha\beta B\frac{x}{s}} \le 2\beta\frac{x}{s}$, then Expression (41) can be simplified as:

$$v \ge (\alpha w + 2\beta)\frac{x}{s} - \alpha B \text{ or } 2\sqrt{\alpha\beta B\frac{x}{s}} - \alpha B + \alpha w\frac{x}{s} \le v \le (\alpha w + 2\beta)\frac{x}{s} - \alpha B,$$

which is essentially $v \ge 2\sqrt{\alpha\beta B\frac{x}{s}} - \alpha B + \alpha w\frac{x}{s}$. If $\alpha B > \beta\frac{x}{s}$, which is equivalent to $2\sqrt{\alpha\beta B\frac{x}{s}} > 2\beta\frac{x}{s}$, Expression (41) can be simplified as $v \ge (\alpha w + \beta)\frac{x}{s}$. □

Proof of Proposition 7

Note that the optimal delivery time follows $T^* = \max\left\{T_1, \min\left\{T_\star, T_2, \frac{x}{s}\right\}\right\}$. In the proof, we derive conditions to simplify $\min\left\{T_\star, T_2, \frac{x}{s}\right\}$. First, it is straightforward to verify the following observations:

(Ob.1) $T_\star \le T_2$ is equivalent to $\frac{\gamma+R}{1-\frac{1}{\alpha}} \le \alpha B - \alpha w\frac{x}{s} + \sqrt{\Delta}$,

(Ob.2) $T_\star \le \frac{x}{s}$ is equivalent to $\frac{\gamma+R}{1-\frac{1}{\alpha}} \le 2\beta\frac{x}{s} - v$,

(Ob.3) $T_2 \le \frac{x}{s}$ is equivalent to $\alpha B + \sqrt{\Delta} \le (2\beta + \alpha w)\frac{x}{s} - v$, where $\Delta = \left(v + \alpha B - \alpha w \frac{x}{s}\right)^2 - 4\alpha\beta B \frac{x}{s}$.

When combining (Ob.1) and (Ob.2), we have $T^* = \max\{T_1, T_\star\}$. In addition, we can show that $T_1 \le T_\star$ (i.e., $T_1 \le T_\star$) is equivalent to $\alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta} \le \frac{\gamma+R}{1-\frac{1}{\alpha}}$. Hence, we have $T^* = T_\star$ if $\alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta} \le \frac{\gamma+R}{1-\frac{1}{\alpha}} \le \min\left\{\alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta}, 2\beta \frac{x}{s} - v\right\}$; likewise, we obtain $T^* = T_1$ if $\frac{\gamma+R}{1-\frac{1}{\alpha}} \le \min\left\{\alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta}, 2\beta \frac{x}{s} - v\right\}$.

When combining (Ob.1) and (Ob.3), we have $T_2 \le \min\{T_\star, \frac{x}{s}\}$ if

$$\alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta} \le \min\left\{\frac{\gamma+R}{1-\frac{1}{\alpha}}, 2\beta \frac{x}{s} - v\right\}.$$

Therefore, $T^* = \max\{T_1, T_2\} = T_2$ in this case.

When combining (Ob.2) and (Ob.3), we have $\frac{x}{s} \le \min\{T_\star, T_2\}$ if

$$2\beta \frac{x}{s} - v \le \min\left\{\frac{\gamma+R}{1-\frac{1}{\alpha}}, \alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta}\right\}.$$

Note that $T_1 \le \frac{x}{s}$ for the problem to be feasible. Hence, $T^* = \frac{x}{s}$ in this case. □

Proof of Proposition 8

The proof is similar to Proposition 7's proof; we omit it for brevity. □

Proof of Proposition 9

First, we prove that the incident probability $G(T^*)$ weakly decreases as R increases. This is equivalent to showing that T^* weakly increases with R. Note that when T^* equals T_1, T_2, or $\frac{x}{s}$, it is independent of R. Hence, it suffices to show that when $T^* = T_\star$, it increases with R. Given the expression of $T_\star$ in Definition (2), $\frac{dT_\star}{dR} \ge 0$ holds automatically.

Second, we show that $\frac{\partial T^*}{\partial B} \le 0$ when $T^* = T_2$ and $v \le (\beta + \alpha w)\frac{x}{s}$. We begin by verifying that there exist customers with $v \le (\beta + \alpha w)\frac{x}{s}$ such that $T^* = T_2$. It suffices to show that the following inequality, which assures that the condition for $T^* = T_2$ is not empty, holds:

$$\alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta} \le 2\beta \frac{x}{s} - v.$$

We claim the inequality above holds under $v \le (\beta + \alpha w)\frac{x}{s}$ because we have:

$$v \le (\beta + \alpha w)\frac{x}{s} \le (2\beta + \alpha w)\frac{x}{s} - \alpha B - \sqrt{\Delta}.$$

The second inequality in the above expression holds because it is equivalent to $v \le (\beta + \alpha w)\frac{x}{s}$. Then, it is straightforward to verify that the interval $2\sqrt{\alpha\beta B \frac{x}{s}} - \alpha B + \alpha w \frac{x}{s} \le v \le (\beta + \alpha w)\frac{x}{s}$ is not empty. Next, we show $\frac{\partial T_2}{\partial B} \le 0$. In particular, we note that $\frac{\partial T_2}{\partial B} \le 0$ is equivalent to $\sqrt{\Delta} \le 2\beta \frac{x}{s} - v - \alpha B + \alpha w \frac{x}{s}$, where its right-hand side is positive.[17] Then, the last inequality can be further simplified as $4\beta^2 \left(\frac{x}{s}\right)^2 - 4\beta \frac{x}{s}\left(v - \alpha w \frac{x}{s}\right) \ge 0$, which is equivalent to $v \le (\beta + \alpha w)\frac{x}{s}$.

Third, by Lemma 9 and the definition of $\mathcal{D}(B)$, it is straightforward to verify that $\mathcal{D}(B)$ expands as B reduces; yet, it remains unchanged as we reduce R. □

[17] This is because $(2\beta + \alpha w)\frac{x}{s} - \alpha B > 2\sqrt{\alpha\beta B \frac{x}{s}} - \alpha B + \alpha w \frac{x}{s}$. So, a customer who falls in the regime of $T^* = T_2$ has $v \le (2\beta + \alpha w)\frac{x}{s} - \alpha B$.

Proof of Theorem 3

Under condition $v \geq (\beta+\alpha)\frac{x}{s}$ in $\mathcal{C}$, we note that only the case of impatient customers holds (i.e., Proposition 7). Then, we need $B \leq \frac{\beta}{\alpha} \cdot \frac{x}{s}$ to ensure $\mathcal{C} \cap \mathcal{D}(B) \neq \emptyset$. Besides, under $\beta < \alpha w$, we have:

$$\alpha B - \alpha w \frac{x}{s} - \sqrt{\Delta} \leq (\beta - \alpha w)\frac{x}{s} - \sqrt{\Delta} < 0.$$

Therefore, the characterization of T^* can be simplified as:

$$T^* = \begin{cases} T_\star \ (< \frac{x}{s}), & \text{if } \frac{\gamma+R}{1-\frac{1}{\alpha}} \leq \min\left\{\alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta}, 2\beta\frac{x}{s} - v\right\} \\ T_2 \ (< \frac{x}{s}), & \text{if } \alpha B - \alpha w \frac{x}{s} + \sqrt{\Delta} \leq \min\left\{\frac{\gamma+R}{1-\frac{1}{\alpha}}, 2\beta\frac{x}{s} - v\right\} \\ \frac{x}{s}, & \text{if } 2\beta\frac{x}{s} - v \leq \min\left\{\frac{\gamma+R}{1-\frac{1}{\alpha}}, \alpha B - \alpha w\frac{x}{s} + \sqrt{\Delta}\right\} \end{cases}.$$

Since $T_\star$ and $\frac{x}{s}$ are independent of B and we have shown $\frac{\partial T_2}{\partial B} < 0$ under $(\beta+\alpha w)\frac{x}{s}$ in the proof of Proposition 9, we have $\frac{\partial T^*}{\partial B} \leq 0$ for any customer $(x,v) \in \mathcal{C} \cap \mathcal{D}(B)$.

Next, we show that $\frac{\partial \pi^*}{\partial B} \leq 0$ for any customer $(x,v) \in \mathcal{C} \cap \mathcal{D}(B)$. By the objective function of Program (22), π, we note that it is independent of B. Hence, $\frac{\partial \pi^*}{\partial B} = 0$ when $T^* = T_\star$ or $T^* = \frac{x}{s}$. For $T^* = T_2$, we have $\frac{\partial \pi^*}{\partial B} = \frac{\partial \pi}{\partial T}|_{T=T_2} \cdot \frac{\partial T_2}{\partial B}$. Note that π is concave and quadratic and is maximized at $T_\star$. Besides, in the case of $T^* = T_\star$, we have $T_2 < T_\star$. Therefore, $\frac{\partial \pi}{\partial T}|_{T=T_2} > 0$, which leads to $\frac{\partial \pi^*}{\partial B} < 0$.

Then, note that $S(x,v;B,R) = \pi^* + u^* T^* + \frac{s}{x} T^* P$. When $T^* = T_\star$ or $T^* = \frac{x}{s}$, it is straightforward to show that $\frac{\partial S}{\partial B} = \frac{\partial u^*}{\partial B} T^* \leq 0$. When $T^* = T_2$, we have $u^* = w$. Hence, $\frac{\partial S}{\partial B} = \frac{\partial \pi^*}{\partial B} + \left(w + \frac{s}{x}P\right)\frac{\partial T^*}{\partial B} < 0$. Therefore, we prove that $\frac{\partial S}{\partial B} \leq 0$ for any customer $(x,v) \in \mathcal{C} \cap \mathcal{D}(B)$; the social surplus of serving any customer in $\mathcal{C} \cap \mathcal{D}(B)$ improves as B reduces. Moreover, by Proposition 9, $\mathcal{C} \cap \mathcal{D}(B)$ expands as B decreases. Therefore, $B^* = 0$ is the optimal solution to Program (23). □

Proof of Theorem 4

First, we note that the customer surplus is zero (i.e., $\sigma^* = 0$) under the platform's optimal policy (Lemma 2); we can simplify Equation (24) as

$$\widehat{S}(x,v;A,R) = a_1 \pi^* + a_3 u^* T^* + (1 - G(T^*))P, \tag{42}$$

where $a_i \geq 0$ for $i \in \{1,3\}$, and $a_1 + a_3 = 1$. In addition, the government's general optimization problem can be formulated as

$$\max_{A \geq 0, R \geq 0} \int_{(x,v)\in\mathcal{D}(A)} (a_1 S_p(x,v;A,R) + a_3 S_d(x,v;A,R)) \mathbb{1}\{\pi^*(x,v;A,R) \geq 0\} F_{\mathcal{C}}(dx,dv), \tag{43}$$

where $S_p(x,v;A,R)$ and $S_d(x,v;A,R)$ denote the pro-platform social surplus (i.e., $a_1 = 1$) and the pro-driver social surplus (i.e., $a_3 = 1$), respectively, characterized by Definition 3.

Second, we observe that to solve Problem (43), it suffices to search for the optimal A and R within $\left[0, \frac{\beta}{\alpha}\right] \times [0, +\infty)$ (i.e., Scenario (1)). The observation can be shown by verifying that Policy (34) is optimal for the problem below with policy space $\left[\frac{\beta}{\alpha}, +\infty\right) \times [0, +\infty)$ (i.e., Scenario (2))

$$\max_{A \geq \frac{\beta}{\alpha}, R \geq 0} \int_{(x,v)\in\mathcal{D}(A)} (a_1 S_p(x,v;A,R) + a_3 S_d(x,v;A,R)) \mathbb{1}\{\pi^*(x,v;A,R) \geq 0\} F_{\mathcal{C}}(dx,dv). \tag{44}$$

To show that Policy (34) is optimal for Problem (44), it is equivalent to showing that Policy (34) (weakly) dominates any policy (A,R) where $A \geq \frac{\beta}{\alpha}$ and $R \geq 0$. By Expression (50), we obtain $\pi^*(x,v;A^\dagger,R^\dagger) \geq 0$ for any customer $(x,v) \in \mathcal{D}(A) = \bar{\mathcal{F}}$, where $\bar{\mathcal{F}}$ is characterized by Definition 1. That is, under Policy (34), the platform finds it profitable to serve all customers in $\bar{\mathcal{F}}$. So, Policy (34) serves no fewer customers compared with any policy with $A \geq \frac{\beta}{\alpha}$ and $R \geq 0$. Next, note that $\left(1-\frac{1}{\alpha}\right)(\beta-\alpha w)\frac{x}{s} \leq P+\gamma$ holds for any $x \in (0,\bar{x}]$, which is given. Then, Lemmas 10 and 11 in Appendix F have shown that for any customer $(x,v) \in \bar{\mathcal{F}}$, objectives $S_p(x,v;A,R)$ and $S_d(x,v;A,R)$ are both maximized at Policy (34) under the constraints of $A \geq \frac{\beta}{\alpha}$, $R \geq 0$ and $\pi^*(x,v;A,R) \geq 0$. Therefore, for customer $(x,v) \in \{(x,v) : \pi^*(x,v;A,R) \geq 0\} \cap \bar{\mathcal{F}}$ under any given policy (A,R), it follows that

$$a_1 S_p(x,v;A,R) + a_3 S_d(x,v;A,R) \leq a_1 S_p(x,v;A^\dagger,R^\dagger) + a_3 S_d(x,v;A^\dagger,R^\dagger). \tag{45}$$

Therefore, policy (A,R) cannot perform better than policy $(A^\dagger,R^\dagger)$ in Problem (44).

Third, we focus on optimizing the problem

$$\max_{0 \leq A \leq \frac{\beta}{\alpha}, R \geq 0} \int_{(x,v)\in\mathcal{D}(A)} (a_1 S_p(x,v;A,R) + a_3 S_d(x,v;A,R)) \mathbb{1}\{\pi^*(x,v;A,R) \geq 0\} F_{\mathcal{C}}(dx,dv), \tag{46}$$

and we have shown that its optimal policy also optimizes Problem (43). In particular, we show that $A^* = 0$ is optimal for Problem (46). We prove the last claim by showing that any policy (A,R), referred to as the original policy, with $0 < A \leq \frac{\beta}{\alpha}$ and $R \geq 0$ is (weakly) dominated by policy $(\tilde{A},R)$, referred to as the alternative policy, where $\tilde{A} = A - \epsilon$, and $\epsilon \downarrow 0$. First, we note that

$$\underline{\mathcal{F}}(A) \cap \{(x,v) : \pi^*(x,v;A,R) \geq 0\} \subseteq \underline{\mathcal{F}}(\tilde{A}) \cap \{(x,v) : \pi^*(x,v;\tilde{A},R) \geq 0\}, \tag{47}$$

where $\underline{\mathcal{F}}(A)$ and $\underline{\mathcal{F}}(\tilde{A})$ are characterized by Definition 1. The condition above holds because as A decreases, the set $\underline{\mathcal{F}}(A)$ expands (Definition 1) and $\pi^*(x,v;A,R)$ increases (Corollary 1). Then, Condition (47) implies that the alternative policy serves no fewer customers than the original policy. Any customer served by the alternative policy but not by the original policy contributes a non-negative value to the total social surplus. Hence, it suffices to show that for any customer (x,v) served by both policies, the alternative policy generates no less social surplus (from the customer) than the original policy does. That is, we aim to show

$$a_1 S_p(x,v;A,R) + a_3 S_d(x,v;A,R) \leq a_1 S_p(x,v;\tilde{A},R) + a_3 S_d(x,v;\tilde{A},R) \tag{48}$$

holds for any customer $(x,v) \in \underline{\mathcal{F}}(A) \cap \{(x,v) : \pi^*(x,v;A,R) \geq 0\}$. In Lemmas 12 and 13 in Appendix F, we have shown that for any customer $(x,v) \in \underline{\mathcal{F}}(A)$, we have $\frac{\partial S_p}{\partial A}(x,v;A,R) \leq 0$ and $\frac{\partial S_d}{\partial A}(x,v;A,R) \leq 0$. Hence, Inequality (48) holds (note that $\tilde{A} < A$ and $\min\{a_1,a_3\} \geq 0$). In sum, we have established that any policy with $A > 0$ is suboptimal. Therefore, the optimal policy for Problem (43) satisfies $A^* = 0$.

Lastly, we focus on proving $R^* < P$ under the optimal policy. Based on the proof of $A^* = 0$ above, it suffices to show that for any policy $(0,R)$ with $R \geq P$, referred to henceforth as the original policy, we can find an alternative policy $(0,\tilde{R})$ with $\tilde{R} = R - \epsilon$ (where $\epsilon \downarrow 0$) that improves the total social surplus.

Under the original policy, we note that $R \geq P > \left(1-\frac{1}{\alpha}\right)(\beta-\alpha w)\frac{x}{s} - \gamma$ holds for any $x \in (0,\bar{x}]$. Then, only Proposition 1's Statement (A) will hold, and the platform's optimal delivery time can be simplified as:

$$T^* = \begin{cases} \frac{v-\alpha w\frac{x}{s}}{\beta}, & \text{if } \frac{\alpha w}{s}x \leq v < \frac{\beta+\alpha w}{s}x \\ \frac{x}{s}, & \text{if } v > \frac{\beta+\alpha w}{s}x \end{cases}.$$

which are independent of R. Next, we show that for any customer within $\underline{\mathcal{F}}(0) = (0, \bar{x}] \times (0, \bar{v}]$ (where $\underline{\mathcal{F}}(0)$ is given by Definition 1), the general social surplus generated from the order, Equation (42), decreases weakly in R. Note that the general social surplus can be equivalently expressed as $\widehat{S}(x, v; 0, R) = a_1 S_p(x, v; 0, R) + a_3 S_p(x, v; 0, R)$. Hence, it suffices to show that for any customer (x, v), $S_p(x, v; 0, R)$, and $S_p(x, v; 0, R)$ (weakly) decrease in R, respectively. We show the last claim as follows.

- For $S_p(x, v; 0, R)$, which is characterized by Equation (52), we have $\frac{\partial S_p}{\partial R}(x, v; 0, R) = -\left(1 - \frac{s}{x} T^*\right) \leq 0$ (because $T^* \leq \frac{x}{s}$). Thus, $S_p(x, v; 0, R)$ decreases in R.
- For $S_d(x, v; 0, R)$, we have $S_d(x, v; 0, R) = -\frac{\beta}{\alpha} \cdot \frac{s}{x} T^{*2} + \left(\frac{v}{\alpha} + P\right) \cdot \frac{s}{x} T^*$, which is independent of R. The claim follows naturally. See Equation (55) in Appendix F for the derivation.

In sum, any policy $(0, R)$ with $R \geq P$ is suboptimal. In other words, $R^* < P$. □

Appendix F: Auxiliary Results

Lemma 10. *Suppose* $\left(1-\frac{1}{\alpha}\right)(\beta-\alpha w)\frac{x}{s}\le P+\gamma$ *holds for any* $x\in(0,\bar{x}]$. *Then, Policy (34) is optimal for*

$$\max_{A\ge 0, R\ge 0} \quad S_p(x,v;A,R), \quad \textit{s.t. Problem (8) is feasible, } \pi^*(x,v;A,R)\ge 0, \textit{ and } A\ge\tfrac{\beta}{\alpha}. \tag{49}$$

Proof. Lemma 3 implies that $\bar{\mathcal{F}}$ from Definition 1 is the customer set such that Problem (8) is feasible under any policy with $A\ge\frac{\beta}{\alpha}$ and $R\ge 0$ (i.e., $\mathcal{D}(A)=\bar{\mathcal{F}}$). In addition, under Policy (34), we have shown $T^*=\frac{x}{s}$, and the platform's profit from serving $(x,v)\in\bar{\mathcal{F}}$ follows:

$$\pi^*(x,v;A^\dagger,R^\dagger)=\gamma+\left(1-\frac{1}{\alpha}\right)\left(v-\beta\frac{x}{s}\right)\ge 0. \tag{50}$$

That is, the platform finds it profitable to serve all customers in $\bar{\mathcal{F}}$.

Then, for any other policy (A,R) with $A\ge\frac{\beta}{\alpha}$ and $R\ge 0$, we aim to show that the policy is weakly dominated by Policy (34) for customer $(x,v)\in\bar{\mathcal{F}}\cap\{(x,v):\pi^*(x,v;A,R)\ge 0\}$. That is,

$$S_p(x,v;A,R)\le S_p(x,v;A^\dagger,R^\dagger). \tag{51}$$

Using the objective function Program (8), we expand $S_p(x,v;A,R)$ as follows (we omit arguments for π^*, b^*, and T^* for brevity):

$$\begin{aligned} S_p(x,v;A,R) &= \pi^*+\frac{s}{x}T^*P \\ &= -\left(1-\frac{1}{\alpha}\right)\beta\cdot\frac{s}{x}T^{*2}+\left(P+R+\gamma+\left(1-\frac{1}{\alpha}\right)v\right)\cdot\frac{s}{x}T^*-R. \end{aligned} \tag{52}$$

By Proposition 2, we know that it is equivalent to show Inequality (51) holds for customers in regions $\bar{\mathcal{F}}\setminus\left(\mathcal{U}_0^{(2)}\cup\mathcal{U}_1^{(2)}\right)$, $\mathcal{U}_0^{(2)}$, and $\mathcal{U}_1^{(2)}$, separately, where $\mathcal{U}_0^{(2)}$ and $\mathcal{U}_1^{(2)}$ denote the unsafe zones under policy (A,R).

- For customer $(x,v)\in\bar{\mathcal{F}}\setminus\left(\mathcal{U}_0^{(2)}\cup\mathcal{U}_1^{(2)}\right)$, we have $T^*=\frac{x}{s}$ under both policies (Proposition 2). Then, Equation (52) implies that

$$S_p(x,v;A,R)=\left(1-\frac{1}{\alpha}\right)\left(v-\beta\frac{x}{s}\right)+\gamma+P=S_p(x,v;A^\dagger,R^\dagger),$$

  which establishes Inequality (51) in this case.

- For customer $(x,v)\in\mathcal{U}_0^{(2)}$, we have $T^*=\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ from Expression (12). By Equation (52), we note that $S_p(x,v;A,R)$ is independent of A given T^*. Then $\frac{\partial S_p}{\partial A}(x,v;A,R)=\frac{\partial S_p}{\partial T^*}(x,v;T^*,R)\cdot\frac{\partial T^*}{\partial A}(x,v;A,R)$. First, we note that $\frac{\partial T^*}{\partial A}(x,v;A,R)=\frac{\alpha}{(\beta-\alpha A)^2}\left(v-(\beta+\alpha w)\frac{x}{s}\right)\ge 0$, because $v\ge(\beta+\alpha w)\frac{x}{s}$ for $(x,v)\in\bar{\mathcal{F}}$. Second, we have

$$\frac{\partial S_p}{\partial T^*}(x,v;A,R)=-2\left(1-\frac{1}{\alpha}\right)\beta\cdot\frac{s}{x}T^*+\left(P+R+\gamma+\left(1-\frac{1}{\alpha}\right)v\right)\frac{s}{x}.$$

  Then, $\frac{\partial S_p}{\partial T^*}(x,v;A,R)\ge 0$ is equivalent to $2\beta\left(1-\frac{1}{\alpha}\right)\frac{v-\alpha(A+w)\frac{x}{s}}{\alpha A-\beta}+P+R+\gamma+\left(1-\frac{1}{\alpha}\right)v\ge 0$. Since $\alpha A-\beta>0$, the last inequality is then equivalent to $2\beta\left(1-\frac{1}{\alpha}\right)\left(v-\alpha(A+w)\frac{x}{s}\right)+(\alpha A-\beta)\left(P+R+\gamma+\left(1-\frac{1}{\alpha}\right)v\right)\ge 0$. By rearranging the terms on the left side of the last inequality, we show that it is equivalent to:

$$v\ge\frac{2\alpha(A+w)\beta\frac{x}{s}-\frac{P+R+\gamma}{1-\frac{1}{\alpha}}(\alpha A-\beta)}{\alpha A+\beta}.$$

To show what the inequality above holds for any $(x,v)\in\bar{\mathcal{F}}$, where $v\geq(\beta+\alpha w)\frac{x}{s}$, it suffices to show that $(\beta+\alpha w)\frac{x}{s}\geq\frac{2\alpha(A+w)\beta\frac{x}{s}-\frac{P+R+\gamma}{1-\frac{1}{\alpha}}(\alpha A-\beta)}{\alpha A+\beta}$. Note that the last inequality can be simplified as $P+R+\gamma\geq(\beta-\alpha w)\frac{x}{s}\left(1-\frac{1}{\alpha}\right)$, which holds because $P+R+\gamma\geq P+\gamma\geq\left(1-\frac{1}{\alpha}\right)(\beta-\alpha w)\frac{x}{s}$ for any $x\in(0,\bar{x}]$. Therefore, we have shown $\frac{\partial S_p}{\partial T^*}(x,v;A,R)\geq 0$. As a result, we obtain $\frac{\partial S_p}{\partial A}(x,v;A,R)=\frac{\partial S_p}{\partial T^*}(x,v;T^*,R)\cdot\frac{\partial T^*}{\partial A}(x,v;A,R)\geq 0$.

Hence, we have $S_p(x,v;A,R)\leq\lim_{A\to+\infty}S_p(x,v;A,R)=\left(1-\frac{1}{\alpha}\right)\left(v-\beta\frac{x}{s}\right)+\gamma+P=S_p(x,v;A^\dagger,R^\dagger)$, which proves Inequality (51) in this case.

- For customer $(x,v)\in\mathcal{U}_1^{(2)}$, we have $T^*=T_\star$ by Proposition 2, where $T_\star$ is given by Definition 2. First, we note that by Equation (52), Inequality (51) can be rewritten as

$$-\left(1-\frac{1}{\alpha}\right)\beta\cdot\frac{s}{x}T_\star^2+\left(P+R+\gamma+\left(1-\frac{1}{\alpha}\right)v\right)\cdot\frac{s}{x}T_\star-R\leq\left(1-\frac{1}{\alpha}\right)\left(v-\beta\frac{x}{s}\right)+\gamma+P,$$

which is equivalent to $\left(1-\frac{1}{\alpha}\right)\left(\beta\frac{x}{s}\left(1-\left(\frac{s}{x}T_\star\right)^2\right)-v\left(1-\frac{s}{x}T_\star\right)\right)\leq(P+R+\gamma)\left(1-\frac{s}{x}T_\star\right)$. Note that $1-\frac{s}{x}T_\star>0$. Hence, the last inequality can be simplified as $\left(1-\frac{1}{\alpha}\right)\left(\beta\frac{x}{s}\left(1+\frac{s}{x}T_\star\right)-v\right)\leq P+R+\gamma$, which is equivalent to $\beta\frac{x}{s}+\beta T_\star=\beta\frac{x}{s}+\beta\frac{\gamma+R+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}\leq v+\frac{P+R+\gamma}{1-\frac{1}{\alpha}}$. The last inequality can be further simplified as

$$v\geq 2\beta\frac{x}{s}-\frac{2P+R+\gamma}{1-\frac{1}{\alpha}}$$

Note that $v\geq(\beta+\alpha w)\frac{x}{s}$ (because $(x,v)\in\bar{\mathcal{F}}$), and $R\geq 0$. It suffices to show:

$$(\beta+\alpha w)\frac{x}{s}\geq 2\beta\frac{x}{s}-\frac{2P+\gamma}{1-\frac{1}{\alpha}},\tag{53}$$

which is equivalent to $2P+\gamma\geq\left(1-\frac{1}{\alpha}\right)(\beta-\alpha w)\frac{x}{s}$. The last inequality holds for any $(x,v)\in\bar{\mathcal{F}}$ under the given condition. Therefore, Inequality (53) holds, which yields Inequality (51).

In sum, Policy (34)'s optimality for Problem (49) has been established. □

Lemma 11. *For any customer $(x,v)\in\bar{\mathcal{F}}$, Policy (34) is optimal to*

$$\max_{A\geq 0,R\geq 0}\quad S_d(x,v;A,R),\quad \text{s.t. Problem (8) is feasible, } \pi^*(x,v;A,R)\geq 0,\text{ and } A\geq\tfrac{\beta}{\alpha}.\tag{54}$$

Proof. Lemma 3 implies that $\bar{\mathcal{F}}$ from Definition 1 is the customer set such that Problem (8) is feasible under any policy with $A\geq\frac{\beta}{\alpha}$ and $R\geq 0$ (i.e., $\mathcal{D}(A)=\bar{\mathcal{F}}$). By Definition 3, $S_d(x,v;A,R)$ can be characterized as (we omit arguments for π^*, b^*, u^*, and T^* for brevity):

$$\begin{aligned}S_d(x,v;A,R)&=u^*T^*+\frac{s}{x}T^*P\\&=\left(sb^*-\left(1-\frac{s}{x}T^*\right)A\right)T^*+\frac{s}{x}T^*P\\&=\left(\frac{s}{\alpha x}(v-\beta T^*)-\left(1-\frac{s}{x}T^*\right)A\right)T^*+\frac{s}{x}T^*P\\&=\left(A-\frac{\beta}{\alpha}\right)\cdot\frac{s}{x}T^{*2}+\left(\frac{s}{\alpha x}v+\frac{s}{x}P-A\right)T^*.\end{aligned}\tag{55}$$

The third equality holds due to $b^*x=\frac{1}{\alpha}(v-\beta T^*)$ (Lemma 2). Then, under Policy (34), Proposition 2 suggests that $T^*=\frac{x}{s}$ for customer $(x,v)\in\bar{\mathcal{F}}$. Then, Expression (50) indicates that $\pi^*(x,v;A^\dagger,R^\dagger)\geq 0$ for any customer $(x, v) \in \bar{\mathcal{F}}$; that is, it is profitable for the platform to serve the entire customer set $\bar{\mathcal{F}}$. Next, for any policy (A, R) with $A \geq \frac{\beta}{\alpha}$ and $R \geq 0$, we aim to show that it is weakly dominated by Policy (34) for customer $(x, v) \in \bar{\mathcal{F}} \cap \{(x, v) : \pi^*(x, v; A, R) \geq 0\}$. That is:

$$S_d(x, v; A, R) \leq S_d(x, v; A^\dagger, R^\dagger). \tag{56}$$

By Proposition 2, we know that it is equivalent to show Inequality (56) holds for customers in regions $\bar{\mathcal{F}} \setminus \left(\mathcal{U}_0^{(2)} \cup \mathcal{U}_1^{(2)}\right)$, $\mathcal{U}_0^{(2)}$, and $\mathcal{U}_1^{(2)}$, separately, where $\mathcal{U}_0^{(2)}$ and $\mathcal{U}_1^{(2)}$ denote the unsafe zones under policy (A, R).

- For customer $(x, v) \in \bar{\mathcal{F}} \setminus \left(\mathcal{U}_0^{(2)} \cup \mathcal{U}_1^{(2)}\right)$, we have $T^* = \frac{x}{s}$ (Proposition 2). Then, it follows that

$$S_d(x, v; A, R) = \frac{1}{\alpha}\left(v - \beta\frac{x}{s}\right) + P = S_d(x, v; A^\dagger, R^\dagger),$$

  which establishes Inequality (56) in this case.

- For customer $(x, v) \in \mathcal{U}_0^{(2)}$, $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A}$ from Expression (12). In this case, we know that the driver's earnings are w (i.e., $u^* = w$) by Proposition 2. Then, $S_d(x, v; A, R) = \left(w + \frac{s}{x}P\right) T^* \leq S_d(x, v; A^\dagger, R^\dagger)$ because $u^*(x, v; A^\dagger, R^\dagger) \geq w$ and $T^*(x, v; A^\dagger, R^\dagger) = \frac{x}{s} \geq T^*$. Therefore, Inequality (56) holds.

- For customer $(x, v) \in \mathcal{U}_1^{(2)}$, we have $T^* = T_\star$ by Proposition 2, where $T_\star$ is characterized by Definition 2 , which is independent of A. Then, Equation (55) implies that

$$\frac{\partial S_d}{\partial A}(x, v; A, R) = -T_\star\left(1 - \frac{s}{x}T_\star\right) < 0.$$

  Then, we have

$$S_d(x, v; A, R) \leq \lim_{A \downarrow \frac{\beta}{\alpha}} S_d(x, v; A, R) = \left(\frac{1}{\alpha}\left(v - \beta\frac{x}{s}\right) + P\right) \cdot \frac{s}{x}T_\star \leq \frac{1}{\alpha}\left(v - \beta\frac{x}{s}\right) + P = S_d(x, v; A^\dagger, R^\dagger).$$

  The second inequality holds because $\frac{s}{x}T_\star \leq 1$. Hence, Inequality (56) follows in this case.

In sum, Policy (34)'s optimality for Problem (54) has been established. ☐

Lemma 12. *Suppose $A \leq \frac{\beta}{\alpha}$ and $R \geq 0$. Then $\frac{\partial S_p}{\partial A}(x, v; A, R) \leq 0$ for any customer $(x, v) \in \underline{\mathcal{F}}(A)$.*

Proof. First, by Equation (52), we note that $S_p(x, v; A, R)$ is independent of A given T^*. Then, $\frac{\partial S_p}{\partial A}(x, v; A, R) = \frac{\partial S_p}{\partial T^*}(x, v; T^*, R) \cdot \frac{\partial T^*}{\partial A}(x, v; A, R)$. When $T^* = \frac{x}{s}$ or $T^* = T_\star$, where $T_\star$ is given by Definition 2, the claim holds because $\frac{\partial T^*}{\partial A}(x, v; A, R) = 0$.

Next, we focus on the case of $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A}$. We begin by noting that

$$\frac{\partial T^*}{\partial A}(x, v; A, R) = \frac{\alpha}{(\beta - \alpha A)^2}\left(v - (\beta + \alpha w)\frac{x}{s}\right) \leq 0, \tag{57}$$

because Proposition 1 suggests that $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A}$ holds when $v < (\beta + \alpha w)\frac{x}{s}$. To see this:

- If $(\beta - \alpha w)\frac{x}{s} \leq \frac{\gamma + R}{1 - \frac{1}{\alpha}}$, which is Statement (A) of Proposition 1, then $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A}$ requires that $v < (\beta + \alpha w)\frac{x}{s}$.

- If $(\beta - \alpha w)\frac{x}{s} > \frac{\gamma + R}{1 - \frac{1}{\alpha}}$, which is Statement (B) of Proposition 1, then $T^* = \frac{v - \alpha(A+w)\frac{x}{s}}{\beta - \alpha A}$ requires that $v < \frac{2\alpha(A+w)\frac{\beta}{s}x + \frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta - \alpha A)}{\alpha A + \beta} < (\beta + \alpha w)\frac{x}{s}$, where the last inequality holds because it is equivalent to $(\beta - \alpha w)\frac{x}{s} > \frac{\gamma + R}{1 - \frac{1}{\alpha}}$.

Hence, we have shown that $\frac{\partial T^*}{\partial A}(x,v;A,R)<0$.

Then, we aim to show $\frac{\partial S_p}{\partial T^*}(x,v;T^*,R)>0$ in this case. In particular, by Equation (52), the last inequality is equivalent to $\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}<\frac{P+R+\gamma+\left(1-\frac{1}{\alpha}\right)v}{2\left(1-\frac{1}{\alpha}\right)\beta}$. Since $P\geq 0$, it suffices to show that the last inequality holds when $P=0$; that is,

$$v\leq\frac{2\alpha(A+w)\frac{\beta}{s}x+\frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta}. \tag{58}$$

Then, we claim that Inequality (58) follows from Proposition 1. To see this:

- If $(\beta-\alpha w)\frac{x}{s}\leq\frac{\gamma+R}{1-\frac{1}{\alpha}}$, which is Statement (A) of Proposition 1, then $T^*=\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ requires that $v<\frac{\beta+\alpha w}{s}x\leq\frac{2\alpha(A+w)\frac{\beta}{s}x+\frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta}$, where the last inequality holds because it is equivalent to $(\beta-\alpha w)\frac{x}{s}\leq\frac{\gamma+R}{1-\frac{1}{\alpha}}$.
- If $(\beta-\alpha w)\frac{x}{s}>\frac{\gamma+R}{1-\frac{1}{\alpha}}$, which is Statement (B) of Proposition 1, then $T^*=\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ requires that $v<\frac{2\alpha(A+w)\frac{\beta}{s}x+\frac{\gamma+R}{1-\frac{1}{\alpha}}(\beta-\alpha A)}{\alpha A+\beta}$.

Hence, Inequality (58) holds, which renders $\frac{\partial S_p}{\partial T^*}(x,v;T^*,R)>0$. Combining the last inequality with Inequality (57), we obtain $\frac{\partial S_p}{\partial A}(x,v;A,R)=\frac{\partial S_p}{\partial T^*}(x,v;T^*,R)\cdot\frac{\partial T^*}{\partial A}(x,v;A,R)\leq 0$. □

Lemma 13. *Suppose $A\leq\frac{\beta}{\alpha}$ and $R\geq 0$. Then $\frac{\partial S_d}{\partial A}(x,v;A,R)\leq 0$ for any customer $(x,v)\in\underline{\mathcal{F}}(A)$.*

Proof. First, by Equation (55), we obtain $\frac{\partial S_d}{\partial A}(x,v;A,R)=-T^*\left(1-\frac{s}{x}T^*\right)\leq 0$ when $T^*=\frac{x}{s}$ or $T^*=T_\star$ (Definition 2). This is because T^* is independent of A.

Next, we focus on the case when $T^*=\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$. Then, by Equation (55), we have (we omit the arguments of T^* for brevity)

$$\begin{aligned}\frac{\partial S_d}{\partial A}(x,v;A,R)&=\frac{\partial S_d}{\partial T^*}(x,v;A,R,T^*)\cdot\frac{\partial T^*}{\partial A}+\frac{\partial S_d}{\partial A}(x,v;A,R,T^*)\\&=\left(-\frac{s}{x}\cdot\frac{v}{\alpha}+A+2w+\frac{s}{x}P\right)\cdot\frac{\alpha}{(\beta-\alpha A)^2}\left(v-(\beta+\alpha w)\frac{x}{s}\right)-T^*\left(1-\frac{s}{x}T^*\right)\\&=\frac{\alpha\left(w+\frac{s}{x}P\right)}{(\beta-\alpha A)^2}\left(v-(\beta+\alpha w)\frac{x}{s}\right)\\&\leq 0.\end{aligned}$$

The last inequality holds because $T^*=\frac{v-\alpha(A+w)\frac{x}{s}}{\beta-\alpha A}$ requires that $v<(\beta+\alpha w)\frac{x}{s}$ (Proposition 1), which follows the same argument as that in the proof of Lemma 12. Hence, we omit it for brevity. Therefore, we have completed the proof. □

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